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NOTES ON MAGNETIC COIL DESIGN

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Introduction

This brief report summarizes work done which addressed the issue of sizing the USTX Ohmic heating solenoid by imposing some physical constraints on the TF and OH coil designs.

TF Coil

The main concern is temperature rise, which is determined by the toroidal field required at the major radius - R_0, and the flat-top duration. The result, given in the Appendix, indicates that a central TF post of 9.5 cm radius is a reasonable starting point. The parameters assumed were :

$R_0 = 0.7$ meters
$B_0 = 0.5$ Tesla
$t_f = 0.5$ seconds
$\Delta T = 50\ °C$

Fixing a 0.5 cm gap between the TF and OH coil then starts the OH coil solenoid at a radius of 10.0 cm.

Plasma Parameters

Setting the design aspect ratio at

$$A_0 = 1.43$$

and using the above major radius, the minor radius is approximately

$$a_p = 0.49 \text{ meters}$$

The inner plasma radius is

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$$R_{min} = 0.21 \text{ meters}$$

The "natural" elongation is assumed given by a regression fit to the numerical results of Kalmykov (Plasma Physics Reports, 20, #10, p. 844 1994)

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$$\kappa = 1.7 / A^{0.57} / l_i^{0.25}$$

Formulas for inductance and flux consumption are given in the Appendix.

During the current ramp-up phase, the resistive flux loss is assumed given by Ejima's results (Nuclear Fusion, 22, #10, p. 1313 1982), where the resistive loop voltage is equal to 2/3 of the voltage drop associated with the internal inductance. If this falls below 1.5 volts, arbitrarily set the resistive drop at 1.5 volts.

The right value for the internal inductance factor l_i is unknown. A value of 0.5 is uniform current density, while 1.0 is roughly parabolic. Chose a value of 0.7, the value used in the 1985 STX proposal. Since volt-second requirements depend only on the final equilibrium, a value greater than 0.5 seems conservative and pushes up the design flux swing.

OH Solenoid

The objective here has been to include the constraints imposed by force and thermal limits. Also, the loss of conducting area due to insulation build and cooling channels is included. Relevant equations are found in the Appendix. Equating force and thermal limits, with no area loss included, yields a 6 (six) layer design. Cooling requirements make this difficult, so that a 4 layer design was used. An insulation build of 0.1 cm per turn is assumed, and an area loss corresponding to a 1/4 inch copper tube included. Force limits dominate. Also, while a design which stresses each layer equally gives a larger flux swing, the design numbers quoted here assume equal turns per layer, so that only the stress on the inner layer counts.

The design code computes self and mutual inductances using the "exact" elliptic integral equation.. These values are always smaller than the ideal solenoid results. A charging current of 80 kiloAmps is supplied by a capacitive store. The initial current ramp is done by resistively clamping the OH solenoid to zero current, then completing the swing with a conventional power supply.

Point Design

Use the values

B_0 = 0.5 Tesla
R_0 = 0.7 meters
A_0 = 1.43
l_i = 0.7

The TF coil sets the solenoid inner radius at 0.10 meters. Allowing a 2.5 cm gap between the inner plasma radius and the outer radius of the solenoid fixes the later at 0.185 meters. The OH solenoid radial thickness is then 0.085 meters (i.e. 2.125 cm per layer).

Running the design code for equal turns per layer gives (for each layer)

$$N = 80 \text{ turns}$$

Other parameters are

$$L_{OH} = 2.31 \text{ mH}$$
$$R_{OH} = 14.67 \text{ m}\Omega$$
$$M_{SP} = 8.97 \ \mu\text{H}$$
$$L_P = 0.48 \ \mu\text{H}$$

The plasma current obtainable is

$$I_P = 1.05 \text{ Megamp}$$

where only a 80 kiloAmp swing of the OH solenoid contributes to forming the plasma. The remaining OH current swing is used to maintain the plasma current.

Appendix

TF Coil

If a toroidal field B_0 is required at a radius R_0, then from Ampere's law the total TF center post ampere-turns is

$$I_{TF} = 5\, B_0\, R_0$$

Assume that this current is carried in a "hollow" center post with inner radius R_i and outer radius R_o. The area utilization is ε, so the current density is

$$J = \frac{I_{TF}}{\varepsilon\pi(R_o^2 - R_i^2)}$$

The center post is assumed to be made of copper. The equation describing heating of the post is

$$\rho C_p \frac{\partial T}{\partial t} = \eta\, J^2$$

where the various thermophysical parameters are temperature dependent, i.e.

$$\rho = \rho_0\,(1 - \alpha_1 T) \quad ; \quad \rho_0 = 8.96\,\frac{gm}{cm^3}$$

$$C_p = C_0\,(1 + \alpha_2 T) \quad ; \quad C_0 = 0.3565\,\frac{J}{gm\ {}^\circ K}$$

$$\eta = \eta_0\,(1 + \beta T) \quad ; \quad \eta_0 = 1.7 \cdot 10^{-6}\ ohm \cdot cm$$

Published values are

$$\alpha_1 = 5.1 \cdot 10^{-5} \frac{1}{°K}$$

$$\alpha_2 = 2.588 \cdot 10^{-4} \frac{1}{°K}$$

$$\beta = 4.3 \cdot 10^{-3} \frac{1}{°K}$$

For moderate temperature rises, only the variation of resistivity need be included. Then the thermal equation becomes

$$\int_0^{\Delta T} \frac{\beta \, dT}{(1 + \beta T)} = \frac{\eta_0 \beta}{\rho_0 C_0 A^2} \int_0^t I_{TF}^2 \, dt$$

where

$$A = \varepsilon \pi (R_o^2 - R_i^2)$$

Define the "action" integral as

$$A_c = \int_0^t I_{TF}^2 \, dt$$

For a linear ramp to I_{TF} in a time $\Delta t / 2$, staying constant at I_{TF} for a time Δt, and then a ramp down similar to the ramp up, one has

$$A_c = \frac{4}{3} I_{TF}^2 \Delta t$$

Solving for the area gives

$$A^2 = \frac{\eta_0 A_c}{\rho_0 C_0} \frac{\beta}{\ln(1 + \beta \Delta T)}$$

For small temperature rises, the logarithm can be expanded giving

$$A^2 = \frac{\eta_0 A_c}{\rho_0 C_0} \frac{1}{\Delta T (1 - \frac{1}{2} \beta \Delta T)}$$

Scaling Estimates :

Ignore R_i and set the temperature rise to 50 °K. Then with $\varepsilon = 2/3$ one gets

$$R_o = 17.43 \sqrt{B_0 R_0 \sqrt{\Delta t}} \quad [\text{cm}]$$

For 0.5 Tesla at 0.7 meters, for 0.5 seconds, the TF post outer radius is

$$R_o = 8.67 \text{ cm}$$

This is rounded up to 9.5 cm to include the area loss due to R_i.

Plasma

R_0 = major radius [meters]
a_p = minor radius [meters]
A_s = aspect ratio = R_0 / a_p
κ = elongation
l_i = normalized internal inductance
I_p = plasma current [megAmps]
t_f = current flat top duration in seconds
V_p = plasma voltage at flat top [about 1.5 volts]

$$L = \text{total plasma inductance} = \mu_0 R_0 \left\{ \ln \frac{1.08 A_s}{\sqrt{\kappa}} + \frac{l_i}{2} \right\}$$

$$\Psi_p = \text{plasma flux consumption} = \mu_0 R_0 I_p \left\{ \ln \frac{1.08\, A_s}{\sqrt{\kappa}} + \frac{5}{6} l_i \right\} + V_p t_f$$

Solenoid

n_l = number of layers (typically 4)

R_i = inner radius of solenoid [meters] - set by TF center post

R_o = outer radius of solenoid [meters] - set by plasma major radius

ΔR = layer width [meters] = $(R_o - R_i) / N_l$

I_s = solenoid current [kiloAmps] - flows in each turn

R_j = inner radius of jth layer = $R_i + (j-1) \Delta R$

N_j = turns per length of jth layer

$B_{j0} = \mu_0 N_j I_s$ = central magnetic field from j^{th} layer

$B_j = \mu_0 I_s \sum_{k=j}^{n_l} N_k$ = magnetic field at inner radius of jth layer

$J_j = \frac{I_s}{\Delta R} N_j$ = current density of jth layer

$\sigma_j = R_j J_j B_j$ = tensile stress at inner radius of jth layer

This last formula can be used to spread out the windings so that the stresses are uniform. For 4 layer case set

$$\sigma_3 = \sigma_4$$

which gives

$$N_3^2 + N_3 N_4 = \frac{R_4}{R_3} N_4^2$$

This is solved to yield

$$N_3 = \left[\sqrt{\frac{R_4}{R_3} + \frac{1}{4}} - \frac{1}{2} \right] N_4 = \alpha_3 N_4 < N_4$$

Similarly

$$N_2 = \left(\sqrt{\frac{R_4}{R_2} + \frac{(1+\alpha_3)^2}{4}} - \frac{(1+\alpha_3)}{2} \right) N_4 = \alpha_2 N_4 < N_4$$

$$N_1 = \left(\sqrt{\frac{R_4}{R_1} + \frac{(1+\alpha_2+\alpha_3)^2}{4}} - \frac{(1+\alpha_2+\alpha_3)}{2} \right) N_4 = \alpha_1 N_4 < N_4$$

For design purposes set

$$R_i = 0.095 \quad \text{and} \quad R_o = 0.185$$

Then

$$\Delta R = 0.0225$$

$$R_1 = 0.095, \quad R_2 = 0.1175, \quad R_3 = 0.14, \quad R_4 = 0.1625$$

which gives

$$\alpha_1 = 0.593037, \quad \alpha_2 = 0.603575, \quad \alpha_3 = 0.687735$$

Specify N_4 and then compute the rest, fractional turns rounded to even integer.

One constraint enters here. If the peak stress must be less than 30 ksi, then

$$\sigma_4 = \mu_0 I_s^2 \frac{R_4}{\Delta R} N_4^2 \leq 30 \times 6.895 \cdot 10^6 = 207 \cdot 10^6 \frac{J}{m^3}$$

or

$$N_4 I_s \leq 12{,}834.54 \sqrt{\frac{\Delta R}{R_4}} \quad \frac{\text{Kamp} \cdot \text{Turns}}{\text{m}}$$

An alternate constraint arises from thermal considerations. The temperature rise for any layer is determined from the equation

$$\frac{\partial T_j}{\partial t} = \frac{\eta}{\rho\, C_p} J_j^2$$

or

$$\Delta T_j = \frac{\eta}{\rho\, C_p} \frac{N_j^2}{\Delta R^2} \int_0^{t_f} I_s^2\, dt \cong \frac{1}{3} \frac{\eta}{\rho\, C_p} \frac{N_j^2}{\Delta R^2} I_s^2\, t_f$$

Again for a 4 layer solenoid, the 4th layer is worst. Fixing the temperature rise at 50 °K or less, and using numbers for copper gives

$$N_4 I_s \leq 5700 \frac{\text{Kamp} \cdot \text{Turns}}{\text{m}}$$

where $t_f = 0.4$ seconds was used.

For detailed calculations, one needs to include area loss associated with insulation between turns, and the cooling channel in each turn. The actual current density is then

$$J_j = \frac{I_S}{((\Delta R - 2\,\delta R)(\frac{1}{N_j} - 2\,\delta Z) - A_{Cooling})}$$

and is used in the above expressions. Here the various quantities are

δR = radial insulation build on each side of winding = 2 mm

δZ = axial insulation build on each side of winding = 1 mm

$A_{Cooling}$ = area loss to cooling = 0.403 cm^2

Solenoid - Plasma Coupling

The way to connect the solenoid and plasma is to note that the flux consumption of the plasma is provided by the (double swing) flux in the center of the solenoid (i.e. treating it as an ideal "infinite length" device, with current reversal). Thus,

$$\Delta\Psi_s = 2 \int B_s \, dA$$

where the integral is over the whole cross-section of the solenoid. Treating the current density as uniform in each layer, the above gives

$$\Delta\Psi_s = 2\,\pi\,\mu_0\,N_4\,I_s \sum_{j=1}^{n_1} \alpha_j\,R_j^2 \left[1 + \frac{\Delta R}{R_j} + \frac{1}{3}\frac{\Delta R^2}{R_j^2} \right] = \beta\,N_4\,I_s$$

Equating this to the consumed flux gives

$$\mu_0\,R_0\,I_p \left\{ \ln \frac{1.08\,A_s}{\sqrt{\kappa}} + \frac{5}{6}\,l_i \right\} + V_p\,t_f = \beta\,N\,I \le 4775\,\beta$$

where the stress-defined inequality has been used.

Capacitor Bank

Capacitor bank charging of OH solenoid :

L_s = solenoid inductance

R_s = solenoid resistance

C_s = bank capacitance

V_0 = initial bank charging voltage

I_0 = desired solenoid charge current

The differential equation to be solved is

$$L_s \frac{d^2 I_s}{dt^2} + R_s \frac{dI_s}{dt} + \frac{1}{C_s} I_s = 0$$

with initial conditions being

$$I_s(0) = 0$$

$$\frac{dI_s(0)}{dt} = \frac{V_0}{L_s}$$

Define

$$\omega_s^2 = \frac{1}{L_s C_s}$$

$$Q = \frac{2\sqrt{\frac{L_s}{C_s}}}{R_s}$$

so the differential equation can be written as

$$\frac{d^2 I_s}{dt^2} + 2\frac{\omega_s}{Q}\frac{dI_s}{dt} + \omega_s^2 I_s = 0$$

If Q >> 1 (always true for us) then the solution can be written as

$$I_s = \frac{V_0}{\omega_s L_s} e^{-\frac{\omega_s t}{Q}} \sin \omega_s t$$

which peaks at

$$t_m \cong \frac{\pi}{2\,\omega_s} = \frac{\pi}{2}\sqrt{L_s C_s}$$

Then

$$I_0 = \frac{V_0}{\omega_s L_s} e^{-\frac{\pi}{2Q}}$$

Note this can be rearranged to give

$$E_0 = \frac{1}{2} C_s V_0^2 = \frac{1}{2} L_s I_0^2 e^{\frac{\pi}{Q}}$$

Once the inductor is charged, the plasma is formed initially by switching a resistor into the capacitor bank circuit so that roughly 20 loop volts is induced at the plasma location for a period of about 5 milliseconds. The resistor can be in series or shunt with the capacitor.

Consider the parallel case for simplicity. The solenoid / plasma equations are

$$L_s \frac{dI_s}{dt} + M_{sp} \frac{dI_p}{dt} + (R_0 + R_s) I_s = 0$$

$$L_p \frac{dI_p}{dt} + M_{sp} \frac{dI_s}{dt} + R_p I_p = 0$$

where

R_0 = crowbar resistance

M_{sp} = solenoid - plasma mutual inductance

During plasma current formation and rise, the Ejima criteria (probably fairly good) gives

$$R_p I_p \cong \frac{2}{3} L_{internal} \frac{dI_p}{dt} \cong \frac{2}{3} L_p \frac{dI_p}{dt}$$

so that the plasma circuit equation reduces to

$$\frac{dI_p}{dt} = -\frac{3}{5}\frac{M_{sp}}{L_p}\frac{dI_s}{dt}$$

or

$$I_p = \frac{3}{5}\frac{M_{sp}}{L_p}\left(I_0 - I_s(t) \right)$$

The solenoid equation becomes

$$\left(L_s - \frac{3}{5}\frac{M_{sp}^2}{L_p} \right)\frac{dI_s}{dt} + \left(R_0 + R_s \right) I_s = 0$$

with solution

$$I_s(t) = I_0 \; e^{-\frac{t}{\tau}}$$

where

$$\tau = \frac{\left(L_s - \frac{3}{5}\frac{M_{sp}^2}{L_p} \right)}{\left(R_0 + R_s \right)}$$

Solenoid Calculations

The ohmic heating solenoid is modelled as four concentric cylindrical shells, each shell consisting of a specified number of turns. Other input parameters consist of the solenoid height and the inner and outer radii of the solenoid. The integral form of Lenz's law is

$$\int \nabla \times \mathbf{E} \cdot d\mathbf{S} = \int \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{S} = -\frac{d\Psi}{dt}$$

Using Ohm's relation

$$\mathbf{E} = \eta \mathbf{J}$$

and the assumption that the current density is locally uniform so that

$$\mathbf{J} = \frac{I_s}{A}\hat{\theta}$$

where A is the cross-sectional area of the winding, then

$$\int \mathbf{E} \cdot d\mathbf{l} = I_s \int \frac{\eta}{A} dl = R_s I_s$$

with R_s the total resistance of the solenoid.

The computation of the flux term is a bit more complicated. By definition,

$$\Psi = L_s I_s = \int \mathbf{B} \cdot d\mathbf{S}$$

The integral over area must include every turn in the solenoid. Assume axisymmetry and write

$$\mathbf{B} = \frac{1}{R}\left(-\frac{\partial(R\psi_s)}{\partial z}\hat{\mathbf{R}} + \frac{\partial(R\psi_s)}{\partial R}\hat{\mathbf{Z}} \right)$$

with

$$R\psi_s = \int G(R,Z;R',Z')\ dI(R',Z')$$

and G(R,Z;R',Z') the free space Green's function of a current loop filament, i.e.

$$G(R,Z;R',Z') = \frac{\mu_0}{\pi}\frac{\sqrt{RR'}}{k}\left((1-\frac{1}{2}k^2)\ K(k) - E(k) \right)$$

$$k^2 = \frac{4RR'}{(R+R')^2 + (Z-Z')^2}$$

where K(k) and E(k) are complete elliptic integrals of the first and second kind, respectively. Representing the solenoid by N turns allows one to write

$$R\psi_s = \sum_{i=1}^{N} \int G(R,Z;R_i,Z_i)\ dI(R_i,Z_i)$$

If each turn of the solenoid is "small" enough, then one can perform a series expansion

$$G(R,Z;R_i,Z_i) = G(R,Z;R_c,Z_c) + \frac{\partial G}{\partial R_c}(R_i-R_c) + \frac{\partial G}{\partial Z_c}(Z_i-Z_c) + \frac{1}{2}\frac{\partial^2 G}{\partial R_c^2}(R_i-R_c)^2 + \cdots$$

where R_c , Z_c are fixed coordinates characterizing each turn. By defining

$$R_c = \frac{1}{I_s}\int R_i\ dI(R_i,Z_i)$$

$$Z_c = \frac{1}{I_s}\int Z_i\ dI(R_i,Z_i)$$

(which for uniform current density corresponds to the center of each turn), then

$$\int G(R,Z;R_i,Z_i)\, dI(R_i,Z_i) = G(R,Z;R_c,Z_c)\, I_s + \cdots$$

with correction terms now dependent on second order derivatives and higher. These are usually negligible. Putting everything together, one arrives at the expression

$$L_s = 2\pi \sum_{i=1}^{N} \sum_{j=1}^{N} G(R_i,Z_i;R_j,Z_j)$$

for the solenoid self-inductance. Note that the double sum gives the N^2 type scaling expected.

Vessel Modeling

The vessel is represented by N current loop filaments which are inductively coupled to each other, as well as to the plasma and OH solenoid. The circuit equations describing the system are

$$L_s \dot{I}_s + M_{sp} \dot{I}_p + \sum_{i=1}^{N} M_{si} \dot{I}_i + R_s I_s = V_s$$

$$M_{sp} \dot{I}_s + L_p \dot{I}_p + \sum_{i=1}^{N} M_{pi} \dot{I}_i + R_p I_p = 0$$

$$M_{si} \dot{I}_s + M_{pi} \dot{I}_p + L_i \dot{I}_i + \sum_{j \neq i} M_{pj} \dot{I}_j + R_i I_i = 0 \quad (i = 1,N)$$

Note that this can be expressed conveniently in matrix form as

$$M \dot{I} = V - R I$$

which is rewritten as

$$\dot{I} = M^{-1} (V - R I)$$

The code computes the self- and mutual- inductances using the same prescription given in the solenoid design, as are the resistances. The current derivative expression is used with a fourth-fifth order Runge-Kutta integrator to time advance the solution starting from some initial state (usually starting with a current charged inductor and no current in vessel or plasma). The plasma resistive term is modelled as either 1.5 volts, or 2/3 of the internal inductive drop, which ever is larger. It appears on the RHS of the equation so that the inductance matrix can be treated as constant in time. The code also computes the stress induced in the solenoid by JxB forces, and calculates the integral of I^2 dt to allow temperature rises in the solenoid to be followed.