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A most interesting problem in quantum field theory is the question whether a particle introduced as a fixed pole in the lowest order of perturbation theory stays "elementary" or whether it becomes a member of a Regge trajectory if higher orders are included.[1-3] It is the purpose of this note to show that vector mesons may become Regge particles in higher orders of renormalizable perturbation theory.

It follows from the general notions of dispersion theory that bosons with spin larger than one must be described by moving poles.[4] Perhaps it is characteristic that interactions involving such particles are not renormalizable. As far as the possibility of renormalization as well as the necessity of reggeization are concerned, the vector mesons are a boundary case. At present, an elementary vector meson[4] cannot be completely excluded on the basis of general arguments. However, there are severe restrictions.

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In an earlier publication[2] we have shown that an elementary vector meson contribution like $B(s)t$ to the high energy limit ($t \rightarrow \infty$) of the elastic amplitude $F(s,t)$ necessarily induces an imaginary term which corresponds to a singularity in the complex angular momentum plane in the s-channel. We found that in the limit $t \rightarrow \infty$ an expression of the form

$$F(s,t) \sim B(s)t \;+\; C(s)t^{\alpha(s)}(\ell n t)^{\beta(s)}$$

is compatible with unitarity in the t-channel provided $\alpha(o) = 1$ and $0 \leqslant \beta(o) \leqslant 1$. Here $B(s)$ is real for $s \leqslant 0$ and $C(s)$ is complex. This result was obtained on the basis of the unitarity condition for forward scattering ($s = 0$). Further restrictions on $\beta(s)$ have been obtained by Yamamoto[5] by integrating the unitarity condition for large values of t in a symmetric way over a cone around the forward direction such that the inelastic contribution to this integral becomes positive definite. For $\beta(s) = $ const. one finds then that only $\beta = 1$ is allowed. With $\beta = 1$ and $\alpha(o) = 1$ we have a double pole of $F(s,\lambda)$ at $\lambda = \alpha(s)$ which implies a logarithmically increasing total cross section in the t-channel.

In view of these restrictions it seems to be more natural to assume that vector mesons are described by Regge poles. Consequently it becomes of special interest to see whether a reggeization is possible in weak coupling perturbation theory. In the following we first consider the pole due to a neutral vector meson in the amplitude $F(s,t)$ for the process $\pi^+\pi^- \rightarrow \pi^+\pi^-$. The partial wave amplitudes $F_\ell(s)$ in the s-channel can be interpolated uniquely by the analytic function $F\,(s,\lambda)$ such that $F\,(s,\ell) = F_\ell(s)$ for odd, integer values of ℓ. If there exists a Regge trajectory associated with a vector meson of mass m with $\mu < m < 2\mu$ (μ = pion mass), then we can write

$$F(s,\lambda) = \frac{\beta(s)}{\lambda - \alpha(s)} + R(s,\lambda), \qquad (1)$$

where $\alpha(m^2) = 1$ and where $R(s,\lambda)$ is not singular on the surface $\lambda = \alpha(s)$. For real values of s in the interval $4\mu^2 \leq s < s_i$ we have the continued elastic unitarity condition which can be evaluated near $\lambda = \alpha(s)$. Taking the limit from above (s + io) for $4\mu^2 \leq s < s_i$ it implies

$$\alpha(s) = \frac{\rho(s)\,\beta^*(s)}{1 - 2i\rho(s)\,R(s,\alpha^*(s))}, \qquad (2)$$

where

$$\rho(s) = \left(\frac{s - 4\mu^2}{s}\right)^{1/2}.$$

The contribution of the pole at $\lambda = \alpha(s)$ to the high energy limit of the absorptive part in the t-channel is given by

$$A_t(s,t) \sim 1/2\; b(s)t^{\alpha(s)}, \qquad (3)$$

where

$$b(s) = \frac{\Gamma(\alpha+3/2)}{\Gamma(\alpha+1)}\; 2\sqrt{\pi}\; q^{-2\alpha}\; \beta(s) \qquad (4)$$

with $4q^2 = s-4\mu^2$. If we also assume the existence of a Sommerfeld-Watson representation we obtain for F(s,t) the contribution

$$F(s,t) \qquad \frac{b(s)}{\sin \pi\alpha(s)}\; 1/2\; (1-e^{-i\pi\alpha(s)})\,t^{\alpha(s)} \qquad (5)$$

For reasons of simplicity we restrict ourselves to the usual renormalizable coupling between pions and neutral vector mesons.

We expand the functions α,β,R, etc. in powers of the coupling constand g^2; for example

$$\alpha(s) = 1 + g^2\alpha_1 + g^4\alpha_2 + g^6\alpha_3 + \dots \tag{6}$$

As a consequence of invariance under charge-conjugation only graphs with intermediate states in the s-channel corresponding to an odd number of vector mesons can contribute to the pole term in Eq. (1). Since the graphs with the exchange of a single vector meson give no contribution to $A_t(s,t)$ and to $F(s,\lambda)$, the lowest order of $b(s)$ or $\beta(s)$ is determined by the sixth order graphs in Fig. 1 c,d and the corresponding crossed diagrams. Hence we have

$$\beta(s) = g^6\beta_3 + g^8\beta_4 + \dots \tag{7}$$

The Born graph in Fig. 1a gives the contribution

$$\frac{g^2\, 4q^2(s)\,(1+t/2q^2)}{m^2 - s} \tag{8}$$

to $F(s,t)$, and in the partial wave projection $F_1(s)$ we have the corresponding term

$$\frac{g^2 4q^2(s)}{3(m^2 - s)} \tag{9}$$

We can expand the expression (5) in powers of g^2 and compare it with the high energy limit of the Born term (8). We find that we must have $\alpha_1(s) \equiv 0$ and from the second order term

$$F(s,t) \sim - g^2 \frac{b_3}{\pi\alpha_2}\, t \tag{10}$$

we obtain the relation

$$\alpha_2(s) = (s-m^2)\ \frac{1}{2\pi} \cdot b_3(s) = \frac{3(s-m^2)}{4q^2(s)}\ \beta_3(s) \tag{11}$$

The same result can be obtained by comparing the expansion of $F_-(s,1)$ with Eq. (9), provided we identify the term $g^2R_1(s,1)$ in the expansion of $R(s,\lambda)$ with the Born term in the t-channel, which gives a contribution regular in s near $s = m^2$ and in λ near $\lambda = 1$. Note that we have imposed here the requirement that the pole term (9) is completely taken up by the Regge trajectory $\lambda = \alpha(s)$ of $F(s,\lambda)$. The possible existence of such a trajectory in perturbation theory is just what we would like to explore.

We are now in a position to consider the implications of the elastic unitarity condition (2). We find that the first terms in the expansions (6) and (7) must satisfy the relations

$$\text{Im}\ \alpha_2(s) = 0, \quad \text{Im}\ \beta_3(s) = 0 \tag{12}$$

and

$$\text{Im}\ \alpha_3(s) = \rho(s)\beta_3(s) \tag{13}$$

for $4\mu^2 \leq s < s_i$, where now $s_i = 9m^2$. In these equations we have made the assumption that $R(s,\alpha^*)$ is of order g^2 or higher. Essentially this implies that we do not allow a secondary trajectory $\lambda = \varphi(s)$ which is degenerate with $\alpha(s)$ in lowest order such that

$$R(s,\lambda) = \frac{\psi(s)}{\lambda - \varphi(s)}$$

has a zero-order contribution for $\lambda = \alpha^*(s)$.

The function $\beta_3(s)$ can be computed from the high energy limits of the sixth order graphs in Figs. 1c, 1d and the related crossed diagrams, or from the corresponding partial wave functions near $\lambda = 1$. We find

$$b_3(s) = \left(\frac{g^2}{4\pi}\right)^3 \frac{1}{90} \int_0^1 dx_1\, dx_2\, dx_3\, dy_1\, dy_2\, dy_3\, dy_4\, \delta(x_1+x_2+x_3-1)$$
$$\times\, \delta(y_2+y_4-1)\,\delta(y_1+y_3-1)\, B(x_i, y_j, s) \tag{14}$$

where

$$B = A_1(A_2 s - A_1 m^2)^{-1}(f_1^{-2} + 2f_2^{-2})$$

with

$$A_1 = x_1 x_2 + x_2 x_3 + x_3 x_1$$
$$A_2 = x_1 x_2 x_3$$
$$f_1 = x_2(y_1 y_2 + y_2 y_3 + y_3 y_4 + y_4 y_1) + x_3 y_1 y_2 + x_1 y_3 y_4$$
$$f_2 = x_2 y_1 + x_3 y_2 - x_1 y_3 y_4$$

The function $\alpha_2(s)$ is then given by Eq. (11). Diagrams like Fig. 1e do not contribute to b_3 because they do not give rise to a pole at $\lambda = 1$. The function $b_3(s)$ is analytic in the cut s-plane with a branch line for real $s \geq 9m^2$. Hence we see that the functions $\alpha_2(s)$ and $b_3(s)$ or $\beta_3(s)$ satisfy the relations (12). It is important to note the absence of the elastic branch line in the lowest order contributions to the trajectory function $\alpha(s)$. This is a typical requirement of the unitarity condition in a situation where there is no coupling to "nonsense" amplitudes.

The sixth order term in the expansion of $\alpha(s)$ should have a discontinuity along the elastic cut $4\mu^2 \leq s < 9m^2$. In order to identify the graphs which are related to $\alpha_3(s)$ we consider a selected number of higher terms in the expansion of the high energy limit (5):

$$F(s,t) \sim -g^2 \frac{b_3}{\pi\alpha_2} t + g^4 \frac{\alpha_3 b_3}{\pi\alpha_2^2} t + \cdots + g^6 \frac{b_3}{\pi}\left(\frac{i\pi}{2} - \ln t\right) t + \cdots + g^{10} \frac{\alpha_2 b_3}{2\pi}(i\pi - \ln t)\, t \ln t + \cdots + g^{12} \frac{\alpha_3 b_3}{2\pi}(i\pi - \ln t)\, t \ln t + \cdots \tag{15}$$

The fourth order term corresponds to the vector meson self-energy graph Fig. 1b. This graph has a two-pion intermediate state, and the discontinuity is consistent with the requirement (13). After the mass renormalization has been performed the function corresponding to this graph has a simple pole at $s = m^2$. Since $\alpha_2(s) \sim$ Const.$(s-m^2)$ for $s \to m^2$, we see from Eq. (15) that $\alpha_3(s)$ must also have a zero at $s = m^2$. Of course, this is just what we expect if $\alpha(s)$ shall be a trajectory through the vector-meson pole.

The tenth order contribution in Eq. (15) is difficult to identify; note that it has no elastic cut. On the other hand it is clear that the twelfth order term proportional to $\alpha_3 b_3$ is connected with the graphs in Figs. 1f,g and related ones. The two-pion discontinuity gives a contribution proportional to b_3^2, as expected. However there are two unresolved questions concerning the contribution from the sum of all these twelfth order graphs:

1. The high energy limit of a given diagram like the ones in Figs. 1f,g may well have a leading term proportional to $t(\lg t)^3$ or to $it(\lg t)^2$ in the absorptive part A_t. Such terms are due to

intermediate states in the s-channel which involve more particles than two pions. If the simple reggeization procedure considered here shall be successful we must assume that these terms cancel in the sum over all twelfth order graphs.

2. Another requirement is that the function $\alpha_3(s)$ obtained in the high energy limit of these graphs must have a zero at $s = m^2$.

Summing up our considerations concerning vector mesons in the pion-pion amplitude we can say that there exists a function

$$F(s,\lambda) \approx \frac{g^6\beta_3(s)}{\lambda-1-g^4\alpha_2(s) - g^6\alpha_3(s)} \tag{16}$$

which satisfies the elastic unitarity equation and which contains the vector meson pole term. Within this framework it provides evidence that the perturbation expansion is compatible with the reggeization of the fixed vector meson pole introduced in the lowest order. We note that our considerations are not applicable to the photon because of the infrared divergences.

The possible transformation of a fixed vector-meson pole into a Regge pole as a consequence of radiative corrections can also be studied in nucleon-antinucleon scattering. The situation is quite analogous to the one discussed above. In lowest order the vector meson pole in the s-channel appears in the three coupled triplet amplitudes[6] $h_{ik}(s,J)$ for J = 1, which satisfy the unitarity equation

$$\operatorname{Im} h_{ik}(s,J) = \rho(s) \sum_{j=1}^{2} h_{ij}(s,J)\, h^*_{jk}(s,J^*) \tag{17}$$

in the elastic region $4M^2 \leq s < 9m^2$, where M is the nucleon mass. For reasons of simplicity we assume here that $\frac{2}{3}M < m < M$, and we consider only the renormalizable coupling between neutral vector mesons and the conserved spinor current. At J = 1 the pole terms are given by

$$h_{ik}(s, J=1) = \frac{4}{3} C_{ik} \frac{g^2 m^2}{m^2 - s}$$

with

$$C_{11} = 1 \qquad C_{12} = \left(s/2m^2\right)^{1/2} \qquad C_{22} = s/2m^2$$

We see that the residue factorizes and hence there is no difficulty with the reggeization as far as the unitarity condition (17) is concerned.

An essential feature in the perturbation expansion of the vector meson trajectory is the fact that in the lowest nonvanishing order the function $\alpha(s) - 1$ does not have the elastic cut, which only appears in the next order. This result is not changed if we add a direct pion-pion coupling. It is also similar to the situation encountered in connection with the possible nucleon trajectory in pion-nucleon scattering.[1] Here we have a vector meson-nucleon and a pion-nucleon coupling. Assuming $m > \mu$, we consider the amplitude

$$f_+(W, j) = \frac{\beta(W)}{j - \alpha(W)}$$

which has to satisfy a decoupled elastic unitarity condition for $(\mu + M)^2 \leq W^2 \leq (m + M)^2$. From the lower order graphs we see that we have the expansions

$$\alpha(W) = \frac{1}{2} + g_V^2\, \alpha_{10}(W) + g_V^2 g_\pi^2 \alpha_{11}(W) + \ldots$$

$$\beta(W) = g_V^2 g_\pi^2\, \beta_{11}(W)$$

where g_V and g_π are the vector-meson-nucleon and pion-nucleon coupling constants respectively. The functions $\alpha_{10}(W)$ and $\beta_{11}(W)$ have a cut for $W^2 \quad (m + M)^2$, but not the elastic branch point at $W^2 = (\mu + M)^2$. For the function $\alpha_{11}(W)$ we obtain an elastic cut with a weight proportional to $\beta_{11}(W)$ for $(\mu + M)^2 \leq W^2 < (m + M)^2$. These properties are in agreement with the elastic unitarity condition for the amplitude $f_+(w, j)$.[7]

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7. In lower orders of perturbation theory we know that the nucleon cannot be on a Regge trajectory if we have only the pseudoscalar pion-nucleon interaction.[1] For scalar mesons with $g\phi^3$ - coupling a more general proof that the pole remains elementary has been given by Tiktopoulos [Phys. Rev. (to be published)]. Essentially he shows that there are no terms in the expansion of F(s,t) which increase as fast t^0 for $t \to \infty$. This implies also that the function $F(s,\lambda)$ has no pole at $\lambda = 0$ for every finite order. We note that here elastic unitarity alone is not enough to exclude a possible reggeization. It again only implies that the lowest, nonvanishing term in the expansion $\alpha(s) = g^{2n}\alpha_n(s) + \cdots$ [illegible] has no elastic cut, and that $\mathrm{Im}\,\alpha_{n+1} = \rho\,\beta_{n+1}$, $\beta_\nu = 0$ for $\nu \leq n$, etc.

Figure Caption

Fig. 1. Some Feynman graphs which are considered in connection with the vector meson trajectory.

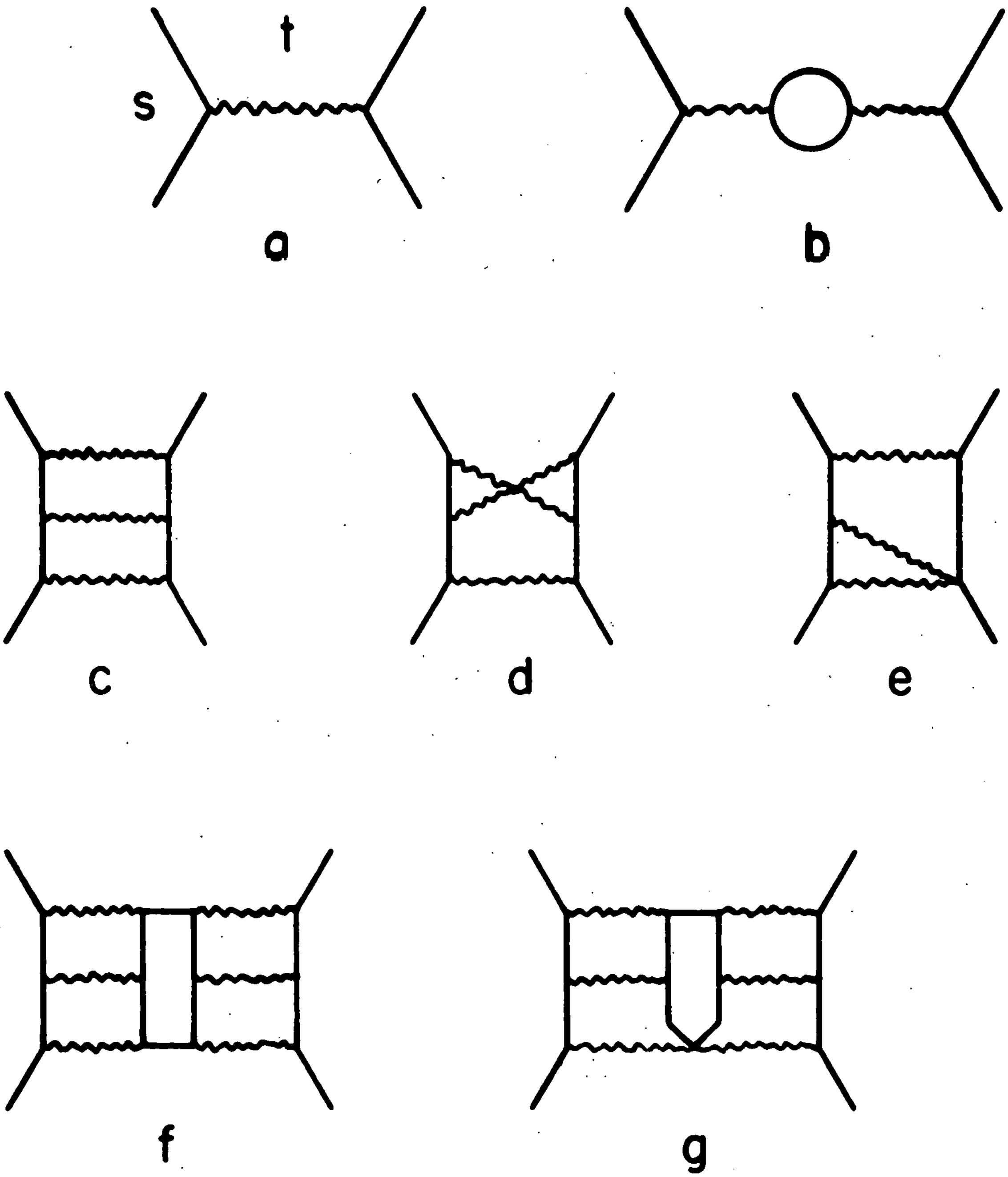

Fig. 1