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# Meson Spectrum from the Bethe-Salpeter equation.

*C. J. Burden<sup>A</sup>, L. Qian<sup>B</sup>, C. D. Roberts<sup>C</sup>, P. C. Tandy<sup>B</sup>  
and M. J. Thomson<sup>D</sup>.*

<sup>A</sup> Department of Theoretical Physics,  
Research School of Physical Sciences and Engineering,  
Australian National University, Canberra, ACT 0200, Australia.

<sup>B</sup> Department of Physics, Kent State University,  
Kent, Ohio, USA 44242.

<sup>C</sup> Physics Division, Argonne National Laboratory,  
Argonne, Illinois, USA 60439.

<sup>D</sup> School of Physics, University of Melbourne,  
Parkville, Victoria 3052, Australia.

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*Abstract:* We present details of a model for calculating the mass spectrum of light-quark mesons and decay constants of the pseudoscalar meson octet from a phenomenological model based on Dyson-Schwinger and Bethe-Salpeter equations. In this model the Bethe-Salpeter kernel is approximated by a separable ansatz obtained from input quark propagators.

## 1 Introduction

The dynamics of the light mesons is a sensitive test of confinement and dynamical chiral symmetry breaking in QCD, with the pion being realised as an almost-Goldstone boson. With this in mind, we present here details of a phenomenological model of light mesons based on the application of approximate Dyson-Schwinger (DS) and Bethe-Salpeter (BS) equations [1].

Central to the success of this model is the fact that the combined rainbow DS equation and homogeneous ladder BS equation have the property that, in the chiral limit, they admit as solutions massless Goldstone pions [2]. This property has been used to advantage in a number of earlier studies [3] of light mesons. However, the model presented here differs from previous models in that

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the only inputs are light quark propagators inspired by established studies of QCD DS equations [4]. The Bethe-Salpeter kernel is then approximated by a separable effective form obtained by inverting the quark DS equation. Similar methods have also been employed in a recent study by Cahill and Gunner [5]. We emphasise that the philosophy behind such calculations is one of attempting to understand physical processes driving observed phenomena, rather than numerical verification of QCD itself.

## 2 The model

We begin with the combination of a rainbow approximation DS equation for each quark propagator [1],

$$S_f^{-1}(p) - i \not{p} - m = \frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \gamma_\mu S_f(q) \Delta(p-q) \gamma_\mu, \quad (1)$$

and the homogeneous ladder approximation to the meson Bethe-Salpeter equation,

$$\Gamma(p, P) = -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \gamma_\mu S_{f_1}(q - \xi P) \Gamma(q, P) S_{f_2}(q + (1-\xi)P) \gamma_\mu. \quad (2)$$

Here  $S_f(p)$  is the full quark propagator of flavour  $f$ , and  $\Gamma(q, P)$  the BS amplitude defined so that external outgoing and incoming quark lines carry momenta  $q + (1-\xi)P$  and  $q - \xi P$  respectively. In the DS equation, the full gluon propagator has been modelled by the form  $g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu} \Delta(p-q)$  which one may describe as a representation of the propagator in a Feynman-like gauge. In the BS equation, the generalised ladder approximation has been employed in which the effect of the full kernel is assumed to be well approximated by the insertion of a single gluon propagator. By rainbow approximation we mean that the full quark-gluon vertex has been replaced by the bare vertex. The metric is Euclidean.

The most general form of the fermion propagators consistent with Lorentz covariance, together with C, P and T, is given in terms of two scalar functions. Here we use either of the two following representations for our quark propagators:

$$S(p) = -i \not{p} \sigma_V(p^2) + \sigma_S(p^2) = \frac{1}{i \not{p} A(p^2) + B(p^2)}. \quad (3)$$

To ensure quark confinement, and therefore the absence of spurious quark production thresholds in the BS amplitudes, it is sufficient [1] that  $\sigma_V$  and  $\sigma_S$  be entire functions in the complex  $p^2$ -plane. Propagator amplitudes with this property are the only input for the present approach. A separable ansatz for the BS kernel is obtained by making the truncation

$$\Delta(p-q) = G(p^2)G(q^2) + p \cdot q F(p^2)F(q^2). \quad (4)$$

This can be thought of as a separable approximation to the first two moments of an expansion in terms of Tschebyshev polynomials in the angular variable  $\hat{p} \cdot \hat{q}$ . Orthogonality of Tschebyshev polynomials with respect to the measure  $d^4q$  ensures that higher moments do not contribute to Eq. (1), whose solution then requires

$$F(q^2) = \frac{1}{a} (A(q^2) - 1), \quad G(q^2) = \frac{1}{b} (B(q^2) - m). \quad (5)$$

Here  $a$  and  $b$  are given by

$$a^2 = \frac{2}{3} g^2 \int \frac{d^4q}{(2\pi)^4} q^2 \sigma_V(q^2) (A(q^2) - 1), \quad (6)$$

$$b^2 = \frac{16}{3} g^2 \int \frac{d^4q}{(2\pi)^4} \sigma_S(q^2) (B(q^2) - m). \quad (7)$$

To treat mesons involving unequal quark masses (e.g. the kaon), the separable ansatz for the BS kernel is generalized to a symmetric combination of the functions  $F$  and  $G$  obtained for the individual quarks in such a way that Eq. (4) is recovered in the limit  $m_{f_2} \rightarrow m_{f_1}$ .

### 3 Solution of the Bethe-Salpeter equation

To illustrate the method of solving the BS equation, we consider the case of equal quark masses:  $f_1 = f_2$  and  $\xi = \frac{1}{2}$  in Eq. (2). The general form of the scalar and pseudoscalar meson amplitudes can be written as [6]

$$\begin{aligned} \Gamma^{\text{scalar}}(q, P) &= g_I(q^2, P^2, q \cdot P) I \\ &+ [g_P(q^2, P^2, q \cdot P) P_\mu + g_u(q^2, P^2, q \cdot P) u_\mu(q)] i\gamma_\mu, \quad (8) \end{aligned}$$

and

$$\begin{aligned} \Gamma^{\text{pseud}}(q, P) &= g_5(q^2, P^2, q \cdot P) \gamma_5 \\ &+ [g_{P5}(q^2, P^2, q \cdot P) P_\mu + g_{u5}(q^2, P^2, q \cdot P) u_\mu(q)] i\gamma_\mu \gamma_5, \quad (9) \end{aligned}$$

where, for later convenience we have defined

$$u_\mu(q) = \frac{P^2 q_\mu - (q \cdot P) P_\mu}{q^2 P^2 - (q \cdot P)^2}, \quad (10)$$

which satisfies  $P \cdot u(q) = 0$ ,  $q \cdot u(q) = 1$ . Note that in Feynman gauge it follows from the Fierz identity that there is no piece proportional to  $[P, \not{q}]$ . For mesons which are even (odd) under charge conjugation,  $g_I$ ,  $g_u$ ,  $g_5$  and  $g_{P5}$  are even (odd) functions and  $g_P$  and  $g_{u5}$  odd (even) functions of  $q \cdot P$ .

Defining  $k_\mu = q_\mu - \frac{1}{2}P_\mu$  and  $l_\mu = q_\mu + \frac{1}{2}P_\mu$ , we multiply Eq. (2) through by  $I$ ,  $P$  or  $\not{p}$  and take traces to project out a set of coupled integral equations for the scalar meson amplitudes:

$$\begin{aligned}
g_I(p, P) &= \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} \Delta(p-q) \times \\
&\quad \{g_I(q, P) [k \cdot l \sigma_V(k^2) \sigma_V(l^2) - \sigma_S(k^2) \sigma_S(l^2)] \\
&\quad + g_P(q, P) [-k \cdot P \sigma_V(k^2) \sigma_S(l^2) - l \cdot P \sigma_V(l^2) \sigma_S(k^2)] \\
&\quad + g_u(q, P) [-\sigma_V(k^2) \sigma_S(l^2) - \sigma_V(l^2) \sigma_S(k^2)] \}, \quad (11)
\end{aligned}$$

$$\begin{aligned}
P^2 g_P(p, P) &= \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} \Delta(p-q) \times \\
&\quad \{g_I(q, P) [-k \cdot P \sigma_V(k^2) \sigma_S(l^2) - l \cdot P \sigma_V(l^2) \sigma_S(k^2)] \\
&\quad + g_P(q, P) [(P^2 k \cdot l - 2k \cdot Pl \cdot P) \sigma_V(k^2) \sigma_V(l^2) + P^2 \sigma_S(k^2) \sigma_S(l^2)] \\
&\quad + g_u(q, P) [-(k+l) \cdot P \sigma_V(k^2) \sigma_V(l^2)] \}, \quad (12)
\end{aligned}$$

$$\begin{aligned}
u^2(p) g_u(p, P) &= \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} \Delta(p-q) \times \\
&\quad \{-g_I(q, P) [\sigma_V(k^2) \sigma_S(l^2) + \sigma_V(l^2) \sigma_S(k^2)] q \cdot u(p) \\
&\quad + g_P(q, P) [-(k+l) \cdot P q \cdot u(p) \sigma_V(k^2) \sigma_V(l^2)] \\
&\quad + g_u(q, P) [(k \cdot lu(p) \cdot u(q) - 2q \cdot u(p)) \sigma_V(k^2) \sigma_V(l^2) \\
&\quad + u(p) \cdot u(q) \sigma_S(k^2) \sigma_S(l^2)] \}. \quad (13)
\end{aligned}$$

Here we have employed the representation Eq. (3) of the quark propagator. The corresponding equations for the pseudoscalar components  $g_5$ ,  $g_{P5}$  and  $g_{u5}$  are obtained from these by making the replacement  $\sigma_S(l^2) \rightarrow -\sigma_S(l^2)$  (but  $\sigma_S(k^2)$  unchanged).

In order to extract on-mass-shell amplitudes, we set  $P = (0, iM)$ ,  $p = (\mathbf{p}, p_4)$  and  $q = (\mathbf{q}, q_4)$ . It is also convenient to rescale the scalar amplitude components via the definitions  $f(|\mathbf{q}|, q_4; M) = g_I(q, P)$ ,  $W(|\mathbf{q}|, q_4; M) = iM g_P(q, P)$  and  $U(|\mathbf{q}|, q_4; M) = g_u(q, p)/|\mathbf{q}|$ . Then from Eqs. (11), (12) and (13) we obtain

$$\begin{aligned}
f(p) &= \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} \Delta(p-q) (T_{ff}f(q) + T_{fW}W(q) + T_{fU}U(q)), \\
W(p) &= \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} \Delta(p-q) (T_{Wf}f(q) + T_{WW}W(q) + T_{WU}U(q)), \\
U(p) &= \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} \Delta(p-q) \frac{\mathbf{p} \cdot \mathbf{q}}{|\mathbf{p}| |\mathbf{q}|} \times \\
&\quad (T_{Uf}f(q) + T_{UW}W(q) + T_{UU}U(q)), \quad (14)
\end{aligned}$$

where  $T$ 's are given by

$$\begin{aligned}
T_{ff}^{\text{scalar}} &= \left( |\mathbf{q}|^2 + q_4^2 + \frac{1}{4}M^2 \right) |\sigma_V|^2 - |\sigma_S|^2, \\
T_{WW}^{\text{scalar}} &= \left( |\mathbf{q}|^2 - q_4^2 - \frac{1}{4}M^2 \right) |\sigma_V|^2 + |\sigma_S|^2, \\
T_{UU}^{\text{scalar}} &= \left( -|\mathbf{q}|^2 + q_4^2 + \frac{1}{4}M^2 \right) |\sigma_V|^2 + |\sigma_S|^2, \\
T_{fW}^{\text{scalar}} &= T_{Wf}^{\text{scalar}} = M\Im(\sigma_V^* \sigma_S) - 2q_4 \Re(\sigma_V^* \sigma_S), \\
T_{fU}^{\text{scalar}} &= T_{Uf}^{\text{scalar}} = -2|\mathbf{q}| \Re(\sigma_V^* \sigma_S), \\
T_{WU}^{\text{scalar}} &= T_{UW}^{\text{scalar}} = -2|\mathbf{q}| q_4 |\sigma_V|^2,
\end{aligned} \tag{15}$$

where

$$\sigma_V = \sigma_V \left( q_4^2 + |\mathbf{q}|^2 - \frac{1}{4}M^2 - iMq_4 \right), \quad \sigma_V^* = \sigma_V \left( q_4^2 + |\mathbf{q}|^2 - \frac{1}{4}M^2 + iMq_4 \right), \tag{16}$$

and similarly for  $\sigma_S$ . For the pseudoscalar states the analogous definitions  $f(|\mathbf{q}|, q_4; M) = ig_5(q, P)$ ,  $W(|\mathbf{q}|, q_4; M) = iMg_{P5}(q, P)$  and  $U(|\mathbf{q}|, q_4; M) = g_{u5}(q, P)/|\mathbf{q}|$  lead once again to the form Eq. (14), but with the  $T$ 's given by

$$\begin{aligned}
T_{ff}^{\text{pseud}} &= \left( |\mathbf{q}|^2 + q_4^2 + \frac{1}{4}M^2 \right) |\sigma_V|^2 + |\sigma_S|^2, \\
T_{WW}^{\text{pseud}} &= \left( |\mathbf{q}|^2 - q_4^2 - \frac{1}{4}M^2 \right) |\sigma_V|^2 - |\sigma_S|^2, \\
T_{UU}^{\text{pseud}} &= \left( -|\mathbf{q}|^2 + q_4^2 + \frac{1}{4}M^2 \right) |\sigma_V|^2 - |\sigma_S|^2, \\
T_{fW}^{\text{pseud}} &= -T_{Wf}^{\text{pseud}} = M\Re(\sigma_V^* \sigma_S) + 2q_4 \Im(\sigma_V^* \sigma_S), \\
T_{fU}^{\text{pseud}} &= -T_{Uf}^{\text{pseud}} = 2|\mathbf{q}| \Im(\sigma_V^* \sigma_S), \\
T_{WU}^{\text{pseud}} &= T_{UW}^{\text{pseud}} = -2|\mathbf{q}| q_4 |\sigma_V|^2.
\end{aligned} \tag{17}$$

The form of the effective kernel  $\Delta(p-q)$  appearing in Eq. (14) is so far unspecified. We now assume the separable form Eq. (4). Carrying out the angular integrals, and taking into account the symmetry properties of the functions  $f$ ,  $W$  and  $U$  under  $q_4 \rightarrow -q_4$  following from charge conjugation symmetry, we arrive at the following sets of integral equations for the positive charge parity states:

Scalar,  $J^{PC} = 0^{++}$  mesons

$$f(p) = \frac{8}{3\pi^3} \int_0^\infty dq_4 \int_0^\infty d|\mathbf{q}| |\mathbf{q}|^2 G(p^2) G(q^2) \times [T_{ff}(q) + T_{fW}(q) + T_{fU}(q)],$$

$$\begin{aligned}
W(p) &= \frac{4}{3\pi^3} \int_0^\infty dq_4 \int_0^\infty d|\mathbf{q}| p_4 q_4 |\mathbf{q}|^2 F(p^2) F(q^2) \times \\
&\quad [T_{Wf}f(q) + T_{WW}W(q) + T_{WU}U(q)], \\
U(p) &= \frac{4}{9\pi^3} \int_0^\infty dq_4 \int_0^\infty d|\mathbf{q}| |\mathbf{p}| |\mathbf{q}|^3 F(p^2) F(q^2) \times \\
&\quad [T_{Uf}f(q) + T_{UW}W(q) + T_{UU}U(q)]. \quad (18)
\end{aligned}$$

Pseudoscalar,  $J^{PC} = 0^{-+}$  mesons

$$\begin{aligned}
f(p) &= \frac{8}{3\pi^3} \int_0^\infty dq_4 \int_0^\infty d|\mathbf{q}| |\mathbf{q}|^2 G(p^2) G(q^2) \times \\
&\quad [T_{ff}f(q) + T_{fW}W(q)], \\
W(p) &= \frac{4}{3\pi^3} \int_0^\infty dq_4 \int_0^\infty d|\mathbf{q}| |\mathbf{q}|^2 G(p^2) G(q^2) \times \\
&\quad [T_{Wf}f(q) + T_{WW}W(q)], \\
U(p) &= 0. \quad (19)
\end{aligned}$$

The  $T_{ff}, T_{fW}, \dots$  are given by Eqs. (15) and (17). An analogous set of equations can be found for the  $C = -1$  states.

The separable form of the kernel allows solutions for these equations in terms of the functions  $F$  and  $G$ . For the  $0^{++}$  meson for instance, one then has that the solution must be of the form

$$\begin{aligned}
f(p) &= \lambda_f G(p^2), \\
W(p) &= \lambda_W p_4 F(p^2), \\
U(p) &= \lambda_U |\mathbf{p}| F(p^2). \quad (20)
\end{aligned}$$

Substituting the above ansatz into the integral equations gives a matrix equation of the form  $\vec{\lambda} = K(M)\vec{\lambda}$  where  $\vec{\lambda} = (\lambda_f, \lambda_W, \lambda_U)^T$  and  $K(M)$  is a  $3 \times 3$  matrix whose elements are numerically tractable double integrals which are determined once  $\sigma_V$  and  $\sigma_S$  are specified. One then adjusts the meson mass  $M$  until one of its eigenvalues equals 1.

For the case of unequal quark masses one finds that the functions  $f$ ,  $W$  and  $U$  are complex and that Eq. (21) generalises to forms such as, for example,

$$f(p) = \lambda_1 G_u(p^2) + \lambda_2 G_s(p^2) + i(\lambda_3 p_4 F_u(p^2) + \lambda_4 p_4 F_s(p^2)). \quad (21)$$

The matrix  $K(M)$  becomes, in general, a  $12 \times 12$  matrix, which reduces to a  $10 \times 10$  matrix when charge symmetry is taken into account.

## 4 Decay constants

Once the Bethe-Salpeter amplitude is determined, pseudoscalar decay constants can be calculated. The decay constant  $f_P$  is defined by (see Eqs. (2.8.35) and

(2.8.24) of ref. [7]):

$$\frac{1}{\sqrt{2}} \langle 0 | \bar{\Psi}_{f_1}(0) \gamma_\mu \gamma_5 \Psi_{f_2}(0) | \Phi(P) \rangle = P_\mu f_P, \quad (22)$$

where  $|\Phi(P)\rangle$  is the pseudoscalar meson state vector,  $\Psi_f$  is the Dirac spinor field corresponding to a quark of flavour  $f$ , and  $\bar{\Psi} \gamma_\mu \gamma_5 \Psi$  contains an implicit sum over colours.

The fourier transform of the fermion-dressed BS amplitude is

$$\begin{aligned} & (2\pi)^4 \delta^4(p - q) S_{f_1}(p - \xi P) \Gamma(p, P) S_{f_2}(p + (1 - \xi)P) \\ &= \int d^4x d^4y e^{iP \cdot \frac{(1-\xi)x + \xi y}{2}} e^{i(x \cdot q - y \cdot p)} \langle 0 | \Psi_{f_1}(x) \bar{\Psi}_{f_2}(y) | \Phi(P) \rangle, \end{aligned} \quad (23)$$

there being one such vertex for each colour. Taking  $N_c \text{tr}(\mathcal{P} \gamma_5 \dots)$  of both sides, integrating out  $p$  and  $q$  and using Eq. (22) then gives

$$P^2 f_P = \frac{N_c}{\sqrt{2}} \int \frac{d^4k}{(2\pi)^4} \text{tr}[\mathcal{P} \gamma_5 S_1(p - \xi P) \Gamma(p, P) S_2(p + (1 - \xi)P)]. \quad (24)$$

Before this formula can be used, the Bethe-Salpeter amplitude must be properly normalised. The canonical normalisation of the mass shell BS amplitude  $\Gamma$  is given by [8]

$$\begin{aligned} 2P_\mu = -N_c \int \frac{d^4k}{(2\pi)^4} \times \\ \{ -\xi \text{tr} [\bar{\Gamma}(k, -P) \partial_\mu^k S_{f_1}(k - \xi P) \Gamma(k, P) S_{f_2}(k + (1 - \xi)P)] \\ + (1 - \xi) \text{tr} [\bar{\Gamma}(k, -P) S_{f_1}(k - \xi P) \Gamma(k, P) \partial_\mu^k S_{f_2}(k + (1 - \xi)P)] \}, \end{aligned} \quad (25)$$

where  $\bar{\Gamma}(k, P)^T = C^{-1} \Gamma(-k, P) C$  defines the corresponding anti-meson amplitude. Note that, in ladder approximation the scattering potential  $V$  is independent of  $P$ , so there is no contribution from a  $\partial V / \partial P$  term.

Finally, note that Eq. (24) can only be used for numerical calculations as it stands away from the chiral limit  $m_q = 0$  because both sides of the equation are of order  $P^2$  as  $P^2 \rightarrow 0$ . A similar calculation of decay constants is given in ref. [9].

## 5 Summary and results.

We have set out a formalism for calculating the masses and decay constants of light-quark mesons in terms of input model quark propagators. The method involves the solution of a ladder approximation Bethe-Salpeter equation in which the kernel is replaced by a separable ansatz. Our calculated BS results are

based on quark propagators that summarize DS equation studies [4] through the approximating analytic forms:

$$\sigma_V(s=p^2) = \frac{1}{2D} \bar{\sigma}_V\left(\frac{s}{2D}\right), \quad \sigma_S(s=p^2) = \frac{1}{\sqrt{2D}} \bar{\sigma}_S\left(\frac{s}{2D}\right), \quad (26)$$

where  $D$  is a mass scale and the dimensionless functions  $\bar{\sigma}_V$  and  $\bar{\sigma}_S$  are given by

$$\bar{\sigma}_S(x) = \frac{\hat{m}}{x + \hat{m}^2} \left(1 - e^{-2(x+\hat{m}^2)}\right) + \frac{1 - e^{-b_1 x}}{b_1 x} \frac{1 - e^{-b_3 x}}{b_3 x} \left(b_0 + b_2 \frac{1 - e^{-\Lambda x}}{\Lambda x}\right) \frac{1 - e^{-(\epsilon_S x)^2}}{(\epsilon_S x)^2} \quad (27)$$

$$\bar{\sigma}_V(x) = \frac{2(x + \hat{m}^2) - e^{-\epsilon_V(x+\hat{m}^2)} + e^{-2(x+\hat{m}^2)}}{2(x + \hat{m}^2)^2}. \quad (28)$$

These forms have been successfully applied elsewhere to studies of the electromagnetic pion and kaon form factors [10]. In order to ensure convergence of the integrals in Eqs. (6) and (7), ultraviolet regulators have been introduced into these functions in the form of gaussian damping factors. The four free parameters, consisting of the bare masses  $\hat{m}_{u/d}$ ,  $\hat{m}_s$  and the regulator parameters  $\epsilon_V$  and  $\epsilon_S$ , enable the previously developed propagator forms to be adapted for the present work and to fit the mass and decay constant of the pion and kaon.

Results arising from the parameter choices in Table 1 are given in Table 2. We find the results are in good agreement with experiment for low lying octet pseudoscalar and vector states. It is of interest to note that a simple separable BS kernel ansatz, determined only by the quark propagators and consistent with the Goldstone mechanism, can produce good results for the non-Goldstone vector mesons. We have also carried out calculations for the scalar mesons and find our calculated masses to be well below those for the observed lightest scalar mesons  $a_0$ ,  $f_0$  and  $K_0^*$ . The general consensus in the literature regarding these states is that they cannot easily be explained as simple  $q\bar{q}$  bound states (see for instance ref. [12] and references therein). As pointed out in ref. [6] the relativistic Bethe-Salpeter formalism does not necessarily disallow scalar or pseudoscalar states with negative charge parity, though such states have not been observed. We find that the separable ansatz yields very heavy  $0^{+-}$  and  $0^{-+}$  states with  $m_{0^{+-}} \sim 10m_{0^{-+}}$  and  $m_{0^{-+}} \sim 2m_{0^{++}}$ , which is possibly beyond the limit of applicability of the separable ansatz. Complete details of our results will be presented elsewhere [13].

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Table 1: Input parameters for the the quark propagator functions  $\sigma_V$  and  $\sigma_S$ .

	up/down quark	strange quark
$D$	0.160 GeV <sup>2</sup>	0.160 GeV <sup>2</sup>
$\hat{m}$	0.00811	0.187
$b_0$	0.131	0.105
$b_1$	2.90	2.90
$b_2$	0.603	0.740
$b_3$	0.185	0.185
$\Lambda$	0.0001	0.0001
$\epsilon_V$	0.1	0.1
$\epsilon_S$	0.482	0.400

Table 2: Meson results

	Calculation (MeV)	Experiment [11] (MeV)
$m_\pi$	138.5 (fit)	$\pi^\pm(140)$ , $\pi^0(135)$
$f_\pi$	92.4 (fit)	$\pi^+(92.4)$
$m_{a_0/f_0}$	715.	$a_0(980)$ , $f_0(975)$
$m_K$	493.9 (fit)	$K^\pm(494)$ , $K^0(498)$
$f_K$	113.0 (fit)	$K^+(113.0)$
$m_{K_0^*}$	1180.	$K_0^*(1430)$
$m_{\rho/\omega}$	736.	$\omega(783)$ , $\rho(770)$
$m_{a_1/f_1}$	1340.	$a_1(1260)$ , $f_1(1285)$
$m_{K^*}$	868.	$K^{*\pm}(891.6)$
$m_\phi$	980.	$\phi(1020)$

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