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INTERNAL LETTER
NORTH AMERICAN AVIATION, INC.

DATE September 9, 1963

TO R. M. Galantine
ADDRESS 726-60
AI-HQ 2

FROM P. M. Magee
ADDRESS 727-71
AI-HQ 5

PHONE 1563

SUBJECT SNAP 10A Core Thermal Stresses During Startup

REF: (1) IL, G. S. Drucker to those listed, "Review of Phase I Results of the SNAP 10A Startup Simulation", July 29, 1963.

(2) IL, W. M. Farr to E. B. Baumeister, "SNAP 2 Reactor Model and Hydrogen Loss", May 29, 1963. Figure 2, "SNAP 10A/2 Overall Thermal Conductance as Function of Temperature", prepared by P. M. Magee.

(3) J. D. Watrous, "Thermal Expansion of SNAP Materials", NAA-SR-6047, July 30, 1961.

Thermal stresses within the SNAP 10A reactor core during the anticipated startup transient have been analyzed by means of a digital computer code which predicts transient temperature distributions within the core. Inputs for the study were curves of reactor power, flow rate and reactor inlet temperature obtained from the system startup analog analysis conducted by G. S. Drucker (Reference 1). There do not appear to be any significant thermal stress problems associated with the FS-1 nominal startup procedure.

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AUTHOR: P. M. Magee

DEPT & GROUP NO: 727-71

TITLE: SNAP 10A Core Thermal Stresses During Startup

PROGRAM: SNAP 10A

SUBACCOUNT TITLE: SNAP 10A Core Thermal Analysis

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STATEMENT OF PROBLEM

Determine the thermal stresses within the reactor core to be encountered during SNAP 10A nominal startup.

ABSTRACT

Thermal stresses within the SNAP 10A reactor core during the anticipated startup transient have been analyzed by means of a digital computer code which predicts transient temperature distributions within the core. Inputs for the study were curves of reactor power, flow rate and reactor inlet temperature obtained from the system startup analog analysis conducted by G. S. Drucker (Reference 1). There do not appear to be any significant thermal stress problems associated with the FS-1 nominal startup procedure.

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Computer Code for Core Transients

A mathematical model of the 10A reactor core has been formulated to determine temperatures within the core and, subsequently, thermal stresses. The model considers one fuel element with its associated lower and upper end caps, upper grid plate and surrounding coolant. Depending on the radial power factor used, the model is representative of fuel elements at various radial locations within the core. In this study, average, centerline and edge-of-core fuel elements have been considered.

The model fuel element is divided into five radial and ten axial sections to give a fifty node representation. Ten coolant and clad nodes are employed. The associated upper end cap and grid plate are each divided into five radial nodes, matching those of the fuel element. Conduction, convection and heat capacity effects are included while radiation is neglected. Conduction from the ends of the fuel to the end caps is included as is conduction from the upper end cap through a stagnant NaK gap to the upper grid plate. Convective heat transfer between the upper grid plate and the NaK in the upper plenum is estimated. A mixing routine is used to relate the bulk fluid temperature in the plenum to that of the coolant leaving the core.

The equations of heat conduction, convection and heat storage are programmed as transient expressions for digital solution. The general form of the equations for the fuel nodes is:

$$MC\frac{dT_f}{d\theta} = q'''dV + \sum k_f A \frac{dT}{dx} - UA(T_f - T_c)$$

where MC = heat capacity of a fuel node

$q'''dV$ = product of internal heat generation rate and the node volume

$\sum k_f A \frac{dT}{dx}$ = rate of heat conducted into the node from adjacent fuel nodes

$UA(T_f - T_c)$ = rate of heat loss to the coolant for a surface node

U = combined heat transfer coefficient for the gas gap, hydrogen barrier, clad and NaK film

The last term, of course, is included only for surface nodes.

To account for radial power peaking, the power generation rate used for the centerline element is 31% greater than the core average and the rate for the edge element is 65% of the average. Heat generation terms for individual nodes along the axis of the fuel element are weighted according to the axial power distribution.

Required inputs to the computer program are reactor power, flow rate and reactor inlet temperature as functions of time. For the cases analyzed, straight-line segments have been fitted to these parameters as determined by G. S. Drucker in his system analog simulation studies.

Two additional sections are being written for the core transient digital model: reactor kinetics and NaK thermo-electric pump. When these sub-routines are incorporated into the program, the only input required will be the reactor inlet temperature. Publication of the code is being delayed until these sections are finished and checked out.

Transient Solutions

The nominal startup conditions are (Reference 1):

Orbit	700 mile ecliptic orbit
Source Level	10^{-11} kw
Temperature Coefficients	$\alpha_f = -(.074 + 6.6 \times 10^{-5}\,\overline{T}_f)$ ¢/°F
	$\alpha_g = -0.05$ ¢/°F
Repetition Time	150 seconds
Initial Flow Rate	6% of full-flow
Initial Temperature	70°F

The most severe period during startup is the initial transient: i.e., the first penetration into the power range. The power, flow and outlet temperature corresponding to the initial transient are shown in Figure 1; the first two figures are approximations to the curves generated on the system analog by G. S. Drucker, 726-60, in July, 1963. The inlet temperature, which is the third required input, remains constant at 70°F until 175 seconds after the start of the power pulse and then slowly increases.

A comparison between the analog and digital studies is provided by the graph of outlet temperature in Figure 1. Agreement is reasonably good; the differences are primarily due to the approximations used as inputs to the digital study and secondarily to differences in the nodal models used.

Average fuel and clad temperatures are plotted in Figure 2. The fuel temperature shown is an average over all 50 nodes. The axial clearance between the fuel and the upper end cap is plotted for elements at the average radial position and at the edge of the core. No axial interference problems are anticipated; the minimum clearance indicated is 0.6 mils. Because of the thermal expansion characteristics of the fuel and the cladding material (Reference 3), axial clearances are somewhat less at the edge of the core than they are at the average radial position.

A maximum temperature gradient in the fuel of the center rod occurs 60 seconds after the power peak. Thermal stress is highest at the fuel surface where a tensile stress of 1,550 psi is induced. Since the yield stress of the fuel is 12,000 psi at 500°F, the fuel will not crack nor yield.

The ceramic barrier is subjected to a maximum stress in tension at the junction of the upper end cap and the clad. This stress is related to the difference between the average end cap temperature and the adjacent clad temperature and to the slope of the clad temperature profile near the end cap. (A complete deri[illegible]s formula is given in Appendix A.)

The maximum temperature difference between the end cap and the clad 0.1 inch away occurs in the centerline element and is 49.5°F (see Figure 3). The corresponding maximum stresses in the clad and barrier are 19,710 and 6,570 psi, respectively. The yield strength of Hastelloy N at 500°F is 40,000 psi. Because the clad stress is relieved by yielding and thermal stress fatigue is not encountered, there appears to be an adequate margin of safety in clad design.

If there were no compensating stresses within the ceramic barrier, one would be very hesitant to recommend proceeding with FS-1 nominal startup. The ultimate strength of the barrier in tension is estimated to be 10,000 psi, too close to the calculated value of 6,570 psi for safety. However, the barrier is in a state of compression due to the bonding procedure used. After the ceramic is coated on the inside of the cladding, the cladding cylinder is fired at 1800°F and allowed to cool. At approximately 1000°F, the barrier is sufficiently solid to maintain an internal stress. As the coefficient of thermal expansion of the Hastelloy N cladding is greater than that of the ceramic, the barrier is put into compression as cooling continues. When the element is reheated to 400°F during startup, a residual compressive stress of 24,900 psi is estimated to exist in the barrier (see Appendix C). This is greater than the maximum tensile stress produced by thermal shock and, consequently, the barrier will remain in compression throughout the FS-1 nominal startup.

Effect of UA Variation

A significant parameter effecting thermal stress is the overall heat transfer conductance UA. While care has been taken in the calculation of the nominal curve for UA (References 1 and 2), the uncertainty in this parameter is large and an investigation into the effect of UA variation on thermal stress is merited.

This study has been made by running a parametric case with UA equal to one-half the nominal value. (This run is labeled #8 by G. S. Drucker. Unfortunately, it assumes an initial temperature of 170°F, not 70°F; but, by comparison with the nominal case, modified to 170°F initial temperature (run 1), valid conclusions can be drawn.) Straight-line approximations were made to the reactor power, flow rate and inlet temperature generated by the systems analog for the case with low UA, and these were used as input to the digital code.

In general, a reduction in UA raises the differences between the fuel and coolant temperatures and increases the maximum clad and barrier stresses. The axial clearance between the fuel and the upper end cap is decreased, and the maximum stress within the fuel is reduced.

With UA one-half nominal, the maximum tensile stresses in the clad and barrier are increased approximately 27%. Because of the large residual compressive stress in the barrier, the barrier material will, as in the nominal case, remain in compression throughout startup. Axial clearance between the fuel and the upper end cap is reduced somewhat to a minimum clearance of 0.48 mils.

IL to: R. M. Galantine
From: P. M. Magee
Subject: SNAP 10A Core Thermal Stresses During Startup

September 9, 1963

Conclusions

There appear to be no significant thermal stress problems associated with the FS-1 nominal startup procedure. Positive axial clearance between the fuel and the upper end cap is maintained throughout the initial power transient. The maximum tensile stress in the clad occurs at the junction of the clad and upper end cap and is 19,710 psi. The ceramic barrier remains safely in compression throughout the initial power transient. Thermal stress in the centerline fuel element reaches a maximum of 1,550 psi.

Reduction of the overall heat transfer conductance, UA, to one-half its nominal value would increase the maximum clad and barrier stresses approximately 27% and decrease the minimum axial clearance between the fuel and the upper end cap to 0.48 mils. These changes are not extreme and can be tolerated during FS-1 startup.

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Appendix A - Thermal Stresses at Junction of End Cap and Cladding

The total thermal stress at the junction of the end cap and the cladding is the sum of two stresses arising from two different constraints: (1) a discontinuity stress due to the junction temperature being different from the average temperature of the end cap, and (2) a moment stress as the result of constraining the cladding to maintain zero slope with respect to the end cap while a temperature gradient is imposed.

1. Discontinuity Stress

If there were no end cap on the cladding, the cylinder would expand in accordance with its temperature profile; at an open end, its diameter would be completely determined by the local temperature. However, the radial expansion of the end cap is fixed by its average temperature. Thus, the cladding at the end of the cylinder is compressed by the attached end cap. The resulting discontinuity stress can be calculated as:

$$\sigma_{dis} = 1.82\ E \alpha \Delta T$$

where
- 1.82 = constant derived for a material with $\mu = 0.3$
- E = elastic modulus of clad
- α = coefficient of expansion of clad
- ΔT = temperature difference at discontinuity (junction temperature minus average end cap temperature)

Nominal run:

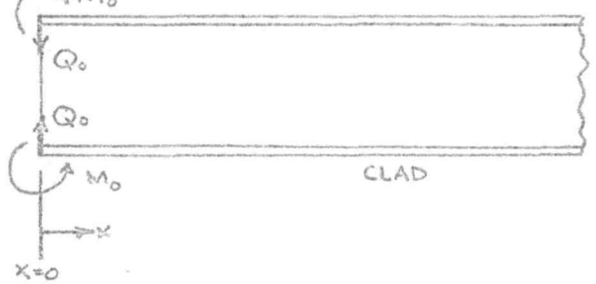
$$\left.\begin{array}{l} \text{Junction temperature} = 328.4°F \\ \text{Average end cap temperature} = 278.9°F \end{array}\right\} \text{maximum difference}$$

$$(\sigma_{dis})_{max} = 1.82\ (193.5)\ (49.5) = 17{,}430 \text{ psi}$$

2. Stress Produced by Axial Temperature Profile Along Cladding

Ref: Timoshenko, Theory of Plates and Shells; Chapter 11, "General Theory of Cylindrical Shells"

The cladding cylinder is constrained by the end cap to maintain zero slope at its end. This boundary condition implies a moment M_o and a shear Q_o, at the junction to the end cap, sufficient to produce a slope equal and opposite to that generated by the cladding temperature profile.



Appendix A

In general, a moment M_o and a shear Q_o applied at $x = 0$ result in a deflection at that point of

$$(W)_{x=0} = -\frac{1}{2\beta^3 D}(\beta M_o + Q_o)$$

and a slope of

$$\left(\frac{dw}{dx}\right)_{x=0} = \frac{1}{2\beta^2 D}(2\beta M_o + Q_o)$$

The temperature distribution along the clad can be approximated for a short distance from the end cap (say, 0.6 inch) by the linear relationship

$$t(x) = t_o + (t_b - t_o)\frac{x}{b}$$

where t_o = temperature at junction to end cap ($x = 0$)
t_b = temperature at distance b from end cap.

This temperature gradient, if unrestrained, would produce a deflection curve

$$W = \alpha a (t_o - t_b)\frac{x}{b}$$

At $x = o$, the deflection would be zero (assuming the end cap is at the uniform temperature t_o) and the slope would be

$$\left(\frac{dw}{dx}\right)_{x=o} = \frac{\alpha a}{b}(t_o - t_b)$$

However, the built-in end imposes zero slope at $x = 0$. Thus, a moment M_o and a shear force Q_o are generated of such magnitude as to produce a slope equal and opposite to that generated by the temperature field.

In this case, the basic equations become

$$0 = -\frac{1}{2\beta^3 D}(\beta M_o + Q_o)$$

$$-\frac{\alpha a}{b}(t_o - t_b) = \frac{1}{2\beta^2 D}(2\beta M_o + Q_o)$$

Solution of these equations yields

$$M_o = 2\beta D\frac{\alpha a}{b}(t_b - t_o)$$

where $$\beta = \left[\frac{3(1-\mu^2)}{a^2 h^2}\right]^{1/4}$$

and $$D = \frac{Eh^3}{12(1-\mu^2)}$$

Appendix A

The maximum stress is given by

$$(\sigma_{mom})_{max} = \frac{6M_o}{h^2}$$

which, for $\mu = 0.3$, reduces to

$$(\sigma_{mom})_{max} = 1.412 \frac{E\alpha}{b} \sqrt{ah} \quad (t_b - t_o)$$

where
- a = radius of fuel element
- b = distance from end cap to location of t_b
- h = thickness of cylinder wall
- E = modulus of elasticity, cylinder wall
- α = coefficient of thermal expansion, cylinder wall

For a SNAP 10A fuel element:

- a = 0.625 inch
- h = 0.018 inch
- b = 0.57 inch (distance from end cap to last cladding node)
- $E\alpha$ = 193.5 psi/°F (Hastelloy N)

Nominal run:

t_b = 373.3°F
t_o = 328.4°F
max. $t_b - t_o$ = 44.9°F

$$(\sigma_{mom})_{max} = 0.263\ (193.5)(44.9) = 2280 \text{ psi}$$

The maximum total cladding stress is given by summing the above stresses.

Nominal run:

$$(\sigma_c)_{max} = 17430 + 2280 = 19{,}710 \text{ psi}$$

3. Relation of Barrier Stress to Clad Stress

When a stress σ_c is produced in the cladding, there is a corresponding strain

$$\varepsilon_c = \frac{\sigma_c}{E_c}$$

This strain is transmitted directly to the ceramic barrier material:

$$\varepsilon_b = \varepsilon_c$$

Appendix A

The corresponding barrier stress is

$$\sigma_b = E_b \varepsilon_b = \frac{E_b}{E_c} \sigma_c$$

where E_b = modulus of elasticity, barrier (10×10^6 psi)

E_c = modulus of elasticity, clad (30×10^6 psi)

Therefore, the maximum barrier stress is

Nominal run:

$$(\sigma_b)_{max} = \frac{1}{3} (19710) = 6570 \text{ psi}$$

Appendix B - Thermal Stress in the Fuel

Thermal stresses at the surface of the unrestrained fuel cylinder were calculated from the equation:

$$\sigma = \frac{E}{1-\mu}\left(\overline{T} - T_s\right)$$

where σ = stress, psi
E = modulus of elasticity of the fuel at 500°F (7.0×10^6 psi)
α = coefficient of expansion of the fuel at 500°F (5.1×10^{-6} in/in°F)
μ = Poisson's ratio for the fuel (0.3)
$\overline{T}$ = volume mean temperature or areal mean temperature of any cross-section of the fuel
T_S = temperature at the surface of the fuel

Nominal run, centerline fuel element:

At 150 seconds after the start of the power transient,

$$\overline{T}\ (\text{nodes } 51\text{-}55) = 476.5°F$$

$$T_S = \underline{446.2}$$

$$\Delta T = 30.3°F$$

$$\sigma = 1{,}550 \text{ psi}$$

The ultimate tensile strength of the fuel is approximately 12,000 psi. Therefore, the fuel will not yield or crack during startup.

Appendix C - Residual Compressive Stress in Barrier

Mechanical properties of the ceramic barrier*:

Tensile strength	10,000 psi
Compressive strength	100,000 psi
Poisson's ratio, μ	0.3
Thermal expansion coefficient (average-room temperature to 1000°F)	3.8×10^{-6} in/in/°F

Coating is plastic above 1000°F.

Thermal expansion coefficient for Hastelloy "N" 6.7×10^{-6} in/in/°F (average-room temperature to 1000°F)

The basic equations expressing the state of two-dimensional stress in a body are

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \mu\sigma_y)$$

and

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \mu\sigma_x)$$

or when solved for stresses,

$$\sigma_x = \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y)$$

and

$$\sigma_y = \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x)$$

The compressive strain in the barrier due to the unequal contraction of the clad and barrier materials on cool-down from 1000°F is

$$\varepsilon_x = \varepsilon_y = -\Delta\alpha\,\Delta T$$

where $\Delta\alpha = \alpha_{clad} - \alpha_{barrier}$

Thus, the compressive stress in the barrier is

$$\sigma_x = -\frac{E\,\Delta\alpha\,\Delta T}{1-\mu}$$

*Personal communication from ... -51

Appendix C

This equation neglects the effects of relative deformations during cool-down due to strength characteristics.

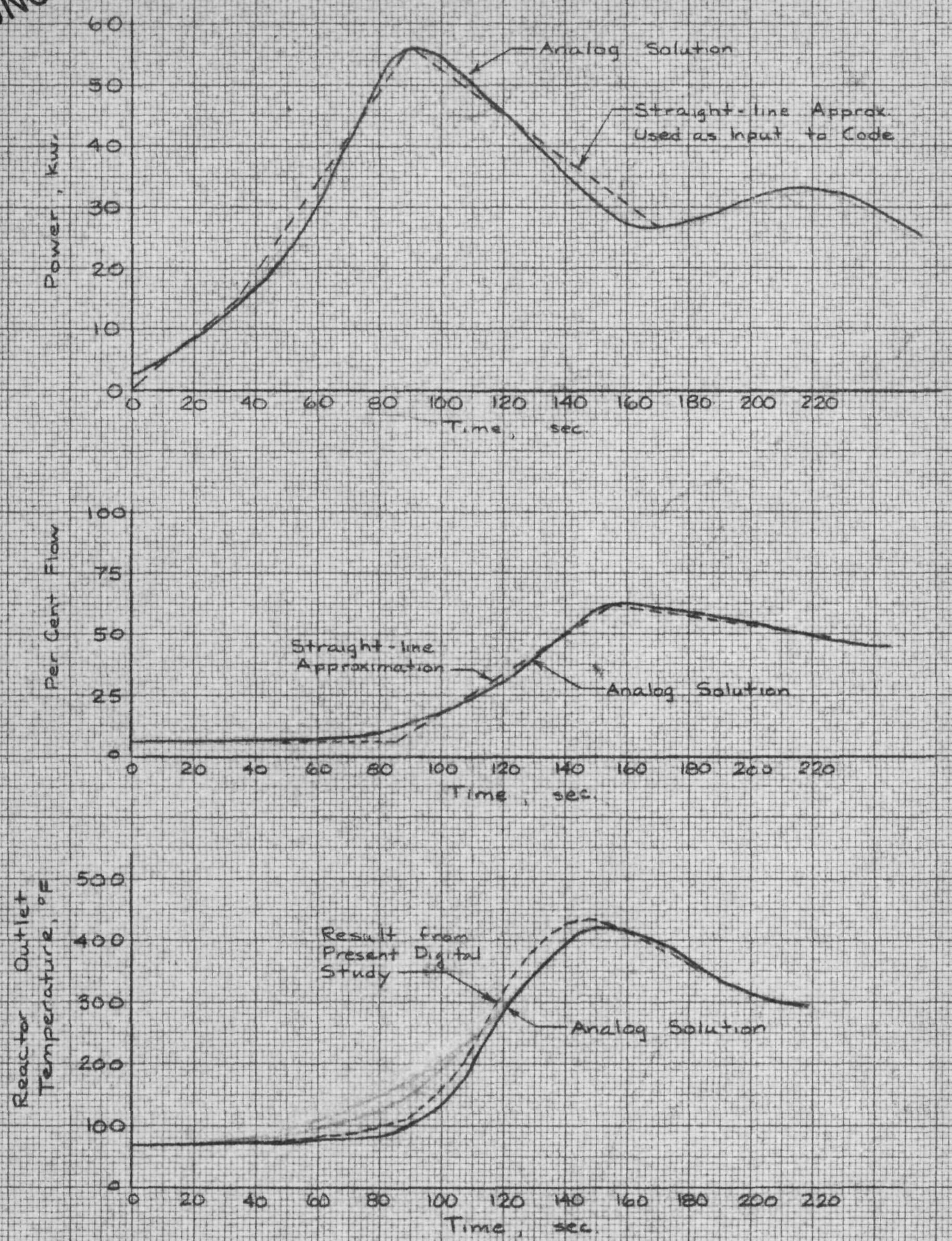
The stress produced by cooling from 1000°F to 100°F is

$$\sigma = -\frac{(10 \times 10^{6})(6.7 - 3.8)\ 10^{-6}\ (900)}{.7} = -37,300 \text{ psi}$$

Cooling from 1000°F to 400°F, the temperature of interest, produces a stress of

$$\sigma = -\frac{(10 \times 10^{6})(6.7 - 3.8)\ 10^{-6}\ (600)}{.7} = -24,900 \text{ psi}$$

FIGURE 1



REACTOR POWER, FLOW & OUTLET TEMPERATURE
DESCRIBING SNAP 10A INITIAL STARTUP TRANSIENT

Inlet temperature = 70°F until 175 seconds.
Analog results were obtained by G.S. Drucker, 726-60, during July 1963.

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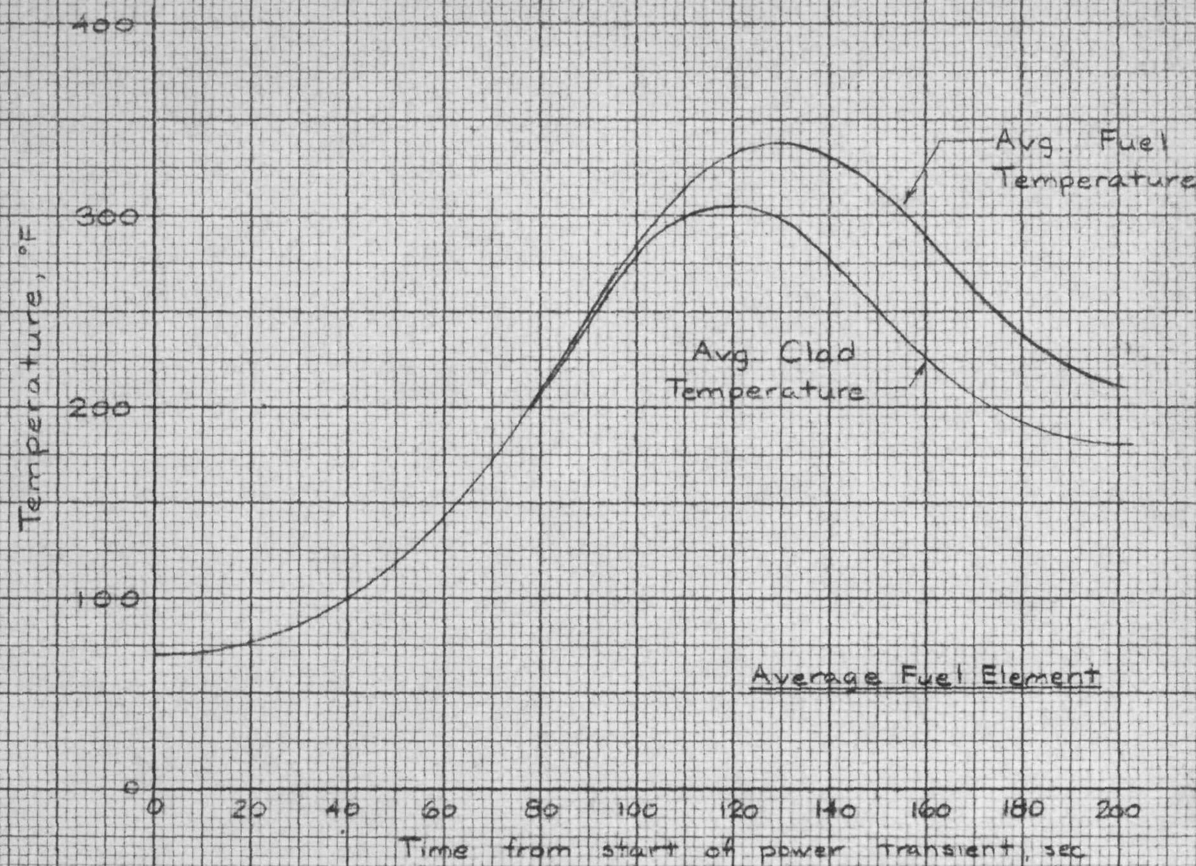
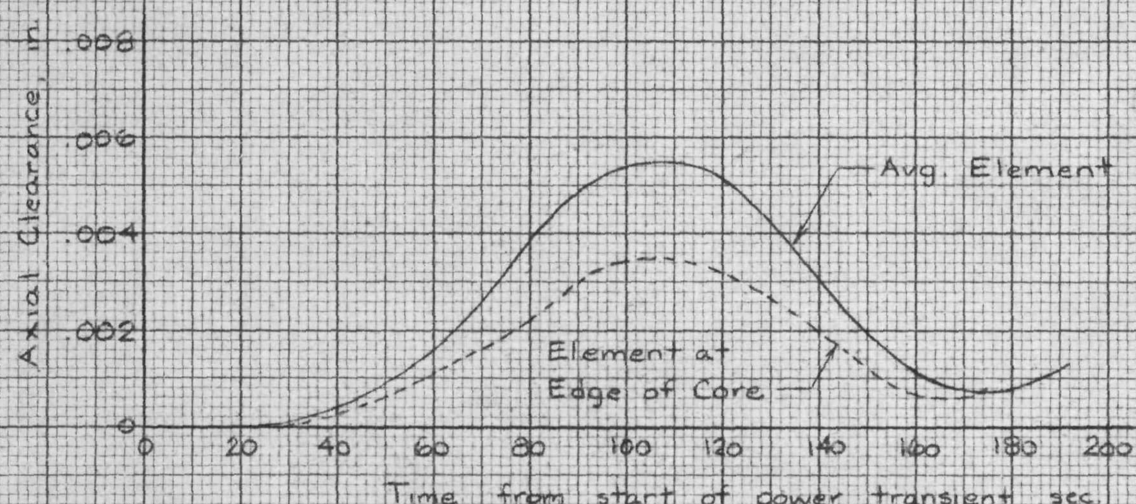
FIGURE 2



AVERAGE TEMPERATURES
DURING INITIAL POWER TRANSIENT
Nominal Run



AXIAL CLEARANCE
DURING INITIAL POWER TRANSIENT
OF SNAP 10A STARTUP
Nominal Run
Zero axial clearance at assembly.

9-12-63

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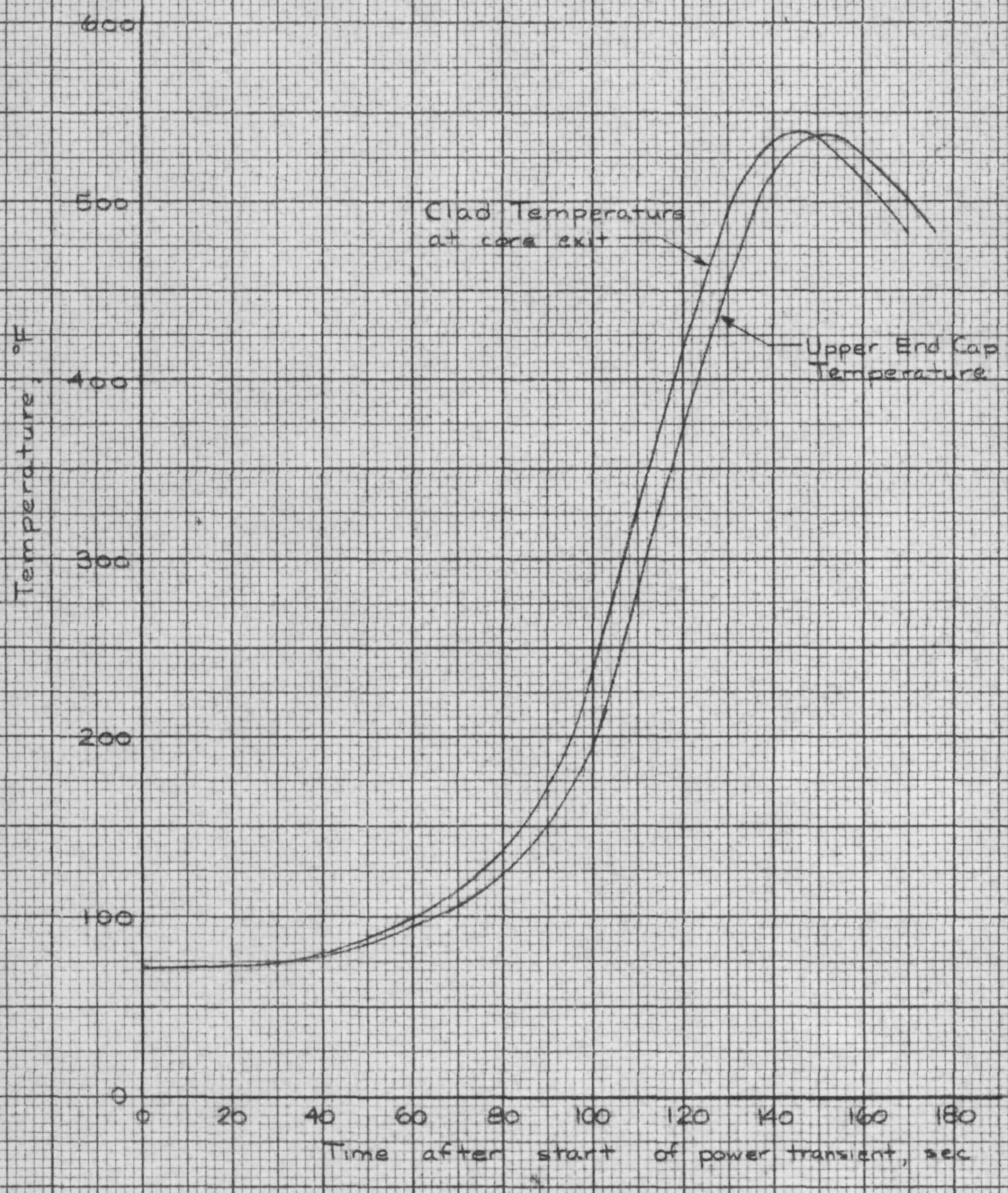
FIGURE 3



CENTERLINE FUEL ELEMENT -
UPPER END CAP & CLAD TEMPERATURES
DURING SNAP 10A INITIAL POWER TRANSIENT
Nominal run

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