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Accelerator Department
BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York

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ISABELLE PROJECT

INTERSECTING STORAGE ACCELERATOR NOTES

SOME REMARKS ON INELASTIC e-p SCATTERING AT ISABELLE

Jay Orear*

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We will assume a 10-GeV e^- ring inside the ISABELLE tunnel with an e-p colliding beam luminosity of $\sim 10^{33}$ cm^{-2} sec^{-1} over the interacting region. When the target proton is stationary the cross section is

$$\frac{d\sigma_o}{dE_o' d\omega_o} = \frac{4\alpha^2 E_o'^2}{\nu Q^4} \left[\cos^2 \frac{\theta_o}{2} + 2 \sin^2 \frac{\theta_o}{2} \; \frac{Q^2 + \nu^2}{Q^2} \; \frac{1}{1+R} \right] F_2 \quad , \tag{1}$$

where $Q^2 = 2EE'(1 - \cos\theta)$ in all reference frames and $\nu = E_o - E_o'$ measured in the frame of the target proton. We use subscript zero for quantities measured in the frame of the target proton. The factor in the brackets is ≈ 1. So far all measurements give $F_2 \approx 0.3$ whenever $2M\nu/Q^2 \geq 3$ (this is called scale invariance). Hence

$$\frac{d^2\sigma_o}{\frac{dE_o'}{E_o'}\, d\omega_o} \approx \frac{1.2\; \alpha^2 E_o'^3}{\nu Q^4} \quad . \tag{2}$$

Karl Berkelman[1] points out that so far experiments have been limited to the region

$$\frac{d\sigma_o}{\frac{dE_o'}{E_o}\, d\omega_o} \geq 12\; \alpha^2 \text{ GeV}^{-2}$$

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*Laboratory of Nuclear Studies, Cornell University.

1. Karl Berkelman, "How Far Can We Go Into the Deep Inelastic?" unpublished report.

or

$$Q^2 \leq \sqrt{\frac{(E_o - \nu)^3}{\nu}} \quad . \tag{3}$$

This limiting region is plotted in Fig. 1 for $E_o = 20$ GeV. It corresponds rather well to the region in $Q^2 - \nu$ space which has been explored by SLAC.

From p. 7 of Ref. 1 it is easily seen using Eq. (2) above that

$$\frac{d\sigma}{dE' d\omega} = 4EE'(E_p + P_p) \frac{1}{4E_o E_o' M} \left(\frac{d\sigma_o}{dE_o' d\omega_o} \right) \approx \frac{EE'(2E_p)}{E_o E_o' M} \left(\frac{1.2\ \alpha^2 E_o'^2}{\nu Q^4} \right),$$

where $E_p = P_p = 216$ GeV/c is the energy of the target proton in ISABELLE. From p. 8 of Ref. 1 we have the relation

$$E' = E - \frac{M\nu - \frac{P_p Q^2}{2E}}{E_p + P_p} \quad .$$

Hence

$$\frac{d\sigma}{\frac{dE'}{E'} d\omega} \approx \frac{2EE'^2 E_p}{E_o\ M} \left(\frac{1.2\ \alpha^2 E_o'}{\nu Q^4} \right) = 2 \frac{E}{E_o} \left[E - \frac{M\nu - P_p Q^2/2E}{2E_p} \right]^2 \frac{E_p}{M} \left(\frac{1.2\ \alpha^2 E_o'}{\nu Q^4} \right) .$$

Also we use from p. 7 of Ref. 1 that $E/E_o = M/2E_p$

$$\frac{d\sigma}{\frac{dE'}{E'} d\omega} \approx 2 \frac{M}{2E_p} \left[E - \frac{M}{2E_p} \nu + \frac{Q^2}{4E} \right]^2 \frac{E_p}{M} \left(\frac{1.2\ \alpha^2 E_o'}{\nu Q^4} \right) \quad . \tag{4}$$

As long as $E' \sim 20$ GeV and the ISABELLE luminosity is comparable to previous e-p inelastic experiments we can use the same limit

$$\frac{d\sigma}{\frac{dE'}{E'} d\omega} \geq 12\ \alpha^2 \quad .$$

(The luminosity in the Cornell experiments was 10^{33} cm^{-2}/sec.) The limiting relation between Q^2 and ν is obtained by equating the above formula to 12 α^2

$$12\,\alpha^2 \approx \left[E - \frac{M}{2E_p}\,\nu + \frac{Q^2}{4E}\right]^2 \left(\frac{1.2\,\alpha^2 E_o'}{\nu Q^4}\right)$$

$$1 \approx \left[10 - \frac{\nu}{460} + \frac{Q^2}{40}\right]^2 \frac{(E_o - \nu)}{10\nu Q^4} \qquad \text{since } E = 10 \text{ GeV and } \frac{2E_p}{M} = 460$$

$$Q^2 = \left[10 - \frac{\nu}{460} + \frac{Q^2}{40}\right] \sqrt{\frac{4600 - \nu}{10\nu}} \qquad \text{since } E_o = \frac{2E_p}{M} \cdot E = 4600 \text{ GeV equivalent}$$

$$\boxed{Q^2 = \frac{\left(10 - \frac{\nu}{460}\right)\sqrt{\frac{4600 - \nu}{10\nu}}}{1 - \frac{1}{40}\sqrt{\frac{4600 - \nu}{10\nu}}}}$$

This curve is also plotted in Fig. 1. We see that in ISABELLE with a 10-GeV electron ring, Q^2 up to 40 GeV^2 might be achieved in inelastic electron-proton scattering and that scale invariance could be checked over a kinematical region $\sim$ 100 times that of SLAC. I wish to thank Karl Berkelman for some helpful conversations.

A Particular Example

Consider point A in Fig. 1. Then $Q^2 = 30$ GeV^2 and $\nu = 50$ GeV. These two parameters correspond to $E' = 10.64$ GeV and $\theta = 31^o$. Substituting into Eq. (4) yields $d\sigma/(dE'/E')d\omega = 7.4 \times 10^{-3}$ $GeV^{-2} = 2.85 \times 10^{-30}$ cm^2. Figure 2 shows the outline of an experimental setup. A 30D72 looks at 1 m of interacting region giving 6^o of bend for 10-GeV scattered electrons. Assume we are interested in a momentum bite $\Delta p/p = \pm 1\%$. The back proportional chamber subtends a solid angle of $\Delta\omega = 3.5 \times 10^{-3}$. Then the event rate would be

$$N_{ep} = \mathcal{L}\, \frac{d\sigma}{\frac{dE'}{E'}\, d\omega}\, \frac{\Delta p'}{p'}\, \Delta\omega$$

$$= 10^{33} \times 2.85 \times 10^{-30} \times 0.02 \times 3.5 \times 10^{-3}$$

$$= 2 \times 10^{-1} \text{ events/sec}$$

$$= 700/\text{h} \quad .$$

Also consider point B at $Q^2 = 100$ GeV^2 and $\nu = 160$ GeV. Now $E' = 12.15$ and $\theta = 54^o$ with the cross section reduced by a factor of 25. Then the event rate would be $\sim$ 30/h.

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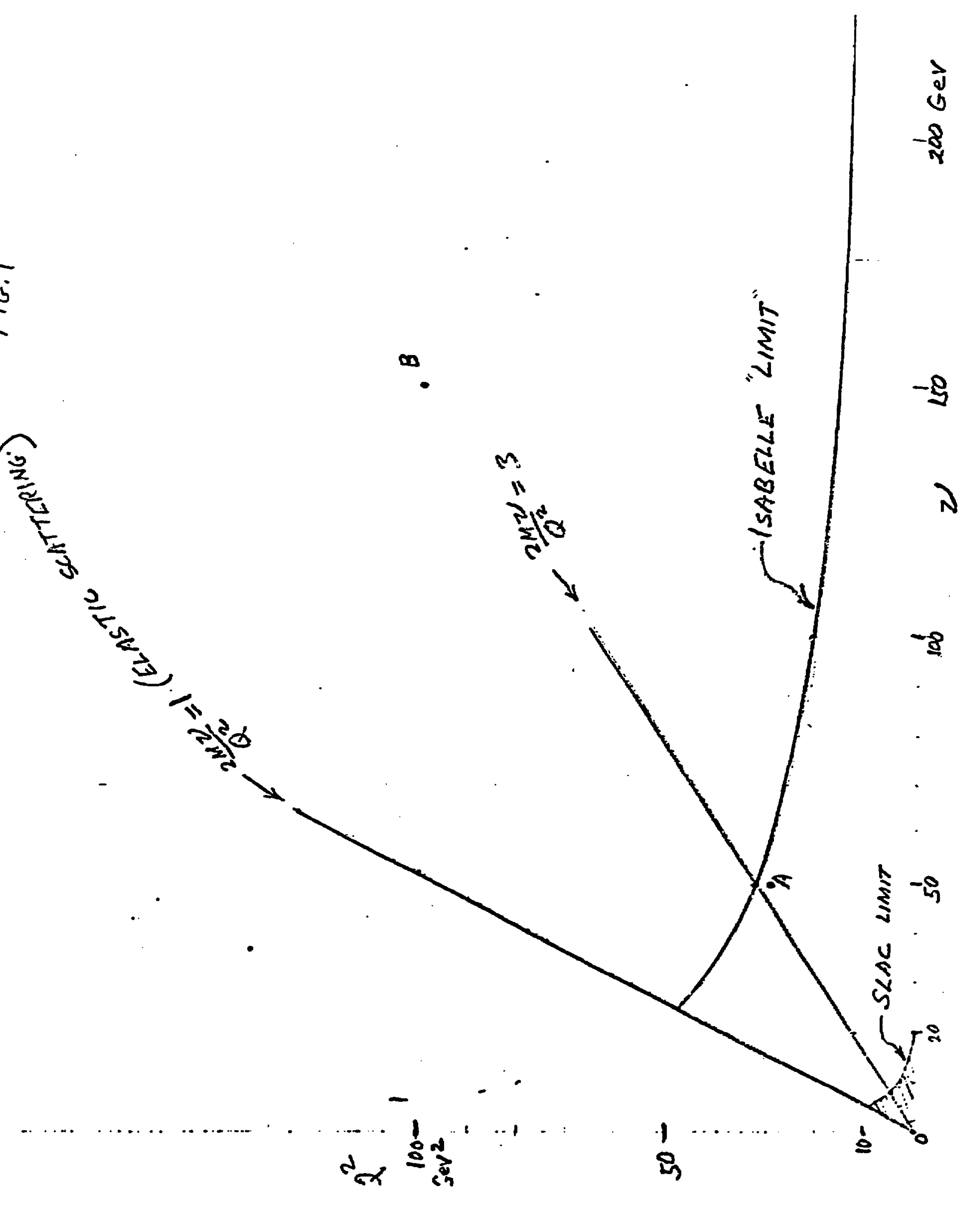
FIG. 1
$\frac{2M\nu}{Q^2} = 1$ (ELASTIC SCATTERING)
$\frac{2M\nu}{Q^2} = 3$
ISABELLE "LIMIT"
SLAC LIMIT
B
A
ν
200 GeV
150
100
50
20
0
10
50
100 GeV^2
Q^2

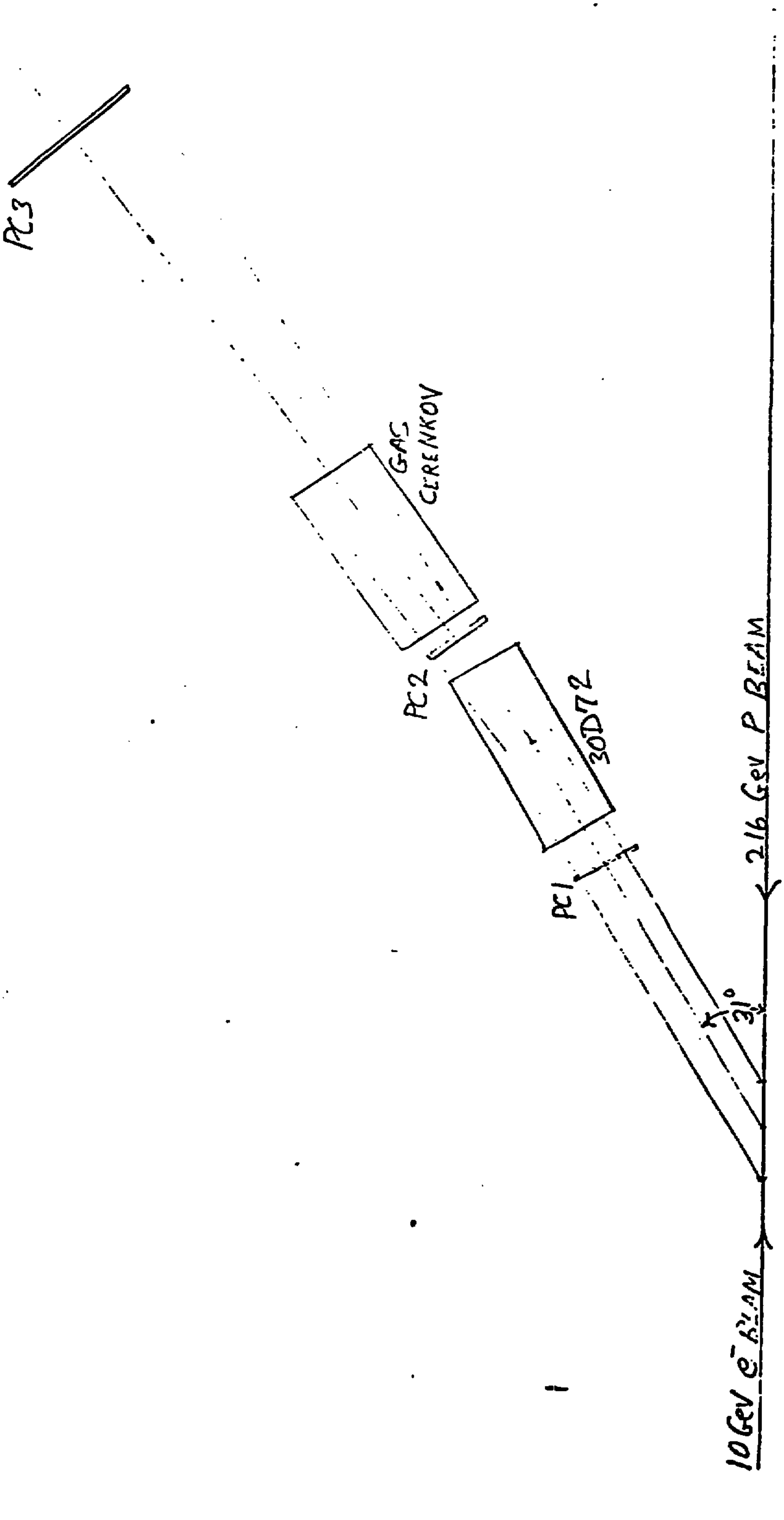

Fig. 2

e-P INELASTIC SCATTERING

to Scale
1" = 5 ft.