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On the Transfer of Heat to Fluids Flowing Through Pipes, Annuli, and Parallel Plates*

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Heat Transfer in Various Shaped Channels

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ABSTRACT

Nusselt numbers have been calculated for heat transfer to fluids flowing through annuli under conditions of uniform heat flux and fully-established velocity and temperature profiles. The following cases were considered: (a) laminar flow, (b) slug flow, (c) turbulent flow with molecular conduction only, and (d) turbulent flow with both molecular and eddy conduction. These Nusselt numbers were determined for two conditions: heat transfer from the inner wall only and heat transfer from the outer wall only. The results were correlated by semi-empirical equations.

The final results obtained on cases (a), (b), and (c) are applicable to any fluid, whereas those obtained on (d) are for liquid metals only.

Wall- and bulk-temperature relationships for the above four cases were also determined. These relationships were treated as dimensionless temperature ratios.

Both the Nusselt numbers and temperature ratios were evaluated over the r_1/r_2 range, zero to unity; the former being the case of the circular pipe, and the latter, the case of infinite parallel plates.

INTRODUCTION

This paper is the third in a series, originating at the Brookhaven National Laboratory, on the general subject of heat transfer to liquid metals flowing through annuli. The first (1) was entitled "Unilateral Heat Transfer to Liquid Metals Flowing in Annuli" and the second (2), "Equations for Bilateral Heat Transfer to a Fluid Flowing in a Concentric Annulus."

The one geometrical variable correlating convective heat transfer and flow behavior in annuli is the radius ratio, r_1/r_2. In the present study, this ratio was varied from zero to unity, the former being the case of a circular pipe, and the latter, that of infinite parallel plates. Thus, whereas the main emphasis of this paper is on heat transfer to fluids flowing in annuli, results on both pipes and parallel plates were also collected.

The present paper deals with a number of miscellaneous topics, all for the case of uniform heat flux with fully-established velocity and temperature profiles. Nusselt numbers are presented in tabular, graphical, and equation form for the following flow conditions: (a) laminar flow, (b) slug flow, (c) turbulent flow with molecular conduction only, and (d) turbulent flow with both molecular conduction and eddy conduction. Also, relationships are given for obtaining the difference between wall temperatures, with heat transfer from either the inner or outer wall.

In the case of turbulent-flow heat transfer, with both molecular and eddy conduction operating, the results are correlated by use of the term $\bar{\psi}$, which is the average value of the ratio of the eddy diffusivity of heat transfer to that of mass transfer. It is recommended that $\bar{\psi}$ be evaluated by the recent correlation of Dwyer (3).

The work summarized in this paper was undertaken with special reference to liquid metals. It happens, however, that several of the mathematical relationships, which were developed, are valid for any fluid. For this reason, the term "liquid metals" was not used in the title.

BASIC HEAT TRANSFER EQUATIONS

The Nusselt number for the case of heat transfer to a fluid flowing through a concentric annulus, under conditions of fully-established flow, uniform heat flux, and heat transfer from the inner wall only, is given (2) by the expression

$$Nu_1 = \frac{2(y-1)\left[\int_{r_1}^{r_2} v r dr\right]^2}{\int_{r_1}^{r_2}\left[\int_{r_1}^{r} \frac{\int_{r}^{r_2} v r dr}{r\left(1 + \frac{\psi \epsilon_M Pr}{\nu}\right)} dr\right] v r dr} \qquad (1)$$

The only simplifying assumption made in the derivation of this equation is that the physical properties were assumed to be independent of temperature – an assumption which is usually quite acceptable for liquid metals. The above equation is valid for either laminar, turbulent, or slug flow. The corresponding equation for the case of heat transfer from the inner wall only is

$$Nu_2 = \frac{2(1-\frac{1}{y})\left[\int_{r_1}^{r_2} vrdr\right]^2}{\int_{r_1}^{r_2}\left[\int_{r}^{r_2}\frac{\int_{r_1}^{r} vrdr}{r\left(1+\frac{\psi\,\epsilon_M Pr}{\nu}\right)}dr\right]vrdr} \tag{2}$$

LAMINAR FLOW

In this case, the variation of linear velocity with radius is given by Lamb's (4) equation,

$$v = \frac{(\Delta p)g_o}{4\mu N}\left[r_1^2 - r^2 + \frac{r_2^2 - r_1^2}{\ln\frac{r_2}{r_1}}\ln\frac{r}{r_1}\right] \tag{3}$$

It can easily be shown that

$$\frac{(\Delta p)g_o}{4\mu N} = \frac{2v_a}{r_1^2\left[y^2 + 1 - \frac{y^2 - 1}{\ln y}\right]} \tag{4}$$

Combining Equations (3) and (4) then gives the equation

$$v = \frac{2v_a}{[y^2 + 1 - \frac{y^2-1}{\ln y}]} [1 - \frac{r^2}{r_1^2} + \frac{y^2-1}{\ln y} \ln \frac{r}{r_1}] \quad (5)$$

which, for a given annulus, gives v as a function of v_a and r, only. Now, combining Equations (1) and (5), remembering that for laminar flow $(\psi \epsilon_M Pr)/\nu = 0$, gives

$$[Nu_L]_1 = \frac{(y-1)(y^2-1)^2 [y^2 + 1 - \frac{y^2-1}{\ln y}]^2 r_1^8}{8\xi_1} \quad (6)$$

where

$$\xi_1 = \int_{r_1}^{r_2} \left[\int_{r_1}^{r} \frac{\int_r^{r_2} zrdr}{r} dr \right] zrdr \quad (7)$$

and

$$z = r_1^2 - r^2 + \frac{r_2^2 - r_1^2}{\ln \frac{r_2}{r_1}} \ln \frac{r}{r_1} \quad (8)$$

From these three equations, it is obvious that $[Nu_L]_1$ is independent of velocity and therefore independent of both Reynolds and Prandtl numbers. It turns out that $[Nu_L]_1$ depends only on the geometrical variable y.

Equation (6) was solved, with the aid of an IBM 7090 computer, for different values of r_1 and y. The results are shown in Table I and Figure 1.

$[Nu_L]_1$ is almost a linear function of y.

The equation for $[Nu_L]_2$, corresponding to Equation (6), is

$$[Nu_L]_2 = \frac{(1-\frac{1}{y})(y^2-1)^2[y^2+1-\frac{y^2-1}{\ln y}]^2 r_1^{\,8}}{8\xi_2} \tag{9}$$

where

$$\xi_2 = \int_{r_1}^{r_2}\left[\int_{r}^{r_2}\frac{\int_{r_1}^{r} zrdr}{r}dr\right]zrdr \tag{10}$$

and where z is defined by Equation (8), as before. Equation (9) was also solved on the computer, and the results are also shown in Table I and Figure 1.

The limiting case of a concentric annulus as 1/y approaches zero is that of a circular pipe, for which the well-known equation for the Nusselt number is

$$[Nu_L]_p = 48/11 \tag{11}$$

The limiting case, as 1/y approaches unity, is that of infinite parallel plates. The Nusselt number for this case will now be derived.

The partial differential equation giving the temperature distribution for a fluid flowing between parallel plates under steady-state conditions is

$$\alpha \frac{\partial^2 t}{\partial y^2} = v \frac{dt}{dx} \tag{12}$$

The linear velocity distribution is given by the equation

$$v = 6v_a\left(\frac{y}{b} - \frac{y^2}{b^2}\right) \tag{13}$$

Substituting Equation (13) into (12) gives

$$\frac{\partial^2 t}{\partial y^2} = \left(\frac{6v_a}{\alpha b}\frac{dt}{dx}\right)y - \left(\frac{6v_a}{\alpha b^2}\frac{dt}{dx}\right)y^2 \tag{14}$$

The terms inside the parentheses are all constant for a particular case, the constancy of the ratio dt/dx being a consequence of the boundary condition that the heat flux is uniform. Integration of Equation (14) in accordance with the additional boundary conditions

$$t = t_1 \text{ at } y = 0$$

and

$$\frac{\partial t}{\partial y} = 0 \text{ at } y = b$$

yields the following expression for t.

$$t = t_1 + \left(\frac{Bb^3}{3} - \frac{Ab^2}{2}\right)y + \frac{A}{6}y^3 - \frac{B}{12}y^4 \tag{15}$$

where

$$A = \frac{6v_a}{\alpha b} \frac{dt}{dx}$$

and

$$B = \frac{6v_a}{\alpha b^2} \frac{dt}{dx}$$

With this we can now evaluate the bulk temperature, which is defined as

$$t_b = \frac{\int_0^b tvdy}{v_a b} \tag{16}$$

Substituting Equation (15) into (16), and simplifying, gives

$$t_b = t_1 + \frac{117}{315} \frac{v_a b^2}{\alpha} \frac{dt}{dx} \tag{17}$$

The Nusselt number can be written as

$$[Nu_L]_{pp} = \frac{hD_e}{k} = \frac{qD_e}{(t_1 - t_b)k} = \frac{v_a C_p \rho b \frac{dt}{dx} D_e}{(t_1 - t_b)k} \tag{18}$$

Finally substituting Equation (17) into (18), and solving, gives

$$[Nu_L]_{pp} = \frac{630}{117} = 5.38 \tag{19}$$

for laminar flow of any fluid through infinite parallel plates under conditions of fully-established flow, uniform heat flux, and heat transfer from one wall only.

Equations (6), (9), (11), and (19) all give consistent results as shown by the curves in Figure 1.

SLUG FLOW

For slug flow, the equations giving the Nusselt number for the cases of heat transfer from the inner wall only, and from the outer wall only, are given in reference (1). For comparison with the laminar-flow results, calculated slug Nusselt numbers are shown in Table I.

MOLECULAR-CONDUCTION HEAT TRANSFER WITH TURBULENT FLOW

The author (3) has previously pointed out that, in the case of liquid metals, it is possible to have convective heat transfer under turbulent-flow conditions with only the mechanism of molecular conduction operating. This happens at the low end of the turbulent flow regime, where the eddy currents are presumably not strong enough to transport a significant amount of heat. In other words, they do not move sufficiently fast to prevent losing essentially all of their heat while in transit. As the flow rate is increased, in a given case, a point is reached where eddy transport begins to assert itself as a contributing mechanism in the total heat transfer process. The higher the thermal conductivity of the liquid metal, the higher the Reynolds number at which this occurs. In the case of flow through annuli, it is estimated (3) that, at a Prandtl number of 0.01, the critical Peclet number is about 300, i.e., the Reynolds number has to rise to about 30,000 before eddy transport becomes significant.

Thus, it is seen that molecular-conduction Nusselt numbers are of considerable practical importance.

Equations (1) and (2) have been solved for the condition that eddy conduction is zero. The results are shown in Table II. They are well represented by the equation

$$[Nu_{m.c.}]_1 = 4.98 + 0.662y \qquad (20)$$

and

$$[Nu_{m.c.}]_2 = 5.60 + 0.195(y-1)^{0.64}(\log_{10} Re - 3.70)^{0.54} \qquad (21)$$

It is interesting to note that $[Nu_{m.c.}]_1$ depends only on y, whereas $[Nu_{m.c.}]_2$ is a function of both y and Re, albeit the effect of Re is slight. This result is a bit surprising. One would expect that both Nusselt numbers would have a slight dependence on the Reynolds number, owing to the increasing steepness of the velocity profile curves, near each wall, as the Reynolds number is increased. From Table II, it is seen that, for an annulus with a y value of 2, $[Nu_{m.c.}]_2$ increases only 3.5% as the

Reynolds number increases 100 fold, from 10^4 to 10^6. The variation of $[Nu_{m.c.}]_2$ with Reynolds number is shown graphically in Figure 2.

Theoretically, both Equations (20) and (21) should give the same result at y = 1. The former gives a value of 5.64, the latter, 5.60. This difference is within the precision of the method of calculation. It is estimated that Equation (20) represents the calculated results to within ±0.2% and Equation (21) to within ± 1.0%.

COMPARISON OF NON-EDDY TYPES OF HEAT TRANSFER

Figure 3 shows a comparison of laminar-flow, slug-flow, and molecular-conduction turbulent-flow Nusselt numbers for flow of fluids through annuli, with heat transfer from the inner wall only. The surprising thing about this graph is the relationship between the three curves. At a y value of 2.68, the three types of Nusselt numbers have identical values. Below this, the laminar, molecular-conduction, and slug values fall in the expected order, i.e., laminar flow gives the lowest Nusselt numbers and slug flow, the highest. Above y = 2.68, they fall unexpectedly in the reverse order. The question is: is this real or apparent. The answer is: it is real.

The magnitude of any one of the three types of Nusselt numbers depends on the relative magnitudes of the temperature and velocity profiles across the flow channel. It is therefore conceivable that at the larger values of y the higher velocities in the central portion of the channel for laminar flow cause the Nusselt number to exceed that for slug flow.

Referring to Figure 3 again, the fact that the molecular-conduction curve always falls between the laminar and slug curves tends to corroborate the correctness of the relative positions of the latter curves. In other words, beyond a y value of 2.68, not only did the slug Nusselt numbers fall below the laminar Nusselt numbers, but the molecular-conduction ones did also.

It will be noticed also that at the limiting condition of $y = 1$, which is the case of parallel plates, the three curves show the correct Nusselt numbers.

Figure 4 shows a comparison of the Nusselt numbers for the three types of non-eddy heat transfer conditions, for the case of heat transfer from the outer wall only. At the limits of $y = 0$ (round pipes), and $y = 1$ (parallel plates), all curves show the correct Nusselt values.

Of all six Nusselt numbers, $[Nu_L]_1$, $[Nu_s]_1$, $[Nu_{m.c.}]_1$, $[Nu_L]_2$, $[Nu_s]_2$, and $[Nu_{m.c.}]_2$, the only one that is a function of the Reynolds number is the last, and then only slightly. All are functions of y; and none, of course, is a function of the Prandtl number.

TURBULENT-FLOW HEAT TRANSFER WITH BOTH MOLECULAR AND EDDY CONDUCTION

In reference (2), equations were given for estimating Nusselt numbers for liquid metals flowing through annuli, under conditions of uniform heat flux and fully-established flow. Those equations were based upon graphical solutions of Equations (1) and (2). Recently, the solutions of Equations (1) and (2) were obtained with the aid of a IBM 709 computer, i.e., the successive integrations were done by machine rather than graphically. The new results are represented by the following equations:

$$[Nu_T]_1 = \alpha_1 + \beta_1(\bar{\psi}\,Pe)^{\gamma_1} \tag{22}$$

where

$$\alpha_1 = 4.58 + 0.742y$$

$$\beta_1 = 0.0290 - 0.00414y + 0.000364y^2$$

$$\gamma_1 = 0.725y^{0.091}$$

and

$$[Nu_T]_2 = \alpha_2 + \beta_2(\bar{\psi}\,Pe)^{\gamma_2} \tag{23}$$

where

$$\alpha_2 = 5.24 + 0.0800y$$

$$\beta_2 = 0.0262 - 0.000953y + 0.0000453y^2$$

$$\gamma_2 = 0.725y^{0.045}$$

For the special case of y = 1 (parallel plates), Equations (22) and (23) reduce to the equation

$$[Nu_T]_{pp} = 5.32 + 0.0253(\bar{\psi}\,Pe)^{0.725} \qquad (24)$$

These three equations for $[Nu_T]_1$, $[Nu_T]_2$, and $[Nu_T]_{pp}$ are more accurate than their counterparts in reference (2), although the difference between the two sets is, in most cases, very slight. The greatest difference – of the order of 5 to 10% – occurs for y values less than 2 at very high Peclet numbers (around 10^4) for both Nu_1 and Nu_2. The constants in Equations (22) and (23) were evaluated over the $\bar{\psi}\,Pe$ range 50 to 10^4 and Prandtl number range 0.005 to 0.05. These two equations are not valid at the limit where $\bar{\psi}\,Pe = 0$. For this condition, the Nusselt numbers must be evaluated by Equations (20) and (21).

It is not strictly true to say that, in semi-empirical equations of the type (22) and (23), the α terms represent the molecular-conduction contribution to heat transfer and the $\beta(\bar{\psi}\,Pe)^{\gamma}$ terms the eddy-conduction contribution. It is an approximation only.

WALL AND BULK TEMPERATURE RELATIONSHIPS

For the case of heat transfer from one wall of an annulus through which a liquid metal is flowing, it may be of practical importance to determine, not only the temperature drop from heated wall to the bulk fluid, but the difference between the two wall temperatures. The first of these temperature differences can be easily obtained from a knowledge of the heat flux and the heat transfer coefficient. The second requires a special calculation. Let us first consider the case of heat transfer from the inner wall only.

The temperature drop between r_1 to r_2 is given by

$$t_1 - t_2 = -\int_{r_1}^{r_2} \frac{\partial t}{\partial r} dr \tag{25}$$

The radial heat transfer rate at any radius, r, must equal the heat transport rate between r and r_2. This can be expressed mathematically as

$$-2\pi r k_{eff} \frac{\partial t}{\partial r} = 2\pi \rho C_p \frac{dt}{dx} \int_{r}^{r_2} vrdr \tag{26}$$

assuming a uniform heat flux and physical properties independent of temperature.

Combining Equations (25) and (26), gives

$$t_1 - t_2 = \rho C_p \frac{dt}{dx} \int_{r_1}^{r_2} \frac{\int_{r}^{r_2} vrdr}{r k_{eff}} dr \tag{27}$$

For correlation purposes, it is convenient to work with the ratio $(t_1-t_b)/(t_1-t_2)$. The numerator in this ratio is given by the equation

$$t_1-t_b = \frac{v_a \rho C_p \frac{dt}{dx} r_1^2 (y-1)(y^2-1)}{k[Nu_T]_1} \tag{28}$$

Dividing Equation (27) by (28), and then substituting Equation (1) into the result, finally gives

$$\frac{t_1-t_b}{t_1-t_2} = \frac{\int_{r_1}^{r_2} \left[\int_{r_1}^{r} \frac{\int_r^{r_2} vrdr}{r\left(1 + \frac{\psi \epsilon_M Pr}{\nu}\right)} dr \right] vrdr}{\int_{r_1}^{r_2} vrdr \int_{r_1}^{r_2} \frac{\int_r^{r_2} vrdr}{r\left(1 + \frac{\psi Pr \epsilon_M}{\nu}\right)} dr} \tag{29}$$

The corresponding equation for the case of heat transfer from the outer wall only is

$$\frac{t_2-t_b}{t_2-t_1} = \frac{\int_{r_1}^{r_2} \left[\int_{r}^{r_2} \frac{\int_{r_1}^{r} vrdr}{r\left(1 + \frac{\psi \epsilon_M Pr}{\nu}\right)} dr \right] vrdr}{\int_{r_1}^{r_2} vrdr \int_{r_1}^{r_2} \frac{\int_{r_1}^{r} vrdr}{r\left(1 + \frac{\psi \epsilon_M Pr}{\nu}\right)} dr} \tag{30}$$

These two equations were used to calculate the respective dimensionless temperature ratios, using an IBM computer, for conditions of constant heat flux and fully-established turbulent flow. The smoothed results are shown in Tables III and IV. Typical sets of curves for $(t_1-t_b)/(t_1-t_2)$ and $(t_2-t_b)/(t_2-t_1)$ are shown in Figures 5 and 6, respectively, for the case of $y = 2.0$.

Since values of (t_1-t_b) and (t_2-t_b) are easily obtained from a knowledge of the heat flux and Nusselt number, values of (t_1-t_2) and (t_2-t_1) are readily obtained from Tables III and IV or from plots such as Figures 5 and 6.

The dimensionless temperature ratios are functions of y, Reynolds number, and Prandtl number, with the exception of $(t_1-t_b)/(t_1-t_2)$ for the case of $Pr = 0$. In that case, it is independent of Reynolds number.

For the case of slug flow, Equations (29) and (30) can be integrated, and then simplified, to give

$$\frac{t_1-t_b}{t_1-t_2} = \frac{\dfrac{y^4 \ln y}{2(y^2-1)} - \dfrac{3y^2+1}{8} + \dfrac{1}{4}}{\dfrac{y^2 \ln y}{2} - \dfrac{y^2-1}{4}} \qquad (31)$$

and

$$\frac{t_2-t_b}{t_2-t_1} = \frac{\frac{y^2-1}{2} - \frac{y^2+1}{4} + \frac{\ln y}{y^2-1}}{\frac{y^2-1}{2} - \ln y} \qquad (32)$$

respectively.

For the case of laminar flow, Equations (29) and (30) can be integrated, using Equation (5) to express v as a function of r. This was done with the aid of the IBM 709 computer, and the results, along with those for slug flow by Equations (31) and (32), are given in Table V.

The calculation of the values in this table for the cases of round pipes and parallel plates are summarized in the Appendix.

Table I

Calculated Nusselt Numbers for Flow Through Annuli

y	r_1, inches	r_2, inches	Laminar Flow $[Nu_L]_1$	Laminar Flow $[Nu_L]_2$	Slug Flow $[Nu_s]_1$	Slug Flow $[Nu_s]_2$
1 (parallel plates)			5.38	5.38	6.00	6.00
1.250	0.500	0.625	5.60	5.26	6.08	6.07
1.500	0.500	0.750	5.78	5.15	6.16	6.14
2.000	0.500	1.000	6.17	5.03	6.36	6.24
2.000	1.000	2.000	6.17	5.03	6.36	6.24
2.000	5.000	10.000	6.17	5.03	6.36	6.24
4.000	0.500	2.000	7.78	4.92	7.57	6.73
6.000	0.500	3.000	9.22	4.875	8.83	7.03
8.000	0.500	4.000	10.57	4.844	10.10	7.21
10.000	0.500	5.000	11.90	4.832	11.29	7.35
∞ (round pipes)			∞	4.364	∞	8.00

Table II

Calculated Molecular-Conduction Nusselt Numbers

for Turbulent Flow of Liquid Metals Through Annuli

		$[Nu_{m.c.}]_2$		
y	$[Nu_{m.c.}]_1$	$Re = 10^4$	$Re = 10^5$	$Re = 10^6$
1.0	(5.64)	(5.60)	(5.60)	(5.60)
2.0	6.30	5.70	5.81	5.90
3.0	6.96	5.77	5.98	6.09
4.0	7.62	5.82	6.10	6.22
5.0	8.29	5.86	6.20	6.33
6.0	8.95	5.90	6.29	6.42

Table III

Values of $(t_1 - t_b)/(t_1 - t_2)$ for Case of Constant Heat Flux, Heat Transfer from the Inner Wall Only, and Fully-Established Flow

r_1/r_2	Pr	Re = 2×10^4	Re = 10^5	Re = 5×10^5	Re = 2×10^6
0.150	0.000	0.850	0.850	0.850	0.850
	.005	.852	.854	.857	.861
	.010	.854	.858	.862	.869
	.02	.857	.863	.870	.879
	.03	.859	.867	.875	.887
	.05	.863	.872	.884	.897
0.250	0.000	0.817	0.817	0.817	0.817
	.005	.818	.819	.822	.827
	.01	.818	.822	.827	.834
	.02	.819	.825	.833	.843
	.03	.819	.827	.837	.849
	.05	.820	.829	.841	.856
0.400	0.000	0.778	0.778	0.778	0.778
	.005	.779	.781	.783	.786
	.01	.780	.784	.788	.792
	.02	.782	.788	.794	.801
	.03	.783	.790	.798	.807
	.05	.785	.793	.804	.816
0.550	0.000	0.745	0.745	0.745	0.745
	.005	.748	.750	.752	.756
	.01	.750	.754	.777	.764
	.02	.754	.759	.765	.774
	.03	.757	.763	.770	.781
	.05	.761	.768	.777	.791
0.700	0.000	0.721	0.721	0.721	0.721
	.005	.726	.727	.729	.733
	.01	.730	.732	.735	.742
	.02	.736	.740	.745	.754
	.03	.740	.745	.752	.762
	.05	.746	.752	.762	.775

Table IV

Values of $(t_2-t_b)/(t_2-t_1)$ for Case of Constant Heat Flux, Heat Transfer from the Outer Wall Only, and Fully-Established Flow

r_1/r_2	Pr	Re = 2×10^4	Re = 10^5	Re = 5×10^5	Re = 2×10^6
0.150	0.000	0.571	0.560	0.554	0.552
	.005	.572	.566	.572	.583
	.010	.574	.570	.580	.593
	.02	.577	.577	.589	.607
	.03	.579	.580	.594	.618
	.05	.581	.584	.601	.640
0.250	0.000	0.590	0.581	0.576	0.575
	.005	.592	.585	.589	.601
	.01	.593	.589	.597	.612
	.02	.597	.594	.606	.625
	.03	.599	.600	.614	.637
	.05	.603	.607	.625	.658
0.400	0.000	0.617	0.610	0.606	0.606
	.005	.619	.612	.619	.629
	.01	.621	.616	.625	.638
	.02	.624	.622	.633	.652
	.03	.628	.628	.642	.664
	.05	.632	.636	.654	.683
0.550	0.000	0.641	0.636	0.634	0.634
	.005	.644	.638	.644	.652
	.01	.646	.642	.651	.663
	.02	.650	.648	.661	.676
	.03	.653	.653	.668	.685
	.05	.658	.663	.679	.700
0.700	0.000	0.661	0.658	0.657	0.657
	.005	.665	.662	.665	.672
	.01	.669	.665	.673	.685
	.02	.673	.671	.682	.701
	.03	.677	.676	.690	.709
	.05	.682	.682	.700	.716

Table V

Wall and Bulk Temperature Relationships for Laminar Flow and Slug Flow Through Annuli

y	r_1, inches	r_2, inches	Laminar Flow $\frac{t_1-t_b}{t_1-t_2}$	Laminar Flow $\frac{t_2-t_b}{t_2-t_1}$	Slug Flow $\frac{t_1-t_b}{t_1-t_2}$	Slug Flow $\frac{t_2-t_b}{t_2-t_1}$
1 (parallel plates)			0.743	0.743	2/3	2/3
1.250	0.500	0.625	0.755	0.725	0.691	0.641
1.500	0.500	0.750	0.771	0.716	0.712	0.623
2.000	0.500	1.000	0.790	0.699	0.745	0.596
2.000	1.000	2.000	0.790	0.699	0.745	0.596
2.000	5.000	10.000	0.790	0.699	0.745	0.596
4.000	0.500	2.000	0.834	0.666	0.811	0.547
6.000	0.500	3.000	0.856	0.652	0.843	0.529
8.000	0.500	4.000	0.870	0.645	0.860	0.519
10.000	0.500	5.000	0.879	0.640	0.873	0.514
∞ (round pipes)			1.000	11/18	1.000	1/2

APPENDIX

When the value of 1/y for annuli is reduced to the lower limit of zero, we have the case of a circular pipe; when it is increased to the upper limit of unity, we have the case of parallel plates.

The ratio $(t_2-t_b)/(t_2-t_1)$ becomes for a pipe $(t_R-t_b)/(t_R-t_0)$. The latter ratio for the case of laminar flow is derived as follows:

$$t_R-t_b = \frac{q}{h} = \frac{Dq}{k[Nu_L]_p} = \frac{DRv_a\rho C_p \frac{dt}{dx}}{2k[Nu_L]_p} \tag{33}$$

$$t_R-t_0 = \int_0^R \frac{\partial t}{\partial r}\, dr \tag{34}$$

Now, writing an equation for pipes, similar to Equation (26) for annuli, gives us

$$2\pi rk\frac{\partial t}{\partial r} = 2\pi\rho C_p \frac{dt}{dx}\int_r^R vrdr \tag{35}$$

Combining Equations (34) and (35), gives

$$t_R-t_0 = \frac{\rho C_p \frac{dt}{dx}}{k}\int_0^R \frac{\int_0^r vrdr}{r}\, dr \tag{36}$$

The velocity distribution across the pipe radius is given by the equation

$$v = 2v_a[1 - (r/R)^2] \tag{37}$$

Finally, substituting this equation into (36), then dividing (33) by (36), and then integrating and simplifying, gives the simple result

$$\left[\frac{t_R - t_b}{t_R - t_0}\right]_{L,p} = \frac{11}{18} \tag{38}$$

For the case of slug flow, dividing Equation (33) by (36), integrating, and simplifying, gives

$$\left[\frac{t_R - t_b}{t_R - t_0}\right]_{s,p} = \frac{1}{2} \tag{39}$$

The ratio $(t_1 - t_b)/(t_1 - t_2)$ for the case of laminar flow between parallel plates is derived as follows:

$$t_1 - t_b = \frac{q}{h} = \frac{D_e q}{k[Nu_L]_{pp}} = \frac{2b^2 C_p \rho \frac{dt}{dx} v_a}{k[Nu_L]_{pp}} \tag{40}$$

$$t_1 - t_2 = \int_0^b -(\partial t/\partial y)dy \tag{41}$$

Now, writing an equation for parallel plates, similar to Equation (35) for pipes, gives

$$-k \frac{\partial t}{\partial y} = \rho C_p \frac{dt}{dx} \int_0^b v dy \tag{42}$$

which, when combined with Equation (41), gives

$$t_1 - t_2 = \frac{C_p \rho \frac{dt}{dx}}{k} \int_0^b \left[\int_y^b v dy \right] dy \tag{43}$$

The velocity distribution across the channel, for laminar flow, is given by the equation

$$v = \frac{3v_a}{2} \left[1 - \frac{(\frac{b}{2} - y)^2}{(\frac{b}{2})^2} \right] \tag{44}$$

Finally, substituting Equation (44) into (43), then dividing (43) into (40), integrating, and simplifying, gives another simple result, i.e.,

$$\left[\frac{t_1 - t_b}{t_1 - t_2} \right]_{L,pp} = \frac{4}{[Nu_L]_{pp}} = \frac{4}{5.38} = 0.743 \tag{45}$$

For the case of slug flow, dividing Equation (40) by (43), integrating, and simplifying, leads finally to the equ

$$\left[\frac{t_1 - t_b}{t_1 - t_2} \right]_{s,pp} = \frac{2}{3} \tag{46}$$

NOMENCLATURE

A	= quantity defined in Equation (15), $^{o}F/ft^{3}$
b	= distance between parallel plates, ft
B	= quantity defined in Equation (15), $^{o}F/ft^{4}$
C_p	= specific heat, $Btu/(lb_m)(^{o}F)$
D	= inside diameter of pipe, ft
D_e	= equivalent diameter = $\frac{4(\text{cross-sectional area})}{\text{wetted perimeter}}$, ft
g_o	= conversion factor, $(lb_m)(ft)/(lb_f)(hr)^2$
k	= molecular thermal conductivity, $Btu/(hr)(ft)(^{o}F)$
k_e	= eddy thermal conductivity, $Btu/(hr)(ft)(^{o}F)$
k_{eff}	= $k + k_e$ = effective thermal conductivity, $Btu/(hr)(ft)(^{o}F)$
L	= length of annular channel, ft
Nu	= Nusselt number, hD_e/k, dimensionless
$[Nu]_1$	= Nusselt number for any type of flow through an annulus, with heat transfer from inner wall only, dimensionless
$[Nu]_2$	= Same as $[Nu]_1$, except with heat transfer from outer wall only
$[Nu_T]_1$	= Nusselt number for turbulent flow of a liquid metal through an annulus, with both molecular and eddy conduction operating, and with heat transfer from the inner wall only, dimensionless
$[Nu_T]_2$	= Same as $[Nu_T]_1$, except with heat transfer from the outer wall only

$[Nu_L]_1$ = Nusselt number for laminar flow of fluid through an annulus, with heat transfer from inner wall only, dimensionless

$[Nu_L]_2$ = Same as $[Nu_L]_1$, except with heat transfer from outer wall only

$[Nu_L]_p$ = Nusselt number for laminar flow through a pipe, dimensionless

$[Nu_L]_{pp}$ = Nusselt number for laminar flow through parallel plates, with heat transfer from one plate only, dimensionless

$[Nu_{m.c.}]_1$ = Nusselt number for turbulent flow through an annulus, with molecular conduction only, and with heat transfer from inner wall only, dimensionless

$[Nu_{m.c.}]_2$ = Same as $[Nu_{m.c.}]_1$, except with heat transfer from the outer wall only

$[Nu_s]_1$ = Nusselt number for slug flow of fluid through an annulus, with heat transfer from inner wall only, dimensionless

$[Nu_s]_2$ = Same as $[Nu_s]_1$, except with heat transfer from outer wall only

$[Nu_s]_p$ = Nusselt number for slug flow through a pipe, dimensionless

$[Nu_s]_{pp}$ = Nusselt number for slug flow between parallel plates, with heat transfer from one plate only, dimensionless

$[Nu_T]_p$ = Nusselt number for turbulent flow through a pipe, with both molecular and eddy conduction operating, dimensionless

$[Nu_T]_{pp}$ = Same as $[Nu_T]_p$, except for flow through parallel plates, with heat transfer from one plate only

Δp = pressure drop over distance L, lb_f/ft^2

Pe	$= \frac{D_e v_a \rho C_p}{k}$ = Peclet number, dimensionless
Pr	$= C_p \mu / k$ = Prandtl number, dimensionless
q	= heat flux, Btu/(hr)(ft)^2
r	= radial distance, ft
r_1	= inner radius of annulus, ft
r_2	= outer radius of annulus, ft
R	= inside radius of pipe, ft
Re	$= \frac{D_e v_a \rho}{\mu}$ = Reynolds number, dimensionless
t	= temperature at any value of r or y, ^{0}F
t_1	= temperature at inner wall of annulus, or at left-hand plate of parallel plates, ^{0}F
t_2	= temperature at outer wall of annulus, or at right-hand plate of parallel plates, ^{0}F
t_b	= bulk fluid temperature, ^{0}F
t_R	= temperature at inner wall of circular pipe, ^{0}F
t_0	= temperature (of fluid flowing in a pipe) at r = 0, ^{0}F
v	= local linear velocity at radius r, or distance ℓ, ft/hr
v_a	= average linear velocity across flow channel, ft/hr
x	= axial distance along flow channel, ft

y = radius ratio for annuli, r_2/r_1, dimensionless; also perpendicular distance from one plate toward the second, in the case of parallel plates, ft

z = function of r, defined by Equation (8), ft^2

Greek Letters

α = $\frac{k}{\rho C_p}$ = molecular diffusivity of heat, ft^2/hr

$\alpha_1, \beta_1, \gamma_1$ = constants defined by Equation (22)

$\alpha_2, \beta_2, \gamma_2$ = constants defined by Equation (23)

ϵ_H = $k_e/\rho C_p$ = eddy diffusivity of heat transfer, ft^2/hr

ϵ_M = μ_e/ρ = eddy diffusivity of momentum transfer, ft^2/hr

μ = molecular dynamic viscosity, $lb_m/(ft)(hr)$

μ_e = eddy dynamic viscosity, $lb_m/(ft)(hr)$

ν = μ/ρ = kinematic viscosity, ft^2/hr

ρ = liquid density, lb_m/ft^3

ψ = ϵ_H/ϵ_M, dimensionless

$\bar{\psi}$ = average value of ψ for use in Equations such as (1) and (22)

ξ_1 = function of r defined by Equation (7), ft^8

ξ_2 = function of r defined by Equation (10), ft^8

Subscripts

L,p refers to laminar flow in a pipe

s,p refers to slug flow in a pipe

L,pp refers to laminar flow between parallel plates

s,pp refers to slug flow between parallel plates

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FIGURE CAPTIONS

Figure 1 Heat transfer rates to fluids flowing through annuli under conditions of uniform heat flux and fully-established laminar flow.

Figure 2 Molecular-conduction Nusselt numbers for fluids flowing through annuli under conditions of uniform heat flux, fully-established flow, and heat transfer from the outer wall only.

Figure 3 Heat transfer to fluids flowing in annuli under conditions of uniform heat flux, fully-established flow, heat transfer from the inner wall only, and absence of eddy conduction.

Figure 4 Heat transfer to fluids flowing in annuli under conditions of uniform heat flux, fully-established flow, heat transfer from the outer wall only, and absence of eddy conduction.

Figure 5 Wall- and bulk-temperature relationships for heat transfer to liquid metals flowing through an annulus under conditions of uniform heat flux, fully-established turbulent flow, and heat transfer from the inner wall only.

Figure 6 Wall- and bulk-temperature relationships for heat transfer to liquid metals flowing through an annulus under conditions of uniform heat flux, fully-established turbulent flow, and heat transfer from the outer wall only.

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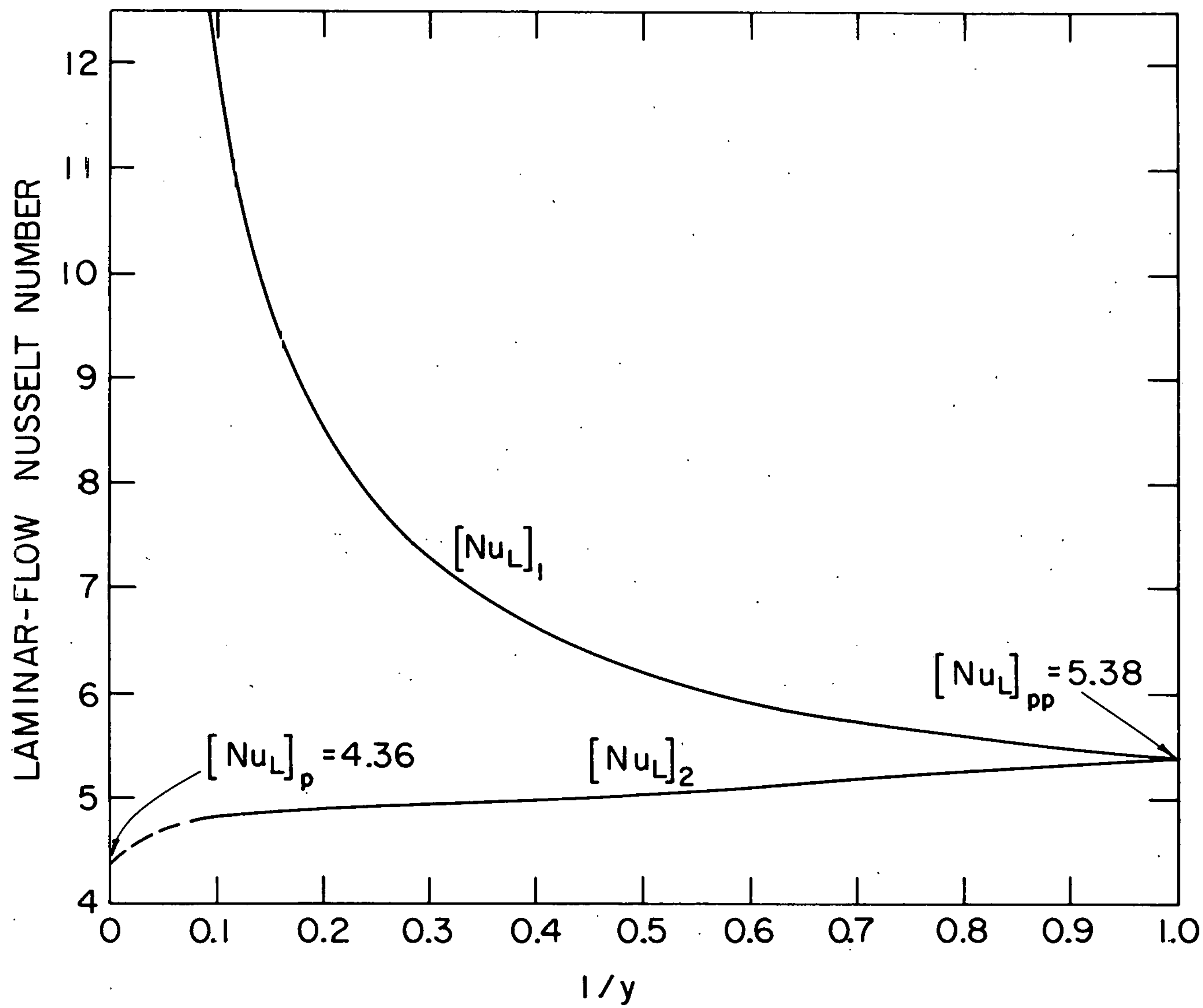
Nuclear Science & Engineering - Dwyer - Fig. 1



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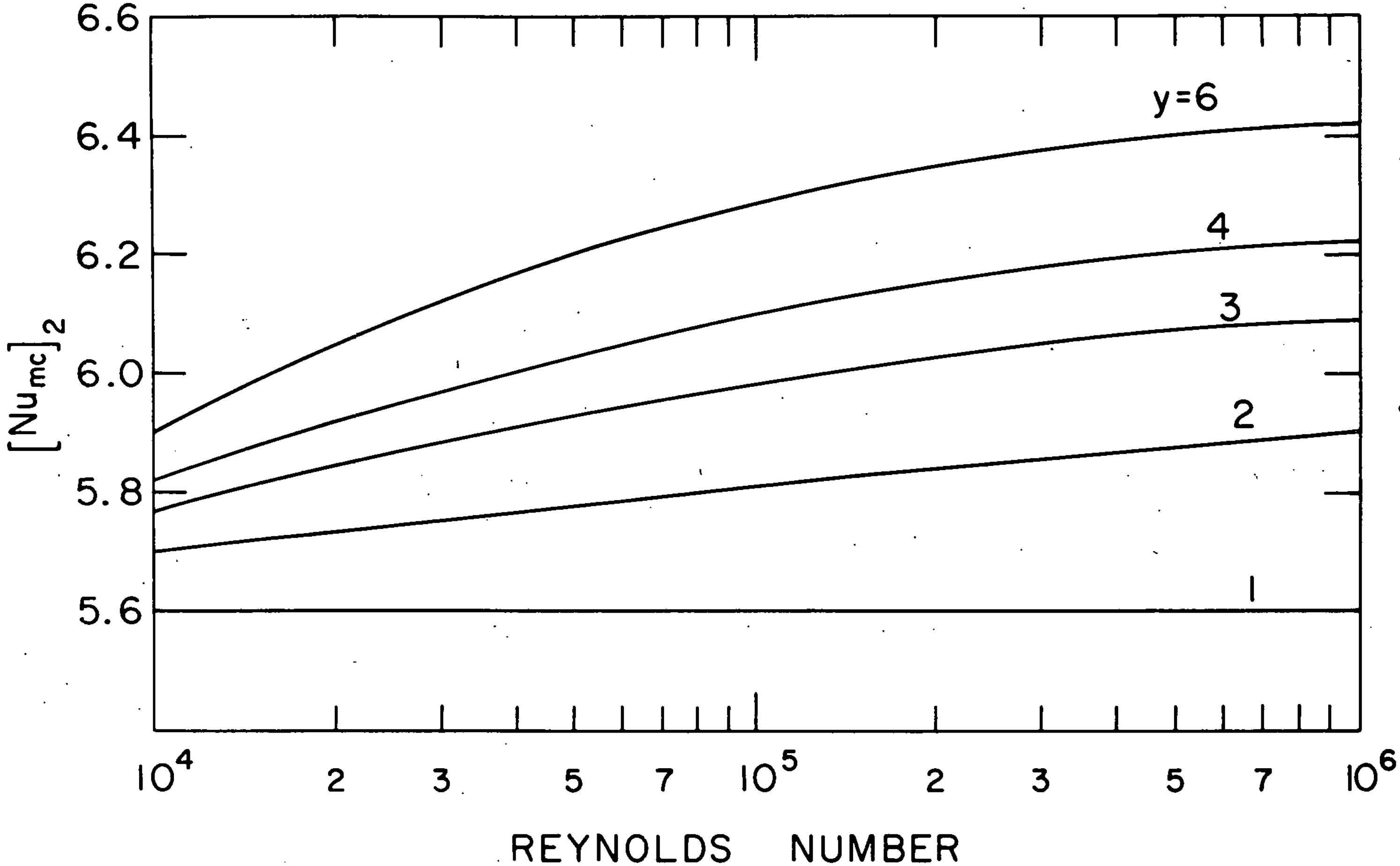
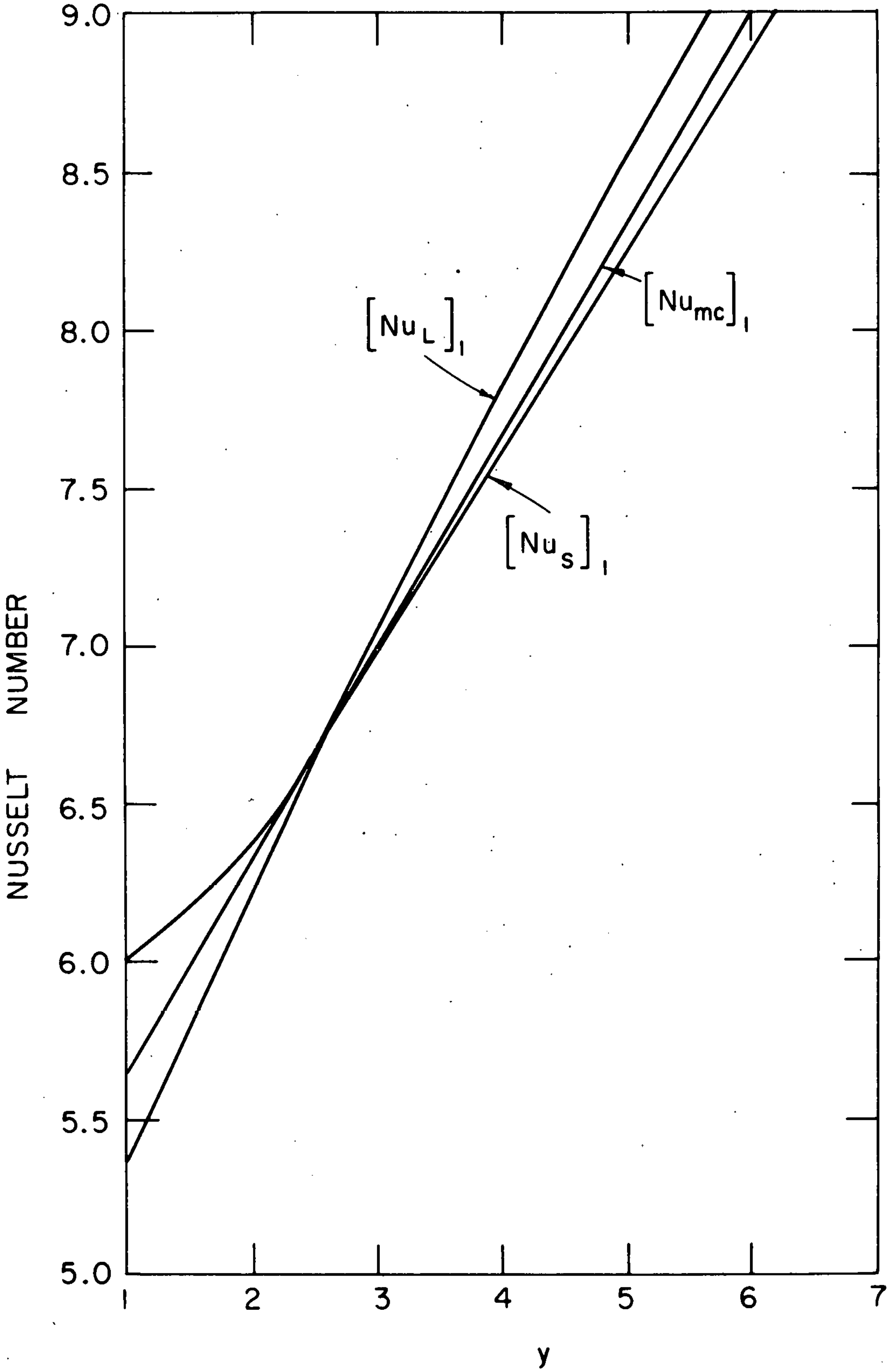
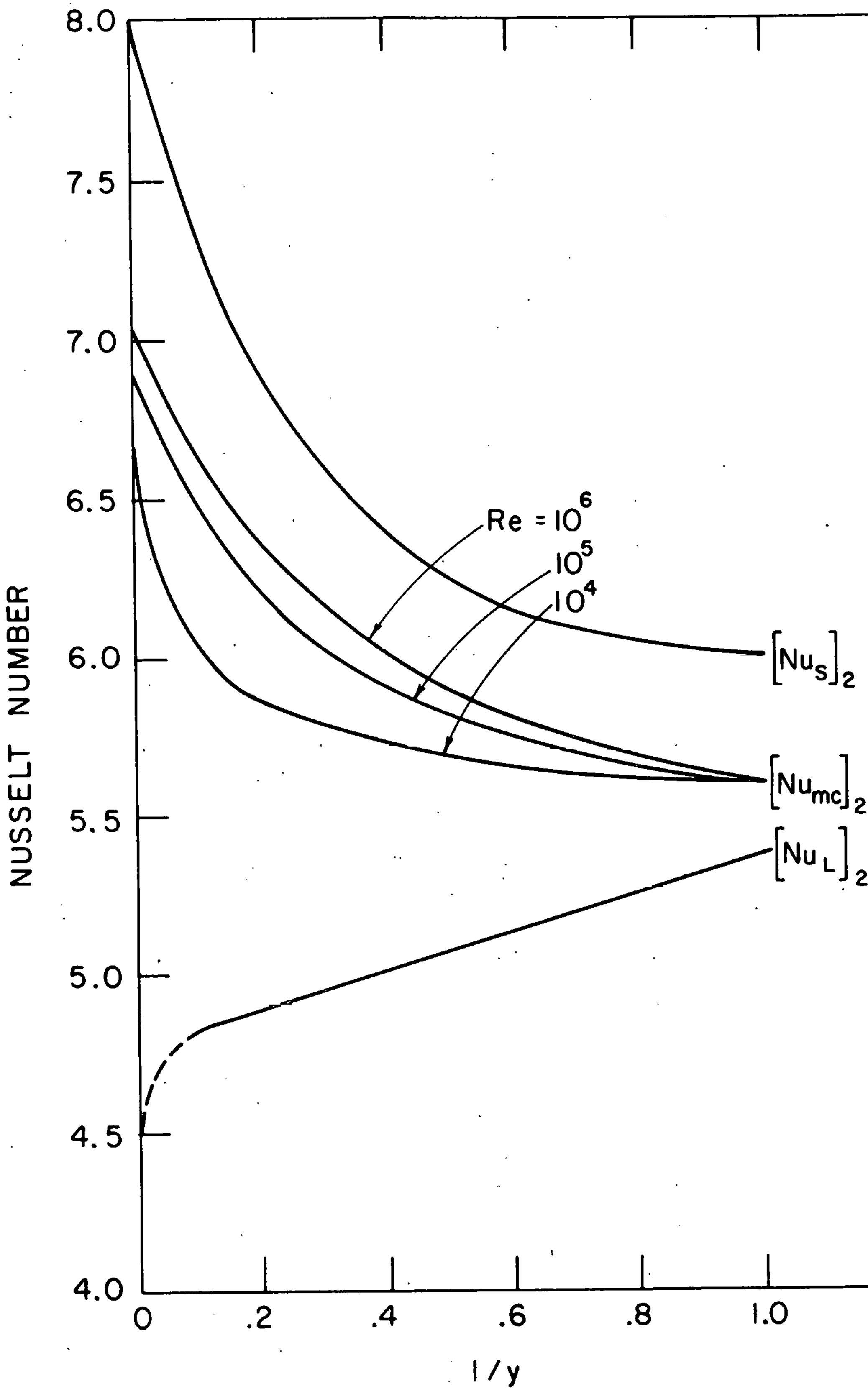
Nuclear Science & Engineering - Dwyer - Fig. 2



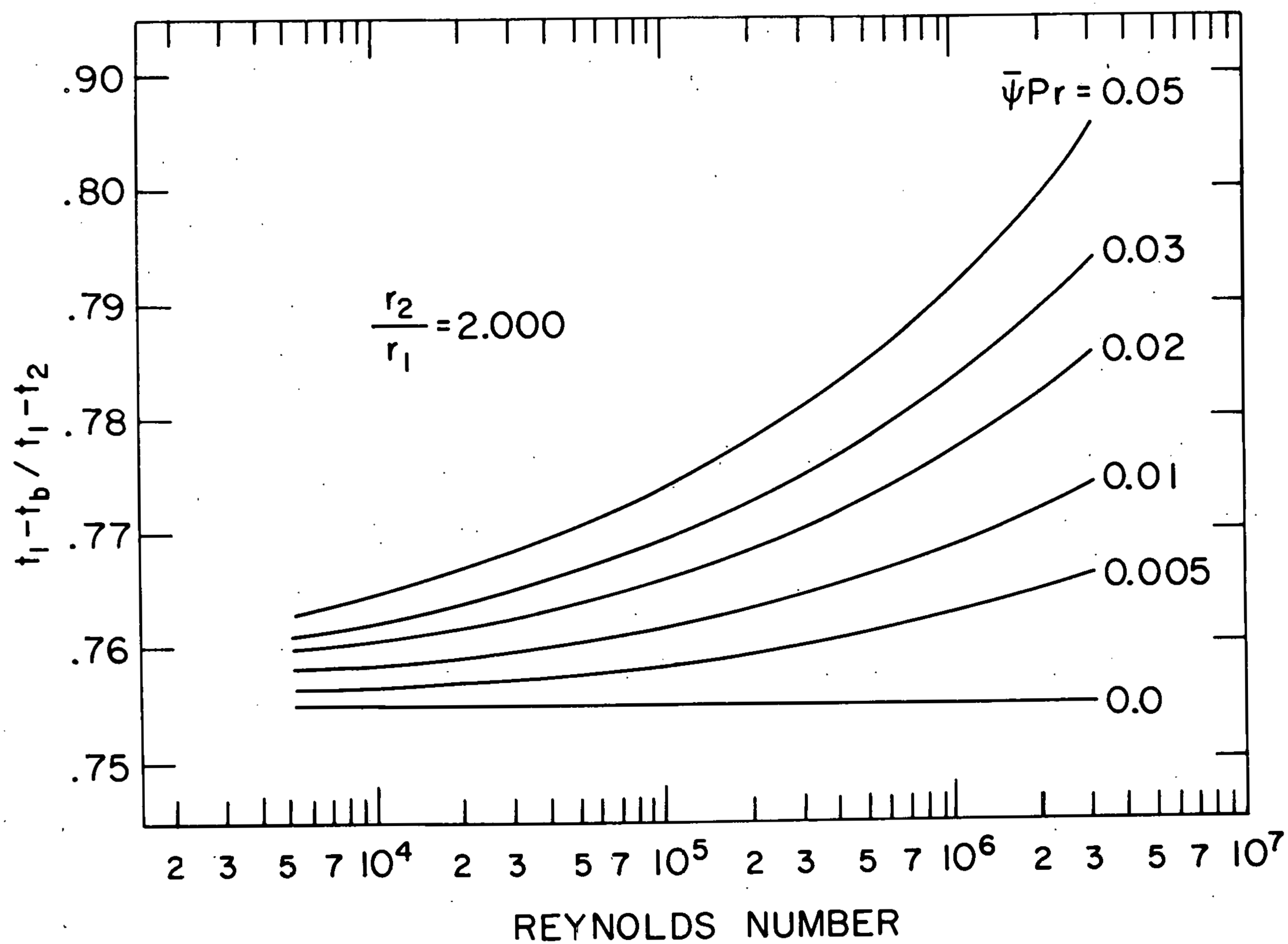


Nuclear Science & Engineering - Dwyer - Fig. 3



Nuclear Science & Engineering - Dwyer - Fig. 4

3C



Nuclear Science & Engineering - Dwyer - Fig. 5

Nuclear Science & Engineering - Dwyer - Fig. 6