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Informal Report

EFFECT OF A NON-UNIFORM INCOHERENT ν-SHIFT ON THE STABILITY CRITERION FOR TRANSVERSE COLLECTIVE OSCILLATIONS

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ABSTRACT

A review is given of the two sufficient stability criteria against growing transverse dipole oscillations self-induced in a particle beam. The effect of a non-uniform incoherent ν-shift is then considered. The first stability criterion places a lower limit on the width, δ, of the ν-distribution, and can be written, $\delta > \delta_o$, where δ_o depends on the chamber and beam geometry, as well as on the chamber conductivity. It is shown that a skewing of the density distribution, by a non-uniform ν-shift of maximum value s_ν at the beam center, changes this condition to: $\delta > \max(\delta_o, s_\nu)$. The second stability criterion can be stated as an upper limit on the ν-density, N, i.e., $N < n_o$, where n_o has a functional dependence qualitatively similar to δ_o, but depends more strongly on chamber conductivity. A skewing of the ν-distribution affects this stability criterion only insofar as it affects the peak value of the density. This depends on the initial distribution of ν-values, unaffected by the ν-shift. If it is peaked and one-humped, the effect will be minimal; while if it is uniform the increase in maximum density could be large. To compensate, an increase in the ν-spread in the beam is required. This procedure will be successful to the extent that it decreases the density maximum. Some comments concerning the CERN experience are made.

1. Introduction

Self-induced beam instabilities have been previously studied.[1,2] Specifically, the self-forces caused by a beam interacting with a resistive wall has been shown to lead to growing transverse oscillations of the dipole moment of the beam.[1] However, if the beam has a sufficiently large spread in ν (accelerator tune), then a threshold is reached, above which growing oscillations cannot be supported by the beam. Thus the study of these processes has led to stability criteria in the form of a minimum beam ν-spread.[1]

Generally, there are two separate sufficient stability conditions. We will go into these in more detail in a later section. At this point, some general characteristics can be given. The first condition can be stated as a lower limit on the width of the ν-distribution. If the conductivity of the chamber is sufficiently large, this condition is dependent mainly on geometric factors (i.e., chamber size and beam size). The second stability criterion puts an upper limit on the maximum density of particles per unit ν. To the extent that the peak density is inversely related to the width of the distribution in ν, then the two conditions are similar in form. They do, however, give different limits, the second being dominated by terms involving the wall conductivity. An interesting conclusion that can be drawn from studying these conditions is this: In the case of large conductivity, the first type of criterion generally leads to a much smaller ν-spread required for stability.[1]

Here, we will consider the influence on these stability criteria when there exists a non-uniform incoherent ν-shift in the beam. Neglecting the uniform component of the incoherent ν-shift, we are left with a ν-shift which is zero at the ends of the distribution and maximum at the center.

The effect on the ν-distribution is, generally speaking, (1) a skewing, with a shift of the peak by a distance given roughly by the maximum ν-shift, i.e., at the beam center; and (2) a change in the maximum ν-density. We will show that the stability conditions are changed as follows: Let δ represent the distribution width and N the maximum value of the ν-density. Then, in the case of a symmetric, one-hump distribution, the first type of stability criterion is

$$\delta > \delta_o \quad ; \qquad (1.1)$$

while, in any case, the second type is

$$N < n_o \quad . \qquad (1.2)$$

The quantities δ_o and n_o are functions of the chamber and beam dimensions as well as the conductivity of the chamber wall. Note that for a one-hump symmetric distribution, $N \sim \frac{1}{\delta}$ and the two stability conditions have the same form. However, for "large" conductivity (1.1) is generally a less stringent requirement on δ, while for "smaller" conductivity, (1.2) becomes the significant criterion. Now, in the presence of an incoherent ν-shift of maximum value s_ν at the beam center, then condition (1.2) remains unchanged in form, while, on the other hand, (1.1) is considerably affected. We find, in fact, that for a triangular distribution, (1.1) becomes, roughly,

$$\delta > \max\,(\delta_o, s_\nu) \quad . \qquad (1.3)$$

In other words, in the event δ_o is small, and s_ν is comparatively large, the incoherent ν-shift will negate the advantage achieved from the criterion (1.1).

The effect on the stability criteria depends, of course, on the unshifted distribution. For a single-humped distribution, the peak density may not be significantly affected. Thus, to satisfy (1.2), no increase in the beam width is required when a non-uniform incoherent ν-shift is present. However, if the peak density is appreciably affected, then an increase in the ν-spread is required to compensate. This would be the case with a uniform distribution, where a non-uniform ν-shift would produce a peaking directly proportional to the skewing. Such appears to be the situation in the CERN intersecting storage accelerators, the ISR, where their technique of energy stacking produces a beam roughly uniform in ν-density.[2] One would therefore expect a sensitivity to such non-uniform ν-shifts. The skewing of the distribution, on the other hand, is not as sensitive to its initial shape. To get a feeling for the effect, a skewed triangle should give a good estimate. The result is simply that condition (1.1) should be replaced by (1.3).

2. The Dispersion Relation and the Stability Criteria

To obtain the dispersion relation for the complex frequency of collective oscillations, we start with the equation for the dipole moment of a particle beam. Let us consider a beam of constant linear density, λ, and a normalized distribution in ν given by $N(\nu)$. If $P_\nu(\theta,t)$ is the local dipole moment per unit ν, with the total dipole moment of the beam given by

$$P(\theta,t) = \int P_\nu(\theta,t)\, d\nu \,, \tag{2.1}$$

then we have for $t \geq 0$, the equation,[3]

$$\ddot{P}_\nu + \nu^2\Omega^2 P_\nu = \lambda N(\nu) \left\{ \frac{e}{m} U P + \frac{e}{m} \frac{W}{\sqrt{\pi}} \int_0^t \frac{P(\theta,t')}{\sqrt{t-t'}}\, dt' \right\} , \tag{2.2}$$

where Ω is the angular frequency of the particles in the beam,

λ is the particle density per unit length,

m is the relativistic mass of the proton, and the time derivative is the hydrodynamic derivative given by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \theta} \quad . \tag{2.3}$$

The quantities U and W, for circular geometry, are

$$U = -\frac{2}{\gamma^2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \quad , \tag{2.4}$$

$$W = \frac{4c\beta^2}{b^3} \quad \frac{1}{\sqrt{4\pi\sigma}} \quad , \tag{2.5}$$

with, a = beam radius,

b = chamber radius,

γ = energy in units of proton rest mass,

β = particle velocity in units of c,

and σ = chamber conductivity in units 1/time.

Expand P and P_ν in a Fourier series in θ and a Fourier integral in t, e.g.

$$P_\nu(\theta,t) = \sum_k \int_C d\omega \; P_\nu(\omega,k) e^{i(k\theta-\omega t)} \quad . \tag{2.6}$$

The contour in the ω-plane is chosen to be above the poles of $P_\nu(\theta,t)$ and $P(\theta,t)$. Thus, the inverse,

$$P_\nu(\omega,k) = \frac{1}{(2\pi)^2} \int_0^\infty dt \int_0^{2\pi} d\theta \; P_\nu(\theta,t) e^{-i(k\theta-\omega t)} \quad , \tag{2.7}$$

is valid in the upper half ω-plane above the contour C.

Although the details are dependent on the initial conditions[4], for reasonable initial (t = 0) disturbances these will not affect the form of the large t behavior of the solution. In fact, the solution of Eq. (2.2) at large t will in general be of the form

$$P\,(\theta,t) \sim e^{i(k\theta-\omega_c t)} \tag{2.8a}$$

for some complex frequency, ω_c, and mode, k. If a solution exists with

$Im(\omega_c) > 0$,

then we have growing collective oscillations of the beam.

Properly using Eqs. (2.1, 2.6 and 2.7), the solution of Eq. (2.2) can be written[4]

$$P(\theta,t) = \sum_k \int_C d\omega \, e^{i(k\theta-\omega t)} \frac{f(\omega,k)}{H(\omega,k)} \tag{2.8b}$$

where $f(\omega,k)$ depends on the initial conditions and $H(\omega,k)$ depends only on the characteristics of the beam and its environment:

$$H(\omega,k) = 1 - \frac{\lambda e}{m\Omega^2} \int \frac{N(\nu)}{\nu^2 - (k-\frac{\omega}{\Omega})^2} \, d\nu \left[U + W \left(\frac{i}{\omega}\right)^{\frac{1}{2}}\right] \; . \tag{2.8c}$$

The function $\left(\frac{i}{\omega}\right)^{\frac{1}{2}}$ is defined to have a positive real part.[3]

Thus, since we are assuming that $f(\omega,k)$ has no poles in the complex ω plane, the time dependence of $P(\theta,t)$ will be determined by the zeros of $H(\omega,k)$.[4] Closing the contour in the lower half ω plane, we obtain a solution of the form (2.8a), where ω_c is a solution of $H(\omega_c,k) = 0$, or

$$\frac{\lambda e}{m\Omega^2} \int \frac{N(\nu)}{\nu^2-(k - \frac{\omega_c}{\Omega})^2} \, d\nu \left[U + W \left(\frac{i}{\omega_c}\right)^{\frac{1}{2}}\right] = 1 \; . \tag{2.9}$$

Note that the ν integration can be simply performed only if $Im(\omega_c) > 0$. As $Im(\omega_c) \to 0$, a singularity in the ν integration results. However, since we know the direction of the approaching pole, the ν integration contour can be suitably deformed to provide the proper analytic continuation.

Equation (2.9) is the dispersion relation for the complex frequency ω_c (we will henceforth drop the subscript c). It can be written as

$$c_o \, (U' + V' \pm iV') \, (2\nu_o) \int \frac{N(\nu)}{\nu^2-(k - \frac{\omega}{\Omega})^2} \, d\nu = 1 \; , \tag{2.10}$$

where

$$c_o = \frac{IRr_p}{e\ \nu_o \Omega \beta^2 \gamma^3} \quad , \tag{2.11}$$

I is the beam current in amps,

R is the radius of the ring,

r_p is the classical proton radius, = 1.54×10^{-18} m,

ν_o is the center of the ν distribution,

$$U' = -\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \quad , \tag{2.12}$$

$$V' = \frac{c\beta^2\gamma^2}{b^3} \sqrt{\frac{2\epsilon_o \rho_e}{|\omega|}} \quad , \tag{2.13}$$

ϵ_o is the free space dielectric constant, $\epsilon_o = \frac{1}{36\pi} \times 10^{-9} \frac{\text{sec}}{\text{ohm-m}}$, and ρ_e is the resistivity of the vacuum chamber wall, in units of ohm-m. The plus sign is applicable when $\omega > 0$, the minus sign when $\omega < 0$. The dispersion relation can be simplified if we assume $N(\nu)$ has a finite range, say, $N(\nu) \neq 0$ only if

$$\nu_o - d \leq \nu \leq \nu_o + d \quad , \tag{2.14}$$

and if we further assume that for all k,

$$d < \left| /k/ - \nu_o \right| \quad . \tag{2.15}$$

First let us restrict our considerations to the question of the stability limit. That is, let us look for solutions with Im $(\omega) = 0$. Then, if we add to the above assumptions the further assumption that $V' > 0$ (these are valid for a reasonable physical system), we can show that a solution exists only if k and ω have the same sign and that the positive (ω,k) equations are equivalent to the negative (ω,k) equations. Furthermore, only the modes k such that $|k| > \nu_o$ can support growing oscillations. Thus, we arrive at the dispersion relation,

$$c_o \left(U' + V' - iV'\right)\left[P\int \frac{N(\upsilon)}{\upsilon-[k-\frac{\omega}{\Omega}]} d\upsilon + i\pi N\left(k - \frac{\omega}{\Omega}\right)\right] = 1 \quad , \tag{2.16}$$

where P signifies that the principal value integral is to be taken,

$k, \omega > 0$,

$k > \upsilon_o$,

and we have replaced ω by

$\omega \approx \Omega\ (k-\upsilon_o)$

everywhere outside the integral.

This last substitution is a good approximation since any solution must satisfy

$$k-\upsilon_o-d < \frac{\omega}{\Omega} < k-\upsilon_o + d \quad , \tag{2.17}$$

and we have assumed that d is small compared to $k-\upsilon_o$.

The final form for the dispersion relation is easily obtained from (2.16):

$$P\int \frac{N(\upsilon)}{\upsilon-(k-\frac{\omega}{\Omega})} d\upsilon + i\pi N\left(k-\frac{\omega}{\Omega}\right) = \frac{1}{c_o}\ \frac{(U' + V') + iV'}{(U' + V')^2 + V'^2} = W_R + iW_I \ . \tag{2.18}$$

This equation also defines the complex quantity, W. Actually, (2.16) represents a mapping for the upper half W plane. (Recall our assumption, $V' > 0$.) If we look for solutions with $V' < 0$, i.e., in order to map the lower half W plane, then we find a similar dispersion relation. The only difference is that k and ω must differ in sign, thus leading to solutions for modes k such that $|k| < \upsilon_o$.

If we map the left-hand side of expression (2.18) onto the W-plane, we obtain a closed curve. This curve divides the plane into two parts. If W lies within the curve, then a solution to the full dispersion relation with Im $(\omega) > 0$ exists. That is, we have instability. If W lies outside the curve, no solution with Im (ω) > 0 exists. Thus, the condition for stability is that W lie outside the boundary curve[4] given by Eq. (2.18).

If we circumscribe a rectangle around this boundary, we can obtain two types of sufficient, though not necessary, stability criteria. Looking at the imaginary parts in Eq. (2.18), we obtain for stability,

$$W_I > \pi N_{max} \quad . \tag{2.19}$$

From the real parts, we have

$$\text{if} \quad W_R > 0, \qquad W_R > \max\left[P\int \frac{N(\upsilon)}{\upsilon-(k-\frac{\omega}{\Omega})}\, d\upsilon\right] \quad , \tag{2.20}$$

$$\text{or,} \quad \text{if} \quad W_R < 0, \qquad W_R < \min\left[P\int \frac{N(\upsilon)}{\upsilon-(k-\frac{\omega}{\Omega})}\, d\upsilon\right] \quad . \tag{2.21}$$

For reasonable distributions, the principal value integral peaks at both ends of the range. Thus, the last two criteria become

$$\text{if} \quad W_R > 0, \qquad W_R > P\int \frac{N(\upsilon)}{\upsilon-\upsilon_o + d}\, d\upsilon \quad , \tag{2.22}$$

$$\text{or,} \quad \text{if} \quad W_R < 0, \qquad W_R < P\int \frac{N(\upsilon)}{\upsilon-\upsilon_o - d}\, d\upsilon \quad . \tag{2.23}$$

To go further one must specify the nature of the distribution. However, an order of magnitude result is obtained by noting that

$$|P\int| \sim \frac{1}{d}$$

and $N_{max} \sim \frac{1}{d}$.

Thus we have the stability criteria from (2.19),

$$d > \pi/W_I \quad , \tag{2.24}$$

and from (2.22) or (2.23),

$$d > \frac{1}{|W_R|} \quad . \tag{2.25}$$

For the triangular distribution (with range ± d), the corresponding conditions are

$$d > \pi/W_I \quad , \tag{2.26}$$

and $$d > \frac{2 \ln 2}{|W_R|} \quad . \tag{2.27}$$

3. <u>Effect of a Non-Uniform ν-Shift</u>

For a given density distribution, $N(\nu)$, we have derived *two* separate sufficient stability criteria. The first is obtained from (2.19) and is

$$N_{max} < W_I/\pi \quad . \tag{3.1}$$

This condition is unaltered in form when the distribution is changed by a non-uniform incoherent ν shift. However, to the extent that the local density may be increased, the condition (3.1) for a fixed environment (i.e., fixed W_R, W_I) might become violated for a large enough ν shift. A symmetrical shift, which is greatest at the beam center, will have the greatest effect on N_{max} if the beam is uniform in density $\{N(\nu) = \text{constant}\}$, while it will have a declining influence as the distribution becomes more peaked and narrower. Of course, for a fixed number of particles, the peak value is roughly related to the inverse of the standard deviation, σ_o. Thus starting out with a small σ_o means a large N_{max} compared to a uniform distribution which has a large σ_o and correspondingly small N_{max}. The point is this: If one is to take account of the non-uniform ν shift, the beam environment (i.e., the value of W_I) should not be designed with an N_{max} taken from a uniform ν distribution, but rather an artifically smaller standard deviation should be assumed for the distribution.

Let us now consider the second stability criterion as represented by Eq. (2.22) or (2.23). We are thus interested in evaluating the integral,

$$R(d) = P\int \frac{N(\rho)}{\rho - d}\, d\rho \quad , \tag{3.2}$$

where the distribution is now centered around zero, and d, the range, can take on positive or negative values. For definiteness, we will assume $d > 0$, although the general results are applicable in the case $d < 0$ as well. In order to see the effect of a non-uniform ν shift, which clearly depends on both the initial distribution and the manner in which ν shifts along the beam, we must specify both these characteristics. Thus, if $N(\nu)$ is the initial ν distribution and the perturbed ν value is given by

$$\bar{\nu} = \nu + f\,(\nu), \tag{3.3}$$

where $f(\nu)$ might for example be

$$f(\nu) = s_\nu \left[1 - \frac{\nu^2}{d^2}\right] \quad , \tag{3.4}$$

then the perturbed ν distribution is given by

$$\bar{N}\,(\bar{\nu}) = N(\nu)\,\frac{d\nu}{d\bar{\nu}} = \frac{N(\nu)}{1 + f'(\nu)} \quad ; \tag{3.5}$$

with the right hand side treated as a function of $\bar{\nu}$, taken from (3.3). We will here, however, avoid the difficulties involved in the choice of the initial distribution, $N(\nu)$, and the ν shift function $f(\nu)$. This we do by simply choosing a perturbed function (and just labeling it $N(\nu)$). With this function we hope to see the effect of a skewing of the ν distribution. Thus, under the influence of the perturbing ν shift of amount s_ν, we assume that the perturbed ν distribution is triangular, but skewed:

$$N(\rho) = \begin{cases} 0 & , \rho < -d \\ \frac{1}{d}\left[1 + \frac{\rho - s_\nu}{d + s_\nu}\right] & , -d < \rho < s \\ \frac{1}{d}\left[1 - \frac{\rho - s_\nu}{d - s_\nu}\right] & , s < \rho < d \\ 0 & , \rho > d \ . \end{cases} \tag{3.6}$$

We then have that

$$R(d) = \frac{2}{d + s_\nu}\left[- \ln 2 + \ln\left(\frac{d-s_\nu}{d}\right)\right] \ . \tag{3.7}$$

The stability criterion can now be obtained from (2.23):

$$|W_R| > \frac{2 \ln 2}{(d + s_\nu)}\left[1 - \frac{\ln\left(\frac{d-s_\nu}{d}\right)}{\ln 2}\right] . \tag{3.8}$$

In the case $s_\nu = 0$, this condition becomes

$$d > d_o = \frac{2 \ln 2}{|W_R|} \ . \tag{3.9}$$

Now, to see how this condition is affected by the perturbing ν shift, we write Eq. (3.8) as

$$d + s_\nu > d_o\left[1 - \frac{\ln\left(1 - \frac{s_\nu}{d}\right)}{\ln 2}\right] \ . \tag{3.10}$$

Let us normalize this by introducing the ratios

$$y = \frac{d}{d_o}, \quad h = s_\nu/d_o \ . \tag{3.11}$$

Then, we have

$$y + h > 1 - \frac{\ln\left(1 - h/y\right)}{\ln 2} \ . \tag{3.12}$$

For $s_\nu > 0$, we clearly must have

$$d > s_\nu \ . \tag{3.13}$$

That this is also a sufficient condition for large h can be seen by writing (3.12) in the equivalent form

$$\frac{h}{y} < 1 - 2^{1-y-h} \quad . \tag{3.14}$$

For large h we can neglect the second term on the right hand side, leading to

$$y > h, \quad \text{or} \quad d > s_\nu \quad . \tag{3.15}$$

For h < 0, we can write (3.12) in the form,

$$1 + \frac{|h|}{y} > 2^{1-y} + |h| \quad . \tag{3.16}$$

This is identical with the condition

$$y > |h| \quad \text{or} \quad d > |s_\nu| \quad . \tag{3.17}$$

For s_ν small compared to d_o, (3.10) clearly leads to the condition

$$d > d_o \quad . \tag{3.18}$$

We thus have that the second stability condition becomes

$$d > \max\ (d_o, s_\nu) \quad . \tag{3.19}$$

There is an interesting implication here. Suppose a beam environment is designed with large conductivity, such that $V' < U'$. Then the stability criterion depends essentially on the geometry of the beam and chamber and the condition $d > d_o$ is much less stringent than the condition $N_{max} < W_I/\pi$. From the above arguments, we can see that this is valid as as long as $|s_\nu|$, the ν shift at the beam center, is small compared to d_o. One must, however, remember that if $|s_\nu| > d_o$, the stability criterion $d > d_o$ becomes $d > |s_\nu|$. Thus, the value of using a regime $V' < U'$ must be questioned if there is appreciable non-uniform ν shift. In such circumstances a more appropriate procedure might be, for example, to use the regime

$$U' + V' \approx 0 \quad ,$$

which from Eqs. (2.18) and (3.1) leads to the stability criterion

$$N_{max} < \frac{1}{\pi c_o V'} \quad . \tag{3.20}$$

For $N_{max} = \frac{1}{d}$ as in the triangular distribution, this becomes

$$d > \pi c_o V' \quad . \tag{3.21}$$

However, N_{max} can increase if the initial distribution in ν is broad and if the rate of change of incoherent ν is sufficiently large. In these cases, we cannot use the simple connection between N_{max} and the range, d (i.e. $N_{max} \sim \frac{1}{d}$). That is, we must obtain N_{max} from

$$N_{max} = \max \{\text{perturbed } N(\nu)\} \tag{3.22}$$

and use it together with (3.20) directly.

4. Conclusions

We have shown in somewhat general terms how a non-uniform incoherent ν shift could affect the stability criteria for transverse collective oscillations. By assuming that the effect of such a ν shift is a skewing of the ν distribution function, we can derive a modified stability condition. If d_o is the minimum required range due to the beam geometry and environment and if s_ν is the maximum ν shift at the beam center, then we conclude that the actual required ν range depends on the relative size of d_o and s_ν. In fact, the required range is just the larger of these two numbers:

$$d > \max \{d_o, s_\nu\} \text{ for stability.}$$

We have also concluded that stability can be maintained by keeping the maximum density in the ν distribution below a prescribed design value.

Of course, a non-uniform incoherent ν shift might increase the density maximum (as compared to the unperturbed value). This effect will be most pronounced in a beam with constant ν density. To the extent that this occurs, we must either raise the design limit on N_{max} or reduce N_{max} itself by external means, such as increasing the ν spread.

The conclusions arrived at here are in rough agreement with the observations of this effect made at the CERN ISR.[5] There is, however, one interesting fact: The observation that beam loss occurs at one or the other edge of the beam. The explanation given is that the beam becomes locally unstable. That is, from our way of looking at it, as the ν distribution skews and the peak density rises, that part of the beam experiencing this density rise becomes locally unstable, leaving the rest of the beam intact. However, it should be recognized that this interesting observation is actually not inconsistent with a "collective growth of the entire beam" even though at first glance this might not be obvious. Let us consider what happens to the different "ν bites" of the beam. We have that the response of the beam to an initial impulse can be written as in Eq. (2.8b). But given the Fourier components of the average beam response, $P(\omega_c,k)$, we can obtain from Eq. (2.2), the response of a particular ν bite. Thus, we have

$$P_\nu(\omega_c,k) = \frac{N(\nu)}{\nu - (k - \frac{\omega_c}{\Omega})} c_o \; (U' + V' - iV') \; P(\omega_c,k) \quad . \qquad (4.1)$$

Now, if $V'>|U'|$ and $N(\nu)$ is roughly rectangular, as is the case at CERN, then we can show from Eq. (2.18) that

$$\mathrm{Re}\;(\omega_c) \approx \Omega\;[k - (\nu_o - d)] \quad . \qquad (4.2)$$

Thus the particles with large amplitude will appear near the lower edge of the ν distribution. Now, as the density function gets skewed and peaked due to the incoherent ν shift, the solution of the dispersion relation will give $Re(\omega_c)$ related to the position of the peak rather than to the lower edge. Thus if the skewing is toward high values of ν, losses will occur preferentially at the upper edge of the ν distribution ("outer" physical edge of beam), while if the skewing is toward low values of ν, losses will initially appear at the "inner" physical edge of beam.

To recapitulate, we have described the effect of a non-uniform incoherent ν shift in terms of a skewing of the ν density distribution. This can then influence the transverse collective dipole instability. We have shown in fact how the stability criteria become more difficult to achieve. This model is consistent with the observation of these effects at the CERN ISR.

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