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HEAT PIPE THEORY

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SPACE POWER NOTE NO. 82 - April 18, 1966

AUTHORS: J. L. Watts and M. A. Janssen

SUBJECT: Heat Pipe Theory

ABSTRACT

The Grover "heat pipe" is a recent innovation which appears to have promising applications in space power plant designs, particularly for a thermionic system. Due to the complexity of the hydrodynamics involved in heat pipe operation, the theory is semi-empirical; and although a number of heat pipes have been successfully constructed and operated, little has been reported along the line of comparing theory and experiment. This note summarizes the theory of heat pipes as it presently exists, the main objective being the presentation of some working formulae and relationships for use in future system studies.

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I. INTRODUCTION

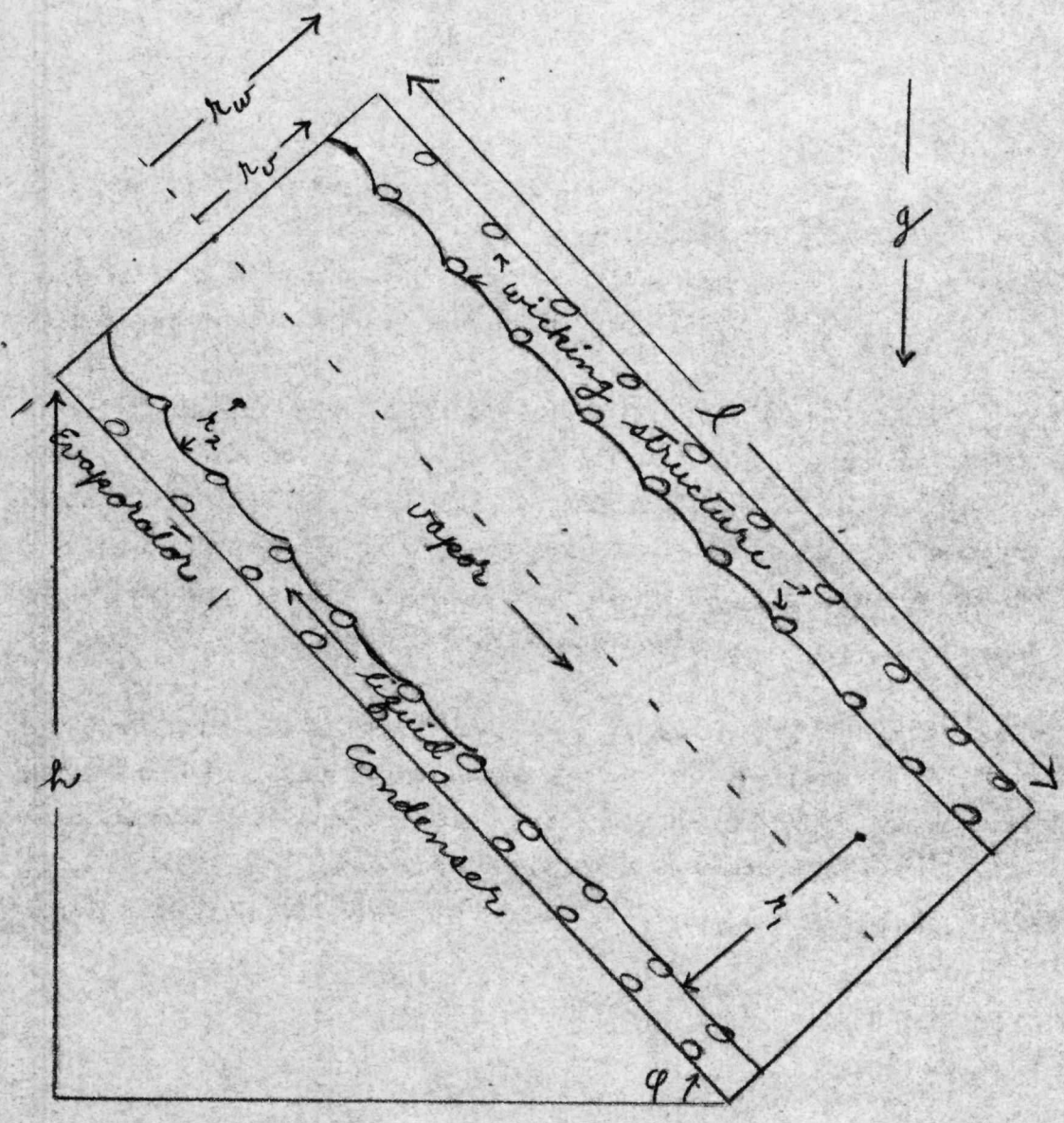
A "heat pipe" is just what its name implies; it is a device for the transfer of thermal energy from one location to another. Evaporation, condensation, and capillary action are employed to transfer heat efficiently and virtually isothermally.

In principle, the heat pipe is simply a form of boiler loop in which the thermal energy is transported down the pipe length by the vapor in the form of the heat of vaporization of the working fluid. Upon condensation, the fluid deposits its heat content in the condenser and is returned to the evaporator by the pumping action of a capillary wick.

A schematic diagram of a heat pipe is found in Figure 1. A simple heat pipe consists of a closed cylinder containing a capillary wicking material which forms an annulus in contact with the inside pipe surface. The dimensions of the heat pipes thus far constructed have outside diameters of approximately 1/2" and lengths of about 15". In all outward appearances, the heat pipe is just a piece of closed tubing.

While the wick usually consists of some type of mesh, it may be nothing more than longitudinal grooves in the inside surface of the pipe. The wick is saturated with some suitable working liquid. The boiling point of the liquid should be in the desired temperature range of operation, and its wetting angle θ must be less than 90° in order for capillary pumping to occur.

Figure 1
Schematic Diagram of a Heat Pipe



The diagram labels are as follows:

g = gravitational acceleration

h = pipe height from horizontal

ϕ = angle pipe makes with horizontal

l = length of pipe

r_1 = radius of meniscus in condenser

r_2 = radius of meniscus in evaporator

r_v = vapor space radius

r_w = outer wick radius

II. HEAT PIPE FORMULAE

The most definitive treatment of heat pipe theory developed thus far was done by T. P. Cotter of Los Alamos[1]. The following summarizes the main results of his paper. For a more nearly complete treatment, the reader is referred to Cotter's paper and to the rather complete bibliography on heat pipes contained at the end of this memo.

The principle involved in heat pipe operation is as simple as the device itself; but due to the unfortunate appearance of complicated hydrodynamic effects in the heat pipe theory, the actual formulation becomes difficult. Analytical solutions are obtainable in only a few limiting cases. Hence heat pipe theory is as yet only approximate and empirical.

The condition necessary for proper heat pipe operation is that the "pumping" pressure due to capillary action be sufficient to overcome the frictional losses encountered as the working fluid circulates throughout the pipe, i.e. the pressure rise due to capillary action must be greater than or equal to the sum of all the pressure losses in the loop:

$$\Delta P_c \geq \Delta P_v + \Delta P_g + \Delta P_l + \Delta P_e \qquad (1)$$

ΔP_c = the pressure rise across the meniscus due to capillary forces; this is the driving force for liquid flow

ΔP_v = pressure drop in the vapor; this is the driving force for vapor flow

ΔP_g = pressure head in liquid due to gravity

ΔP_l = pressure drop in liquid due to viscous drag

ΔP_e = pressure drop at liquid-vapor interface due to evaporation and condensation.

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[1] T. P. Cotter, "Theory of Heat Pipes", LA-3246-MS (1965).

The formulation of the Δp's in terms of the fluid properties and pipe geometry follows.

II.A. Δp_c FORMULATION

The only pressure rise in the system is due to the Δp_c term which is a result of the capillary effect. From Moore[2], the pressure rise across a concave meniscus is given by

$$\Delta p_c = \frac{2\gamma}{r_m}$$

where γ = the surface tension and r_m = the radius of the meniscus. Considering the capillary shown in Figure 2, it is seen that at maximum rise

$$r_m = \frac{r}{\cos \theta}$$

where r = the radius of the capillary pore and θ = the wetting angle. Thus in the evaporator section where $r_2 = r_m$,

$$\Delta p_c = \frac{2\gamma \cos \theta}{r} \qquad (2)$$

Since r_1 is essentially infinite due to condensation and pooling, Δp_c (condenser) = 0.

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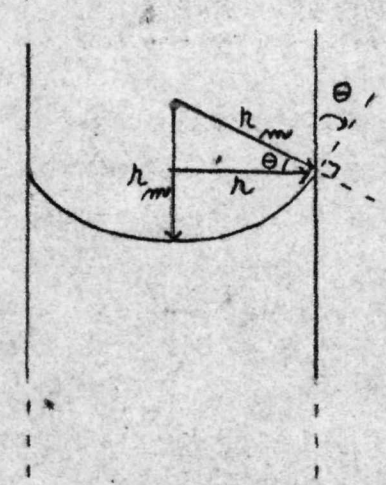
2 W. J. Moore, Physical Chemistry, Prentice Hall, 3rd Ed. p. 729 (1962).

Figure 2
Capillary Pore



II.B. Δp_v FORMULATION

The hydrodynamic equation of motion for both liquid and vapor can be expressed by the Navier-Stokes equation

$$\Delta p = \rho \vec{g} + \eta \nabla \cdot \nabla \vec{v} - \rho \vec{v} \cdot \nabla \vec{v} \qquad (3)$$

where p = pressure in fluid, ρ = density of fluid, η = viscosity, and $\vec{v}$ = the velocity. The implications, however, are vastly different for the liquid and vapor cases.

In the vapor case, the $\rho \vec{g}$ term is negligible; and in the liquid case the authors prefer to consider it as a separate term which is simply the liquid pressure head, Δp_g.

In general for both vapor and liquid, equation (3) gives rise to a set of non-linear differential equations which can be solved analytically only in certain limiting conditions. The type of solution obtained depends upon the value of the Reynolds number

$$R = \frac{\rho V \mathcal{L}}{\eta} \tag{4}$$

where V = the characteristic velocity and $\mathcal{L}$ = the characteristic length.

When the radial Reynolds number is large ($R_r \gg 1$), the inertial term in eq. (3) dominates; and the solution for the vapor pressure gradient becomes[3]

$$\frac{d P_v}{dz} \approx - \frac{S \dot{m}_v}{4 \rho_v r_v^4} \frac{d \dot{m}_v(z)}{dz} \tag{5}$$

where z = axial distance, $\dot{m}_v(z)$ = local axial vapor flow rate, r_v = radius of vapor chamber; and S = 1 for evaporation and $4/\pi^2$ for condensation.

For the vapor

$$R_r = \frac{\rho_v V_v r_v}{\eta_v}$$

In practice, $R_r \gg 1$ is the situation most likely to occur. A calculation of R_r for a typical heat pipe follows.

The total radial vapor flow rate

$$\dot{m}_v = \frac{Q_e}{L}$$

where Q_e = total rate of heat flow into evaporator
and L = heat of vaporization per unit mass. But

$$V_v = \frac{\dot{m}_v}{\rho_v A_v} = \frac{Q_e}{L \rho_v 2 \pi r_v l_e}$$

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[3] B. W. Knight and B. B. McInteer, "Laminar Incompressible Flow in Channels with Porous Walls", LADC-5309.

where A_v = cross-sectional area of flow, and l_e = the evaporator length.

Finally,

$$R_r = \frac{Q_e}{2\pi l_e L \eta_v} \tag{6}$$

For a sodium pipe with the following parametric values

$$Q_e = 7\times10^3 \text{ watts} = 7\times10^{10} \text{ dyne cm/sec}$$

$$l_e = 12 \text{ cm}$$

$$L = 10^3 \text{ cal/gm} = 4.2\times10^{10} \text{ dyne cm/gm}$$

$$\eta_v = 2\times10^{-4} \text{ gm/cm sec}$$

$$R_r = 110 >> 1$$

In the case $R_r << 1$, the solution of the Navier-Stokes equation yields for the axial pressure gradient of the vapor the approximate relationship[4]

$$\frac{dp_v}{dz} \approx -\frac{8\eta_v \dot{m}_v(z)}{\pi \rho_v r_v^4} \tag{7}$$

The axial vapor flow rate will be equal in magnitude to the axial liquid flow rate but opposite in direction. Thus

$$\dot{m}_v(z) = \dot{m}_l(z) = \frac{Q(z)}{L}$$

where $Q(z)$ = the integrated rate of heat transfer to position z. If the heat is added uniformly in the evaporator and removed uniformly in the condenser then

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[4] S. W. Yuan and A. B. Finkelstein, "Laminar Flow with Injection and Suction Through a Porous Wall", ASME 78, p. 719 (1956).

$$Q(z) = L\dot{m}_v(z) = L\dot{m}_l(z) = \begin{cases} \frac{z}{l_e} \, Q_e & 0 \leq z \leq l_e \\ \\ \frac{l-z}{l-l_e} \, Q_e & l_e \leq z \leq l \end{cases} \tag{8}$$

where l = length of pipe, l_e = length of evaporator section, and Q_e = total rate of heat transfer.

Using (8) to perform the integration over z in (5) and (7),

$$\Delta p_v = \begin{cases} \dfrac{(1 - \frac{4}{\pi^2}) \, Q_e^2}{8 \rho_v r_v^4 L^2} & R_r >> 1 \\ \\ \dfrac{4 \eta_v l \, Q_e}{\pi \rho_v r_v^4 L} & R_r << 1 \end{cases} \tag{9}$$

Obviously, other assumptions or conditions may be made regarding how the heat is transferred to the pipe. For example, a simple transport section could be included in which no heat is added or removed. However in that case, it was found that the efficiency of a pipe of given length was reduced when it contained such a transport section.

II.C. Δp_g FORMULATION

The pressure drop due to gravity is negligible in the vapor and is simply the pressure head in the liquid. Hence

$$\Delta p_g = \rho_l g h$$

$$\Delta p_g = \rho_l g l \sin\varphi \tag{10}$$

where ρ_l = density of liquid, g = gravitational acceleration, φ = angle pipe makes with horizontal, and l = pipe length.

II.D. Δp_l FORMULATION

Assuming the radial and tangential components of fluid velocity in a capillary pore to be zero, the Navier-Stokes equation yields for the liquid pressure gradient a form of Darcy's law

$$\frac{dp_l}{dz} = - \frac{b \eta_l \dot{m}_l(z)}{\pi (r_w^2 - r_v^2) \rho_l e r^2} \tag{11}$$

where b = a constant (8 for unobstructed capillaries and 10-20 for wicks), η_l = liquid viscosity, $\dot{m}_l(z)$ = the axially dependent flow rate, r_w = outside wick radius, r_v = vapor radius, ρ_l = liquid density, e = fraction of wick filled with liquid, and r = capillary pore radius.

$$\Delta p_l = \frac{b \eta_l Q_e}{2\pi (r_w^2 - r_v^2) \rho_l e L r^2} \tag{12}$$

II.E. Δp_e FORMULATION

The pressure drop at the liquid-vapor interface due to condensation or evaporation is completely determined by requiring conservation of mass and momentum in the fluid crossing this surface. The mass of liquid reaching the interface, per unit time, per unit surface area, is $\rho_l V_l$, where V_l is the liquid velocity normal to the surface. Similarly, the mass of vapor leaving the interface is $\rho_v V_v$. Thus

$$\rho_l V_l = \rho_v V_v$$

Since momentum must be conserved, the difference in momentum between these two fluid "particles" must equal the pressure drop at the interface, or

$$\rho_v V_v^2 - \rho_l V_l^2 = \Delta p \text{ interface}$$

Thus $$\Delta p_e = \rho_v V_v (V_v - V_l) \approx \rho_v V_v^2$$

But, if Q_v is the heat input per unit time, per unit area of interface, then

$$Q_v = \dot{m}_v L = \rho_v V_v L,$$

and $$V_v^2 = \frac{Q_v^2}{\rho_v^2 L^2}$$

thus $$\Delta p_{\text{interface}} = \frac{Q_v^2}{\rho_v L^2}$$

Comparison with equation (9), where $Q_v^2 << Q_e^2$, or substitution of some likely values shows this Δp term to be much smaller in magnitude than the remainder of pressure losses around the system. Thus, for all practical purposes, the net contribution of evaporation or condensation is negligible, or

$$\Delta p_e \approx 0 \qquad (13)$$

Substituting equations (2), (9), (10), (12), and (13) for the Δp terms into equation (1) gives the final equation relating all the heat pipe parameters.

$$\frac{2\gamma\cos\theta}{r} \geq \begin{cases} \dfrac{\left(1 - \dfrac{4}{\pi^2}\right) Q_e^2}{8 \rho_v r_v^4 L^2} & R_r >> 1 \\ \dfrac{4 \eta_v l Q_e}{\pi \rho_v r_v^4 L} & R_r << 1 \end{cases} + \rho_l g l \sin\varphi$$

$$+ \frac{b \eta_l l Q_e}{2\pi (r_w^2 - r_v^2) \rho_l e L r^2} . \qquad (14)$$

III. OPTIMIZATION

For most cases of interest, $R_r >> 1$; and the heat pipe will be operating in free space ($g = 0$). Equation (14) may then be written as follows:

$$A Q_e^2 + B \frac{Q_e}{r^2} = \frac{C}{r} \qquad (15)$$

where the equality sign applies at optimum performance and where

$$A = \frac{1 - \frac{4}{\pi^2}}{8 \rho_v r_v^4 L^2}, \quad B = \frac{b \eta_1 1 Q_e}{2\pi(r_w^2 - r_v^2) \rho_1 e L r^2}, \text{ and}$$

$$C = 2 \gamma \cos \theta.$$

To optimize Q_e with respect to the capillary pore radius, r, differentiate eq. (15) with respect to r and set $d Q_e/dr$ equal to zero. The condition for optimum is found to be that

$$B \frac{Q_e}{r^2} = A Q_e^2 \qquad (16)$$

or

$$\Delta p_1 = \Delta p_v$$

Thus at optimum the liquid drag term equals the vapor drag term.

Upon substituting (16) into equation (15),

$$r_{opt} = \frac{C}{2A Q_{e_{max}}^2} \qquad (17)$$

Or
$$r_{opt} = \frac{2 B Q_{e_{max}}}{C} \tag{18}$$

In order to obtain $Q_{e_{max}}$, substitute (18) into (15); and the resulting equation for $Q_{e_{max}}$ is

$$Q_{e_{max}} = \left(\frac{C^2}{4 A B}\right)^{\frac{1}{3}} \tag{19}$$

Again substituting (19) into (18) gives r_{opt} in terms of fluid and pipe geometry parameters only.

$$r_{opt} = \left(\frac{2 B^2}{A C}\right)^{\frac{1}{3}} \tag{20}$$

Further optimizing $Q_{e_{max}}$ with respect to the vapor radius, r_v, yields the result

$$\frac{r_v}{r_w} = \sqrt{\frac{2}{3}} \tag{21}$$

At this point, all the formulae for a thorough heat pipe study have been developed. The derivation of the formula for the virtually negligible heat pipe temperature drop is found in Appendix I.

IV. RESULTS AND CONCLUSIONS

Experimental verification of the foregoing theory is at present rather limited. Calculations have been made for several heat pipes built thus far, and as yet there has been no disagreement between theory and experiment to the extent that when theory predicted the pipes would fail they did fail and when theory predicted the pipes would work they did work.

LASL is presently conducting several experiments in which heat pipes are being run to their limit in order to determine the exact point of failure which can be readily checked by theory.

Also E. E. Alexander and G. St. Leger-Barter of LRL have constructed several heat pipes and have carried out a number of experiments. Alexander has reported his results and numerical calculations based upon Cotter's theory in SPN No. 79.

V. LIST OF SYMBOLS

$$A = \frac{(1 - \pi^{\frac{4}{2}})}{8 \rho_v r_v^4 L^2}$$

A_v = cross-sectional area of flow

$$B = \frac{b \eta_l l Q_e}{2 \pi (r_w^2 - r_v^2) \rho_l e L r^2}$$

b = a constant (8 for unobstructed capillaries and 10-20 for wicks)

$C = 2 \gamma \cos \theta$

ΔP = liquid vapor pressure drop

Δp_c = pressure rise across meniscus

Δp_e = pressure drop at liquid-vapor interface due to evaporation

Δp_g = liquid pressure head

Δp_l = pressure drop in liquid due to viscous drag

Δp_v = pressure drop in vapor

ΔT_v = heat pipe temperature difference

e = fraction of wick filled with fluid

η = viscosity

η_v = vapor viscosity

φ = angle of heat pipe to horizontal

γ = surface tension

g = gravitational acceleration

h = height of evaporator end of heat pipe from horizontal

L = heat of vaporization per unit mass

$\mathcal{L}$ = characteristic length to be used in Reynold's number calculation

l = length of heat pipe

l_e = length of evaporator section

$\dot{m}_v$ = total radial or axial vapor flow rate

$\dot{m}_v(z)$ = local axial vapor flow rate

$\dot{m}_l(z)$ = local axial liquid flow rate

P = liquid vapor pressure

p = pressure

p_v = pressure in vapor

p_l = pressure in liquid

Q_e = total heat input per unit time

Q_v = total heat flux or heat transferred per unit time per unit area of heat pipe surface

$Q(z)$ = integrated rate of heat transfer to position z

$\mathcal{R}$ = universal gas constant

R = Reynold's number

R_r = radial Reynold's number

r = radius of capillary pore

r_1 = radius of meniscus in condenser

r_2 = radius of meniscus in evaporator

r_m = radius of meniscus

r_{opt} = optimum capillary pore radius

r_v = radius of vapor space

r_w = outer wick radius

ρ = density

ρ_l = density of liquid

ρ_v = density of vapor

S = constant (1 in evaporator, $\frac{4}{\pi^2}$ in condenser)

T = absolute temperature

θ = liquid wetting angle

$\mathcal{V}$ = characteristic velocity to be used in Reynold's number calculation

v = velocity

V_l = liquid velocity

V_v = vapor velocity

z = axial heat pipe coordinate with origin at evaporator end

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APPENDIX I

CALCULATION OF TEMPERATURE DROP

It has been stated that in practice the temperature drop from the evaporator to the condenser section of a heat pipe is negligible (on the order of a degree or less). However, it may be of interest to the reader to see how this temperature drop, ΔT, can be calculated theoretically.

From kinetic gas theory, the local condensation rate is given by

$$\frac{d\dot{m}_l}{dz} = -\frac{d\dot{m}_v}{dz} = \frac{\alpha r_v (p_v - P(T))}{\sqrt{\mathcal{R}T/2\pi M}}$$

where $\alpha \simeq 1$ is the probability of condensation of an impinging molecule, p_v = the pressure in the vapor, and $P(T)$ = the vapor pressure of the liquid at temperature T, $\mathcal{R}$ = the universal gas constant, and M = the molecular weight of the fluid.

Upon substituting in the above equation the boundary conditions at $z = 0$ and $z = 1$ and taking the difference,

$$\Delta P = P(T_v(1)) - P(T_v(0)) = \Delta p_v + \frac{1 Q_e \sqrt{RT/2\pi M}}{l_e (1-l_e) L \alpha r_v}$$

Using the Clausius-Clapeuyron equation, the temperature difference is then

$$\Delta T_v = T_v(1) - T_v(0) \simeq \frac{RT^2 \Delta P}{ML\, p(T)}$$

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