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Testing Triplet Models

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The radiative decays of mesons can serve as tests of quark models. As shown by Okubo /1/ and by Suura and Young /2/ , such tests favor the Han-Nambu model /3/ over the Gell-Mann Zweig model /4/. In particular, the predicted $\pi^\circ \to \gamma\gamma$ rate agrees with experiment in the former case and is about a factor 9 too small in the latter. Recently, Gell-Mann proposed a modification of the original model with three non-integral charged triplets instead of one /5/. This evades the statistics problem in constructing qqq baryon states and fixes up the $\pi^\circ \to \gamma\gamma$ rate prediction of the old model by introducing an extra factor 3 into the amplitude. The purpose of this note is to present several tests which can distinguish these two models, which we shall call HN /3/ and GM /5/ respectively. We shall not be concerned with the question whether the GM model is equivalent to a model with quarks satisfying rank three parastatistics /6/. For us, the model has three GMZ triplets instead of one triplet.

We shall call the triplets S, U and B and write the electromagnetic current as

$$J_\mu^{em} = \bar{q}_S \gamma_\mu Q_S q_S + \bar{q}_U \gamma_\mu Q_U q_U + \bar{q}_B \gamma_\mu Q_B q_B \tag{1}$$

where $Q_S = Q_U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $Q_B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ for the HN model

and $Q_S = Q_U = Q_B = Q \equiv \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}$ for the GM model. We

introduce two sets of 3x3 matrices λ_a, ρ_a (a 0,1,...,8) for the ordinary (p, n, λ) SU_3 space and the charmed SU_3' space (S, U, B). Introducing further a nine component spinor (q_S, q_U, q_B), we write (1) in the form of a direct product

$$\mathcal{J}_\mu^{em} = \bar{q}\gamma_\mu \hat{Q} q$$
$$\hat{Q} = \frac{1}{2}\lambda_3 \otimes I + \frac{1}{2\sqrt{3}}\lambda_8 \otimes I + \beta\frac{1}{\sqrt{3}} I \otimes \rho_8 \tag{2}$$

where $\beta = 1$ for HN and $\beta = 0$ for GM. The difference between the two models is the presence of the SU_3 singlet and SU_3' octet current $\mathcal{J}_\mu' \equiv (1/\sqrt{3})\bar{q}\gamma_\mu I \otimes \rho_8 q$ in the HN model. This charm octet current does not contribute to single photon amplitudes if we assume charm symmetry and that the low lying hadrons are charm singlets /7/. With this assumption both models give the same results for $\pi^0 \to \gamma\gamma$, $\omega(\rho) \to \pi^0\gamma$ and the $\gamma\gamma$ decay of the octet state η_8. They differ in their predictions regarding the two photon vertices of SU_3 singlet states.

From SU_3, the $\eta_8 \to \gamma\gamma$ amplitude is $T_{\eta_8} = T_{\pi^0}/\sqrt{3}$ where T_{π^0} is the $\pi^0 \to \gamma\gamma$ amplitude. The normalization is that of Ref. /2/. For the singlet $\eta_0 \to \gamma\gamma$ charge counting gives an amplitude proportional to $[(2/3)^2 + (1/3)^2 + (1/3)^2] \times 3 = 2$ for GM and $1+1+1+1 = 4$ for HN. There is a factor two difference in the amplitudes. We find

$$T_{\eta_0} = \frac{1}{\sqrt{3}} T_{\pi^0} \begin{pmatrix} 4\sqrt{2} \\ \\ 2\sqrt{2} \end{pmatrix} \begin{array}{l} \text{HN} \\ \\ \text{GM} \end{array} \tag{3}$$

Including the mixing angle $\theta \simeq 11^\circ$ from the quadratic mass formula we get (see Ref. /2//*/)

/*/ Further references can be found here.

$$\Gamma(\eta \to \gamma\gamma) = \begin{pmatrix} 0.76 \text{ keV} \\ 0.39 \text{ keV} \end{pmatrix} \quad ; \quad \Gamma(\eta' \to \gamma\gamma) = \begin{pmatrix} 25.6 \text{ keV} \\ 6 \text{ keV} \end{pmatrix} \qquad (4)$$

The upper (lower) values refer to the HN (GM) model. The η' decay offers a severe test of the models. We now consider SU_3 breaking. In the context of Ref. /2/ this amounts to shifting the λ quark mass. We find that the amplitudes and predicted $\gamma\gamma$ widths are changed to

$$T_{\eta_8} \to \frac{1}{\sqrt{3}} T_{\pi^0} \begin{pmatrix} 1.5 \\ 1.2 \end{pmatrix} \qquad (3')$$

and

$$\Gamma(\eta \to \gamma\gamma) = \begin{pmatrix} 1.1 \text{ keV} \\ 0.5 \text{ keV} \end{pmatrix} \qquad (4')$$

with the qualitative conclusions unchanged.

We can consider the predictions of the quark loop model for the decay of the $\epsilon(750)$ meson into two photons. If the ϵ is taken to be built up out of non-strange quarks, charge counting gives the ratio of the amplitudes in the HN and GM models to be $[1+1+1]/3\cdot(4/9+1/9) = 9/5$. If there is an ϵ' meson which is pure $\lambda\bar{\lambda}$, the corresponding ratio for it is 3 . The width predictions of the models require a detailed discussion which we shall not enter into here.

Predictions for $\gamma\gamma$ vertices can be tested in the two photon process $e^- + e^- \to e^- + e^- +$ meson . In particular, the $\eta' \to \gamma\gamma$ width from the HN model leads to large η' production cross sections at accessible energies /8/.

In order to bolster our claim that two photon amplitudes can be used to test the HN and GM models, and to further exploit the possibilities of the two photon process in

$e^- + e^- \to e^- + e^- +$ hadrons, we turn to the pseudoscalar vertices with <u>both</u> photons far from the mass shell. This can be studied in the above reaction when the electrons are scattered at relatively large angles. This has already been studied by P. Zerwas and one of us /9/. In the limit where k^2 and q^2 (the virtual photon masses) get large and negative one can expect light-cone dominance of the pseudoscalar-γ-γ vertex

$$T_{\mu\nu} = i \int d^4x \, e^{iQ\cdot x} \langle \pi^0, p | T^* J_\mu^{em}(x/2) \, J_\nu^{em}(-x/2) | 0 \rangle \qquad (5)$$

$$Q \equiv (k-q)/2 \qquad p = k + q$$

One can then substitute into (5) the expressions for the time ordered product obtained from (1) or (2) by formal manipulation on the light cone of the free quark fields as was done for the GMZ model by Fritzsch and Gell-Mann /10/. One finds a scaling law of the off-shell vertex which takes the form $T_\pi(k^2,q^2) \to m_\pi \hat{f}_\pi(\xi) Q^{-2}$ where $\xi = Q\cdot p/Q^2$; similar results hold for the other pseudoscalars. By carrying out the integral in (5) over the bilocal operator which results from the formal manipulations together with its singular (at $x^2 = 0$) coefficient, one can write $\hat{f}_\pi(\xi)$ as

$$\hat{f}_\pi(\xi) = \int_{-1}^{1} du \, \frac{\varphi(u)}{1+\xi u} \qquad -1 < \xi < 1 \qquad (6)$$

for $|Q^2| \to \infty$, $|Q\cdot p| \to \infty$ and ξ fixed. We now note that the equal time limit of the bilocal algebra gives the $U_6 \times U_6$ commutator

$$[J_\mu^{em}(x), J_\nu^{em}(0)]_{ETC} = 2i \, \epsilon_{\mu\nu 0}{}^{\sigma} \, J_\sigma^{5,\hat{Q}^2}(0) \, \delta^3(x) \qquad (7)$$

where $J_\mu^{5,\hat{Q}^2} = \bar{q}\gamma_\mu\gamma_5\hat{Q}^2 q$ in the free quark model, with $\hat{Q}$ given by (2). This commutator--and, indeed, the whole bilocal algebra--is independent of the difference between the GM and GMZ models, provided the physical weak current is identified with $\bar{q}(\lambda_a/2)\otimes\mathbb{1}\,\gamma_\mu\gamma_5 q$; in the HN model this would not correspond to the choice which gives finite radiative corrections to the weak decays of the composite hadrons /11/. With this identification of the current, we find a sum rule for the quantity $\int_{-1}^{+1} du\, \varphi(u)$ in terms of (7) which leads at once to the relations /9/

$$\hat{f}_{\pi^0}(0) = \frac{2}{3} f_\pi \quad ; \quad \hat{f}_{\eta_8}(0) = \frac{2}{3}\frac{1}{\sqrt{3}} f_\pi \ ; \quad \hat{f}_{\eta_0}(0) = \frac{4}{3}\sqrt{\frac{2}{3}}(1+\beta)\, f_\pi \qquad (8)$$

We have used SU_3 to set the various axial decay constants equal. The form of this result is already familiar: the values of the 3 and 8 components of the octet are the same in the HN and GM models and the singlet amplitude is twice as great in the HN as in the GM model. Note that no extra factor 3 appears in going from the GMZ to GM models. From (8), the mixing angle, and the cross section of Ref. /9/, one can find production cross sections for the reaction $e^- + e^- \to e^- + e^- + (\pi^0, \eta, \eta')$. Provided the scaling behavior sets in for $q^2, k^2 \lesssim -0.5\ \mathrm{GeV}^2$, the cross sections should be big enough to measure /9/. In particular, the η' cross section should be comparatively larger than for the η .

We can find one further test, also experimentally realizable in the two-photon part of the reaction $e^- + e^- \to e^- + e^- +$hadrons, where the electrons are scattered at non-zero angles. We sum over <u>all</u> hadrons and take the hadronic discontinuity of the forward $\gamma\gamma$ amplitude for **far** off-shell phtons, $W_{\mu\nu\mu'\nu'}$, given by ($\tilde{T}^*$ is the anti-time-ordered-product):

$$W_{\mu\nu\mu'\nu'} = \int d^4x\, d^4y\, d^4z\, e^{-i[Qx-Qy+Pz]} \langle 0| \left[T^* J_\mu^{em}(x/2)\, J_\nu^{em}(-x/2),\ \tilde{T}^* J_{\mu'}^{em}(y/2+z)\, J_{\nu'}^{em}(-y/2+z) \right] |0\rangle \qquad (9)$$

and we take the limit $|Q^2| \to \infty$ followed by $s = p^2 \to \infty$. In this limit we can expect light-cone dominance of (9) where x, y, and z are lightlike and collinear. Arguments of this sort can be found in the literature and we shall not repeat them /11/. If (5) can be reduced to a function singular at $x^2 = 0$ times a matrix element of a finite bilocal operator, then it is natural that (9) can be similarly written in terms of a commutator of bilocal operators times singular functions. The bilocal operators resulting from the commutation can, in the free quark model, be written as certain finite normal ordered products plus disconnected c-number functions singular on the light cone. The result of the algebra is the same as the Wick reduction of the amplitude for the box graph in massless quantum electrodynamics, apart from the problem of how to count the quark charges correctly. This leads to a characterization of (9) in terms of the absorptive part of the box graph , analogous to the usual expression for the absorptive part of the photon propagator with a massless quark loop. This latter gives the familiar prediction /13/ for σ ($e^+ e^- \to$ hadrons):

$$\lim_{s\to\infty} \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = R = \begin{cases} 2/3 & \text{GMZ} \\ 2 & \text{GM} \\ 2 & \text{HN} \end{cases} \qquad (10)$$

The ratio R is just $\sum_i e_i^2 = 2/3$ in the GMZ model and three times this in the GM model. As we have mentioned, the single photon amplitude is the same in the HN and GM models provided one stays below the threshold for production of charmed states. We shall understand the limit to mean $s_t \gg s \gg m_\pi^2$ in this case, where s_t is the threshold just mentioned. The corresponding result for (9) is very different from (10). For the two-photon part of the electron-electron reaction we have

$$\lim_{s\to\infty} \lim_{|Q^2|\to\infty} \frac{\sigma_{2\gamma}(e+e \to e+e+\text{hadrons})}{\sigma_{2\gamma}(e+e \to e+e+\mu^+\mu^-)} = T = \begin{cases} 2/9 & \text{GMZ} \\ 2/3 & \text{GM} \\ 2 & \text{HN} \end{cases} \tag{11}$$

where we have to take the previously mentioned double limit and s is the hadron $(\text{mass})^2$ in both (10) and (11). Here we have $\sum_i e_i^4 = 2/9$ for the GMZ model, three times this for the GM model and $3\,\mathrm{tr}\left[\left(\tfrac{2}{9} + Q^2\right)^2\right]$ for the HN model. In this last case we have to insure that the produced hadron state is a charm singlet. If this is not done and we assume that all the states of the model are produced, then we have $T = R = 4$. If it turns out experimentally that the prediction (10) works, then (11) is a further test to discriminate the HN and GM models.

We close with some comments on our results and on our assumption that the hadrons are SU_3' singlets in the HN model. Our tests fall into two classes: those with on-shell and off-shell photons. In the former case the pseudoscalar vertices provide decisive tests if one accepts the idea that these vertices are described by quark loops (which gives the same result for $\pi^0 \to \gamma\gamma$ as the Adler-Schwinger anomaly).

The latter test with off-shell photons appears independent of this assumption. There is a further test in the ratio T which depends in an essential way on the correctness of predictions for the disconnected parts of the bilocal algebra. Neither former test depends on this. It is amusing to note that in the on-shell vertices for the GM model we found an extra factor 3 /5/, but that no such extra factor appears in the off-shell vertices determined by the $U_6 \times U_6$ limit of the bilocal algebra. This point can also be tested. The difference between the on-shell and off-shell vertices may mean that there is an appreciable vector meson pole contribution to these vertices in the GM and HN models and an appreciable fall of the vertices between $k^2 = q^2 = 0$ and the scaling limit behavior./14/

It is tempting in the HN model to assume that the current ∂'_μ is dominated by an ω' meson which is an SU_3 singlet and SU_3' octet. This meson might then be responsible for binding quarks in hadrons. It might also lead to a strong mixing of the eighth component of a charm octet into the otherwise charm singlet hadron states, in analogy to $\omega-\phi$ mixing. If this were so, all our predictions would be strongly affected. If δ is the mixing angle, then T_{π^0} becomes

$$T_{\pi^0} = \frac{\alpha}{\pi} \frac{m_\pi}{f_\pi} \left(\cos^2\delta + 2\sqrt{2} \cos\delta \sin\delta \right) \tag{12}$$

which multiplies our old T_{π^0} by a factor ~ 1.5 if δ is even 10°; it does not require much mixing to destroy the agreement of the model with $\Gamma(\pi^0 \to \gamma\gamma)$. One can also work out limits on decays like $\omega \to \pi^0\gamma$ and on the vector meson-photon couplings in the presence of such a mixing. The effects are large for quite small mixing angles, and the known couplings or known bounds then require that any mixing be very small. This supports our assumption that the hadrons are <u>exact</u> charm singlet states, and places constraints on any theory which introduces such charmed mesons or charm symmetry breaking interactions.

We wish to point out in closing that nothing in the quark loop model requires the quarks to be physical particles which can be removed from the hadrons /15/.

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