ANL/PHY/CP--92357

CONF-970140--1

A CONTINUUM ORDER PARAMETER FOR DECONFINEMENT

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Abstract

Dyson-Schwinger equations are presented as a nonperturbative tool for the study and modelling of QCD at finite-T. An order parameter for deconfinement, applicable for both light and heavy quarks, is introduced. In a simple Dyson-Schwinger equation model of two-flavour QCD, coincident, 2nd-order chiral symmetry restoration and deconfinement transitions occur at $T \simeq 150\,\text{MeV}$, with the same critical exponent, $\beta \simeq 0.33$.

1. Introduction. The Dyson-Schwinger equations [DSEs] provide a nonperturbative, continuum approach to solving a quantum field theory; familiar examples are: the gap equation in superconductivity; the Bethe-Salpeter equation, which describes relativistic 2-body bound states, such as mesons composed of light quarks; and the covariant Fadde'ev equation, which describes relativistic 3-body bound states, such as baryons. The DSEs are a system of coupled integral equations, whose solutions are the Schwinger functions (Euclidean propagators), and a weak coupling expansion of the DSEs reproduces all of the diagrams of perturbation theory. Therefore, in any modelling of QCD in this approach, one has a tight constraint on the behaviour of the solution of the DSEs at large spacelike-q^2.[1] The DSEs thereby provide a means of extrapolating what is known about the QCD Schwinger functions at large-q^2 into the small-q^2 (infrared) regime.

The nonperturbative nature of the DSEs entails that they provide a natural framework for the study of confinement, dynamical chiral symmetry breaking [DCSB] and observable effects of bound state substructure. In recent years there have been many successful applications of the framework to the calculation of exclusive processes at zero temperature. The approach is distinguished by the feature that it unifies the treatment of both hard and soft physics; i.e., once a model for the infrared behaviour of the connected gluon 2-point function (gluon propagator) is chosen, one can calculate observables on the entire range of accessible energies and momentum transfers, as illustrated in Refs. [2].

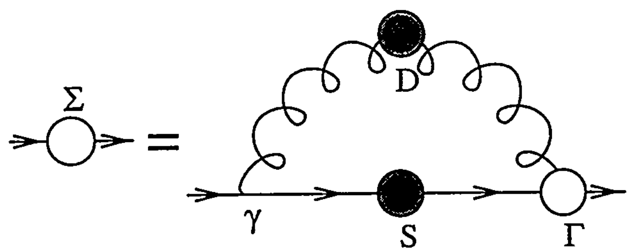

Figure 1: Dyson-Schwinger equation for the quark self energy (QCD gap equation): D is the dressed gluon propagator; Γ is the dressed quark-gluon vertex; the quark propagator $S(p) = 1/[i\gamma\cdot p + \Sigma(p)]$, $\Sigma(p) = i\gamma\cdot p[A(p^2)-1] + B(p^2)$, is obtained as the solution of this nonlinear integral equation.

The phenomenological success of the approach is founded on the important qualitative observation that the gluon vacuum polarisation diagram, tied to the existence of the 3-gluon vertex, generates a significant enhancement of the gluon propagator for $q^2 < 1\,\mathrm{GeV}^2$ with an integrable singularity at $q^2 = 0$.[3] Without fine-tuning, this ensures quark confinement and DCSB, because the gluon propagator is the primary element of the kernel in the DSE for the quark self energy, represented diagrammatically in Fig. 1.

2. Dynamical Chiral Symmetry Breaking. The quark condensate is defined via: $\langle\bar{q}q\rangle_\mu \equiv -\int_\mu^\Lambda \frac{d^4p}{(2\pi)^4}\mathrm{tr}\,[S(p)]$. One aspect of DCSB is the statement that, when the current-quark mass is zero, one nevertheless has $\langle\bar{q}q\rangle_\mu \not\equiv 0$. In terms of the dressed quark mass function, $M(p^2) = B(p^2)/A(p^2)$, this is equivalent to the statement that, when the current-quark mass is zero, the quark DSE in Fig. 1 yields $M(p^2) \not\equiv 0$, Fig. 2. DCSB is more than simply a nonzero quark condensate, however. It is also a mass-enhancement mechanism with observable consequences in QCD. One means of quantifying this is the ratio $M_f^E/m_f(\mu)$, where $m_f(\mu)$ is the current-quark mass and M_f^E, the Euclidean constituent quark mass, is the solution of $p^2 = M_f(p^2)$.

flavour	u/d	s	c	b	t
$\frac{M_f^E}{m_f(\mu)}$	400	20	5	2.5	$\to 1$

(1)

Eq. (1) indicates that the dynamical enhancement of the mass is extremely important for the light quarks and, although it diminishes with increasing current-quark mass, it remains significant even for the b-quark. The magnitude of $\langle\bar{q}q\rangle_\mu$ and this ratio are sensitive to details of the gluon propagator.

3. Quark Dyson-Schwinger Equation. The Matsubara formalism is the natural framework for nonperturbative studies at finite-T. In this case the

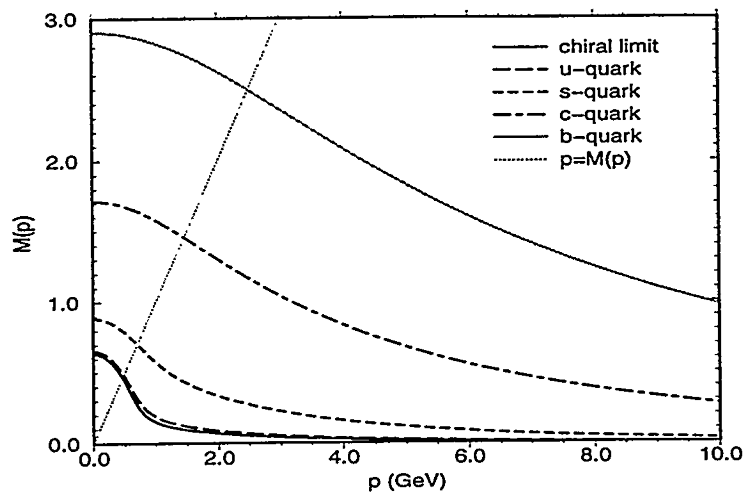

Figure 2: $M_f(p^2)$, f labels the quark flavour, obtained as the solution of the quark DSE using the one parameter gluon propagator of Ref. [4], which drives DCSB since $M(p^2) \not\equiv 0$ in the chiral limit.

renormalised dressed quark propagator is specified by [7]

$$S^{-1}(p,\omega_n) = i\vec{\gamma}\cdot\vec{p}A(p,\omega_n) + i\gamma_4\,\omega_n C(p,\omega_n) + B(p,\omega_n) \tag{2}$$

$$= Z_2^A i\vec{\gamma}\cdot\vec{p} + Z_2(i\gamma_4\omega_n + m_{\rm bm}) + \Sigma'(p,\omega_n) \tag{3}$$

where: $\omega_n = (2n+1)\pi T$; $m_{\rm bm}$ is the quark bare mass; Z_2^A and Z_2 are wave function renormalisation constants; and the regularised self energy is

$$\Sigma'(p,\omega_n) = i\vec{\gamma}\cdot\vec{p}\,\Sigma'_A(p,\omega_n) + i\gamma_4\omega_n\Sigma'_C(p,\omega_n) + \Sigma'_B(p,\omega_n)\,. \tag{4}$$

The quark DSE is a system of three coupled nonlinear integral equations:

$$\Sigma'_{\mathcal{F}}(p,\omega_k) = \int_{l,q}^{\bar\Lambda} \frac{4}{3}\, g^2\, D_{\mu\nu}(p-q,\omega_k-\omega_l)\frac{1}{4}{\rm tr}\left[\mathcal{P}_{\mathcal{F}}\gamma_\mu S(q,\omega_l)\Gamma_\nu(q,\omega_l;p,\omega_k)\right] \tag{5}$$

where $\mathcal{F} = A,B,C$; $\mathcal{P}_A \equiv -(Z_1^A/\vec{p}^2)i\vec{\gamma}\cdot\vec{p}$, $\mathcal{P}_B \equiv Z_1$, $\mathcal{P}_C \equiv -(Z_1/\omega_k)i\gamma_4$; Z_1 and Z_1^A are vertex renormalisation constants; and $\int_{l,q}^{\bar\Lambda} \equiv T\sum_{l=-\infty}^{\infty}\int^{\bar\Lambda}\frac{d^3q}{(2\pi)^3}$. The renormalisation conditions are $S^{-1}(p,\omega_0)|_{p^2+\omega_0^2=\mu^2} = i\vec{\gamma}\cdot\vec{p} + i\gamma_4\,\omega_0 + m_R$. Given $D_{\mu\nu}$, the gluon propagator, and Γ_μ, the quark-gluon vertex, it is straightforward to solve these equations numerically.

4. Confinement. The question of confinement can be addressed by studying the analytic properties of quark and gluon propagators. The absence of a

Lehmann (or spectral) representation for these 2-point functions is a sufficient condition for confinement since it ensures the absence of quark and gluon production thresholds in colour-singlet → singlet $\mathcal{S}$-matrix amplitudes. In perturbation theory it is impossible for interactions to eliminate the Lehmann representation for a 2-point function, however, as elucidated in Refs. [5, 6], the nonlinearity of the nonperturbative fermion DSE makes this possible. Therefore the analytic structure of the propagators in QCD cannot be assumed but must be calculated. The possible nonexistence of a Lehmann representation complicates, and may even preclude, a real-time formulation of the finite-T theory.

The presence or absence of a Lehmann representation can be studied using

$$\Delta_{B_0}(x) \equiv T \sum_{n=-\infty}^{\infty} \frac{1}{4\pi x}\frac{2}{\pi}\int_0^\infty dp\, p\, \sin(px)\, \sigma_{B_0}(p,\omega_n) \equiv \frac{T}{2\pi x}\sum_{n=0}^{\infty} \Delta_{B_0}^n(x)\,. \quad (6)$$

For a free fermion $\sigma_B(p,\omega_n) = M/(\omega_n^2+p^2+M^2)$ and $\Delta^n_{B_0}(x) = M\mathrm{e}^{-x\sqrt{\omega_n^2+M^2}}$, illustrating that the $n=0$ term dominates the sum in Eq. (6). In this case the mass function $M(x,T) \equiv -\frac{d}{dx}\left(\ln|\Delta^0_{B_0}(x)|\right) = \sqrt{\pi^2T^2+M^2}$ and one observes: 1) $M(x,T)$ isolates the poles in the propagator; and 2) finite-T effects only become important for $T \sim \frac{M}{\pi}$, where, in an interacting theory, M is most naturally identified with M_f^E. From Fig. 2 one therefore expects that light quarks only feel the effects of temperature for $T \sim 150\,\mathrm{MeV}$.

An alternative example is provided by $D(p,\Omega_n) = \frac{p^2+\Omega_n^2+m^2}{(p^2+\Omega_n^2+m^2)^2+b^4}$, which has complex conjugate poles shifted from the real-p^2 axis by a distance b^2 and hence no Lehmann representation. In this case $\Delta^0_D(x) = \mathrm{e}^{-mx}\cos(bx)$, and one notes that complex conjugate poles are signalled by zeros in $\Delta^0_D(x)$ or poles in $M(x,T)$; i.e., these features signal confinement. This observation has been employed successfully in Ref. [6].

5. Two-flavour DSE Model of QCD. A minimal, finite-T extension of the one-parameter DSE model of QCD described in Ref. [4], is introduced in Ref. [7]. It is specified by the finite-T gluon propagator

$$g^2 D_{\mu\nu}(p,\Omega_n) = P^L_{\mu\nu}(p,\Omega_n)\Delta_F(p,\Omega_n) + P^T_{\mu\nu}(p)\Delta_G(p,\Omega_n)\,;\ \Omega_n = 2n\pi T \quad (7)$$

$$P^T_{\mu\nu}(p) \equiv \begin{cases} 0;\ \mu \text{ and/or } \nu = 4, \\ \delta_{ij} - \dfrac{p_i p_j}{p^2};\ \mu,\nu = 1,2,3 \end{cases} \quad (8)$$

with $P^T_{\mu\nu}(p) + P^L_{\mu\nu}(p,p_4) = \delta_{\mu\nu} - p_\mu p_\nu/\sum_{\alpha=1}^4 p_\alpha p_\alpha$; $\mu,\nu = 1,\ldots,4$; $\Delta_F(p,\Omega_n) \equiv$

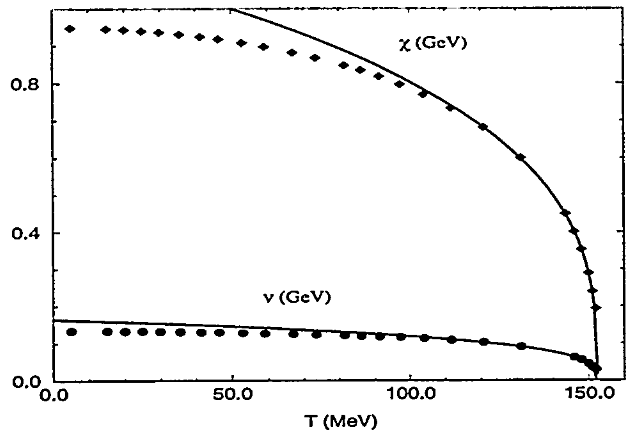

Figure 3: Chiral symmetry, $\chi(T)$, and deconfinement, $\nu(T)$, order parameters.

$\mathcal{D}(p,\Omega_n;m_D)$ and $\Delta_G(p,\Omega_n) \equiv \mathcal{D}(p,\Omega_n;0)$;

$$\mathcal{D}(p,\Omega_n;m) \equiv 4\pi^2 d\left[\frac{2\pi}{T}m_t^2\delta_{0n}\delta^3(p) + \frac{1-\mathrm{e}^{[-(\mathrm{p}^2+\Omega_\mathrm{n}^2+\mathrm{m}^2)/(4\mathrm{m}_\mathrm{t}^2)]}}{\mathrm{p}^2+\Omega_\mathrm{n}^2+\mathrm{m}^2}\right]. \quad (9)$$

$m_D^2 = \bar{c}T^2$, $\bar{c} = 4\pi^2 dc$, $c = (N_c/3 + N_f/6)$, is the T-dependent "Debye-mass" obtained in perturbation theory, which vanishes at $T=0$. There is no simple analogue in $\Delta_G(p,\Omega_n)$.

The single parameter in Eq. (9) is m_t, which characterises the boundary between the infrared and ultraviolet regimes of the model. Requiring that the model provide a good description of π and ρ observables at $T=0$ fixes the value of $m_t = 0.69\,\mathrm{GeV}$. This corresponds to a length-scale of $1/m_t = 0.29\,\mathrm{fm}$.

5.1 *Results.* For $T \sim 0$ the numerical solution of Eq. (5) yields $\Delta_{B_0}^0(x)$ with zeros; i.e., a confined quark. An order parameter for deconfinement is $\nu = 1/r_1^z$, where $x = r_1^z$ is the position of the first zero in $\Delta_{B_0}^0(x)$. Deconfinement is observed if, for some $T = T_c^\nu$, $\nu(T_c^\nu) = 0$; then the poles have coalesced on the real axis and the quark propagator has developed a Lehmann representation. This confinement order parameter is valid for both light and heavy quarks. As discussed in Sec. 2, the quark condensate and the scalar piece of the fermion self energy are equivalent order parameters for DCSB. For simplicity we use the latter; i.e., $\chi \equiv B_{m(\mu)=0}(0,\omega_0)$. The T-dependence of these order parameters is shown in Fig. 3, which illustrates that the model has coincident $[T_c \simeq 150\,\mathrm{MeV}]$, 2nd-order chiral symmetry restoration and deconfinement transitions with the same critical exponent, $\beta_\chi^\nu \simeq 0.33$. Analysing the fits, the difference between this critical exponent and that of the $N = 4$ Heisenberg magnet, $\beta_H \approx 0.37$, is statistically insignificant. However, the transitions cannot be described by a mean-field critical exponent. The transition temperature agrees with that obtained in recent lattice-QCD simulations of 2-flavour QCD.

f_π and m_π are insensitive to temperature until $T \simeq 0.7\,T_c$, which is illustrated by the fact that, even at $T = 0.9\,T_c$, $\Gamma_{\pi\to\mu\nu}$ is only reduced by 20%. However, as one reaches T_c there is a dramatic effect: the pion pole contribution to the quark-antiquark $\mathcal{T}$-matrix is eliminated; i.e., the pion, as a true quark-antiquark bound state, disappears from the spectrum: quark-antiquark correlations above T_c are too weak to bind.

6. Closing Remarks. Although the DSEs have been widely used in the study and modelling of hadronic observables at $T = 0$, their application at finite-T is in its early stages. As a Poincaré invariant, continuum framework that allows the study of both DCSB and confinement, the common domain between DSEs and lattice-QCD simulations is large. DSE studies provide a complement to lattice-QCD, which, once constrained on the common domain, can be used to explore QCD in those regions currently inaccessible to lattice simulations, such as finite density and the effects of temperature on bound state properties.

Acknowledgments. This work was supported by the US Department of Energy, Nuclear Physics Division, under contract number W-31-109-ENG-38.

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