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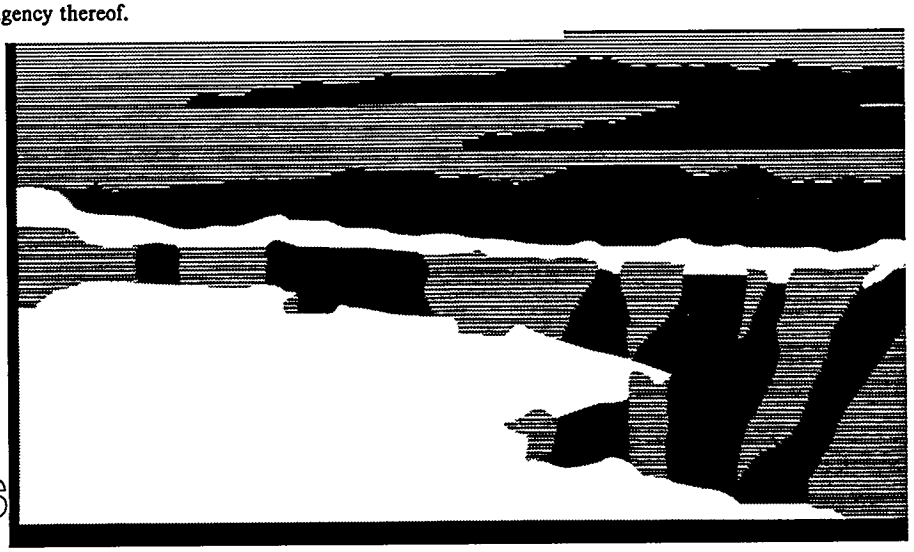

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From Landau's Hydrodynamical Model to Field Theory Models of Multiparticle Production: A Tribute to Peter Carruthers on his 61st Birthday

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(November 21, 1996)

We review the assumptions and domain of applicability of Landau's Hydrodynamical Model. By considering two models of particle production, pair production from strong electric fields and particle production in the linear σ model, we demonstrate that many of Landau's ideas are verified in explicit field theory calculations.

PACS numbers:

I. INTRODUCTION

In the early 1970's Peter Carruthers [1] [2] (at the prodding of one of his graduate students Minh Duong Van) realized that Landau's Hydrodynamical model [3] explained the single particle rapidity distribution $dN/d\eta$ of pions produced in the inclusive reaction P-P $\rightarrow \pi$ + X extremely well.(see Fig. 1) He then asked me to think about the effect of adding thermal fluctuations and with the help of Graham Frye and Edmond Schonberg at Yeshiva University (and also some help from Mitchell Feigenbaum) we found the correct covariant method of adding thermal fluctuations to the hydrodynamic flow which is now called the Cooper-Frye-Schonberg formula [4], [5]. Based on these ideas we were able to fit both the transverse momentum distribution as well as the rapidity distribution extremely well (see Fig. 2).

The hydrodynamical model, which was considered "heretical" when applied to proton-proton collisions,as opposed to Feynman scaling models, was resurrected by Bjorken [7] in 1983 to describe relativistic heavy ion collisions. The use of the hydrodynamical model to study heavy-ion collisions is now a sophisticated mini-industry using 3-dimensional fluid codes and sophisticated equations of state based on lattice QCD. The effects of resonance decays have also been recently included. A recent fit to single particle inclusive spectra for Pb-Pb collisions at 160 AGeV and S-S collisions at 200AGev [8] is shown in Fig. 3.

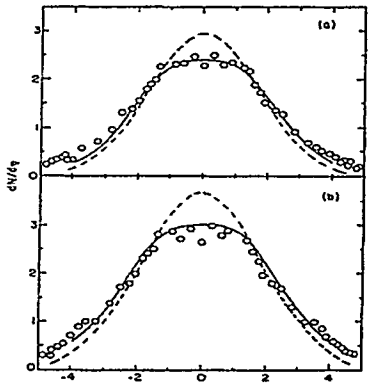

FIG. 1. Comparison of the experimental C.M. rapidity distribution of outgoing pions from Proton Proton collisions at the ISR at (a) p_{ISR} = 15.4 GeV/c and (b) p_{ISR} = 26.7 GeV/c compared with the no adjustable parameter result of the Landau Model from [2]

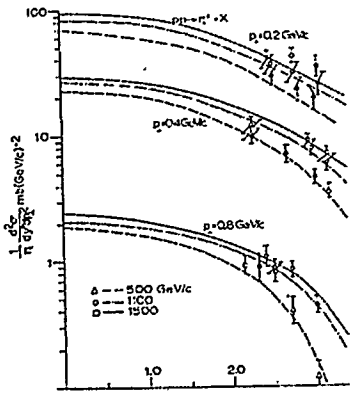

FIG. 2. Comparison of both transverse momentum and rapidity distributions for an ISR experiment with the Cooper-Frye-Schonberg reinterpretation of the Landau Model. from [6]

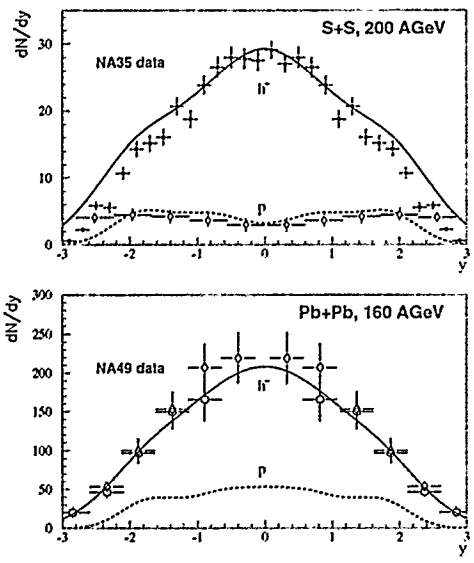

FIG. 3. Fit of recent Pb+Pb and S+S data of the NA49 and NA35 collaborations by Schlei et. al. [8]

After the initial successes of the hydrodynamical model, Carruthers and Zachariasen [9] [10] made the first transport approach to particle production based on the covariant Wigner-transport equation. This further work of Pete's inspired me and my collaborators David Sharp [11] and Mitchell Feigenbaum [12] to study particle production in $\lambda\phi^4$in a mean field approximation using the formalism espoused by Carruthers and Zachariasen. Our efforts in the 70's were hampered by two factors: first we were not sure what the appropriate field theory was, and secondly computers were not large enough or fast enough to perform accurate simulations of time evolution problems. This covariant transport theory approach turned out not to be the most convenient way to study initial value problems and would later be replaced by a non-covariant approach where the time was singled out.

Starting in the mid-80's two new approaches were taken to understand time evolution problems in field theory, both related in spirit to the covariant transport approach of Carruthers and Zachariasen which utilized a mean field approximation to close the coupled equations for the distribution functions. The first method was to assume a Gaussian ansatz in a time dependent variational method in the Schrodinger picture, [13] [14]. The second method was to directly study the time evolution of the Green's functions in the Heisenberg picture in a leading order in large-N approximation [15] [16] [17].

Both these methods, which are mean-field approximations, lead to a well posed initial value problem for the time evolution of a field theory. In the mean-field approximation, we discovered that one has to solve at least 10,000 equations for the Fourier modes of the quantum field theory in order to be in the continuum limit (i.e. for the coupling constant to run according to the continuum renormalization group). Thus, although much of the formalism was worked out by me and my collaborators in the mid and late 80's, [18] it was not until the advent of parallel computation that numerical algorithms were fast enough to make these calculations practical. The first initial simulation attempts were presented in Santa Fe in 1990 at a workshop on intermittency that Pete asked me to help organize [19]. These first simulations took weeks of dedicated machine time. With the advent of the connection machine CM-5 at the ACL at Los Alamos simulations can now be done in a few hours finally making serious studies possible.

In the past few years, we have been able to consider two aspects of Relativistic Heavy Ion Collisions. The first aspect is connected with the production of the quark gluon plasma. The model we used was the popping of quark-antiquark pairs out of the vacuum due to the presence of Strong Chromoelectric fields. The mechanism we used was based on Schwinger's calculation of pair production from strong Electric Fields [20]. Using this model and assuming boost invariant kinematics we were able to show

that many of Landau's assumptions were verified–namely that from the energy flow alone one could determine the particle spectrum, and that the fluid rapidity spectra was the same as the particle rapidity spectra when one is in a scaling regime. Also the hydrodynamical prediction for the dependence of the entropy density as a function of the proper time was verified at long times. Numerical evidence for this will be displayed below.

The second problem we considered was the dynamics of a non-equilibrium chiral phase transition. In this case because of the phase transition, one can obtain single particle spectra which are different from a local equilibrium flow such as that given by Landau's model. We found that when the evolution proceeds through the spinodal regime, where the effective mass becomes negative, low momentum modes grow exponentially for short periods of proper time. This leads to an enhancement of the low momentum spectrum over what would have been found in an equilibrium evolution. While considering this second problem we realized that the Cooper-Frye-Schonberg formula was valid even in a field theory evolution. provided one interprets the field theory interpolating number density as the single particle phase space distribution function of a classical transport theory [17].

It is safe to say that much of my career was stimulated by Pete encouraging me to understand multiparticle production in high energy collisions and by him freely sharing all of his intuition about this subject. It has taken 20 years to show that our original thinking back in 1973 was mostly correct!!

Landau's model [3] of multiparticle production was based on very few assumptions. He first made an assumption about the initial condition of the fluid. Namely. he assumed that after a high energy collision some substantial fraction ($\approx 1/2$) of the kinetic energy in the center of mass frame was dumped into a Lorentz contracted disc with transverse size that of the smallest initial nuclei. He then assumed the flow of energy was describable by the relativistic hydrodynamics of a perfect fluid having an ultrarelativistic equation of state $p = \frac{\varepsilon}{3}$.(This assumption was later modified by later workers as knowledge first from the bag model and then from lattice QCD became available). The motion of the fluid is described by a collective velocity field $u^\mu(x,t)$. Once the collective variables, energy density, pressure and collective velocity are postulated, along with the equation of state. the equations of motion follow from the conservation of energy-momentum. The same assumption of a perfect relativistic fluid is used in Cosmology. One solves the Einstein equations for the curvature assuming an energy momentum tensor that is a perfect fluid. (see [21])

The rest of the dynamics is embodied in the initial and final conditions. We stated already that the initial condition is to assume that the initial energy density distribution is constant in a Lorentz contracted pancake. In our field theory calculations using strong fields. we will assume that all the energy density is in the initial semiclassical electric (chromoelectric) field. and equate the energy density $E^2(x,t)$ with Landau's $\varepsilon(x,t)$. Thus we will not attempt to derive this initial condition but will retain this assumption. However various event generators do find similar energy densities to those obtained from a hydrodynamical scaling expansion. The final boundary condition is to state that when the energy density ε reduces to a critical value ε_c which is approximately one physical pion/ pion compton wavelength[3] in a comoving frame, then there are no more interactions and one then calculates the spectrum at that "freeze out" surface. The original method of Landau to determine the particle distribution was to identify the pion velocity with the collective velocity and assume that the number of particles in a bin of particle rapidity was equal to the energy in that bin divided by the energy of a single pion having that rapidity. An alternative was to assume the number distribution was proportional to the entropy distribution. This ansatz was later modified by myself and my collaborators by assuming there was a local thermal distribution of pions in the comoving frame at temperature T_c described by

$$g(x,k) = g_\pi\{\exp[k^\mu u_\mu/T_c] - 1\}^{-1}. \tag{1.1}$$

The particle distribution was then given by:

$$E\frac{d^3N}{d^3k} = \frac{d^3N}{\pi dk_\perp^2 dy} = \int g(x,k)k^\mu d\sigma_\mu \tag{1.2}$$

where σ_μ is the surface defined by $\varepsilon = \varepsilon_c$. We will verify that both ideas can be justified by our field theory calculations.

Let us now let us look at the hydrodynamics of a perfect fluid. In the rest frame (comoving frame) of a perfect relativistic fluid the stress tensor has the form:

$$T_{\mu\nu} = \text{diagonal}\,(\epsilon, p, p, p) \tag{1.3}$$

Boosting by the relativistic fluid velocity four vector $u^\mu(x.t)$ one has:

$$T_{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu} \tag{1.4}$$

From a hydrodynamical point of view, flat rapidity distributions seen in multiparticle production in p-p as well as A-p and A-A collisions are a result of the hydrodynamics being in a scaling regime for the longitudinal flow. (More exact 3-D numerical simulations with sophisticated equations of state have now been performed. The interested reader can see for example [8]).

That is for $v = z/t$ (no size scale in the longitudinal dimension) the light cone variables τ, η:

$$z = \tau\sinh\eta; t = \tau\cosh\eta \tag{1.5}$$

become the fluid proper time $\tau = t(1-v^2)^{1/2}$ and fluid rapidity:

$$\eta = 1/2\ln[(t-x)/(t+x)] \Rightarrow 1/2\ln[(1-v)/(1+v)] = \alpha \tag{1.6}$$

Letting $u^0 = \cosh\alpha; u^3 = \sinh\alpha$, we have when $v = zt$ that $\eta = \alpha$, the fluid rapidity. If one has an effective equation of state $p = p(\varepsilon)$ then one can formally define temperature and entropy as follows:

$$\varepsilon + p = Ts; d\varepsilon = Tds; \ln s = \int d\varepsilon/(\varepsilon + p) \qquad (1.7)$$

Then the equation:

$$u^\mu \partial^\nu T_{\mu\nu} = 0$$

becomes:

$$\partial^\nu (s(\tau) u_\nu) = 0 \qquad (1.8)$$

Which in 1 + 1 dimensions becomes

$$ds/d\tau + s/\tau = 0 \text{ or } s\tau = \text{constant} \qquad (1.9)$$

The assumption of Landau's hydrodynamical model is that the two projectiles collide in the center of mass frame leaving a fixed fraction (about 1/2) of their energy in a a Lorentz contracted disc (with the leading particles going off). The initial energy density for the flow can then be related to the center of mass energy and the volume of the Lorentz contracted disk of energy. It is also assumed that the flow of energy is unaffected by the hadronization process and that the fluid rapidity can be identified in the out regime with particle rapidity. Thus after hadronization the number of pions found in a bin of fluid rapidity can be obtained from the energy in a bin of rapidity by dividing by the energy of a single pion having that rapidty. When the comoving energy density become of the order of ε_c we are in the out regime. This determines a surface defined by

$$\varepsilon_c(\tau_f) = m_\pi / V_\pi \qquad (1.10)$$

On that surface of constant τ,

$$\begin{aligned} \frac{dN}{d\eta} &= \frac{1}{m_\pi u^0}\frac{dE}{d\eta} = \frac{1}{m_\pi \cosh\alpha}\int T^{0\mu}\frac{d\sigma_\mu}{d\eta} \\ d\sigma_\mu &= A_\perp(dz, -dt) = 4\pi a^2 \tau_f(\cosh\eta, -\sinh\eta) \\ \frac{dN}{d\eta} &= \frac{A_\perp}{m_\pi \cosh\alpha}[(\varepsilon + p)\cosh\alpha\cosh(\eta - \alpha) - p\cosh\eta] \end{aligned} \qquad (1.11)$$

where $A_\perp$ is the transverse size of the system at freezeout. If we are in the scaling regime where $\eta = \alpha$ then

$$\frac{dN}{d\eta} = \frac{A_\perp}{m_\pi}\varepsilon(\tau_f).$$

which is a flat distribution in fluid rapidity. At finite energy, where scaling is not exact, only the central region is flat and instead one gets a distribution which is approximately Gaussian in rapidity. Exact numerical simulations (see for example [5]) show that the isoenergy curves do indeed follow a constant τ curve in the central region, so that the scaling result does apply for particle production in the central rapidity region at high but finite center of mass energy.

In Landau's model one needed an extra assumption to identify the collective fluid rapidity α with particle rapidity $y = 1/2\ln[(E_\pi + p_\pi)/(E_\pi - p_\pi)]$, where p_π is the longitudinal momentum of the pion. What results from our field theory simulations of both the production of a fermion-anti fermion pairs from strong Electric fields (the Schwinger mechanism [20]) as well as in the production of pions following a chiral phase tranisition in the σ model, is that if we make the kinematical assumption that the quantum expectation values of measurables are solely a function of τ we will obtain a flat rapidity distribution for the distribution of particles. We can prove, using a coordinate transformation, that the distribution of particles in fluid rapidity is exactly the same as the distribution of particles in particle rapidity. Furthermore we also will find that it is a good approximation to use eq. 1.11 to determine the spectra of particles. For the full single particle distribution $E\frac{dN}{d^3p}$ we will find that the Cooper-Frye formula is valid with the identification of the interpolating number density with the single particle distribution function of classical transport theory.

II. PRODUCTION AND TIME EVOLUTION OF A QUARK-ANTIQUARK PLASMA

Our model for the production of the quark-gluon plasma begins with the creation of a flux tube containing a strong color electric field. If the energy density of the chromoelectric field gets high enough (see below) the quark-anti quark pairs can be popped out of the vacuum by the Schwinger mechanism [20]. For simplicity, here we discuss pair production (such as electron-positron pairs) from an abelian Electric Field and the subsequent quantum back-reaction on the Electric Field. The extension to quark anti-quark pairs produced from a chromoelectric field is straightforward. The physics of the problem can be understood for constant electric fields as a simple tunneling process. If the electric field can produce work of at least twice the rest mass of the pair in one compton wavelength, then the vacuum is unstable to tunnelling. This condition is:

$$eE\frac{\hbar}{mc} \geq 2mc^2$$

which leads to a critical electric field of order $\frac{2m^2c^3}{\hbar e}$

The problem of pair production from a constant Electric field (ignoring the back reaction) was studied by J. Schwinger in 1951 [20] . The WKB argument is as follows: One imagines an electron bound by a potential well of order $|V_0| \approx 2m$ and submitted to an additional electric potential eEx . The ionization probability is proportional to the WKB barrier penetration factor:

$$\exp[-2\int_o^{V_o/e} dx\{2m(V_o-|eE|x)\}^{1/2}] = \exp(-\frac{4}{3}m^2/|eE|)$$

In his classic paper Schwinger was able to analytically solve for the effective Action in a constant background electric field and determine an exact pair production rate:

$$w = [\alpha E^2/(2\pi^2)]\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\exp(-n\pi m^2/|eE|).$$

By assuming this rate could be used when the Electric field was slowly varying in time, the first back reaction calculations were attempted using semi classical transport methods. Here we directly solve the field equations in the large-N approximation [16]. We assume for simplicity that the kinematics of ultrarelativistic high energy collisions results in a boost invariant dynamics in the longitudinal (z) direction (here z corresponds to the axis of the initial collision) so that all expectation values are functions of the proper time $\tau = \sqrt{t^2 - z^2}$.We introduce the light cone variables τ and η, which will be identified later with fluid proper time and rapidity . These coordinates are defined in terms of the ordinary lab-frame Minkowski time t and coordinate along the beam direction z by

$$z = \tau\sinh\eta \quad , \quad t = \tau\cosh\eta\,. \tag{2.1}$$

The Minkowski line element in these coordinates has the form

$$ds^2 = -d\tau^2 + dx^2 + dy^2 + \tau^2 d\eta^2\,. \tag{2.2}$$

Hence the metric tensor is given by

$$g_{\mu\nu} = \mathrm{diag}(-1,1,1,\tau^2). \tag{2.3}$$

The QED action in curvilinear coordinates is:

$$S = \int d^{d+1}x\,(\det V)[-\frac{i}{2}\bar{\Psi}\tilde{\gamma}^\mu\nabla_\mu\Psi + \frac{i}{2}(\nabla_\mu^\dagger\bar{\Psi})\tilde{\gamma}^\mu\Psi - im\bar{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}]. \tag{2.4}$$

where

$$\nabla_\mu\Psi \equiv (\partial_\mu + \Gamma_\mu - ieA_\mu)\Psi \tag{2.5}$$

Varying the action leads to the Heisenberg field equation:

$$(\tilde{\gamma}^\mu\nabla_\mu + m)\,\Psi = 0\,, \tag{2.6}$$

$$\left[\gamma^0\left(\partial_\tau + \frac{1}{2\tau}\right) + \gamma_\perp\cdot\partial_\perp + \frac{\gamma^3}{\tau}(\partial_\eta - ieA_\eta) + m\right]\Psi = 0\,, \tag{2.7}$$

and the Maxwell equation: $E = E_z(\tau) = -\dot{A}_\eta(\tau)$

$$\frac{1}{\tau}\frac{dE(\tau)}{d\tau} = \frac{e}{2}\left\langle[\bar{\Psi},\tilde{\gamma}^\eta\Psi]\right\rangle = \frac{e}{2\tau}\left\langle[\Psi^\dagger,\gamma^0\gamma^3\Psi]\right\rangle. \tag{2.8}$$

We expand the fermion field in terms of Fourier modes at fixed proper time: τ,

$$\Psi(x) = \int[d\mathrm{k}]\sum_s[b_s(\mathrm{k})\psi^+_{\mathrm{ks}}(\tau)e^{ik\eta}e^{i\mathrm{p}\cdot\mathrm{x}} + d_s^\dagger(-\mathrm{k})\psi^-_{-\mathrm{ks}}(\tau)e^{-ik\eta}e^{-i\mathrm{p}\cdot\mathrm{x}}]. \tag{2.9}$$

The $\psi^\pm_{\mathrm{ks}}$ then obey

$$\left[\gamma^0\left(\frac{d}{d\tau} + \frac{1}{2\tau}\right) + i\gamma_\perp\cdot\mathrm{k}_\perp + i\gamma^3\pi_\eta + m\right]\psi^\pm_{\mathrm{ks}}(\tau) = 0, \tag{2.10}$$

Squaring the Dirac equation:

$$\psi^\pm_{\mathrm{ks}} = \left[-\gamma^0\left(\frac{d}{d\tau} + \frac{1}{2\tau}\right) - i\gamma_\perp\cdot\mathrm{k}_\perp - i\gamma^3\pi_\eta + m\right]\chi_s\frac{f^\pm_{\mathrm{ks}}}{\sqrt{\tau}}. \tag{2.11}$$

$$\gamma^0\gamma^3\chi_s = \lambda_s\chi_s \tag{2.12}$$

with $\lambda_s = 1$ for $s = 1,2$ and $\lambda_s = -1$ for $s = 3,4$, we then get the mode equation:

$$\left(\frac{d^2}{d\tau^2} + \omega^2_{\mathrm{k}} - i\lambda_s\dot{\pi}_\eta\right)f^\pm_{\mathrm{ks}}(\tau) = 0, \tag{2.13}$$

$$\omega^2_{\mathrm{k}} = \pi^2_\eta + \mathrm{k}^2_\perp + m^2; \quad \pi_\eta = \frac{k_\eta - eA}{\tau}. \tag{2.14}$$

The back-reaction equation in terms of the modes is

$$\frac{1}{\tau}\frac{dE(\tau)}{d\tau} = -\frac{2e}{\tau^2}\sum_{s=1}^{4}\int[d\mathrm{k}](\mathrm{k}^2_\perp + m^2)\lambda_s|f^+_{\mathrm{ks}}|^2, \tag{2.15}$$

A typical proper time evolution of E and j is shown in fig. 4. Here an initial value of $E = 4$ was chosen.

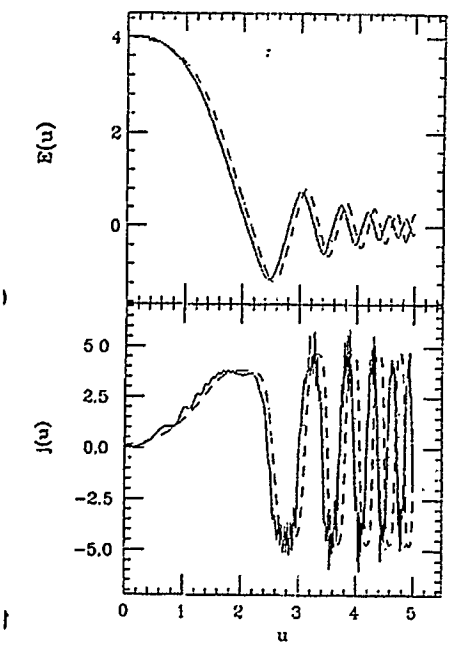

FIG. 4. Proper time evolution of E and j as a function of $u = \ln(\frac{\tau}{\tau_0})$ for an initial $E = 4$.

A. Spectrum of Particles

To determine the number of particles produced one needs to introduce the adiabatic bases for the fields:

$$\Psi(x) = \int [d\mathrm{k}] \sum_s [b_s^0(\mathrm{k};\tau) u_{\mathrm{k}s}(\tau) e^{-i\int \omega_\mathrm{k} d\tau} + d_s^{(0)\dagger}(-\mathrm{k};\tau) v_{-\mathrm{k}s}(\tau) e^{i\int \omega_\mathrm{k} d\tau}] e^{i\mathrm{k}\cdot\mathrm{x}}. \quad (2.16)$$

The operators $b_s(\mathrm{k})$ and $b_s^{(0)}(\mathrm{k};\tau)$ are related by a Bogolyubov transformation:

$$b_r^{(0)}(\mathrm{k};\tau) = \sum \alpha_{\mathrm{k}r}^s(\tau) b_s(\mathrm{k}) + \beta_{\mathrm{k}r}^s(\tau) d_s^\dagger(-\mathrm{k})$$
$$d_r^{(0)}(-\mathrm{k};\tau) = \sum \beta_{\mathrm{k}r}^{*s}(\tau) b_s(\mathrm{k}) + \alpha_{\mathrm{k}r}^{*s}(\tau) d_s^\dagger(-\mathrm{k}) \quad (2.17)$$

One finds that the interpolating phase space number density for the number of particles (or antiparticles) present per unit phase space volume at time τ is given by:

$$n(\mathrm{k};\tau) = \sum_{r=1,2} \langle 0_{in}| b_r^{(0)\dagger}(\mathrm{k};\tau) b_r^{(0)}(\mathrm{k};\tau)|0_{in}\rangle = \sum_{s,r} |\beta_{\mathrm{k}r}^s(\tau)|^2 \quad (2.18)$$

This is an adiabatic invariant of the Hamiltonian dynamics governing the time evolution of the one and two point functions, and is therefore the logical choice as the particle number operator. At $\tau = \tau_0$ it is equal to our initial number operator. If at later times one reaches the out regime because of the decrease in energy density due to expansion it becomes the usual out state phase space number density. Although this does not happen for the above pair production in the Mean field approximation,(because we have not allowed the electric field to dissipate due to the production of real photons), reaching an out regime does happen in the σ model if the energy density decreases as a result of an expansion into the vacuum.

The phase space distribution of particles (or antiparticles) in light cone variables is

$$n_\mathrm{k}(\tau) = f(k_\eta, k_\perp, \tau) = \frac{d^6 N}{\pi^2 dx_\perp^2 dk_\perp^2 d\eta dk_\eta}. \quad (2.19)$$

A typical spectrum is shown in fig. 5 which shows the effect of the Pauli- exclusion principle. The raw results and also the results of averaging over typical experimental momentum bins are shown. This latter result compares well with a transport approach including Pauli-blocking effects (see [16])

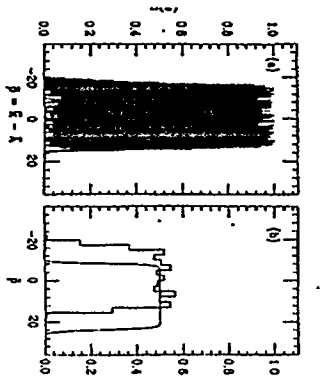

FIG. 5. Comoving spectra of fermion pairs, before and after binning for an initial electric field $E = 4$ at τ=400

We now need to relate this quantity to the spectra of electrons and positrons produced by the strong electric field (the production of electrons and positrons from a strong electric field is our prototype model for the production of the quark gluon plasma from strong chromoelectric fields). We introduce the particle rapidity y and $m_\perp = \sqrt{k_\perp^2 + m^2}$ defined by the particle 4-momentum in the center of mass coordinate system

$$k_\mu = (m_\perp \cosh y, k_\perp, m_\perp \sinh y) \tag{2.20}$$

The boost that takes one from the center of mass coordinates to the comoving frame where the energy momentum tensor is diagonal is given by $\tanh\eta = v = z/t$. so that one can define the "fluid" 4-velocity in the center of mass frame as

$$u^\mu = (\cosh\eta, 0, 0, \sinh\eta) \tag{2.21}$$

We then find that the variable

$$\omega_k = \sqrt{m_\perp^2 + \frac{k_\eta^2}{\tau^2}} \equiv k^\mu u_\mu \tag{2.22}$$

has the meaning of the energy of the particle in the comoving frame. The momenta k_η that enters into the adiabatic phase space number density is one of two momenta canonical to the variables defined by the coordinate transformation to light cone variables. Namely the variables

$$\tau = (t^2 - z^2)^{1/2} \qquad \eta = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$$

have as their canonical momenta

$$k_\tau = Et/\tau - k_z z/\tau \quad k_\eta = -Ez + tk_z. \tag{2.23}$$

To show this we consider the metric $ds^2 = d\tau^2 - \tau^2 d\eta^2$ and the free Lagrangian

$$L = \frac{m}{2} g_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} \tag{2.24}$$

Then we obtain for example

$$\begin{aligned} k_\tau = m\frac{d\tau}{ds} &= m[(\frac{\partial\tau}{\partial t})_z\frac{dt}{ds} + (\frac{\partial\tau}{\partial z})_t\frac{dz}{ds}] \\ &= \frac{Et - k_z z}{\tau} = k^\mu u_\mu \end{aligned} \tag{2.25}$$

The interpolating phase-space density f of particles depends on k_η, $k_\perp$, τ, and is η-independent. In order to obtain the physical particle rapidity and transverse momentum distribution, we change variables from (η, k_η) to (z, y) at a fixed τ where y is the particle rapidity defined by (2.20). We have

$$E\frac{d^3N}{d^3k} = \frac{d^3N}{\pi dy\, dk_\perp^2} = \int \pi dz\, dx_\perp^2\, J\, f(k_\eta, k_\perp, \tau) \tag{2.26}$$

where the Jacobian J is evaluated at a fixed proper time τ

$$\begin{aligned} J = \begin{vmatrix} \partial k_\eta/\partial y & \partial k_\eta/\partial z \\ \partial\eta/\partial y & \partial\eta/\partial z \end{vmatrix} &= \begin{vmatrix} \frac{\partial k_\eta}{\partial y} & \frac{\partial\eta}{\partial z} \end{vmatrix} \\ = \frac{m_\perp\cosh(\eta - y)}{\cosh\eta} &= \frac{\partial k_\eta}{\partial z}|_\tau. \end{aligned} \tag{2.27}$$

We also have

$$k_\tau = m_\perp\cosh(\eta - y); \quad k_\eta = -\tau m_\perp \sinh(\eta - y). \tag{2.28}$$

Calling the integration over the transverse dimension the effective transverse size of the colliding ions $A_\perp$ we then obtain that:

$$\frac{d^3N}{\pi dy\, dk_\perp^2} = A_\perp \int dk_\eta f(k_\eta, k_\perp, \tau) \equiv \frac{d^3N}{\pi d\eta\, dk_\perp^2} \tag{2.29}$$

This quantity is independent of y which is a consequence of the assumed boost invariance. Note that we have proven using the property of the Jacobean, that the distribution of particles in partical rapidity is the same as the distribution of particles in fluid rapidity!! verifying that in the boost-invariant regime that Landau's intuition was correct.

We now want to make contact with the Cooper- Frye Formalism. First we note that the interpolating number density depends on k_η and $k_\perp$ only through the combination:

$$\omega_k = \sqrt{m_\perp^2 + \frac{k_\eta^2}{\tau^2}} \equiv k^\mu u_\mu \tag{2.30}$$

Thus $f(k_\eta, k_\perp) = f(k_\mu u^\mu)$ and so it depends on exactly the same variable as the comoving thermal distribution!. We also have that a constant τ surface (which is the freeze out surface of Landau) is parametrized as:

$$d\sigma^\mu = A_\perp(dz, 0.0, dt) = A_\perp d\eta(\cosh\eta, 0, 0, \sinh\eta) \tag{2.31}$$

We therefore find

$$k^\mu d\sigma_\mu = A_\perp m_\perp \tau\cosh(\eta - y) = A_\perp |dk_\eta| \tag{2.32}$$

Thus we can rewrite our expression for the field theory particle spectra as

$$\frac{d^3N}{\pi dy\, dk_\perp^2} = A_\perp \int dk_\eta f(k_\eta, k_\perp, \tau) = \int f(k^\mu u_\mu, \tau) k^\mu d\sigma_\mu \tag{2.33}$$

where in the second integration we keep y and τ fixed. Thus with the replacement of the thermal single particle distribution by the interpolating number operator, we get via the coordinate transformation to the center of mass frame the Cooper-Frye formula.

Schwinger's pair production mechanism leads to an Energy Momentum tensor which is diagonal in the$(\tau, \eta, x_\perp)$ coordinate system which is thus a comoving one. In that system one has:

$$T^{\mu\nu} = \text{diagonal}\,\{\varepsilon(\tau), p_\parallel(\tau), p_\perp(\tau), p_\perp(\tau)\} \tag{2.34}$$

We thus find in this approximation that there are two separate pressures, one in the longitudinal direction and one in the transverse direction which is quite different from the thermal equilibrium case. However only the longitudinal pressure enters into the "entropy" equation.

Only the longitudinal pressure enters into the "entropy" equation

$$\varepsilon + p_{\|} = Ts \tag{2.35}$$

$$\frac{d(\varepsilon\tau)}{d\tau} + p_{\|} = Ej_{\eta}$$

$$\frac{d(s\tau)}{d\tau} = \frac{Ej_{eta}}{T}$$

In the out regime we find as in the Landau Model

$$s\tau = constant$$

as is seen in fig.6.

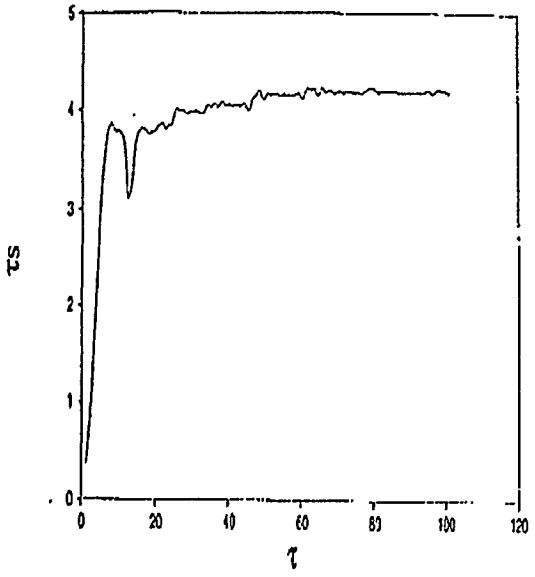

FIG. 6. Time evolution of $s\tau$ as a function of τ

Here we have used 2.35 and the thermodynamic relation:

$$d\varepsilon = Tds$$

to calculate the entropy from the energy density and longitudinal pressure. An alternative effective entropy can be determined from the diagonal part of the full density matrix in the adiabatic number basis. The energy density as a function of proper time is shown in fig.7.

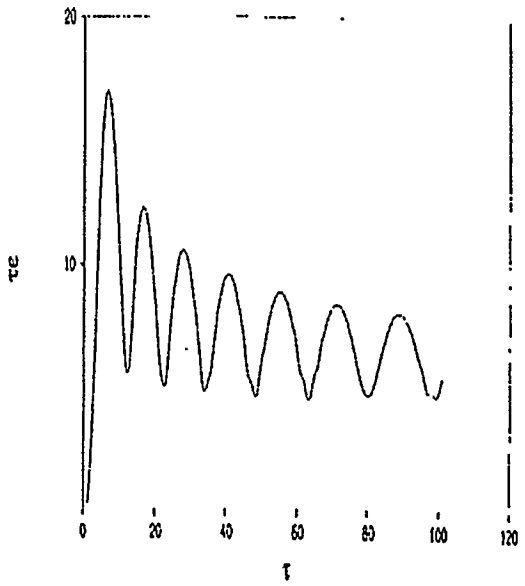

FIG. 7. Time evolution of $\varepsilon\tau$ as a function of τ

For our one-dimensional boost invariant flow we find that the energy in a bin of fluid rapidity is just:

$$\frac{dE}{d\eta} = \int T^{0\mu} d\sigma_{\mu} = A_{\perp}\tau \cosh\eta\varepsilon(\tau) \tag{2.36}$$

which is just the $(1+1)$ dimensional hydrodynamical result.Here however ε is obtained by solving the field theory equation rather than using an ultrarelativistic equation of state. This result does not depend on any assumptions of thermalization. We can ask if we can directly calculate the particle rapidity distribution from the ansatz:

$$\frac{dN}{d\eta} = \frac{1}{m\cosh\eta}\frac{dE}{d\eta} = \frac{A_{\perp}}{m}\varepsilon(\tau)\tau. \tag{2.37}$$

We see from fig. 8. that this works well even in our case where we have ignored rescattering, so that one does not have an equilibrium equation of state.

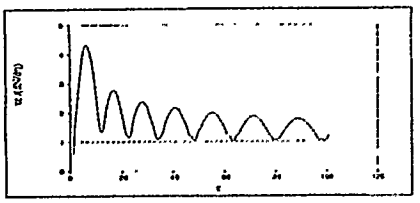

FIG. 8. The ratio of the approximate rapidity distribution $(\frac{E}{m\cosh y}\frac{dN}{dy})$ and exact rapidity distribution as a function of τ.

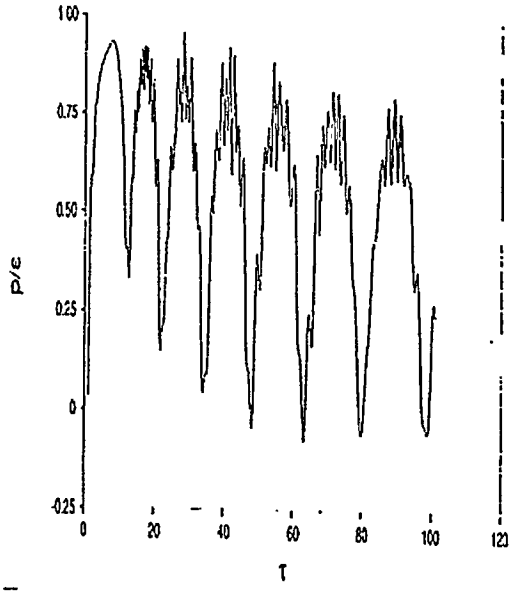

FIG. 9. Proper time evolution of $\frac{p}{\epsilon}$

In the field theory calculation the expectation value of the stress tensor must be renormalized since the electric field undergoes charge renormalization. We can determine the two pressures and the energy density as a function of τ. Explicitly we have in the fermion case.

$$\varepsilon(\tau) =< T_{\tau\tau} >= \tau\Sigma_s \int [dk] R_{\tau\tau}(k) + E_R^2/2$$

where

$$R_{\tau\tau}(k) = 2(p_\perp^2 + m^2)(g_0^+|f^+|^2 - g_0^-|f^-|^2) - \omega$$
$$-(p_\perp^2 + m^2)(\pi + e\dot{A})^2/(8\omega^5\tau^2)$$
$$p_\parallel(\tau)\tau^2 =< T_{\eta\eta} >= \tau\Sigma_s \int [dk]\lambda_s \pi R_{\eta\eta}(k) - \frac{1}{2}E_R^2\tau^2 \quad (2.38)$$

where

$$R_{\eta\eta}(k) = 2|f^+|^2 - (2\omega)^{-1}(\omega + \lambda_s\pi)^{-1} - \lambda_s e\dot{A}/8\omega^5\tau^2$$
$$-\lambda_s e\dot{E}/8\omega^5 - \lambda_s\pi/4\omega^5\tau^2 + 5\pi\lambda_s(\pi + e\dot{A})^2/(16\omega^7\tau^2) \quad (2.39)$$

and

$$p_\perp(\tau) =< T_{yy} >=< T_{xx} >$$
$$= (4\tau)^{-1}\sum_s \int [dk]\{p_\perp^2(p_\perp^2 + m^2)^{-1}R_{\tau\tau} - 2\lambda\pi p_\perp^2 R_{\eta\eta}\}$$
$$+E_R^2/2. \quad (2.40)$$

Thus we are able to numerically determine the dynamical equation of state $p_i = p_i(\varepsilon)$ as a function of τ. A typical result is shown in fig. 9.

III. DYNAMICAL EVOLUTION OF A NON-EQUILIBRIUM CHIRAL PHASE TRANSITION

Recently there has been a growing interest in the possibility of producing disoriented chiral condensates (DCC's) in a high energy collision [22–24]. This idea was first proposed to explain CENTAURO events in cosmic ray emperiments where there was a deficit of neutral pions [25]. It was proposed that a nonequilibrium chiral phase transition such as a quench might lead to regions of DCC [24]. To see whether these ideas made sense we studied numerically [26]the time evolution of pions produced following a heavy ion collision using the linear sigma model, starting from the unbroken phase. The quenching (if present) in this model is due to the expansion of the initial Lorentz contracted energy density by free expansion into vacuum. Starting from an approximate equilibrium configuration at an initial proper time τ in the disordered phase we studied the transition to the ordered broken symmetry phase as the system expanded and cooled. We determined the proper time evolution of the effective pion mass, the order parameter $< \sigma >$ as well as the pion two point correlation function. We studied the phase space of initial conditions that lead to instabilities (exponentially growing long wave length modes) which can lead to disoriented chiral condensates.

We showed that the expansion into the vacuum of the initial energy distribution led to rapid cooling. This caused the system, initially in quasi local thermal equilibrium to progress from the unbroken chiral symmetry phase to the broken symmetry phase vacuum. This expansion is accompanied by the exponential growth of low

momentum modes for short periods of proper time for a range of initial conditions. This exponential growth of long wave length modes is the mechanism for the production of disordered chiral condensates. Thus the production of DCC's results in an enhancement of particle production in the low momentum domain. Whether such an instability occurs depends on the size of the initial fluctuation from the initial thermal distribution. The relevant momenta for which this exponential growth occurs are the transverse momenta and the momenta $k_\eta = -Ez+tp$ conjugate to the fluid rapidity variable $\eta = \tanh^{-1}(z/t)$. We also found that the distribution of particles in these momenta had more length scales than found in local thermal equilibrium. When there is local thermal equilibrium, the length scales are the mass of the pion and the temperature which is related to the changing energy density, both of which depend on the proper time τ.

When we reexpress the number density in the comoving frame in terms of the physically measurable transverse distribution of particles in the collision center of mass frame, we find that there is a noticable distortion of the transverse spectrum, namely an enhancement of particles at low transverse momentum, when compared to a local equilibrium evolution. We will consider two cases, one in which there is exponential growth of low momentum modes due to the effective pion mass going negative during the expansion, and one where the initial fluctuations do not lead to this exponential growth. Both situations will be compared to a purely hydrodynamical boost invariant calculation based on local thermal equilibrium. In determining the actual spectra of secondaries, we find that the adiabatic number operator of our large-N calculation replaces the relativistic phase space density $g(x,p)$ of classical transport theory in determining the distribution of pariticles in rapidity and transverse momentum. This makes it easy to compare our results with the hydrodynamical calculation in the boost invariant approximation which assumes the final pions are in local thermal equilibrium in the comoving frame.

The model we use to discuss the chiral phase transition is the linear sigma model described by the Lagrangian:

$$L = \frac{1}{2}\partial\Phi\cdot\partial\Phi - \frac{1}{4}\lambda(\Phi\cdot\Phi - v^2)^2 + H\sigma. \qquad (3.1)$$

The mesons form an $O(4)$ vector $\Phi = (\sigma, \pi_i)$ This can be written in an alternative form by introducing the composite field: $\chi = \lambda(\Phi\cdot\Phi - v^2)$.

$$L_2 = -\frac{1}{2}\phi_i(\Box + \chi)\phi_i + \frac{\chi^2}{4\lambda} + \frac{1}{2}\chi v^2 + H\sigma \qquad (3.2)$$

The effective action to leading order in large N is given by [26]

$$\Gamma[\Phi,\chi] = \int d^4x[L_2(\Phi,\chi.H) + \frac{i}{2}N\mathrm{tr}\,\ln G_0^{-1}] \qquad (3.3)$$

$$G_0^{-1}(x,y) = i[\Box + \lambda(x)]\,\delta^4(x-y)$$

Varying the action we obtain:

$$[\Box + \chi(x)]\pi_i = 0 \quad [\Box + \chi(x)]\sigma = H, \qquad (3.4)$$

where here and in what follows, π_i,σ and χ refer to expectation values. Varying the action we obtain

$$\chi = -\lambda v^2 + \lambda(\sigma^2 + \pi\cdot\pi) + \lambda N G_0(x,x). \qquad (3.5)$$

If we assume boost invariant kinematics [5] [7] which result in flat rapidity distributions, then the expectation value of the energy density is only a function of the proper time. The natural coordinates for boost invariant ($v = z/t$) hydrodynamical flow are the fluid proper time τ and the fluid rapidity η defined as

$$\tau \equiv (t^2 - z^2)^{1/2}, \qquad \eta \equiv \frac{1}{2}\log(\frac{t-z}{t+z}).$$

To implement boost invariance we assume that mean (expectation) values of the fields Φ and χ are functions of τ only. We then get the equations:

$$\begin{aligned}&\tau^{-1}\partial_\tau\,\tau\partial_\tau\,\Phi_i(\tau) + \chi(\tau)\,\Phi_i(\tau) = H\delta_{i1}\\ &\chi(\tau) = \lambda\big(-v^2 + \Phi_i^2(\tau) + NG_0(x,x;\tau,\tau)\big),\end{aligned} \qquad (3.6)$$

To determine the Green's function $G_0(x,y;\tau,\tau')$ we introduce the auxiliary quantum field $\phi(x,\tau)$ which obeys the sourceless equation:

$$\Big(\tau^{-1}\partial_\tau\,\tau\partial_\tau\, - \tau^{-2}\partial_\eta^2 - \partial_\perp^2 + \chi(x)\Big)\phi(x,\tau) = 0. \qquad (3.7)$$

$$G_0(x,y;\tau,\tau') \equiv < T\{\phi(x,\tau)\,\phi(y,\tau')\} > .$$

We expand the quantum fields in an orthonormal basis:

$$\phi(\eta, x_\perp, \tau) \equiv \frac{1}{\tau^{1/2}}\int [d^3\mathrm{k}]\big(\exp(ikx) f_\mathrm{k}(\tau)\,a_\mathrm{k}\, + h.c.\big)$$

where $kx \equiv k_\eta\eta + \vec{k}_\perp\vec{x}_\perp$, $[d^3\mathrm{k}] \equiv dk_\eta d^2k_\perp/(2\pi)^3$. The mode functions and χ obey:

$$\ddot{f}_\mathrm{k} + (\frac{k_\eta^2}{\tau^2} + \vec{k}_\perp^2 + \chi(\tau) + \frac{1}{4\tau^2})f_\mathrm{k} = 0. \qquad (3.8)$$

$$\chi(\tau) = \lambda\Big(-v^2 + \Phi_i^2(\tau) + \frac{1}{\tau}N\int [d^3\mathrm{k}]|f_\mathrm{k}(\tau)|^2\,(1 + 2\,n_\mathrm{k})\Big). \qquad (3.9)$$

We notice that when χ goes negative, the low momentum modes with

$$\frac{k_\eta^2 + 1/4}{\tau^2} + \vec{k}_\perp^2 < |\chi|$$

grow exponentially. However these modes then feed back into the χ equation and this exponential growth then gets damped. It is these growing modes that lead to the

possiblity of growing domains of DCC's as well as a modification of the low momentum distribution of particles from a thermal one. The parameters of the model are fixed by physical data. The PCAC condition is

$$\partial_\mu A^i_\mu(x) \equiv f_\pi m_\pi^2 \pi^i(x) = H\pi^i(x).$$

In the vacuum state $\chi_0\sigma_0 = m_\pi^2\sigma_0 = H$, so that $\sigma_0 = f_\pi$= 92.5 MeV. The vacuum gap equation is

$$m_\pi^2 = -\lambda v^2 + \lambda f_\pi^2 + \lambda N \int_0^\Lambda [d^3\mathrm{k}] \frac{1}{2\sqrt{k^2+m_\pi^2}}.$$

This leads to the mass renormalized gap equation:

$$\chi(\tau) - m_\pi^2 = -\lambda f_\pi^2 + \lambda \Phi_i^2(\tau)$$
$$+\frac{\lambda}{\tau} N \int [d^3\mathrm{k}]\{|f_\mathrm{k}(\tau)|^2\ (1+2\ n_\mathrm{k}) - \frac{1}{2\sqrt{k^2+m^2}}\}. \quad (3.10)$$

λ is chosen to fit low energy scattering data as discussed in [17].

If we assume that the inititially (at $\tau_0 = 1$) the system is in local thermal equilibrium in a comoving frame we have

$$n_k = \frac{1}{e^{\beta_0 E_k^0} - 1} \quad (3.11)$$

where $\beta_0 = 1/T_0$ and $E_k^0 = \sqrt{\frac{k_\eta^2}{\tau_0^2} + \vec{k}_\perp^2 + \chi(\tau_0)}$.

The initial value of χ is determined by the equilibrium gap equation for an initial temperature of $200MeV$ and is $.7fm^{-2}$ and the initial value of σ is just $\frac{H}{\chi_0}$. The phase transition in this model occurs at a critical temperature of $160MeV$. To get into the unstable domain, we then introduce fluctuations in the time derivative of the classical field. We varied the value of the initial proper time derivative of the sigma field expectation value and found that for $\tau_0 = 1fm$ there is a narrow range of initial values that lead to the growth of instabilities. namely $.25 < |\dot{\sigma}| < 1.3$.

Fig.10 displays the results of the numerical simulation for the evolution of χ (3.8)–(3.9). We display the auxiliary field χ in units of fm^{-2} , the classical fields Φ in units of fm^{-1} and the proper time in units of fm ($1fm^{-1} = 197MeV$) for two simulations, one with an instability ($\dot{\sigma}|_{\tau_0} = -1$) and one without ($\dot{\sigma}|_{\tau_0} = 0$).

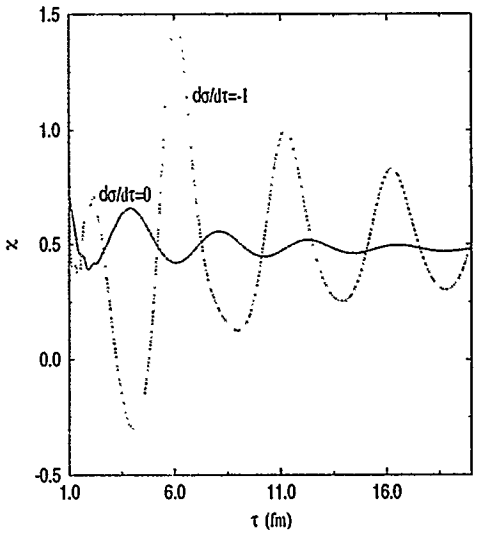

FIG. 10.Proper time evolution of the χ field for two different initial values of $\dot{\sigma}$.

We notice that for both initial conditions, the system eventually settles down to the broken symmetry vacuum result as a result of the expansion. The evolution of the quantities σ and π_1 are displayed for various initial conditions in fig.11.

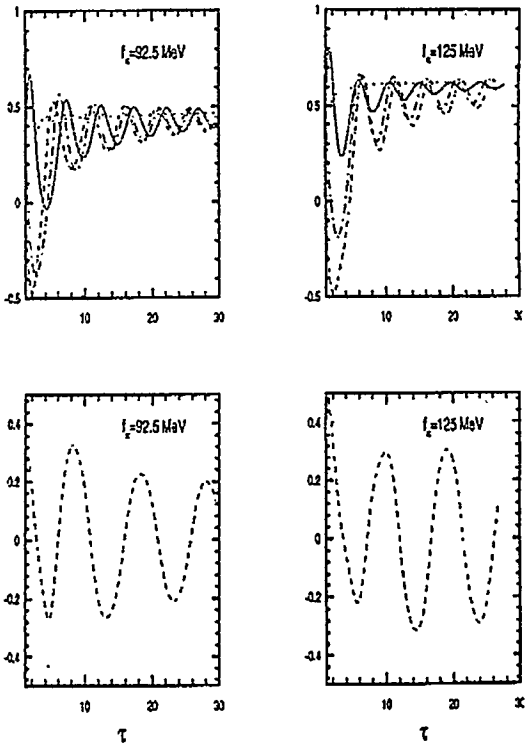

FIG. 11Proper time evolution of the σ field and π field for four different initial conditions with $f_\pi = 92.5MeV, 125MeV$.

To determine the spectrum of particles we introduce the interpolating number density which is defined by expanding the fields in terms of mode functions f_k^0 which are first order in an adiabatic expansion of the mode equation.

$$f_k^0 = \frac{e^{-iy_k(\tau)}}{\sqrt{2\omega_k}}; \quad dy_k/dt = \omega_k, \tag{3.12}$$

where $\omega_k(\tau) \equiv (k_\eta^2/\tau^2 + \vec{k}_\perp^2 + \chi(\tau))^{1/2}$. This leads to the alternative expansion of the fields:

$$\phi(\eta, x_\perp, \tau) \equiv \frac{1}{\tau^{1/2}} \int [d^3\mathrm{k}](\exp(ikx) f_\mathrm{k}^0(\tau)\ a_\mathrm{k}(\tau)\ + h.c.) \tag{3.13}$$

The two sets of creation and annihilation operators are connected by a Bogoliubov transformation:

$$a_k(\tau) = \alpha(k,\tau) a_k + \beta(k,\tau) a_{-k}^\dagger. \tag{3.14}$$

α and β can be determined from the exact time evolving mode functions via:

$$\alpha(k,\tau) = i(f_k^{0*}\frac{\partial f_k}{\partial \tau} - \frac{\partial f_k^{0*}}{\partial \tau} f_k)$$
$$\beta(k,\tau) = i(f_k^0 \frac{\partial f_k}{\partial \tau} - \frac{\partial f_k^0}{\partial \tau} f_k). \tag{3.15}$$

In terms of the initial distribution of particles $n_0(k)$ and β we have:

$$n_k(\tau) \equiv f(k_\eta, k_\perp, \tau) = < a_k^\dagger(\tau) a_k(\tau) >$$
$$= n_0(k) + |\beta(k,\tau)|^2 (1 + 2n_0(k)). \tag{3.16}$$

$n_k(\tau)$ is the adiabatic invariant interpolating phase space number density which becomes the actual particle number density when interactions have ceased. When this happens the distribution of particles is

$$f(k_\eta, k_\perp, \tau) = \frac{d^6N}{\pi^2 dx_\perp^2 dk_\perp^2 d\eta dk_\eta}. \tag{3.17}$$

We now need to relate this quantity to the physical spectra of particles measured in the lab. At late τ our system relaxes to the vacuum and λ becomes the square of the physical pion mass m^2. As before, we introduce the outgoing pion particle rapidity y and $m_\perp = \sqrt{k_\perp^2 + m^2}$ defined by the particle 4-momentum in the center of mass coordinate system. The boost that takes one from the center of mass coordinates to the comoving frame where the energy momentum tensor is diagonal is given by $\tanh\eta = v = z/t$, so that one can define the "fluid" 4-velocity in the center of mass frame as

$$u^\mu = (\cosh\eta, 0, 0, \sinh\eta) \tag{3.18}$$

The variable

$$\omega_k = \sqrt{m_\perp^2 + \frac{k_\eta^2}{\tau^2}} \equiv k^\mu u_\mu \tag{3.19}$$

has the meaning of the energy of the particle in the comoving frame. As discussed earlier the variables τ, η have as their canonical momenta

$$k_\tau = Et/\tau - k_z z/\tau \quad k_\eta = -Ez + tk_z. \tag{3.20}$$

Changing variables from (η, k_η) to (z, y) at a fixed τ we have

$$E\frac{d^3N}{d^3k} = \frac{d^3N}{\pi dy\, dk_\perp^2} = \int \pi dz\ dx_\perp^2\ J\ f(k_\eta, k_\perp, \tau)$$
$$= A_\perp \int dk_\eta f(k_\eta, k_\perp, \tau) \tag{3.21}$$

This is again converted into the Cooper-Frye form by noting that a constant τ surface is parametrized as:

$$d\sigma^\mu = A_\perp(dz, 0, 0, dt) = A_\perp d\eta(\cosh\eta, 0, 0, \sinh\eta) \tag{3.22}$$

Thus

$$k^\mu d\sigma_\mu = A_\perp m_\perp \tau \cosh(\eta - y) = A_\perp |dk_\eta| \tag{3.23}$$

and

$$\frac{d^3N}{\pi dy\, dk_\perp^2} = A_\perp \int dk_\eta f(k_\eta, k_\perp, \tau) = \int f(k_\eta, k_\perp, \tau) k^\mu d\sigma_\mu \tag{3.24}$$

This reconfirms the idea that the interpolating phase space number density plays the role of a classical transport phase space density function, as was found in our calculation of pair production from strong electric fields [16]

We wish to compare our nonequilibrium calculation with the results of the hydrodynamical model in the same boost-invariant approximation. In the hydrodynamical model of heavy ion collisions [5], the final spectra of pions is given by a combination of the fluid flow and a local thermal equilibrium distribution in the comoving frame.

$$E\frac{d^3N}{d^3k} = \frac{d^3N}{\pi dk_\perp^2 dy} = \int g(x,k) k^\mu d\sigma_\mu \tag{3.25}$$

Here $g(x,k)$ is the single particle relativistic phase space distribution function. When there is local thermal equilibrium of pions at a comoving temperature $T_c(\tau)$ one has

$$g(x,k) = g_\pi \{\exp[k^\mu u_\mu / T_c] - 1\}^{-1}. \tag{3.26}$$

In Figures 12 and 13 we compare the boost invariant hydrodynamical result for the transverse momentum distribution using critical temperatures of $T_c = 140, 200$ MeV to the two nonequilibrium cases represented in figure 10. Figure 12 pertains to the initial condition

$\dot{\sigma}|_{\tau_0} = -1$ In this case there is a regime where the effective mass becomes negative and we see a noticable enhancement of the low transverse momentum spectra. We have normalized both results to give the same total center of mass energy E_{cm}. Figure 13 corresponds to the initial condition $\dot{\sigma}|_{\tau_0} = 0$. Here we notice that there is a little enhancement at low transverse momenta.

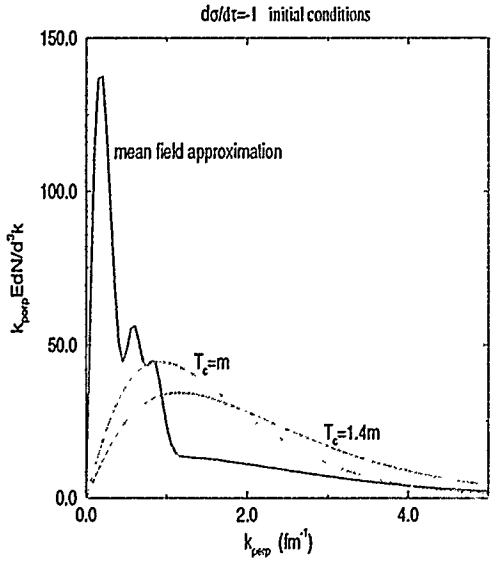

FIG. 12. Single particle transverse momentum distribution for $\dot{\sigma} = -1$ initial conditions compared to a local equilibrium Hydrodynamical calculation with boost invariance.

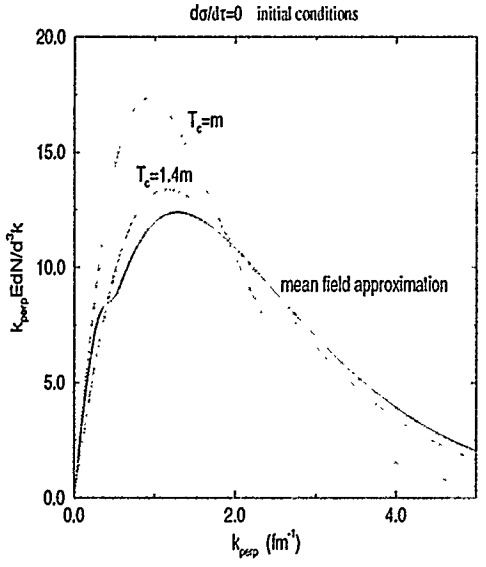

FIG. 13. Single particle transverse momentum distribution for $\dot{\sigma} = 0$ initial conditions compared to a local equilibrium Hydrodynamical calculation with boost invariance.

So we see that a non-equilibrium phase transition taking place during a time evolving quark-gluon or hadronic plasma can lead to an enhancement of the low momentum transverse momentum distribution. In particular, if a Centauro type event is not accompanied by such an enhancement one would be suspicious of ascribing this event to the production of disoriented chiral condensates as a result of a rapid quench. For the case when we have massless goldstone pions in the σ model ($H = 0$) [27],then χ goes to zero at large times. In that case we find the amusing fact that the equation of state becomes $p = \varepsilon/3$ at late times. This is true even though the final particle spectrum is far from thermal equilibrium. This equation of state is shown in Fig 14.

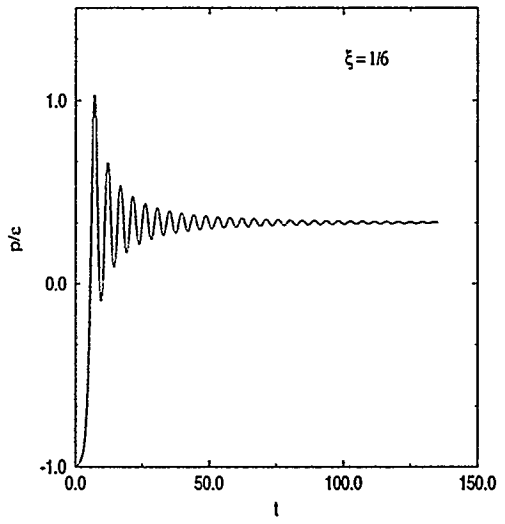

FIG. 14. Equation of state $\frac{p}{\varepsilon}$ as a function of τ for the massless σ model where we start from a quench.

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