CONF-9511165--2 SAND95-0941C

ROBUST MODEL ERROR LOCALIZATION FOR DAMAGE DETECTION AND FINITE ELEMENT MODEL UPDATE

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FEB 11 1997
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ABSTRACT

A new method for identifying the location of finite element model errors given test-identified frequencies and mode shape data is presented. The new method builds on the concept of the modal force error vector, which is the undamped impedance of the given finite element model at each identified frequency multiplied by the corresponding identified mode shape. In order to mitigate the problems associated with reducing analytical models to the set of measurement degrees of freedom, a mode shape projection algorithm is utilized. The projection algorithm is a linear least-squares method which can be controlled to minimize bias caused by model errors. The localization indicator is then defined by the modal force error and a degree of freedom-dependent normalization based on the variance of the identified frequencies and mode shapes. The performance of the method in localizing structural damage is examined using experimental data.

INTRODUCTION

The development of accurate predictive analytical models for structural dynamics traditionally involves the problem of model reconciliation to dynamic testing. This is because, despite advancements in finite element theory, model construction (e.g. meshing algorithms), visualization and high performance computing, there are still significant modeling errors introduced by assumptions of uniform material behavior, joint compliance, element formulations, etc. In order to address the reconciliation of analytical models to dynamic testing, efficient testing methods and algorithms have been developed to adjust model parameters to "fit" the test identified modal parameters. These algorithms can be interpreted as optimization methods; that is, an objective is minimized or maximized with respect to a set of variable parameters.

When the model being adjusted has the correct mathematical form, but inaccurate parameters, parameter estimation algorithms yield excellent results, with the following caveats. First, there must be a sufficient number of test-identified parameters upon which a least-squares estimate of the parameters can be based. Second, the parameters which are in error must be among those being estimated. Finally, the parameters being varied must be as independent as possible in terms of their sensitivity to the data. Unfortunately, these requirements are at odds with one another. For example, if all primary model parameters are allowed to vary, there will not be a sufficient

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number of test-identified parameters available to obtain a least-squares estimate. Furthermore, there will likely be a high degree of correlation between the parameters, further limiting the confidence in the estimate. Therefore, we are usually limited to the variation of a few key model parameters to account for the observable errors. The process of selecting these parameters can be termed *model error localization.* In this context, the problem of updating models is a two-stage iterative approach. The first stage is *localization* or selection of model parameters to be estimated; the second stage is the *estimation* of those parameters to optimize a particular metric.

Similarly, in detecting damage in structures using dynamic response data, these two tasks are generally described as finding the *location* and *extent* of the damage. Damage detection usually involves determining location and extent indicators for a structure relative to some baseline condition of the structure, represented either by a previous set of dynamic response data or by the response parameters of an analytical model of the structure which is assumed to be accurate and reflects a particular condition of health. Using damage localization, problem areas can be identified in order to direct more detailed structural inspections. Similarly, in adaptive structures technology, damage or error localization indicators can be used to monitor adaptive structural systems for health or to identify sensor systems which are no longer functioning properly.

In this paper, a new method for identifying the location of finite element model errors, or equivalently damage, given test-identified frequencies and mode shape data is presented. The present model error localization approach is based on the Sensitivity-Based Element-By-Element (SB-EBE) model update theory (Farhat and Hemez, 1993). This algorithm determines parameter estimates by a minimization of modal force errors for a set of modes. The modal force error vector is the undamped impedance of the given finite element model at each identified frequency multiplied by the corresponding identified mode shape. The minimization leads to both a mode shape projection algorithm and physical model parameter update using the projected mode shapes plus the experimental frequencies and the nominal stiffness and mass matrices of the analytical model.

A key component of this model update procedure is the so-called "zooming" feature, whereby a small number of potential model parameters are chosen for updating based upon the degrees of freedom (d.o.f.) exhibiting the largest modal force errors, and the parameters which are localized to those degrees of freedom. Clearly, the "zooming" feature is an example of model error localization for finite element updating. Kaouk and Zimmerman use a similar approach in defining a "damage vector," which is again the modal force error generalized to utilize the damped modal parameters (Kaouk and Zimmerman, 1994). This common concept of using the modal force error vector for localization was proposed in earlier work (Ojalvo and Pilon, 1988).

The present technique builds on the same concept of the modal force error, but introduces additional algorithmic components to increase robustness of the localization in the presence of model errors, differences in localized stiffness, and uncertainty in the identified parameters. In order to compute the modal force error with respect to the d.o.f. of the model, a mode shape projection algorithm is utilized. The projection is formulated as a least-squares problem using the model and the equations of motion at the identified frequency to solve for the displacements at the unmeasured d.o.f. A key component of the present technique, however, is control of the errors introduced by the projection algorithm. This is accomplished by partitioning out rows of the analytical model matrices associated with the largest modal force errors. An alternative approach investigated is the use of a normalization of the functional underlying the projection algorithm.

The model error localization indicator is then defined by the modal force error vector and a d.o.f.-dependent normalization based on the variance of the identified modal parameters. This normalization is critical to understanding the localization effects caused by random errors in the identification process and the relative dynamic stiffness of the model. That is, areas of the model at which forces tend to localize due to sensitivity in the model formulation itself will be normalized so that they do not mask errors in less sensitive locations. This normalization allows for statistical confidence in the identified modal properties to be incorporated into the localization analysis, such that model parameters sensitive to the most uncertain test parameters will be deemphasized. Finally, this paper investigates the trade-off between dynamic model reduction and mode shape projection within the context of model error localization. This is an important consideration as traditional approaches have considered only a choice between these techniques, rather than a judicious combination to minimize the ambiguity of the results.

The remainder of the paper is organized as follows. The general theory for error localization via computation of the modal force error is first presented, followed by details of the projection algorithm and techniques for controlling bias due to localized model errors. The present error localization indicator is then defined using a statistical normalization of the modal force error. The variance of common model correlation measures such as the Modal Assurance Criteria are also examined, so that the variance measures for the indicator normalization can be properly interpreted. Finally, the performance of the present method is examined using experimental data.

THEORETICAL DEVELOPMENT

We define the modal force error vector R_i for mode i as

$$R_i = (K - \omega_{E_i}^2 M)\begin{Bmatrix} \phi_{m_i}^E \\ \phi_{o_i} \end{Bmatrix} \tag{1}$$

where K and M are the stiffness and mass matrices from the model, respectively, ω_{E_i} is the identified radial frequency for mode i (rad/s), $\phi_{m_i}^E$ is the identified mode shape at the sensor d.o.f., and ϕ_{o_i} is the partition of the mode shape corresponding to the unmeasured d.o.f. in the model. If the correct stiffness and mass matrices are given as

$$\begin{aligned} K_c &= K + \Delta K \\ M_c &= M + \Delta M \end{aligned} \tag{2}$$

we have

$$((K + \Delta K) - \omega_{E_i}^2 (M + \Delta M))\begin{Bmatrix} \phi_{m_i}^E \\ \phi_{o_i} \end{Bmatrix} = 0 \qquad \Rightarrow \qquad -R_i = (\Delta K - \omega_{E_i}^2 \Delta M)\begin{Bmatrix} \phi_{m_i}^E \\ \phi_{o_i} \end{Bmatrix} \tag{3}$$

Thus, the modal force error vector R_i contains information on both magnitude and location of the model errors $[\Delta K, \Delta M]$.

Unfortunately, the d.o.f. at which the mode shape is sampled in test is typically much smaller than the number of d.o.f. in the finite element model which defines K and M. Therefore, to apply Eqn. 1, either the model must be reduced to the measured d.o.f., or the measured portion of the mode shapes must be expanded to the displacement d.o.f. basis of the model.

PROJECTION OF EXPERIMENTAL MODE SHAPES

There have been many algorithms proposed for expanding experimental mode shapes into the d.o.f. of a finite element model (see Imregun and Ewins, 1993, for reviews of various techniques). The algorithm presented in this paper is based on the Sensitivity-Based Element-By-Element (SB-EBE) model update theory (Farhat and Hemez, 1993), which incorporates a mode shape projection theory based on a minimization of the modal force error given in Eqn. 1.

PROJECTION ALGORITHM

We seek a estimate of the unmeasured partition of the mode shape ϕ_{o_i} which minimizes the magnitude of the impedance residual, viz.

$$\min_{\phi_{o_i}} R_i^T R_i \Rightarrow \min_{\phi_{o_i}}(\phi_{m_i}^T Z_{mi}^T Z_{mi}\phi_{mi} + \phi_{o_i}^T[2Z_{oi}^T Z_{mi}\phi_{mi} + Z_{oi}^T Z_{oi}\phi_{oi}]) \tag{4}$$

where $Z_i = K - \omega_{E_i}^2 M$ is the impedance of the model for experimental mode i. This leads to the following least-squares solution for ϕ_{oi}:

$$\phi_{oi} = -(Z_{oi}^T Z_{oi})^{-1} Z_{oi}^T Z_{mi}\phi_{mi}$$

$$\begin{Bmatrix}\phi_{mi} \\ \phi_{oi}\end{Bmatrix} = \begin{bmatrix} I \\ -(Z_{oi}^T Z_{oi})^{-1} Z_{oi}^T Z_{mi}\end{bmatrix}\phi_{mi} = P_i\phi_{mi} \tag{5}$$

where P_i is the mode shape projection for mode i.

It is known, however, that when the model is in error, R_i should be nonzero even when ϕ_{o_i} is correctly determined; in fact, R_i should hopefully have a small number of (possibly) large nonzero entries. We can partition Eqn. 1 as

$$\begin{bmatrix} Z_{m_i}^A & Z_{o_i}^A \\ Z_{m_i}^B & Z_{o_i}^B \end{bmatrix}\begin{Bmatrix}\phi_{m_i}^E \\ \phi_{o_i}\end{Bmatrix} = \begin{Bmatrix} R_i^A \\ R_i^B\end{Bmatrix} \tag{6}$$

where A and B refer to a partitioning of the equations into the highest and lowest magnitudes of the entries of R_i. Then, a least-squares estimate for ϕ_{o_i} is given by

$$\phi_{o_i} = -((Z_{o_i}^B)^T(Z_{o_i}^B))^{-1}(Z_{o_i}^B)^T(Z_{m_i}^B)\phi_{m_i}^E \tag{7}$$

so long as the number of B equations is greater than the number of unmeasured d.o.f. in Z. The choice of the equation set B upon which the least-squares solution is defined is not trivial. The primary motivation for partitioning the equations is to improve the solution for ϕ_{o_i} over that obtained using the full set of equations, given the assumption that the errors in the model are not distributed uniformly among the d.o.f. but rather are localized. It should be noted that delegating the equilibrium equation for a particular d.o.f. to set A does not impede our ability to find model errors associated with that d.o.f. Indeed, it will tend to enhance the modal force error at those d.o.f. in set A since the projection matrix will not be "designed" to minimize those errors.

A generalization of the above partitioning can be obtained by introducing a weighting function to the optimization given by Eqn. 4. In the spirit of statistical estimation, we can select an inverse weighting by the variance of the modal force error R_i. This variance reflects the uncertainty of the modal force error as a linear function of the errors in the identified modal parameters used to compute R_i. If we define the covariance matrix of R_i as Q_i, and the covariance matrix of the measured mode shape ϕ_{mi} as $Q_{\phi_{mi}}$, then

$$Q_i = Z_i P_i Q_{\phi_{mi}} P_i^T Z_i^T \tag{8}$$

Using Eqn. 8 and Eqns. 4 and 5, we obtain

$$\min(R_i^T Q_i^{-1} R_i)$$

$$\therefore P_i = \begin{bmatrix} I \\ -(Z_{oi}^T Q_i^{-1} Z_{oi})^{-1} Z_{oi}^T Q_i^{-1} Z_{mi} \end{bmatrix} \tag{9}$$

Note that a nonlinearity has been introduced, because the modal force error covariance matrix Q_i is a function of the projection matrix P_i. This can be handled in a cursory manner by predicting Q_i based on only the measured partition of Z_i, computing an estimate of P_i, correcting Q_i, and finally computing a new projection P_i based on the corrected covariance matrix.

MODEL REDUCTION

An alternative to the mode shape projection algorithm detailed above is to condense the model d.o.f. down to the set of measured d.o.f. There are a number of established techniques for model reduction, such as Guyan reduction (Guyan, 1965) and the Improved Reduced System (IRS) model (O'Callahan, 1989). The difficult trade-off in model reduction, given that the set of reduced d.o.f. are given as a consequence of the experiment design, is between the accuracy of the reduction and the sensitivity of the transformation to model errors.

A reasonable compromise is to reduce the model to the measurement d.o.f., assess the accuracy of the reduced model in terms of its ability to predict the modal parameters of the full-order model, and then add a minimum number of additional d.o.f. to the reduction in order to ensure that the reduced model predicts the analytical modes to within the uncertainty of the experimental parameters. The best choices of additional d.o.f. are either other displacements which would be useful in localizing model errors, or generalized d.o.f. such as the fixed interface modes (FIM) of the Craig-Bampton component mode synthesis technique (Craig and Bampton, 1968).

COMPUTATION OF THE LOCATION INDICATOR FOR MODEL ERROR

The computation of the impedance residual can now be written as

$$R_i = Z_i P_i \phi_{m_i}^E \tag{10}$$

and an estimate of the variance of the entries in R_i due to assumed zero-mean gaussian noise on each of the entries of $\phi_{m_i}^E$ is given by

$$\sigma_\phi^{\ 2}(R_i) = Z_i P_i \Sigma_\phi P_i^T Z_i^T \tag{11}$$

where the noise covariance matrix for the elements of the measured mode shape $\phi_{m_i}^E$ is given by

Σ_ϕ. In addition, we can consider additional variance due to uncorrelated frequency uncertainty, although the frequency uncertainty is typically smaller than the mode shape uncertainty, relative to their nominal values. It can be reasonably expected (subject to the noise models assumed above and knowledge of the modal parameter variances) that an accurate analytical model will have impedance residuals $R_i < 3\sigma(R_i)$.

We define the indicator as the impedance residual estimate vector R_i normalized by the standard deviations of the estimates $\sigma(R_i)$,

$$\hat{R}_i(j) = \frac{R_i(j)}{\sigma(R_i(j))} \qquad \sigma(R_i(j)) = \sqrt{Z_i(j,:)P_i\Sigma_\phi P_i^T Z_i(j,:)^T} \tag{12}$$

Therefore, $\hat{R}_i$ can be viewed as a normalized modal force error vector, which indicates degree to which the estimated modal force error from the actual modal data exceeds the normal level of force error due to uncertainty in the modal parameters.

MODAL PARAMETER VARIANCE BASED ON RECONCILIATION CRITERIA

Since we have accepted standards for model update convergence (e.g. level of Modal Assurance Criteria, error in frequency estimates), these can be used to determine the modal variances which in turn are used to arrive at the Model Error Localization Indicator $\hat{R}_i$.

We can determine the variance of the mode shape error Σ_ϕ by determining the expected value of the Modal Assurance Criteria (MAC) as a function of Σ_ϕ. The MAC is defined as

$$MAC(\phi_i, \phi_j) = \frac{(\phi_i^T \phi_j)^2}{(\phi_i^T \phi_i)(\phi_j^T \phi_j)} \tag{13}$$

Now assume that the two mode shapes are identical, except for added noise to ϕ_j. It can be shown that, if the noise is random and of equal magnitude across the measured d.o.f. such that the covariance matrix of the mode shape is $\Sigma_\phi = \sigma_{n(i)}^2 I$ and the dimension of ϕ is N_m, then

$$E[MAC] \approx 1 - \frac{(N_m - 1)\sigma_{n(i)}^2}{\phi^T \phi} \tag{14}$$

This relation can then be used in reverse, by supposing the expected value of the MAC given an ensemble of tests, each of which yields an estimate of the mode shapes. For example, if we assume the expected value of the MAC is 0.99, then

$$\sigma_{n(i)}^2 = \frac{0.01\phi^T\phi}{N_m - 1} \Rightarrow \Sigma_\phi = \left(\frac{0.01\phi^T\phi}{N_m - 1}\right) I \Rightarrow \sigma(R_i) = \sqrt{\left(\frac{0.01\phi^T\phi}{N_m - 1}\right)\mathrm{diag}(Z_i P_i P_i^T Z_i^T)} \tag{15}$$

EXPERIMENTAL RESULTS: INTERSTATE 40 RIO GRANDE BRIDGE

The model error localization algorithm detailed in the present paper has been implemented and checked on numerical data. Due to space considerations, those results will not be given here. Instead, the results below detail the application of the algorithm to damage detection of a highway bridge. The bridge in question is one span of Interstate 40 over the Rio Grande in Albuquerque, New Mexico. As part of a research effort by Los Alamos National Laboratory and New Mexico

State University, with the support of Sandia National Laboratories, an older section of the bridge, slated for destruction, was instrumented and modal tests performed while one of the supporting beams of the roadbed was intentionally damaged. A total of 5 modal tests were performed, with the bridge in its "pre-damage" condition and at progressively stages of damage. In each test, the first 6 modes of the bridge were identified using 26 accelerometers equally spaced on the roadbed above the two I-beams which provide the longitudinal bending support.

The corresponding modes of a finite element model of the bridge are shown in Figure 1. This model, composed of beam and plate elements, has 2027 displacement degrees of freedom. Because the number of model d.o.f. exceeds the test measured d.o.f. by almost two orders of magnitude, a significant amount of model reduction and/or mode shape projection is necessary to compute an error indicator. In this case, model reduction alone will not suffice. This can be seen in Table I. Here the modes of two reduced-order models are compared to the full-order model. The Guyan-reduced model, which includes just the 26 measured d.o.f., exhibits considerable errors, to the point where some modes of the full-order model are not present in the reduced model. A second model, using a Craig-Bampton d.o.f. basis comprised of the 26 measured d.o.f. augmented by 50 fixed-interface modal displacements (modes of the full-order model with the measured d.o.f. fixed-to-ground), is sufficient to capture the lower modes of the full-order model. To utilize this model, however, mode shape projection must be employed, to determine the displacements of the experimental modes for the unmeasured fixed interface d.o.f.

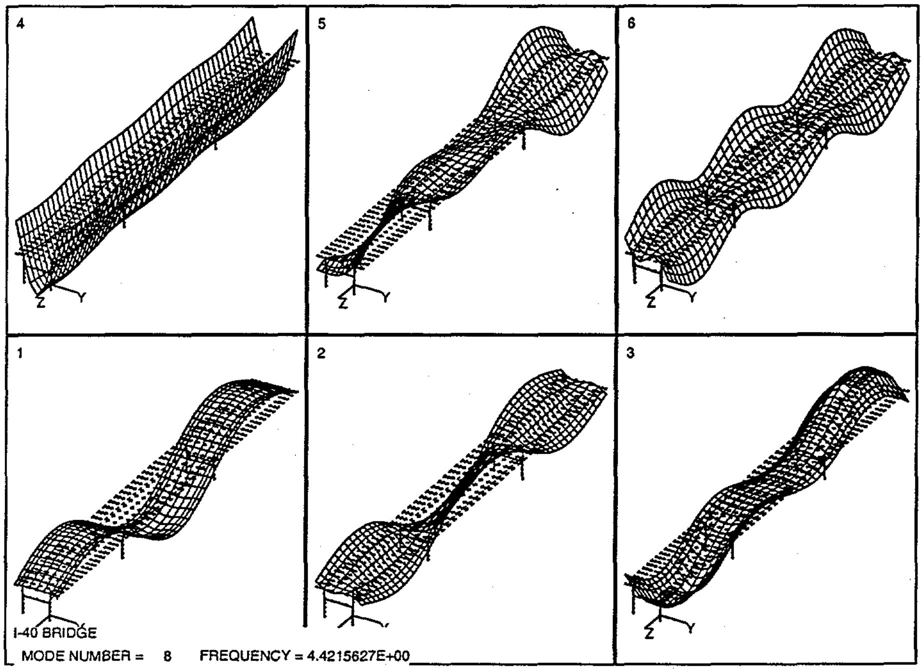

Figure 1: I-40 Rio Grande Bridge: Finite Element Modes

The results of the mode shape projection analysis for the undamaged modal test vectors is given in Table II. These tables compare and contrast the basic mode shape projection and the generalized weighted least-squares projection proposed in a preceding section of this paper. These results are determined for both the full-order model (i.e. projecting the 26 sensor d.of. to the 2027 model d.o.f.) and for the reduced-order Craig-Bampton (C-B) model using the 50 fixed interface

TABLE I: COMPARISON OF MODEL MODES FOR TWO LEVELS OF MODEL REDUCTION

Full-Order Model		Guyan Reduction to Sensor d.o.f.				C-B Model to Sensor d.o.f. + 50 FIM			
Mode #	Freq (Hz)	Mode #	Freq (Hz)	% Freq Error	MAC	Mode #	Freq (Hz)	% Freq Error	MAC
1	2.384	1	2.419	1.46	1.0000	1	2.384	0.01	1.0000
2	2.914	2	2.980	2.28	1.0000	2	2.918	0.16	1.0000
3	3.483	5	4.499	29.14	0.9848	3	3.483	0.00	1.0000
4	3.523	3	3.680	4.45	1.0000	4	3.523	0.02	1.0000
5	3.910	4	4.118	5.31	1.0000	5	3.919	0.23	1.0000
6	4.046	3	3.680	-9.06	0.9993	6	4.046	0.00	1.0000
7	4.358	5	4.499	3.23	1.0000	7	4.359	0.03	1.0000
8	4.422	6	4.812	8.83	0.9999	8	4.433	0.26	1.0000
9	5.077	1	2.419	-52.36	0.9283	9	5.077	0.00	1.0000
10	5.504	6	4.812	-12.57	0.9132	10	5.504	0.01	1.0000

TABLE II: MAC: MODEL VS. PROJECTION OF UNDAMAGED VECTORS

Mode #	Measured MAC	Full-Order Model n=2027		C-B Model (n=67)	
		Basic Projection	Weighted Projection	Basic Projection	Weighted Projection
1	0.9974	0.0002	0.8454	0.0001	0.9975
2	0.9928	0.0141	0.9146	0.0314	0.9931
3	0.9933	0.0101	0.7415	0.0315	0.9942
4	0.9778	0.0190	0.0665	0.2887	0.0806
5	0.9855	0.0185	0.9842	0.0165	0.9756
6	0.9823	0.0536	0.9882	0.0158	0.9853

modal displacements (i.e. projecting the 26 sensor d.o.f. to 76 total model d.o.f.). One problem in evaluating the projections using experimental data is that we do not know the correct responses for the unmeasured d.o.f. One method of evaluation, however, is to compare the MAC between the projected experimental mode shape and model mode shape to the MAC determined by just the measured portions of the two mode shapes. It can be supposed that, if the measured d.o.f. of the model are a reasonable sample of the full mode shape, then the MAC determined by the measured partition will be representative of the MAC between the full mode shapes. Based on this supposition, we can make the following observations.

First, note that the weighted projection is crucial in determining projected mode shapes which are reasonable with respect to the analytical mode shapes. Furthermore, the mode shapes projected into the d.o.f. of the C-B model are more reliable than the projection into the full-order finite element model. This can be seen particularly with the undamaged vector case. Here the MAC between the measured partition of the model's modes and the test modes are quite high, indicating that the model can accurately predict the experimental mode shapes, at least from the point of view of the measured d.o.f. The projected mode shapes for the full-order model, however, have significantly lower modal assurance criteria, which would be indicative of either significant modeling error or significant error in the experimental mode shape. The C-B model, on the other hand, retains the higher MAC of the measured partitions. Since both models are equivalent in terms of their ability to predict these modes (as was seen in Table I) it is reasonable to attribute these differences to our ability to project the mode shapes into these different displacement sets. Finally, note that the project of mode 4 for all of the models is significantly in error. The cause of these errors is not evident in either the data or the model, but it is likely these is some model form error which is not observable from the measured d.o.f.

The results of the model error localization are shown in Figures 2 through 4. In Figure 2, a comparison of the modal force error vector and the error indicator, which is the force vector normalized by its standard deviation, is shown for the undamaged and full damage cases for mode 1. The measured d.o.f. showing large force errors for both cases are at sensors 1 and 14, which are at the supported ends of the bridge and far away from the actual damage. The error indicator, on the other hand, shows that none of the d.o.f. have a significant level of error in the undamaged condition, while in the damaged condition many d.of. exhibit indications of damage. In fact, d.o.f. 20, associated with sensor 20, shows the highest error indicator and is directly above the location on the support beam where the structural damage was introduced. Figure 3 shows a composite error indicator (root-sum-square of the 6 modess) for the undamaged condition and for damage cases 2 through 4. Note again the clear error indicator associated with d.o.f. 20 in damage case 4. Also, there is a consistent indicator of damage or model error associated with d.o.f. 10-12 for all of the cases. This is associated with the undamaged support beam and is not in the same area of the bridge as the damage. Finally, Figure 4 shows the composite model error indicator for the 4 damage cases divided by the pre-damage error indicator. This gives the best indicator for the damage, and shows that the damage is not detectable in any of the prior partial damage conditions.

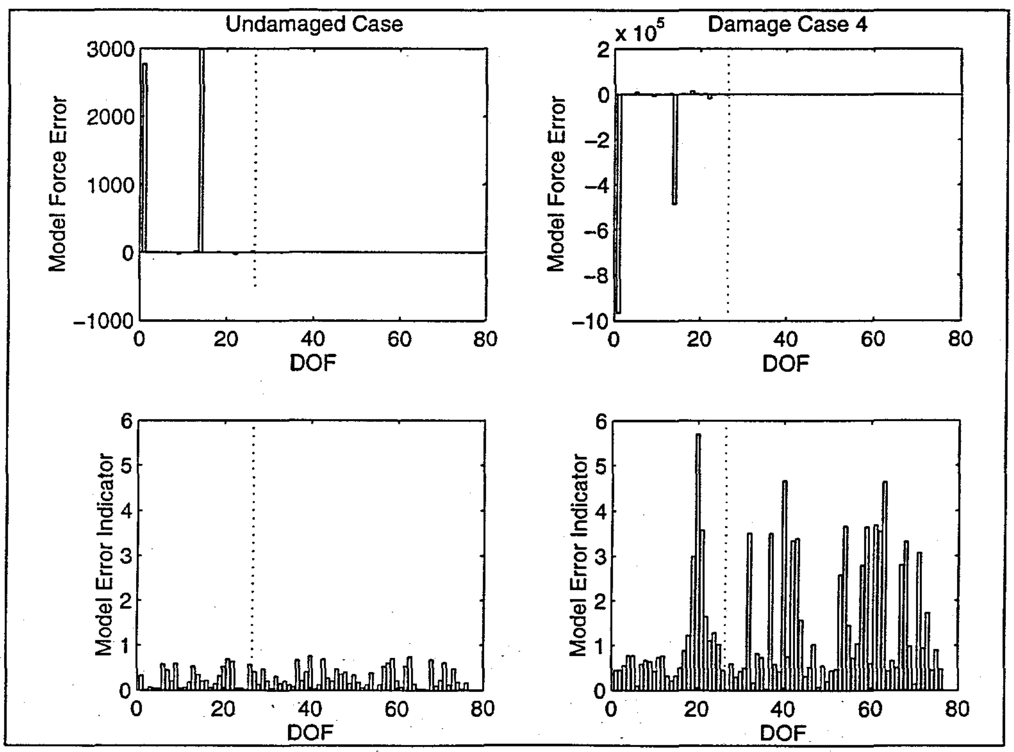

Figure 2: Comparison of Modal Force and Indicator: Results for Mode 1

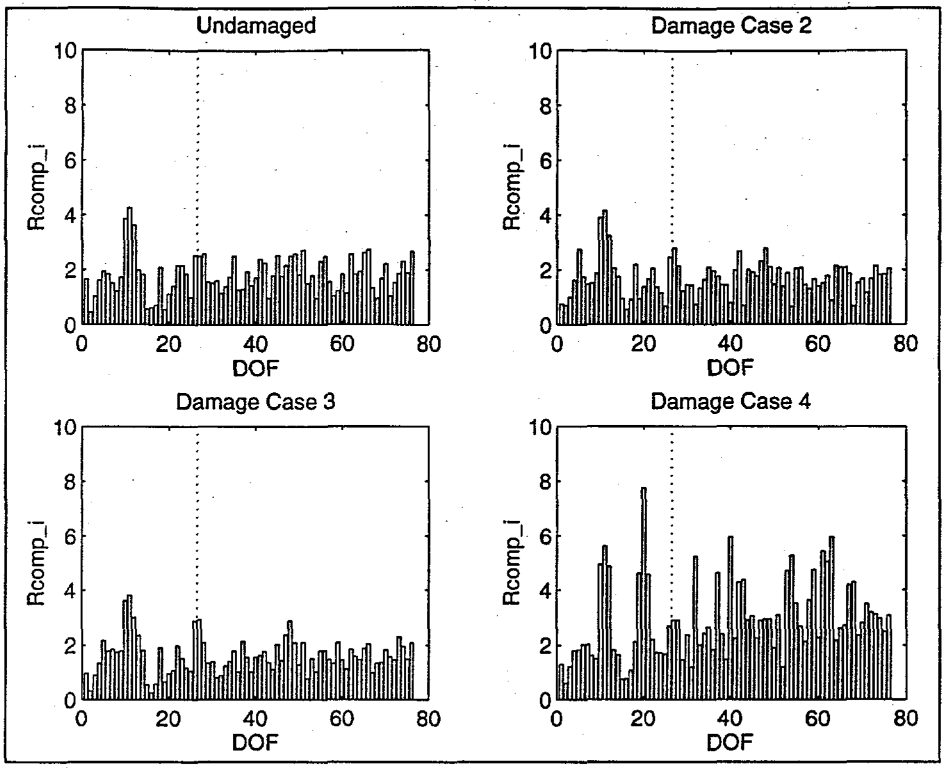

Figure 3: Composite Error Indicator

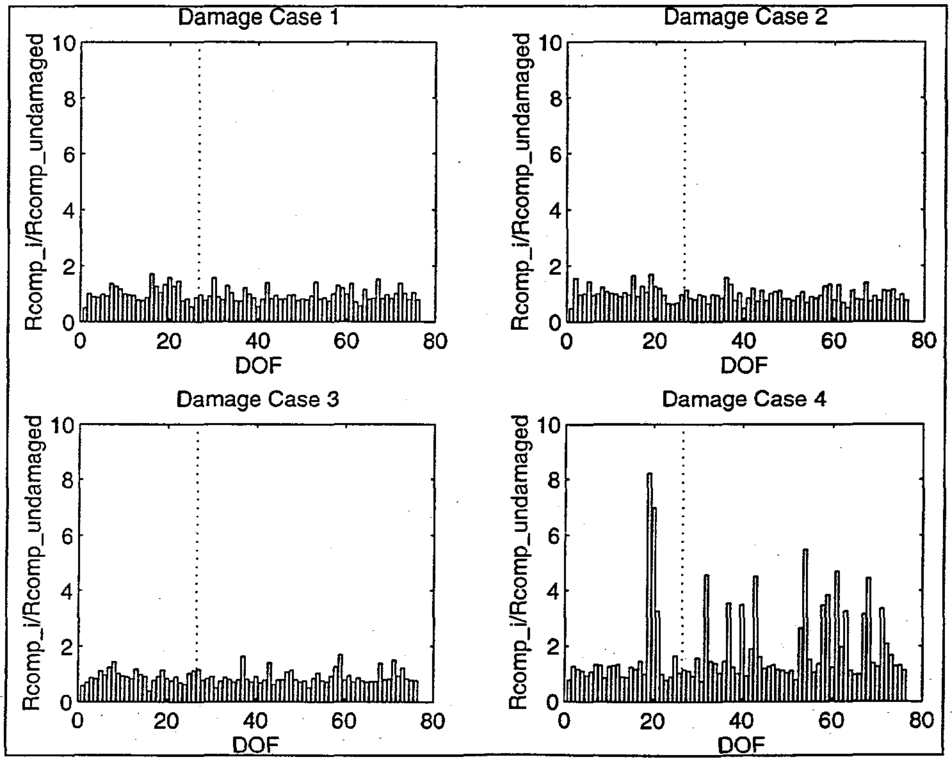

Figure 4: Damage Indicator: Ratio of Error Indicators for Damage Levels

CONCLUSIONS

A method for localizing modeling errors using experimental modal parameters has been presented. The method is robust in the sense that it incorporates the variance of the experimental data used in the localization indicator, and can find errors which would otherwise be masked by stiff areas of the structure. The method can utilize a mix of model reduction and mode shape projection, and a new mode shape projection algorithm is derived which also incorporates statistical measures to reduce bias caused by imperfect experimental data. The method has been successfully applied to damage detection in a highway bridge and is currently being implemented for use as a pre-processor in test-analysis model reconciliation.

ACKNOWLEDGMENTS

This work was supported by the Department of Energy under contract DE-AC04-94AL85000. The author wishes to thank Dr. John Red-Horse of Sandia National Laboratories and Prof. Francois Hemez of Ecole Centrale Paris for their help and encouragement.

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BIOGRAPHY

Kenneth Alvin received his B.S. in Aerospace Engineering from Iowa State University in 1983. From 1983 to 1989 he was a project engineering in Structural Analysis at Harris Corporation Aerospace Systems Division. He received his M.S. in 1990 and Ph.D. in 1993 in Aerospace Engineering Sciences from University of Colorado at Boulder. Since 1994 he has been a Research Fellow in Structural Dynamics at Sandia National Laboratories. His research interests are in system identification, structural dynamics and finite element methods.