GEAP-4580
AEC RESEARCH AND
DEVELOPMENT REPORT
MARCH 22, 1965

FLUZ
A COMPUTER CODE FOR ANALYZING THE REACTOR RESPONSE TO SMALL REACTIVITY OSCILLATIONS

P. GREEBLER
B.A. HUTCHINS
M.D. KELLEY

U.S. ATOMIC ENERGY COMMISSION
CONTRACT AT(04-3)-189
PROJECT AGREEMENT 10

ATOMIC POWER EQUIPMENT DEPARTMENT
GENERAL ELECTRIC
SAN JOSE, CALIFORNIA

DISCLAIMER

GEAP-4580
AEC Research and
Development Report
March 22, 1965

FLUZ - A COMPUTER CODE FOR ANALYZING THE REACTOR RESPONSE TO SMALL REACTIVITY OSCILLATIONS

P. Greebler
B. A. Hutchins
M. D. Kelley

Approved: ______________________
F. J. Leitz, Project Engineer
Fast Ceramic Reactor Program

Prepared for the
U. S. Atomic Energy Commission
under
Contract AT(04-3)-189, Project Agreement 10

Printed in U.S.A. Available from the
Clearing House for Federal Scientific and Technical Information
National Bureau of Standards, U.S. Department of Commerce
Springfield, Virginia
Price: $3.00 per copy

ATOMIC POWER EQUIPMENT DEPARTMENT
GENERAL ELECTRIC
SAN JOSE, CALIFORNIA

2336-TIO-2
135-WDH-5/65

LEGAL NOTICE

TABLE OF CONTENTS

SECTION I

INTRODUCTION

One method of demonstrating the stability of a power reactor is to impress the critical reactor with a sinusoidal oscillating reactivity and measure its power response as a function of frequency. The computer code FLUZ synthesizes such a test for oscillations of small amplitudes. The same technique may be used to experimentally measure different coefficients of reactivity of a given reactor. By varying the period of the impressed reactivity it is possible to change, due to finite heat transfer times, the relative importance of various reactivity feedback mechanisms, such as the fuel Doppler effect or coolant density.

FLUZ computes to the first order the amplitudes and phase angles of the oscillating power and feedback reactivity of a fast reactor which is oscillating about criticality due to an impressed sinusoidal reactivity of given amplitude and frequency. By varying this amplitude and frequency as well as the steady-state power, FLUZ can be used to simulate the power response of the reactor being studied to the magnitudes of various coefficients of reactivity. A comparison of the FLUZ results with experimental results from oscillation tests on the actual reactor can be used to reveal the magnitudes of the various reactivity coefficients.

The oscillating power in FLUZ is expressed in terms of the prompt and delayed components of the reactivity. Linear feedback reactivity is calculated using pre-determined coefficients of reactivity and computed time-dependent material temperatures. Average material temperatures are obtained for each of m axial regions, considering axial heat transfer to occur only by flow of the coolant.* Heat transfer from fuel to coolant and structure is based upon an average coolant channel containing a cylindrical fuel rod with annuli of cladding and coolant. A flat radial power distribution across the fuel rod is assumed.

*See footnote in Section 2.4 which describes an important limitation on the application of FLUZ in its present form.

SECTION II

DESCRIPTION OF THE PHYSICAL MODEL

The physical model used in FLUZ must provide for the computation of (1) the reactor power as a function of both the impressed and feedback reactivities (kinetics equations), (2) the feedback reactivity as a function of the material temperatures, and (3) the material temperatures as functions of the reactor power. The response of a reactor which is subjected to an oscillatory impressed reactivity will depend upon these three functions. In the linear approximations of these functions, the power, feedback reactivity, and material temperatures are all assumed to oscillate with the fundamental frequency of the impressed reactivity, differing only in amplitude and phase angle. This approximation greatly simplifies the computations and provides sufficiently accurate results if the amplitude of the oscillating power is small compared to the steady-state power.

Two quantities which are normally measured during oscillatory tests are the power and feedback reactivity transfer functions, designated by $G^{(i,\omega)}$ and $H^{(i,\omega)}$, respectively. The power transfer function can be defined by the equation

$$G^{(i\omega)} = \frac{P_1(i\omega)/P_0}{k_1/\beta} , \qquad (2\text{-}1)$$

where

$P_1(i\omega)$ = Amplitude of the average oscillating power density over and above P_0 (based on the steady-state volume of fuel),

P_0 = Magnitude of the steady state average power density corresponding to k = 1;

k_1 = Amplitude of the impressed reactivity, and

β = Delayed neutron fraction.

The term $G(i\omega)$ relates the fractional oscillating power to the amplitude of the impressed reactivity in dollars (units of prompt criticality). Associated with the power transfer function is the zero-power transfer function, which applies when there is very little heating of reactor materials and the amplitude of the feedback reactivity, $k_f(i\omega)$, is very small. This is given by

$$G_0(i\omega) = \frac{G(i\omega)}{1 + \frac{k_f(i\omega)}{k_1}} \quad . \tag{2-2}$$

The feedback reactivity transfer function is an expression of the link between the feedback reactivity and the oscillating power. That is,

$$H(i\omega) = \frac{k_f(i\omega)/\beta}{P_1(i\omega)/P_0} = \frac{G(i\omega) - G_0(i\omega)}{G_0(i\omega)\ G(i\omega)} \quad . \tag{2-3}$$

For comparison of the results from FLUZ with experimental data, it is necessary to obtain the amplitudes of the complex quantities $P_1(i\omega)$, $G(i\omega)$, $G_0(i\omega)$, $H(i\omega)$, and $k_f(i\omega)$. This can be done by taking

$$|X| = \left| \left[X^{(r)}\right]^2 + \left[X^{(i)}\right]^2 \right|^{1/2} \quad , \tag{2-4}$$

where $X^{(r)}$ is the real part and $X^{(i)}$ is the imaginary part for each quantity. The phase angles of these amplitudes, relative to the impressed oscillating reactivity, can be obtained by recognizing that the ratio of the imaginary to real parts of each complex amplitude is the tangent of its phase angle, since k_1 is assumed to have a zero phase.

2.1 Kinetics Equations

The total oscillating neutron multiplication factor may be expressed as the sum of the steady-state neutron multiplication factor, impressed reactivity, and feedback reactivity:

$$k(t) = k_0 + k_1 e^{i\omega t} + k_f(i\omega)e^{i\omega t} \quad . \tag{2-5}$$

The average power density in the fuel is the sum of the steady state power and the oscillating power:

$$P(t) = P_0 + P_1(i\omega)e^{i\omega t} \quad . \tag{2-6}$$

In Equations (2-5) and (2-6) k_1, k_f $(i\omega)$, P_0, and P_1 $(i\omega)$ are as defined previously, and k_0 (constant power neutron multiplication factor) equals 1, ω is the frequency of steady oscillations, and t is time.

Although the quantities $k_f(i\omega)$ and $P_1(i\omega)$ are time independent, they take the form of complex numbers due to the fact that they are not in phase with the impressed reactivity.

The basic kinetics equation relates the time rate of change of the neutron density to the excess prompt reactivity and the delayed neutron source. It takes the form

$$\frac{dn(t)}{dt} = \frac{k(1-\beta)-1}{\ell} + \sum_{i=1}^{I} \lambda_i C_i(t) \quad . \qquad (2\text{-}7)$$

The first term in Equation (2-7) is due to prompt neutrons only, thus it is a function of the delayed neutron fraction, β, and the prompt neutron lifetime, ℓ. The second term gives the neutron source due to the delayed neutron precursors, C_i, where the λ_i's are the precursor decay constants. Expressing Equation (2-7) in terms of the average power density in the fuel is more useful in FLUZ. Since the prompt term is based upon the neutron level which already exists at time t, it can just be multiplied by the average power density, P(t). The second term, however, must be multiplied by the number of fissions produced per second per neutron produced. Equation (2-7) then becomes

$$\frac{dP(t)}{dt}\left(\frac{\text{fissions}}{\text{cm}^3\text{-sec}}\right) = P(t)\left[\frac{k(t)(1-\beta)-1}{\ell}\right] + \frac{k(t)}{\nu \ell}\sum_{i=1}^{I} \lambda_i C_i(t), \qquad (2\text{-}8)$$

where

ν = Number of neutrons produced per fission.

The delayed neutron precursors also vary sinusoidally as

$$C_i(t) = C_{i,0} + C_{i,1}(i\omega)e^{i\omega t} \quad , \qquad (2\text{-}9)$$

with $C_{i,0}$ representing the precursor for the steady power level and $C_{i,1}(i\omega)$ the complex amplitude of the oscillating precursor. (Since it is important to keep P(t) proportional to the total reactor power, neither P(t) nor the $C_i(t)$'s vary directly with the fuel density, but are based upon the steady-state fuel volume.) Furthermore, the time rate of change in the precursors can be expressed as the production rate minus the decay rate. That is,

$$\frac{dC_i(t)}{dt} = \left[\nu\beta_i P_1(i\omega) = \lambda_i C_{i,1}(i\omega) \right] e^{i\omega t} , \qquad (2\text{-}10)$$

where β_i is the delayed neutronfraction associated with the i^{th} precursor.

By proper substitution of k, P, and C_i from Equations (2-5), (2-6), and (2-9), into Equations (2-8) and (2-10), both for steady-state power ($k_1 = 0$) and for steady-state oscillations ($k_1 > 0$), the complex amplitude of the oscillating power density can be expressed as

$$P_1 = \frac{\left[\dfrac{k_1 + k_f(i\omega)}{\beta} \right] P_0}{i\omega \left(\dfrac{\ell}{\beta} + \displaystyle\sum_{i=1}^{I} \dfrac{\beta_i/\beta}{\lambda_i + i\omega} \right)} , \qquad (2\text{-}11)$$

where all terms of order higher than $e^{i\omega t}$ have been dropped (linear approximation).

All of the quantities in Equations (2-1), (2-2), (2-3), and 2-11) are known with the exception of the complex amplitude of the feedback reactivity, $k_f(i\omega)$. When $k_f(i\omega)$ has been determined, the amplitudes of the complex quantities P_1 $(i\omega)$, $G(i\omega)$, $G_0(i\omega)$, $H(i\omega)$ and $k_f(i\omega)$ can be obtained.

2.2 Feedback Reactivity

The feedback reactivity for a cylindrical reactor, $k_f(i\omega)$ $e^{i\omega t}$, results from (1) the Doppler effect in the fuel, (2) changes in material densities due to expansion and contraction, (3) axial and radial core expansion, and (4) fuel rod bowing. Considering the four basic fast reactor materials of fuel, cladding, coolant, and structure, the feedback reactivity for a core divided into m_{max} axial regions is given by

$$k_f(i\omega)\, e^{i\omega t} = \sum_{m=1}^{m_{max}} \left\{ \left(\frac{\partial k}{\partial T_f}\right)_{Dop}^{(m)} \Delta T_f^{(m)} - \left(N'_{f,0}\frac{\partial k}{\partial N'_{f,0}}\right)^{(m)} \frac{\Delta N_f'^{(m)}}{N'_{f,0}} - \left(N'_{cl,0}\frac{\partial k}{\partial N'_{cl,0}}\right)^{(m)} \frac{\Delta N_{cl}'^{(m)}}{N'_{cl,0}} - \left(N'_{co,0}\frac{\partial k}{\partial N'_{co,0}}\right)^{(m)} \frac{\Delta N_{co}'^{(m)}}{N'_{co,0}} - \left(N'_{s,0}\frac{\partial k}{\partial N'_{s,0}}\right)^{(m)} \frac{\Delta N_s'^{(m)}}{N'_{s,0}} + \left(H_{c,0}\frac{\partial k}{\partial H_{c,0}}\right) \frac{\Delta H_c^{(m)}}{H_{c,0}} + \left(R_{c,0}\frac{\partial k}{\partial R_{c,0}}\right) \frac{\Delta R_c^{(m)}}{R_{c,0}} + \left(\frac{\partial k}{\partial d_c}\right)^{(m)} \Delta d_c^{(m)} \right\} \quad (2\text{-}12)$$

The quantities $N'_{f,0}$; $N'_{cl,0}$; $N'_{co,0}$; and $N'_{s,0}$ are homogenized nuclear densities of the fuel, cladding, coolant, and structure, respectively. $H_{c,0}$ is the core height, $R_{c,0}$ the core radius, and d_c a number specifying the degree of fuel bowing. Primes on the nuclear densities indicate homogenized values, while the subscript zeros refer to values at the constant-power-density level, P_0.

The factors in the parentheses are reactivity coefficients for the fuel Doppler effect; fuel, cladding, coolant, and structural nuclear density effects; axial core expansion; radial core expansion and degree of fuel rod bowing; in that order. These coefficients, with the exceptions of the axial and radial core expansion coefficients, refer to the total core reactivity response to a change only in axial region m.

The quantities involving "Δ", in Equation (2-12), are fractional differences between the values at time t and the values at constant power density, P_0, of the basic quantities which affect the reactivity. By considering the response of core materials to changes in temperature they may all be expressed as functions of the average material temperatures. These difference quantities can be represented by the general quantity, $\Delta X^{(m)}$, in the generalized form

$$X^{(m)} = X_0^{(m)} + X_1^{(m)}(i\omega)\, e^{i\omega t} = X_0^{(m)} + \Delta X^{(m)} \qquad (2\text{-}13)$$

where the zero subscript refers to the constant-power value and the unit subscript to the amplitude of the oscillating quantity (that over and above the steady-state value). For the Doppler term, $\Delta T_f^{(m)}$ can be substituted directly into Equation (2-12). However, relating the other "Δ" terms to average material temperatures requires a consideration of the design of the reactor core and how it responds to material temperature changes.

The change in core height, $\Delta H_c^{(m)}$, is determined only by axial expansion of the fuel, since the effective reactor height is always that of the fuel. Thus, assuming axial expansion in fuel is additive from region to region, the time dependent core height is

$$H_c = \sum_{m=1}^{M} \Delta Z^{(m)} \left(1 + \alpha_f T_{f,1}^{(m)}\, e^{i\omega t}\right), \qquad (2\text{-}14)$$

where

$\Delta Z^{(m)}$ = Height of region m,

α_f = Linear expansion coefficient of fuel, and

$T_{f,1}^{(m)}$ = Amplitude of the oscillating component of the average fuel temperature.

The change in radius, $\Delta R_c^{(m)}$, with time is assumed to be governed by structural expansion, giving

$$R_c^{(m)} = R_{c,0} \left(1 + \alpha_s T_{s,1}^{m}\, e^{i\omega t}\right) \qquad (2\text{-}15)$$

for the radius of axial region m at time t. In this case α_s is the linear expansion coefficient of the structure and $T_{s,1}^{(m)}$ is the amplitude of the oscillating component of the average structural temperature. Since every axial region might not contain structure which causes core radial expansion, provision is made in FLUZ to specify which regions will cause core radial expansion. In these regions, with $\delta_{R_c}^{(m)} = 1$, Equation (2-15) applies. For the other regions where $\delta_{R_c}^{(m)} = 0$, the core radius is taken to be the linearly interpolated value using the radii of the closest regions for which $\delta_{R_c}^{(m)} = 1$. That is,

$$R_c^{(m)} = R_{c,o}\left[\left[1 + \alpha_s T_{s,1}^{(m_1)} e^{i\omega t}\right] + \alpha_s e^{i\omega t}\left[T_{s,1}^{(m_2)} - T_{s,1}^{(m_1)}\right]\left\{\frac{\frac{1}{2}\left[\Delta Z^{(m_1)} + \Delta Z^{(m)}\right] + \sum_{m'=m_1+1}^{m-1} \Delta Z^{(m')}}{\frac{1}{2}\left[\Delta Z^{(m_1)} + \Delta Z^{(m_2)}\right] + \sum_{m'=m_1+1}^{m_2-1} \Delta Z^{(m')}}\right\}\right] \tag{2-16}$$

when $\delta_{R_c}^{(m)} = 0$. Region numbers m_1 and m_2 refer to the nearest regions less than and greater than m which have $\delta_{R_c}^{(m)} = 1$. The top and bottom regions of the core must have $\delta_{R_c}^{(m)} = 1$.

The fuel-rod-bowing term is simply expressed in terms of the fuel and cladding temperatures by using bowing coefficients. So Δd_c can be written

$$\Delta d_c = \left(\frac{\partial d_c}{\partial T_f}\right)^{(m)} \Delta T_f^{(m)} + \left(\frac{\partial d_c}{\partial T_{cl}}\right)^{(m)} \Delta T_{cl}^{(m)}, \tag{2-17}$$

where the temperature coefficients of bowing must be supplied as input.

To express the difference quantities for the nuclear densities in terms of average material temperature changes, two things must be considered. First there is the change in nuclear densities due to the volumetric expansion of the materials. Second, the homogenized nuclear densities will change due to changes in dimensions, i. e., volume fractions. Using the form of Equation (2-13), the nuclear density of material n in region m is given by

$$N_n'^{(m)} = N_{n,0}^{(m)} V_{n,0} + \Delta N_n'^{(m)} \tag{2-18}$$

where $V_{n,0}$ is the volume fraction of material n and $N_{n,0}$ is the nuclear density of non-homogenized material n at constant power density P_0. However, the time varying quantity can be given by

$$N_n'^{(m)} = N_n^{(m)} V_n^{(m)}$$

$$= N_{n,0}^{(m)} \left(1 - 3\alpha_n T_{n,1}^{(m)} (i\omega)\, e^{i\omega t} \right) V_n^{(m)} , \tag{2-19}$$

where $T_{n,1}^{(m)}$ is the amplitude of the oscillating component of the average temperature of material n, and α_n is its linear expansion coefficient. Since the reactor height is determined by the fuel height, the volume fractions are proportional only to the cross sectional areas of the materials.

For fuel, cladding, and structure, the time varying volume fractions become

$$V_n^{(m)} = \frac{R_{c,0}^2 V_{n,0} \left(1 + 2\alpha_n T_{n,1}^{(m)} (i\omega)\, e^{i\omega t}\right)}{\left(R_c^{(m)}\right)^2} \tag{2-20}$$

where $\left(R_c^{(m)}\right)^2$ is the instantaneous radius of the reactor in region m, given by Equations (2-15) and (2-16).

Since the coolant is a fluid, it will occupy any expansion voids into which it can flow. However, restrictions on the flow of coolant depend upon the particular design being considered. In FLUZ, the volume fraction of the coolant is expressed as

$$V_{co}^{(m)} = \frac{R_{c,0}^{(m)} V_{co,0} - R_{c,0}^{2} (\Delta A)^{(m)} + \delta_{co} \left[\left(R_c^{(m)} \right)^2 - R_{c,0}^2 \right]}{\left(R_c^{(m)} \right)^2} . \qquad (2\text{-}21)$$

The second term in the numerator is the decrease in coolant volume due to radial expansion of the fuel and cladding, and is given by

$$\Delta A^{(m)} = \left(V_{f,0} 2\alpha_f T_{f,1}^{(m)} + V_{cl,0} 2\alpha_{cl} T_{cl,1}^{(m)} \right) e^{i\omega t} , \qquad (2\text{-}22)$$

when $\delta_f = 1$, and

$$\Delta A^{(m)} = \left(V_{f,0} + V_{cl,0} \right) 2\alpha_{cl} T_{cl,1}^{(m)} e^{i\omega t} , \qquad (2\text{-}23)$$

when $\delta_f = 0$. The expression in Equation (2-22) represents the condition in which the fuel expands more than the cladding, thus stretching the cladding. Equation (2-23) is for the opposite condition in which the cladding expansion is greater than that of the fuel, leaving a gap between the two. In this case the displacement of coolant is determined by the cladding expansion alone. The user has the option of choosing δ_f to be 0 or 1.

The third term in the numerator of Equation (2-21) is determined by the magnitude of the input quantity δ_{co}, which may have any value. If the reactor is designed in such a way that expansion of the structure does not increase the coolant flow area, δ_{co} should be zero. If the coolant, under all conditions, occupies all of the reactor volume not occupied by fuel, cladding, or structure, and structure expansion determines the core radius in all axial regions, then δ_{co} should be given the value $\left(1 - V_{s,0}\right)$.

The feedback reactivity may be expressed in terms of the material temperatures by substituting the difference quantities obtained from Equations (2-14) through (2-23) into Equation (2-12). Retaining only those terms which contain $e^{i\omega t}$ to the first power permits the reduction of Equation (2-12) to a steady-state equation involving only the amplitudes of the oscillating components of the average temperature, $T^{(m)}_{f,1}$, $T^{(m)}_{cl,1}$, $T^{(m)}_{co,1}$, and $T^{(m)}_{s,1}$. The derivation of these quantities is described in Section 2.3, Material Temperatures.

The equations used in FLUZ are slight modifications of those given above to differentiate between radial and axial expansion coefficients in fuel, cladding, and structure. This permits the user to consider special core designs in which one or all of these materials do not expand axially as solid, continuous volumes of materials. This distinction between coefficients involves Equations (2-14), (2-15), (2-16), (2-19), (2-20), (2-22), and (2-23).

2.3 Material Temperatures

In Section 2.1 above, it is shown how the sinusoidal component of the reactor power can be expressed in terms of the impressed and feedback reactivities. Section 2.2 then relates feedback reactivity to core material temperatures. The present section describes how the material temperatures are related to the reactor power.

To determine the reactor material temperatures as a function of reactor power, it is necessary to define a specific physical model for heat transfer calculations. The model used in FLUZ is shown in Figure 2-1. An average coolant channel is considered, with the fuel in the

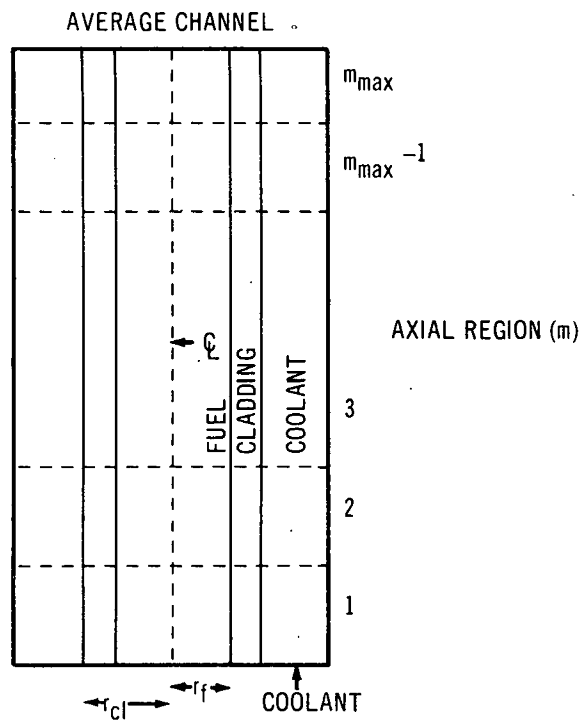


Figure 2-1. Average Channel Heat Transfer Model for FLUZ Calculations

form of a rod, and cladding and coolant forming concentric annuli about it. This average channel is composed of m_{max} axial regions. Heat is transferred from one axial region to another only by movement of the coolant.

An effective conductance, $\bar{h}$, between the fuel surface and the average temperature is used and the heat capacity of the cladding is taken to be zero. The value of $\bar{h}$ is computed from the cladding thermal conductivity, k_{cl}; the thermal conductance across the fuel - cladding gap, h_{gap}; and the thermal conductance from the cladding to the coolant average temperature (including coolant surface effects), h_{co}. Thus

$$\bar{h} = \frac{1}{3600} \left(\frac{1}{h_{gap}} + \frac{r_{cl} - r_f}{k_{cl}} + \frac{1}{h_{co}} \right)^{-1} \tag{2-24}$$

where the quantities r_{cl} and r_f are the outside radii of the cladding and fuel, respectively, as shown in Figure 2-1.

With this value of $\bar{h}$, the oscillating coolant temperature in axial region m can be obtained as a function of the oscillating fuel surface temperature, $T^{(m)}_{surf,1} e^{i\omega t}$, and the coolant temperature at the inlet to region m, $T^{(m)}_{in,1} e^{i\omega t}$, by balancing the temperature and heat content of the coolant. The steady-state heat terms cancel, so the linearized balance equation becomes

$$\frac{d}{dt} T^{(m)}_{co,1} e^{i\omega t} C_{co} \rho_{co} V_{co,0} \Delta Z^{(m)} + V_{co,0}\, 2 \left(T^{(m)}_{co,1} - T^{(m)}_{in,1} \right) v e^{i\omega t} C_{co} \rho_{co} =$$

$$= A^{(m)} P_1 e^{i\omega t} C_h F_{\gamma,co} V_{f,0}\, \Delta Z^{(m)} + \frac{r_f + r_{cl}}{r_{cl}^2} \left(V_{f,0} + V_{cl} \right) \Delta Z^{(m)} \bar{h} \left(T^{(m)}_{surf,1} - T^{(m)}_{co,1} \right) e^{i\omega t}$$

(2-25)

where

C_{co} = Specific heat of coolant (Btu/lb-°F),

v = Velocity of flow of coolant (ft/sec),

C_h = Conversion factor from fissions/cm^3 to Btu/ft^3,

ρ_{co} = Steady-state density of the coolant material (lb/ft^3),

$F_{\gamma, co}$ = Probability that fission energy will be dissipated in the coolant due to radiation (gamma and neutron), and

$A^{(m)}$ = Ratio of power density in region m to core average power density.

Within the fuel a heat balance equation in terms of the radial fuel temperature distribution, $T_f^{(m)}(r)$, for an element of volume at radius r, with a thickness Δr, can be written

$$r \Delta r C_f \rho_f \frac{dT_f^{(m)}(r)}{dt} = C_h F_f A^{(m)} P r \Delta r + k_{fuel} \left(r_1 \frac{dT_1^{(m)}(r)}{dr} - r_2 \frac{dT_2^{(m)}(r)}{dr} \right)$$

$$= C_h F_f A^{(m)} P r \Delta r + k_{fuel} \Delta r \left(\frac{dT_f^{(m)}(r)}{dr} + r \frac{d^2 T_f^{(m)}(r)}{dr^2} \right) . \qquad (2\text{-}26)$$

In Equation (2-26) it is assumed that there is no axial heat transfer and the radial power distribution is constant (flat) across the fuel. The quantities C_f and ρ_f are the fuel counterparts of C_{co} and ρ_{co}, F_f is the probability that the fission energy will be dissipated in the fuel, and k_{fuel} is the thermal conductivity of the fuel. The subscripts 1 and 2 within the parentheses refer to the inner and outer radii of the element of volume at radius r.

The use of (1) $T_f^{(m)}(r)$ and P as defined by Equations (2-13) and (2-6), respectively; (2) the condition that $dT_f^{(m)}(r)/dt = 0$ during steady state; and (3) the following two substitutions,

$$y = T_{f,1}^{(m)}(r) - \frac{C_h F_f P_1 A^{(m)}}{i \omega \rho_f C_f} \qquad (2\text{-}27)$$

and

$$x = \sqrt{-i\omega \frac{\rho_f C_f}{k_{fuel}}}\ r, \tag{2-28}$$

puts Equation (2-26) in the form

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + y = 0. \tag{2-29}$$

The solutions to this differential equation are the zero-order Bessel functions of x. The general solution is

$$y = AJ_0(x) + BY_0(x)$$

$$= T_{f,1}^{(m)}(r) - \frac{C_h F_f P_1 A^{(m)}}{i\omega\ \rho_f\ C_f} \tag{2-30}$$

The Bessel function $J_0(x)$ can be readily evaluated for particular values of r (See Appendix A - Evaluation of Bessel Functions).

Since $Y_0(x) \rightarrow \infty$ as $x \rightarrow 0$, B must be zero in order to have a finite value of y at x = 0. Solving Equation (2-30) for $T_{f,1}^{(m)}(r)$ and substituting r_f for r gives the surface fuel temperature $T_{surf,1}^{(m)}$ with the coefficient A as the only unknown. That is,

$$T_{surf,1}^{(m)} = -i\frac{C_h F_f A^{(m)}}{\omega\ \rho_f\ C_f}\ P_1 + AJ_0\left(\sqrt{i}\sqrt{\frac{\omega\ \rho_f\ C_f}{k_{fuel}}}\ r_f\right). \tag{2-31}$$

The surface boundary condition for Equation (2-30) is given by the balance equation for the excess heat transfer across the fuel surface,

$$2r_f k_{fuel} \left(\frac{dT_{f,1}^{(m)}(r)}{dr} \right)_{r_f} e^{i\omega t} = \bar{h} \left(T_{surf,1}^{(m)} - T_{co,1}^{(m)} \right) \left(r_f + r_{cl} \right) e^{i\omega t} . \tag{2-32}$$

By recognizing that

$$\sqrt{-i}\ \frac{dJ_0(x)}{dx} = -\sqrt{-i}\ J_1(x) ,$$

and substituting $T_{f,1}^{(m)}(r)$ and $T_{surf,1}^{(m)}$ from Equations (2-30) and (2-31), the constant A can be evaluated in terms of the temperature $T_{co,1}^{(m)}$.

Using this expression for A in Equation (2-31) and obtaining expressions for the energy fractions $F_{\gamma,co}$ and F_f would reduce the unknowns to $T_{surf,1}^{(m)}$, $T_{in,1}^{(m)}$ and $T_{co,1}^{(m)}$ with the two independent equations, (2-25) and (2-31).

The probability that fission energy will be dissipated in the coolant due to radiation, $F_{\gamma,co}$, can be approximated by assuming that both neutron and gamma-ray energy absorption rates are proportional to the absorber mass. This gives

$$F_{\gamma,co} = \frac{F_\gamma V_{co,0} \left[\rho_{co} + \delta_\gamma \frac{\rho_{cl} V_{cl,0} + \rho_s V_{s,0}}{V_{co,0}} \right]}{\rho_f V_{f,0} + \rho_{cl} V_{cl,0} + \rho_{co} V_{co,0} + \rho_s V_{s,0}} \tag{2-33}$$

where

F_γ = Fraction of fission energy released to gamma rays and neutrons, and

δ_γ = 1 or 0, depending upon whether it is assumed the radiation energy dissipated in the cladding and structure is transferred immediately to the coolant.

Then F_f, the fraction of fission energy which is deposited in the fuel, can be approximated by

$$F_f = \left(1-F_\gamma\right) + \frac{F_\gamma V_{f,0}\rho_f}{\rho_f V_{f,0} + \rho_{cl} V_{cl,0} + \rho_{co,0} + \rho_s V_{s,0}} . \tag{2-34}$$

If axial region 1 is taken to be at the inlet end of the core, the oscillating component of the inlet temperature, $T_{in,1}^{(m=1)}$, can be set at zero, since there is assumed to be no feedback outside the core. Thus, Equations (2-25) and (2-31) can be used to determine the ratios

$$\frac{T_{surf,1}^{(m=1)}}{P_1} \text{ and } \frac{T_{co,1}^{(m=1)}}{P_1} .$$

For subsequent regions the inlet temperatures can be computed from the coolant temperatures in the previous regions,

$$T_{in,1}^{(m)} \doteq T_{in,1}^{(m-1)} + 2\left(T_{co,1}^{(m-1)} - T_{in,1}^{(m-1)}\right)$$

$$= \sum_{m'=1}^{m-1} (-1)^{m'+1}\, 2T_{co,1}^{(m-m')} . \tag{2-35}$$

The oscillating component of the average fuel temperature $\left(T_{f,1}^{(m)}\right)$ in each axial region may now be obtained by considering the total heat balance in the fuel rod,

$$r_f^2 C_f \rho_f \frac{dT_{f,ave}^{(m)}}{dt} = C_h F_f r^2 A^{(m)} P - \left(r_f + r_c\right) \bar{h}\left(T_{surf}^{(m)} - T^{(m)}\right) . \tag{2-36}$$

However, Equation (2-36) reduces to

$$i\omega r_f^2 C_f \rho_f T_{f,1}^{(m)} = C_h F_f r_f^2 A^{(m)} P_1 - \left(r_f + r_{cl}\right) \bar{h} \left(T_{surf,1}^{(m)} - T_{co,1}^{(m)}\right) \qquad (2\text{-}37)$$

when the steady state terms are removed. Equation (2-37) is used to compute

$$\frac{T_{f,1}^{(m)}}{P_1} .$$

The amplitudes of the oscillating components of the average cladding and coolant temperatures are taken to be the mid-cladding temperature and a factor times $T_{co,1}^{(m)}$, respectively. The equations are

$$\frac{T_{cl,1}^{(m)}}{P_1} = \left[1 - 3600\,\bar{h}\left(\frac{r_{cl}-r_f}{2k_{clad}} + \frac{1}{h_{co}}\right)\right]T_{co,1}^{(m)} + 3600\,\bar{h}\left[\frac{r_{cl}-r_f}{2k_{clad}} + \frac{1}{h_{co}}\right]T_{surf,1}^{(m)} \qquad (2\text{-}38)$$

and

$$\frac{T_{s,1}^{(m)}}{P_1} = \delta_s \, \frac{T_{co,1}^{(m)}}{P_1} . \qquad (2\text{-}39)$$

These ratios of amplitudes of the oscillating components of the average fuel materials to the amplitude of the oscillating power density are used in conjunction with Equations (2-12) and (2-13) to get the ratio of

$$\frac{k_f(i\omega)}{P_1(i\omega)} .$$

This, in turn, is used in Equation (2-3) to determine the feedback transfer function, $H(i\omega)$, and the power transfer function, $G(i\omega)$. From Equation (2-2), then, the ratio $P_1(i\omega)/P_0$ and P_1 may be obtained, followed by computations of k_f, $T^{(m)}_{f,1}$, $T^{(m)}_{cl,1}$, $T^{(m)}_{co,1}$, $T^{(m)}_{s,1}$ and $T^{(m)}_{surf,1}$.

2.4 Applications and Limitations*

FLUZ was designed for fast reactor studies and consequently incorporates certain assumptions which make it less applicable to thermal reactors. Also, some feedback and temperature calculations must, of necessity, assume specific designs for the average coolant channel and core structure. Options have been included to allow for some variation in these designs but the number of choices was limited. In the following paragraphs some of the more important assumptions and design choices are described.

1. A constant thermal conductivity in fuel is assumed. It is suggested that the average value during steady-state operation at the power level P_0 be used.

2. The heat capacities of the cladding and structure are ignored. These could be approximated by increasing the heat capacity of the coolant.

3. A constant radial power distribution across the fuel rod is assumed. This may be a poor approximation for a thermal reactor.

4. Presence of a moderating material separate from the coolant is ignored in the heat transfer calculations.

5. The linearity assumption of feedback reactivity and temperature relationships limits the application of FLUZ to cases where $P_1 \ll P_0$.

6. Either fuel or cladding expansion must determine fuel rod radial expansion at all times; a combination is not possible. If $\delta_f = 0$, the inside cladding radius is assumed to be greater than the fuel radius at all times. If $\delta_f = 1$, radial expansion of the fuel is assumed to be greater than that of the cladding for all temperature combinations. It is not possible to have the fuel rod expansion be due to cladding expansion for part of the cycle and due to fuel expansion during the remainder of the cycle.

---

*In addition to the limitations listed here, a numerical instability has been found which further limits application of the code. For some cases involving more than one axial section, this instability causes the oscillating component of the coolant temperature to alternate in sign from one section to the next. This effect was found to be insignificant for those cases where the coolant temperature coefficient is unimportant. It is recommended, therefore, that the code in its present form be used either with only a single axial section or with more than one axial section only when the influence of the coolant temperature effect may be disregarded.

7. In a given axial region the core radial expansion must be due to either structural expansion within the region, or due to expansion in other regions at all times. Similar to the choice of δ_f, a combination of core expansion due to structural expansion within the region during part of the cycle and core expansion due to expansion outside the region during the remainder of the cycle is not permitted.

8. Radial expansion in the top and bottom axial regions must be due to structural expansion within those regions, i.e., $\delta_{R_c}^{(m=1)}$ and $\delta_{R_c}^{(m=m_{max})}$ must be 1.

9. Any fraction of the increased core volume due to radial expansion may be filled with coolant. If $\delta_{co} = 0$, the oscillating component of the coolant volume fraction will have an amplitude of

$$\mathcal{V}_{co,1}^{(m)} = \frac{\left(V_{co}^{(m)}\right)^{initial} - \Delta V_{fuel}^{(m)} - \Delta V_{clad}^{(m)}}{V_{total}^{(m)}} .$$

This corresponds to a case where the total coolant flow area does not change with radial core expansion. In the other extreme, if the coolant occupies all of the increased core volume, the $\mathcal{V}_{co,1}^{(m)}$ becomes

$$\mathcal{V}_{co,1}^{(m)} = \frac{\left(V_{co}^{(m)}\right)^{initial} + \Delta V_{total}^{(m)} - \Delta V_{fuel}^{(m)} - \Delta V_{clad}^{(m)} - \Delta V_{structure}^{(m)}}{V_{total}^{(m)}} .$$

This condition is achieved if $\delta_{co} = 1 - \mathcal{V}_{s,0}$ and $\delta_{R_c}^{(m)} = 1$ for all m.

10. Separate radial and axial thermal expansion coefficients may be used for fuel, cladding, and structure. This option permits the user to include special reactor designs, such as fuel which is segmented to reduce axial expansion.

11. Reactivity coefficients must be computed in such a way as to be consistent with the feedback reactivity, Equation (2-12). The mass coefficients of reactivity should be computed by changing only the density of the material concerned while holding the reactor size constant. The mass and bowing coefficients should be computed individually for each axial region. The Doppler coefficient may or may not be computed individually for each axial region. The core axial and radial expansion coefficients are geometry effects only and should be computed for the whole core, holding all compositions constant.

12. Feedback effects outside of the core are ignored.

The computer running time for a series of FLUZ cases may be estimated by using the following formula:

$$t(\text{min}) = 0.124 + 0.0145 \sum_{p=1}^{P} \left[1 + 0.763 \left(m_{max}^{(p)} - 1 \right) \right] (NR)^{(p)} \quad ,$$

where $m_{max}^{(p)}$ = Number of axial regions in case p, and

$(NR)^{(p)}$ = Number of angular velocities, ω, in case p.

SECTION III

CODE DESCRIPTION

3.1 Input Quantities

NR	Number of angular velocities
m_{max}	Number of axial regions
δ_f	= 0 if expansion of fuel < expansion of clad = 1 if expansion of fuel > expansion of clad
δ_{Dop}	= 1 if $\left(\frac{\partial k}{\partial T_f}\right)^{(m)}_{Dop}$ has been calculated for each region = 0 if $\left(\frac{\partial k}{\partial T_f}\right)^{(m)}_{Dop} = \left(\frac{\partial k}{\partial T_f}\right)^{\text{Total core}}_{Dop}$
δ_{co}	Fraction of increased volume of core which is filled in with coolant (maximum = 1) = 0 when radial expansion of core results only in increased void in core
δ_γ	= 1 if gamma and neutron heating in cladding and structure is treated as heating of coolant (approximation for low frequency oscillations) = 0 if gamma and neutron heating in cladding and structure is not treated as heating of coolant
$\delta^{(m)}_{R_c}$	= 0 if expansion of structure in region m <u>does</u> <u>not</u> cause radial expansion of core = 1 if expansion of structure in region m <u>does</u> cause radial expansion of core
β_j	j^{th} group delayed neutron fraction
λ_j	j^{th} group decay constant (sec^{-1})
P_0	Steady-state power density in fuel (at k = 1.0) $(\text{fissions/sec-cm}^3)$
k_1	Amplitude of impressed oscillating reactivity (Δk)
ℓ	Neutron lifetime (sec)
F_γ	Fraction of fission energy radiated in γ-rays and neutrons
v	Velocity of coolant (ft/sec)

C_h	Conversion factor $\left(\frac{\text{Btu/sec-ft}^3}{\text{fissions/sec-cm}^3}\right)$
$\left(H_{c,0} \frac{\partial k}{\partial H_{c,0}}\right)$	Axial expansion coefficient of reactivity (geometry effect only) ($\Delta k/k/\Delta ft/ft$)
$\left(R_{c,0} \frac{\partial k}{\partial R_{c,0}}\right)$	Radial expansion coefficient of reactivity (geometry effect only) ($\Delta k/k/\Delta ft/ft$)
$\left(\frac{\partial k}{\partial T_f}\right)^{(m)}_{Dop}$	Doppler coefficient of reactivity for region m ($\Delta k/k/°F$)
$\Delta Z^{(m)}$	Height of m^{th} region (ft)
$A^{(m)}$	Ratio of average power density in region m to average in reactor
ω	Angular velocity of impressed reactivity (up to 20 values) (radians/sec)
$V_{f,0}$	Volume fraction of fuel in steady state reactor
ρ_f	Mass density of fuel (lb/ft^3)
k_{fuel}	Conductivity of fuel (Btu/h-ft-°F)
h_{gap}	Heat conductance fuel-to-cladding gap ($Btu/h\text{-}ft^2\text{-}°F$)
r_f	Outer radius of fuel rod (ft)
C_f	Specific heat of fuel (Btu/lb-°F)
$\alpha_{f,ax}$	Fuel axial coefficient of thermal expansion ($\Delta ft/ft/°F$)
$\alpha_{f,r}$	Fuel radial coefficient of thermal expansion ($\Delta ft/ft/°F$)
$\left(N'_{f,0} \frac{\partial k}{\partial N'_{f,0}}\right)^{(m)}$	Fuel mass coefficient of reactivity for region m($\Delta k/k/\Delta \rho/\rho$)
$\left(\frac{\partial k}{\partial d_c}\right)^{(m)}$	Fuel deflection (bowing) coefficient of reactivity for region m ($\Delta k/k/ft$)
$\left(\frac{\partial d_c}{\partial T_f}\right)^{(m)}$	Fuel temperature coefficient of deflection (bowing) for region m (ft/°F)
$V_{cl,0}$	Volume fraction of cladding in steady state reactor

ρ_{cl}	Mass density of cladding $\left(lb/ft^3\right)$
k_{clad}	Heat conductivity of cladding (Btu/h-ft-°F)
h_{co}	Heat conductance cladding-to-coolant gap (Btu/h-ft^2-°F)
r_{cl}	Outer radius of cladding (ft)
$\alpha_{cl, ax}$	Cladding axial coefficient of thermal expansion (Δft/ft/°F)
$\alpha_{cl, r}$	Cladding radial coefficient of thermal expansion (Δft/ft/°F)
$\left(N'_{cl,0} \frac{\partial k}{\partial N'_{cl,0}}\right)^{(m)}$	Cladding mass coefficient of reactivity for region m (Δk/k/Δρ/ρ)
$\left(\frac{\partial d_c}{\partial T_{cl}}\right)^{(m)}$	Cladding temperature coefficient of deflection (bowing) for region m (ft/°F)
$V_{co,0}$	Volume fraction of coolant in steady state reactor
ρ_{co}	Mass density of coolant $\left(lb/ft^3\right)$
C_{co}	Specific heat of coolant (Btu/lb-°F)
α_{co}	Coolant coefficient of thermal expansion (Δft/ft/°F)
$\left(N'_{co,0} \frac{\partial k}{\partial N'_{co,0}}\right)^{(m)}$	Coolant mass coefficient of reactivity for region m (Δk/k/Δρ/ρ)
$V_{s,0}$	Volume fraction of structure in steady state reactor
ρ_s	Mass density of structure $\left(lb/ft^3\right)$
$\alpha_{s, ax}$	Structure axial coefficient of thermal expansion (Δft/ft/°F)
$\alpha_{s, r}$	Structure radial coefficient of thermal expansion (Δft/ft/°F)
$\left(N'_{s,0} \frac{\partial k}{\partial N'_{s,0}}\right)^{(m)}$	Structure mass coefficient of reactivity for region m (Δk/k/Δρ/ρ)
δ_s	Ratio of amplitude of structure temperature to amplitude of coolant temperature

3.2 Card Formats

Input data for FLUZ must be entered in fixed field formats, and data sheets are available for this use. However, FORTRAN sheets may be used for input by following the instructions given below. A listing of the descriptions of the input quantities is provided in Section 3.1.

As many as 20 angular velocities (ω) may be included in one problem. A maximum of 20 axial nodes (regions) may be used. Overlay cases may be run by putting a "(" in column 1 of the case card, followed by the instruction card plus any input data which are to be changed. An overlay case refers to the case immediately preceeding it. Each case to be run must have a case card, an instruction card, data cards and a case end card. A LAST card must follow the final case of a run.

1. Case Card

Columns	Description
1	) for an independent case (for a dependent case
2-5	FLUZ
6-8	User's initials
10-14	Case number
15-20	000000
21-24	Charge number
28-34	Date
57	Y

2. Instruction Card

1-4	FLUZ
5-7	User's initials
8-9	D*
10-14	Case number
15-20	000000
23-24	NR, Number of angular velocities
27-28	m_{max}, Number of axial regions

3. Data Cards

Columns 1-14 of all data cards are identical to columns 1-14 on the instruction card. Columns 15-18 contain the vector location of the first input quantity on the card. A card sequence number may, or may not, be put in columns 19-20 at the discretion of the user. Each data card contains 4 input quantities in the fixed fields defined by columns 22-33, 35-46, 48-59, and 61-72.

All data must be in floating point form with a decimal point included. If an exponential form is used, the exponent must be at the extreme right side of the field.

The following list of data cards gives the vector location needed for each card plus the input quantities which go on that card. Four quantities are read from each card and stored in the vector locations indicated by the card. Thus, omission of a quantity will result in a zero being stored in that location of memory. Quantities which are axial region dependent are superscripted by an m. If more than four regions are to be considered, the additional data cards needed for these quantities must be provided. Each region dependent quantity forms an array in the order of ascending region number. The vector location of each additional card needed for an array must be 4 greater than the vector location of the preceding card.

The content of data cards is as shown in Table 3-1.

4. Case End Card

Columns	Description
1-14	Same as Data Cards
15-20	999999

5. LAST Card

Columns	Description
1	)
2-5	LAST
9	*

TABLE 3-1

DATA CARD CONTENT

Vector Location, Columns 15-18	Sequence Number, Columns 19-20	Data Columns: 22 through 33	35 through 46	48 through 59	61 through 72
0805	- -	δ_f	δ_{Dop}	δ_{co}	δ_γ
0809	- -	$\delta_{R_c}^{(m=1)}$	$\delta_{R_c}^{(m=2)}$	$\delta_{R_c}^{(m=3)}$	$\delta_{R_c}^{(m=4)}$
0829	- -	β_1	β_2	β_3	β_4
0833	- -	β_5	β_6	β_7	β_8
0837	- -	λ_1	λ_2	λ_3	λ_4
0841	- -	λ_5	λ_6	λ_7	λ_8
0845	- -	P_0	k_1	ℓ	F_γ
0849	- -	v	C_h	$(H_{c,0}\ \partial k/\partial H_{c,0})$	$(R_{c,0}\ \partial k/\partial R_{c,0})$
0853	- -	$(\partial k/\partial T_f)_{Dop}^{(m=1)}$	$(\partial k/dt_f)_{Dop}^{(m=2)}$	$(\partial k/\partial T_f)_{Dop}^{(m=3)}$	$(\partial k/\partial T_f)_{Dop}^{(m=4)}$
0873	- -	$\Delta Z^{(m=1)}$	$(\Delta Z)^{(m=2)}$	$(\Delta Z)^{(m=3)}$	$(\Delta Z)^{(m=4)}$
0893	- -	$A^{(m=1)}$	$A^{(m=2)}$	$A^{(m=3)}$	$A^{(m=4)}$
0913	- -	ω_1	ω_2	ω_3	ω_4
thru					
0929	- -	ω_{17}	ω_{18}	ω_{19}	ω_{20}
0933	- -	$\bar{V}_{f,0}$	ρ_f	k_{fuel}	h_{gap}
0937	- -	r_f	C_f	$\alpha_{f,ax}$	$\alpha_{f,r}$
0941	- -	$(N'_{f,0}\partial k/\partial N'_{f,0})^{(m)}$	←	for m=1 through 4	→
0961	- -	$\partial k/\partial d_c\ ^{(m)}$	←	for m=1 through 4	→
0981	- -	$\partial d_c/\partial T_f\ ^{(m)}$	←	for m=1 through 4	→
1001	- -	$\bar{V}_{cl,0}$	ρ_{cl}	k_{clad}	h_{co}
1005	- -	r_{cl}	$\alpha_{cl,ax}$	$\alpha_{cl,r}$	
1009	- -	$(N'_{cl,0}\partial k/\partial N'_{cl,0})^{(m)}$	←	for m=1 through 4	→

TABLE 3-1 (continued)

Vector Location Columns 15-18	Sequence Number, Columns 19-20	Data Columns			
		22 through 33	35 through 46	48 through 59	61 through 72
1029	- -	$(\partial d_c/\partial T_{cl})^{(m)}$	←	for m=1 through 4	→
1049	- -	$\bar{V}_{co,0}$	ρ_{co}	C_{co}	α_{co}
1053	- -	$(N'_{co,0}\partial k/\partial N'_{co,0})^{(m)}$	←	for m=1 through 4	→
1073	- -	$\bar{V}_{s,0}$	ρ_s	$\alpha_{s,ax}$	$\alpha_{s,r}$
1077	- -	$(N'_{s,0}\partial k/\partial N'_{s,0})^{(m)}$	←	for m=1 through 4	→
1097	- -	δ_s			

3.3 Coded Equations

$$\bar{h} = \frac{1}{3600\left[\frac{1}{h_{gap}} + \frac{r_{cl}-r_f}{k_{clad}} + \frac{1}{h_{co}}\right]} \tag{3-1}$$

$$\beta = \sum_{j=1}^{8} \beta_j \tag{3-2}$$

$$G_0^{(r)} = \frac{\left[1 - \frac{1}{\beta}\left(\sum_{j=1}^{8} \frac{\beta_j \lambda_j^2}{\lambda_j^2 + \omega^2}\right)\right]}{\left\{\left[1 - \frac{1}{\beta}\left(\sum_{j=1}^{8} \frac{\beta_j \lambda_j^2}{\lambda_j^2 + \omega^2}\right)\right]^2 + \left[\frac{\omega}{\beta} \sum_{j=1}^{8} \frac{\beta_j \lambda_j}{\lambda_j^2 + \omega^2} + \frac{\omega}{\beta} \ell\right]^2\right\}} \tag{3-3}$$

$$G_0^{(i)} = - \frac{\left[\frac{\omega}{\beta} \sum_{j=1}^{8} \frac{\beta_j \lambda_j}{\lambda_j^2 + \omega^2} + \frac{\omega}{\beta} \ell\right]}{\left\{\left[1 - \frac{1}{\beta}\left(\sum_{j=1}^{8} \frac{\beta_j \lambda_j}{\lambda_j^2 + \omega^2}\right)\right]^2 + \left[\frac{\omega}{\beta} \sum_{j=1}^{8} \frac{\beta_j \lambda_j}{\lambda_j^2 + \omega^2} + \frac{\omega}{\beta} \ell\right]^2\right\}} \tag{3-4}$$

$$H_{c,0} = \sum_{m=1}^{m_{max}} \Delta Z^{(m)} \tag{3-5}$$

$$\Delta B = \left(\delta_f \bar{V}_{cl,0} + \bar{V}_{f,0} + \bar{V}_{cl,0} - \delta_f \bar{V}_{f,0} - \delta_f \bar{V}_{cl,0}\right) \tag{3-6}$$

$$Z = \sum_{m=1}^{m_{max}} \Delta Z^{(m)} \left(A^{(m)}\right)^2 \qquad (3\text{-}7)$$

$$X_f = 60 \left(\frac{\omega \rho_f C_f}{k_{fuel}} \right)^{1/2} r_f \qquad (3\text{-}8)$$

$$\alpha_1 = \left(\frac{2k_{fuel} X_f}{3600 \sqrt{2}} \right) \left(-\mathrm{ber}_1 X_f - \mathrm{bei}_1 X_f \right) + \left(r_f + r_{cl} \right) \bar{h} \, \mathrm{ber}_0 X_f \qquad (3\text{-}9)^*$$

$$\mu_1 = \left(r_f + r_{cl} \right) \bar{h} \, \mathrm{bei}_0 X_f + \left(\frac{2k_{fuel} X_f}{3600 \sqrt{2}} \right) \left(\mathrm{ber}_1 X_f - \mathrm{bei}_1 X_f \right) \qquad (3\text{-}10)^*$$

$$D_\gamma = \rho_f \overline{V}_{f,0} + \rho_{cl} \overline{V}_{cl,0} + \rho_{co} \overline{V}_{co,0} + \rho_s \overline{V}_{s,0} \qquad (3\text{-}11)$$

$$F_{\gamma,co} = \frac{F_\gamma \overline{V}_{co,0}}{D_\gamma} \left[\rho_{co} + \delta_\gamma \left(\frac{\rho_{cl} \overline{V}_{cl,0} + \rho_s \overline{V}_{s,0}}{\overline{V}_{co,0}} \right) \right] \qquad (3\text{-}12)$$

$$F_f = \left(1 - F_\gamma \right) + \frac{F_\gamma \overline{V}_{f,0} \rho_f}{D_\gamma} \qquad (3\text{-}13)$$

$$E_{co}^{(m)} = 2v \frac{C_{co} \rho_{co}}{\Delta Z^{(m)}} + \left(r_f + r_{cl} \right) \bar{h} \frac{\left(\overline{V}_{f,0} + \overline{V}_{cl,0} \right)}{r_{cl}^2 \overline{V}_{co,0}} \qquad (3\text{-}14)$$

$$F_{co}^{(m)} = \omega C_{co} \rho_{co} \qquad (3\text{-}15)$$

*See Appendix A for evaluation of ber and bei functions.

$$G_{co}^{(m)} = A^{(m)} C_h \, F_{\gamma, co} \, \frac{\mathcal{V}_{f,0}}{\mathcal{V}_{co,0}} \tag{3-16}$$

$$D_{co}^{(m)} = \left(E_{co}^{(m)}\right)^2 + \left(F_{co}^{(m)}\right)^2 \tag{3-17}$$

$$X_{co}^{(m)} = \frac{G_{co}^{(m)} E_{co}^{(m)}}{D_{co}^{(m)}} \tag{3-18}$$

$$Y_{co}^{(m)} = \frac{G_{co}^{(m)} F_{co}^{(m)}}{D_{co}^{(m)}} \tag{3-19}$$

$$S_{co}^{(m)} = \frac{2v \, \dfrac{C_{co} \rho_{co}}{\Delta Z^{(m)}} \, E_{co}^{(m)}}{D_{co}^{(m)}} \tag{3-20}$$

$$W_{co}^{(m)} = \frac{2v \dfrac{C_{co} \rho_{co}}{\Delta Z^{(m)}} F_{co}^{(m)}}{D_{co}^{(m)}} \tag{3-21}$$

$$U_{co}^{(m)} = \frac{\left(r_f + r_{cl}\right) \dfrac{\left(\mathcal{V}_{f,0} + \mathcal{V}_{cl,0}\right)}{r_{cl}^2 \mathcal{V}_{co,0}} \bar{h} \, E_{co}^{(m)}}{D_{co}^{(m)}} \tag{3-22}$$

$$V_{co}^{(m)} = \frac{\left(r_f + r_{cl}\right) \left(\dfrac{\mathcal{V}_{f,0} + \mathcal{V}_{cl,0}}{r_{cl}^2 \mathcal{V}_{co,0}}\right) \bar{h} \, F_{co}^{(m)}}{D_{co}^{(m)}} \tag{3-23}$$

$$X_A^{(m)} = \frac{\left[\bar{h}\,\alpha_1\, X_{co}^{(m)} + \mu_1 \frac{\bar{h}\, C_h F_f A^{(m)}}{\omega \rho_f C_f} - \mu_1\, \bar{h}\, Y_{co}^{(m)}\right]\left(r_f + r_{cl}\right)}{\alpha_1^2 + \mu_1^2} \tag{3-24}$$

$$Y_A^{(m)} = \frac{\left[\bar{h}\,\alpha_1\, Y_{co}^{(m)} - \alpha_1 \frac{\bar{h}\, C_h F_f A^{(m)}}{\omega \rho_f C_f} + \mu_1\, \bar{h}\, X_{co}^{(m)}\right]\left(r_f + r_{cl}\right)}{\alpha_1^2 + \mu_1^2} \tag{3-25}$$

$$S_A^{(m)} = \frac{\left[\bar{h}\,\alpha_1\, S_{co}^{(m)} - \mu_1\, \bar{h}\, W_{co}^{(m)}\right]\left(r_f + r_{cl}\right)}{\alpha_1^2 + \mu_1^2} \tag{3-26}$$

$$W_A^{(m)} = \frac{\left[\bar{h}\,\alpha_1\, W_{co}^{(m)} + \mu_1\, \bar{h}\, S_{co}^{(m)}\right]\left(r_f + r_{cl}\right)}{\alpha_1^2 + \mu_1^2} \tag{3-27}$$

$$U_A^{(m)} = \frac{\left[\bar{h}\,\alpha_1\, U_{co}^{(m)} - \mu_1\, \bar{h}\, V_{co}^{(m)}\right]\left(r_f + r_{cl}\right)}{\alpha_1^2 + \mu_1^2} \tag{3-28}$$

$$V_A^{(m)} = \frac{\left[\bar{h}\,\alpha_1\, V_{co}^{(m)} + \mu_1\, \bar{h}\, U_{co}^{(m)}\right]\left(r_f + r_{cl}\right)}{\alpha_1^2 + \mu_1^2} \tag{3-29}$$

$$D_{surf} = \left[1 - U_A^{(m)}\, \mathrm{ber}_0\, X_f - V_A^{(m)}\, \mathrm{bei}_0\, X_f\right]^2 + \left[U_A^{(m)}\, \mathrm{bei}_0\, X_f - V_A^{(m)}\, \mathrm{ber}_0\, X_f\right]^2 \tag{3-30}$$

$$X_{surf}^{(m)} = \frac{\left[X_A^{(m)} ber_0 X_f + Y_A^{(m)} bei_0 X_f\right]\left[1 - U_A^{(m)} ber_0 X_f - V_A^{(m)} bei_0 X_f\right]}{D_{surf}^{(m)}}$$

$$+ \frac{\left[\frac{C_h F_f A^{(m)}}{\omega \rho_f C_f} - X_A^{(m)} bei_0 X_f + Y_A^{(m)} ber_0 X_f\right]\left[U_A^{(m)} bei_0 X_f - V_A^{(m)} ber_0 X_f\right]}{D_{surf}^{(m)}} \qquad (3\text{-}31)$$

$$Y_{surf}^{(m)} = \frac{-\left[X_A^{(m)} ber_0 X_f + Y_A^{(m)} bei_0 X_f\right]\left[U_A^{(m)} bei_0 X_f - V_A^{(m)} ber_0 X_f\right]}{D_{surf}^{(m)}}$$

$$+ \frac{\left[\frac{C_h F_f A^{(m)}}{\omega \rho_f C_f} - X_A^{(m)} bei_0 X_f + Y_A^{(m)} ber_0 X_f\right]\left[1 - U_A^{(m)} ber_0 X_f - V_A^{(m)} bei_0 X_f\right]}{D_{surf}^{(m)}} \qquad (3\text{-}32)$$

$$S_{surf}^{(m)} = \frac{\left[S_A^{(m)} ber_0 X_f + W_A^{(m)} bei_0 X_f\right]\left[1 - U_A^{(m)} bei_0 X_f - V_A^{(m)} ber_0 X_f\right]}{D_{surf}^{(m)}}$$

$$+ \frac{\left[W_A^{(m)} ber_0 X_f - S_A^{(m)} bei_0 X_f\right]\left[U_A^{(m)} bei_0 X_f - V_A^{(m)} ber_0 X_f\right]}{D_{surf}^{(m)}} \qquad (3\text{-}33)$$

$$W_{surf}^{(m)} = \frac{-\left[S_A^{(m)} ber_0 X_f + W_A^{(m)} bei_0 X_f\right]\left[U_A^{(m)} bei_0 X_f - V_A^{(m)} ber_0 X_f\right]}{D_{surf}^{(m)}}$$

$$+ \frac{\left[W_A^{(m)} ber_0 X_f - S_A^{(m)} bei_0 X_f\right]\left[1 - U_A^{(m)} ber_0 X_f - V_A^{(m)} bei_0 X_f\right]}{D_{surf}^{(m)}} \qquad (3\text{-}34)$$

$$A_{cp}^{(m)} = X_{co}^{(m)} + U_{co}^{(m)} X_{surf}^{(m)} - V_{co}^{(m)} Y_{surf}^{(m)} \tag{3-35}$$

$$B_{cp}^{(m)} = Y_{co}^{(m)} + V_{co}^{(m)} X_{surf}^{(m)} + U_{co}^{(m)} Y_{surf}^{(m)} \tag{3-36}$$

$$A_{ct}^{(m)} = S_{co}^{(m)} + U_{co}^{(m)} S_{surf}^{(m)} - V_{co}^{(m)} W_{surf}^{(m)} \tag{3-37}$$

$$B_{ct}^{(m)} = W_{co}^{(m)} + V_{co}^{(m)} S_{surf}^{(m)} + U_{co}^{(m)} W_{surf}^{(m)} \tag{3-38}$$

$$A_{fp}^{(m)} = \frac{\left(r_f + r_{cl}\right)\bar{h}}{\omega r_f^2 C_f \rho_f} \left[Y_{surf}^{(m)} - B_{cp}^{(m)}\right] \tag{3-39}$$

$$B_{fp}^{(m)} = \frac{C_h F_f A^{(m)}}{\omega C_f \rho_f} - \frac{\bar{h}\left(r_f + r_{cl}\right)}{\omega r_f^2 C_f \rho_f} \left[X_{surf}^{(m)} - A_{cp}^{(m)}\right] \tag{3-40}$$

$$A_{ft}^{(m)} = \frac{\left(r_f + r_{cl}\right)\bar{h}}{\omega r_f^2 C_f \rho_f} \left[W_{surf}^{(m)} - B_{ct}^{(m)}\right] \tag{3-41}$$

$$B_{ft}^{(m)} = -\frac{\bar{h}\left(r_f + r_{cl}\right)}{\omega r_f^2 C_f \rho_f} \left[S_{surf}^{(m)} - A_{ct}^{(m)}\right] \tag{3-42}$$

$$\Delta C_f^{(m)} = \left(\frac{\partial k}{\partial T_f}\right)_{Dop}^{(m)} \left\{ \delta_{Dop} + \left(1 - \delta_{Dop}\right) \frac{\Delta Z^{(m)} \left[A^{(m)}\right]^2}{Z} \right\}$$

$$- \left[\left(N'_{f,0} \frac{\partial k}{\partial N'_{f,0}}\right)^{(m)} \alpha_{f,ax} + \delta_f \frac{V_{f,o}}{V_{co,0}} 2\alpha_{f,r} \left(N'_{co,0} \frac{\partial k}{\partial N'_{co,0}}\right)^{(m)}\right]$$

$$+ \left(H_{c,0} \frac{\partial k}{\partial H_{c,0}}\right)^{(m)} \frac{\alpha_{f,ax} \Delta Z^{(m)}}{H_{c,0}} + \left(\frac{\partial k}{\partial d_c}\right)^{(m)} \left(\frac{\partial d_c}{\partial T_f}\right)^{(m)} \tag{3-43}$$

$$\Delta C_{cl}^{(m)} = - \left[\left(N'_{cl,0} \frac{\partial k}{\partial N'_{cl,0}} \right)^{(m)} \alpha_{cl,ax} + \left(N'_{co,0} \frac{\partial k}{\partial N'_{co,0}} \right)^{(m)} \frac{\Delta B}{V_{co,0}} 2\alpha_{cl,r} \right]$$

$$+ \left(\frac{\partial k}{\partial d_c} \right)^{(m)} \left(\frac{\partial d_c}{\partial T_{cl}} \right)^{(m)} \tag{3-44}$$

$$\Delta C_{co}^{(m)} = - \left(N'_{co,0} \frac{\partial k}{\partial N'_{co,0}} \right)^{(m)} 3\alpha_{co} \tag{3-45}$$

$$\Delta C_{s}^{(m)} = - \left\{ \left(N'_{s,0} \frac{\partial k}{\partial N'_{s,0}} \right)^{(m)} \left[\alpha_{s,ax} + \delta_{R_c}^{(m)} 2\alpha_{s,r} \right] \right.$$

$$+ \left(N'_{f,0} \frac{\partial k}{\partial N'_{f,0}} \right)^{(m)} 2\delta_{R_c}^{(m)} \alpha_{s,r} + \left(N'_{cl,0} \frac{\partial k}{\partial N'_{cl,0}} \right)^{(m)} 2\delta_{R_c}^{(m)} \alpha_{s,r}$$

$$\left. + \left(N'_{co,0} \frac{\partial k}{\partial N'_{f,0}} \right)^{(m)} \left(1 - \frac{\delta_{co}}{V_{co,0}} \right) 2\alpha_{s,r} \delta_{R_c}^{(m)} \right\}$$

$$+ \left(R_{c,0} \frac{\partial k}{\partial R_{c,0}} \right) \frac{\alpha_{s,r} \delta_{R_c}^{(m)} \Delta Z^{(m)} \left[A^{(m)} \right]^2}{Z} \tag{3-46}$$

When $\delta_{R_c}^{(m)} = 0$ for a given m, $m_1^{(m)}$, and $m_2^{(m)}$, are determined, where $m_1^{(m)}$ and $m_2^{(m)}$ are the closest regions to m in which $\delta_{R_c}^{(m)} = 1$, with $m_1^{(m)} < m$ and $m_2^{(m)} > m$. It is a requirement that at least $\delta_{R_c}^{(m=1)} = 1$ and $\delta_{R_c}^{(m=m_{max})} = 1$. The value of $m_1^{(m)}$ is found by successively taking the regions (m-n) where n = 1, 2, 3, . . . and testing for $\delta_{R_c}^{(m-n)} = 1$. The value of $m_2^{(m)}$ is found by successively taking the regions (m + n) where n = 1, 2, 3 . . . and testing for $\delta_{R_c}^{(m+n)} = 1$.

$$\Delta L^{(m)} = \frac{1/2\left\{\Delta Z\left[m_1^{(m)}\right] + \Delta Z^{(m)}\right\} + \sum_{m'=m_1^{(m)}+1}^{m-1} \Delta Z^{(m')}}{1/2\left\{\Delta Z\left[m_1^{(m)}\right] + \Delta Z\left[m_2^{(m)}\right]\right\} + \sum_{m'=m_1^{(m)}+1}^{m_2^{(m)}-1} \Delta Z^{(m')}} \tag{3-47}$$

$$\Delta C_s^{\left[m_2^{(m)}\right]} = -\left\{\left(N'_{f,0}\frac{\partial k}{\partial N'_{f,0}}\right)^{(m)} + \left(N'_{cl,0}\frac{\partial k}{\partial N'_{cl,0}}\right)^{(m)}\right.$$

$$+ \left(N'_{co,0}\frac{\partial k}{\partial N'_{co,0}}\right)^{(m)}\left(1 - \frac{\delta_{co}}{V_{co,0}}\right)$$

$$+ \left(N'_{s,0}\frac{\partial k}{\partial N'_{s,0}}\right)^{(m)}\left.\right\}\left[1 - \delta_{R_c}^{(m)}\right]2\alpha_{s,r}\,\Delta L^{(m)}$$

$$+ \left(R_{c,0}\frac{\partial k}{\partial R_{c,0}}\right)\frac{\left[1-\delta_{R_c}^{(m)}\right]\Delta L^{(m)}\,\alpha_{s,r}\,\Delta Z^{(m)}\left[A^{(m)}\right]^2}{Z} \tag{3-48}$$

$$\Delta C_s^{\left[m_1^{(m)}\right]} = - \left[\left(N'_{f,0} \frac{\partial k}{\partial N'_{f,0}} \right)^{(m)} + \left(N'_{cl,0} \frac{\partial k}{\partial N'_{cl,0}} \right)^{(m)} \right.$$

$$+ \left(N'_{co,0} \frac{\partial k}{\partial N'_{co,0}} \right)^{(m)} \left(1 - \frac{\delta_{co}}{V_{co,0}} \right)$$

$$\left. + \left(N'_{s,0} \frac{\partial k}{\partial N'_{s,0}} \right)^{(m)} \right] \left[1 - \Delta L^{(m)} - \delta_{R_c}^{(m)} + \delta_{R_c}^{(m)} \Delta L^{(m)} \right] 2\alpha_{s,r}$$

$$+ \left(R_{c,0} \frac{\partial k}{\partial R_{c,0}} \right) \frac{\left[1 - \delta_{R_c}^{(m)} \right] \left[1 - \Delta L^{(m)} \right] \alpha_{s,r} \Delta Z^{(m)} \left[A^{(m)} \right]^2}{Z} \tag{3-49}$$

$$\left[\frac{T_{co,1}^{(1)}}{P_1} \right]^r = A_{cp}^{(1)} \tag{3-50}$$

$$\left[\frac{T_{co,1}^{(1)}}{P_1} \right]^i = -B_{cp}^{(1)} \tag{3-51}$$

$$\left[\frac{T_{surf,1}^{(1)}}{P_1} \right]^r = + X_{surf}^{(1)} \tag{3-52}$$

$$\left[\frac{T_{surf,1}^{(1)}}{P_1} \right]^i = - Y_{surf}^{(1)} \tag{3-53}$$

$$\left[\frac{T_{f,1}^{(1)}}{P_1}\right]^r = A_{fp}^{(1)} \tag{3-54}$$

$$\left[\frac{T_{f,1}^{(1)}}{P_1}\right]^i = - B_{fp}^{(1)} \tag{3-55}$$

$$\left[\frac{T_{in,1}^{(m)}}{P_1}\right]^r = 2 \sum_{k'=1}^{m-1} (-1)^{k'+1} \left[\frac{T_{co,1}^{(m-k')}}{P_1}\right]^r \tag{3-56}$$

$$\left[\frac{T_{in,1}^{(m)}}{P_1}\right]^i = 2 \sum_{k'=1}^{m-1} (-1)^{k'+1} \left[\frac{T_{co,1}^{(m-k')}}{P_1}\right]^i \tag{3-57}$$

$$\left[\frac{T_{co,1}^{(m)}}{P_1}\right]^r = A_{cp}^{(m)} + A_{ct}^{(m)} + A_{ct}^{(m)} \left[\frac{T_{in,1}^{(m)}}{P_1}\right]^r + B_{ct}^{(m)} \left[\frac{T_{in,1}^{(m)}}{P_1}\right]^i \tag{3-58}$$

$$\left[\frac{T_{co,1}^{(m)}}{P_1}\right]^i = - B_{cp}^{(m)} + A_{ct}^{(m)} \left[\frac{T_{in,1}^{(m)}}{P_1}\right]^i - B_{ct}^{(m)} \left[\frac{T_{in,1}^{(m)}}{P_1}\right]^r \tag{3-59}$$

$$\left[\frac{T_{surf,1}^{(m)}}{P_1}\right]^r = X_{surf}^{(m)} + S_{surf}^{(m)} \left[\frac{T_{in,1}^{(m)}}{P_1}\right]^r + W_{surf}^{(m)} \left[\frac{T_{in,1}^{(m)}}{P_1}\right]^i \tag{3-60}$$

$$\left[\frac{T_{surf,1}^{(m)}}{P_1}\right]^i = -Y_{surf}^{(m)} + S_{surf}^{(m)} + \left[\frac{T_{in,1}^{(m)}}{P_1}\right]^i - W_{surf}^{(m)} \left[\frac{T_{in,1}^{(m)}}{P_1}\right]^r \tag{3-61}$$

$$\left[\frac{T_{f,1}^{(m)}}{P_1}\right]^r = A_{fp}^{(m)} + A_{ft}^{(m)}\left[\frac{T_{in,1}^{(m)}}{P_1}\right]^r + B_{ft}^{(m)}\left[\frac{T_{in,1}^{(m)}}{P_1}\right]^i \tag{3-62}$$

$$\left[\frac{T_{f,1}^{(m)}}{P_1}\right]^i = -B_{fp}^{(m)} + A_{ft}^{(m)}\left[\frac{T_{in,1}^{(m)}}{P_1}\right]^i - B_{ft}^{(m)}\left[\frac{T_{in,1}^{(m)}}{P_1}\right]^r \tag{3-63}$$

$$\left[\frac{T_{cl,1}^{(m)}}{P_1}\right]^r = \left[1 - 3600\,\bar{h}\left(\frac{r_{cl}-r_f}{2k_{clad}} + \frac{1}{h_{co}}\right)\right]\left[\frac{T_{co,1}^{(m)}}{P_1}\right]^r + 3600\,\bar{h}\left(\frac{r_{cl}-r_f}{2k_{clad}} + \frac{1}{h_{co}}\right)\left[\frac{T_{surf,1}^{(m)}}{P_1}\right]^r \tag{3-64}$$

$$\left[\frac{T_{cl,1}^{(m)}}{P_1}\right]^i = \left[1 - 3600\,\bar{h}\left(\frac{r_{cl}-r_f}{2k_{clad}} + \frac{1}{h_{co}}\right)\right]\left[\frac{T_{co,1}^{(m)}}{P_1}\right]^i + 3600\,\bar{h}\left(\frac{r_{cl}-r_f}{2k_{clad}} + \frac{1}{h_{co}}\right)\left[\frac{T_{surf,1}^{(m)}}{P_1}\right]^i \tag{3-65}$$

$$\left[\frac{T_{s,1}^{(m)}}{P_1}\right]^r = \delta_s\left[\frac{T_{co,1}^{(m)}}{P_1}\right]^r \tag{3-66}$$

$$\left[\frac{T_{s,1}^{(m)}}{P_1}\right]^i = \delta_s\left[\frac{T_{co,1}^{(m)}}{P_1}\right]^i \tag{3-67}$$

$$\left(\frac{k_f}{P_1}\right)^r = \sum_{m=1}^{m_{max}} \left[\Delta C_f^{(m)} \left[\frac{T_{f,1}^{(m)}}{P_1}\right]^r + \Delta C_{cl}^{(m)} \left[\frac{T_{cl,1}^{(m)}}{P_1}\right]^r + \Delta C_{co}^{(m)} \left[\frac{T_{co,1}^{(m)}}{P_1}\right]^r + \Delta C_s^{(m)} \left[\frac{T_{s,1}^{(m)}}{P_1}\right]^r + \Delta C_s^{\left[m_2^{(m)}\right]} \left\{\frac{T_{s,1}^{\left[m_2^{(m)}\right]}}{P_1}\right\}^r + \Delta C_s^{\left[m_1^{(m)}\right]} \left\{\frac{T_{s,1}^{\left[m_1^{(m)}\right]}}{P_1}\right\}^r \right] \tag{3-68}$$

$$\left(\frac{k_f}{P_1}\right)^i = \sum_{m=1}^{m_{max}} \left[\Delta C_f^{(m)} \left[\frac{T_{f,1}^{(m)}}{P_1}\right]^i + \Delta C_{cl}^{(m)} \left[\frac{T_{cl,1}^{(m)}}{P_1}\right]^i + \Delta C_{co}^{(m)} \left[\frac{T_{co,1}^{(m)}}{P_1}\right] + \Delta C_s^{(m)} \left[\frac{T_{s,1}^{(m)}}{P_1}\right]^i + \Delta C_s^{\left[m_2^{(m)}\right]} \left\{\frac{T_s^{\left[m_2^{(m)}\right]}}{P_1}\right\}^i + \Delta C_s^{\left[m_1^{(m)}\right]} \left\{\frac{T_{s,1}^{\left[m_1^{(m)}\right]}}{P_1}\right\}^i \right] \tag{3-69}$$

For each m for which $\delta_{R_c}^{(m)} = 0$ values of $m_1^{(m)}$, and $m_2^{(m)}$ have been previously determined. Thus the quantities $\left\{\frac{T_{s,1}^{\left[m_1^{(m)}\right]}}{P_1}\right\}^r$, $\left\{\frac{T_{s,1}^{\left[m_1^{(m)}\right]}}{P_1}\right\}^i$, $\left\{\frac{T_{s,1}^{\left[m_2^{(m)}\right]}}{P_1}\right\}^r$, and $\left\{\frac{T_{s,1}^{\left[m_2^{(m)}\right]}}{P_1}\right\}^i$ can be directly transferred from the values in the regions corresponding to $m_1^{(m)}$, and $m_2^{(m)}$. For regions in which $\delta_{R_c}^{(m)} = 1$, $\Delta C_s^{\left[m_2^{(m)}\right]}$ and $\Delta C_s^{\left[m_1^{(m)}\right]}$ are zero so any values for the temperature to power ratios can be used.

$$H^r(i\omega) = \left(\frac{P_0}{\beta}\right)\left(\frac{k_f}{P_1}\right)^r \tag{3-70}$$

$$H^i(i\omega) = \left(\frac{P_0}{\beta}\right)\left(\frac{k_f}{P_1}\right)^i \tag{3-71}$$

$$\alpha_g = \left[1 + H^i_{i\omega}\, G^i_0 - H^r_{i\omega}\, G^r_0\right] \tag{3-72}$$

$$\beta_g = \left[H^i_{i\omega}\, G^r_0 + H^r_{i\omega}\, G^i_0\right] \tag{3-73}$$

$$G^r(i\omega) = \frac{G^r_0\ \alpha_g + G^i_0\ \beta_g}{\alpha_g^2 + \beta_g^2} \tag{3-74}$$

$$G^i(i\omega) = \frac{G^r_0\ \beta_g + G^i_0\ \alpha_g}{\alpha_g^2 + \beta_g^2} \tag{3-75}$$

$$P^r_1 = \frac{k_1}{\beta}\ P_0 G^r(i\omega) \tag{3-76}$$

$$P^i_1 = \frac{k_1}{\beta}\ P_0 G^i(i\omega) \tag{3-77}$$

$$\left[T^{(m)}_n\right]^r = P^r_1\left[\frac{T^{(m)}_n}{P_1}\right]^r - P^i_1\left[\frac{T^{(m)}_n}{P_1}\right]^i \tag{3-78}$$

$$\left[T_n^{(m)}\right]^i = P_1^r \left[\frac{T_n^{(m)}}{P_1}\right]^i + P_1^i \left[\frac{T_n^{(m)}}{P_1}\right]^r \tag{3-79}$$

where n = f
= cl
= co
= s
= surf

$$k_f^r = P_1^r \left(\frac{k_f}{P_1}\right)^r - P_1^i \left(\frac{k_f}{P_1}\right)^i \tag{3-80}$$

$$k_f^i = P_1^r \left(\frac{k_f}{P_1}\right)^i + P_1^i \left(\frac{k_f}{P_1}\right)^r \tag{3-81}$$

$$\left|T_n^{(m)}\right| = \left\{ \left\{ \left[T_n^{(m)}\right]^r \right\}^2 + \left\{ \left[T_n^{(m)}\right]^i \right\}^2 \right\}^{1/2} \tag{3-82}$$

$$\phi_{t,\,n}^{(m)} = \text{arc tan} \left\{ \frac{\left[T_n^{(m)}\right]^i}{\left[T_n^{(m)}\right]^r} \right\} \tag{3-83}$$

for each n = f
= cl
= co
= s
= surf

$$\left|G(i\omega)\right| = \left[G^r\,(i\omega)^2 + G^i\,(i\omega)^2\right]^{1/2} \tag{3-84}$$

$$\phi_G = \text{arc tan}\left[\frac{G^i(i\omega)}{G^r(i\omega)}\right] \tag{3-85}$$

$$\left| G_0 \right| = \left({G_0^r}^2 + {G_0^i}^2\right)^{1/2} \tag{3-86}$$

$$\phi_{G_0} = \text{arc tan}\left(\frac{G_0^i}{G_0^r}\right) \tag{3-87}$$

$$\left| H(i\omega) \right| = \left[H^r(i\omega)^2 + H^i(i\omega)^2 \right]^{1/2} \tag{3-88}$$

$$\phi_h = \text{arc tan}\left[\frac{H^i(i\omega)}{H^r(i\omega)}\right] \tag{3-89}$$

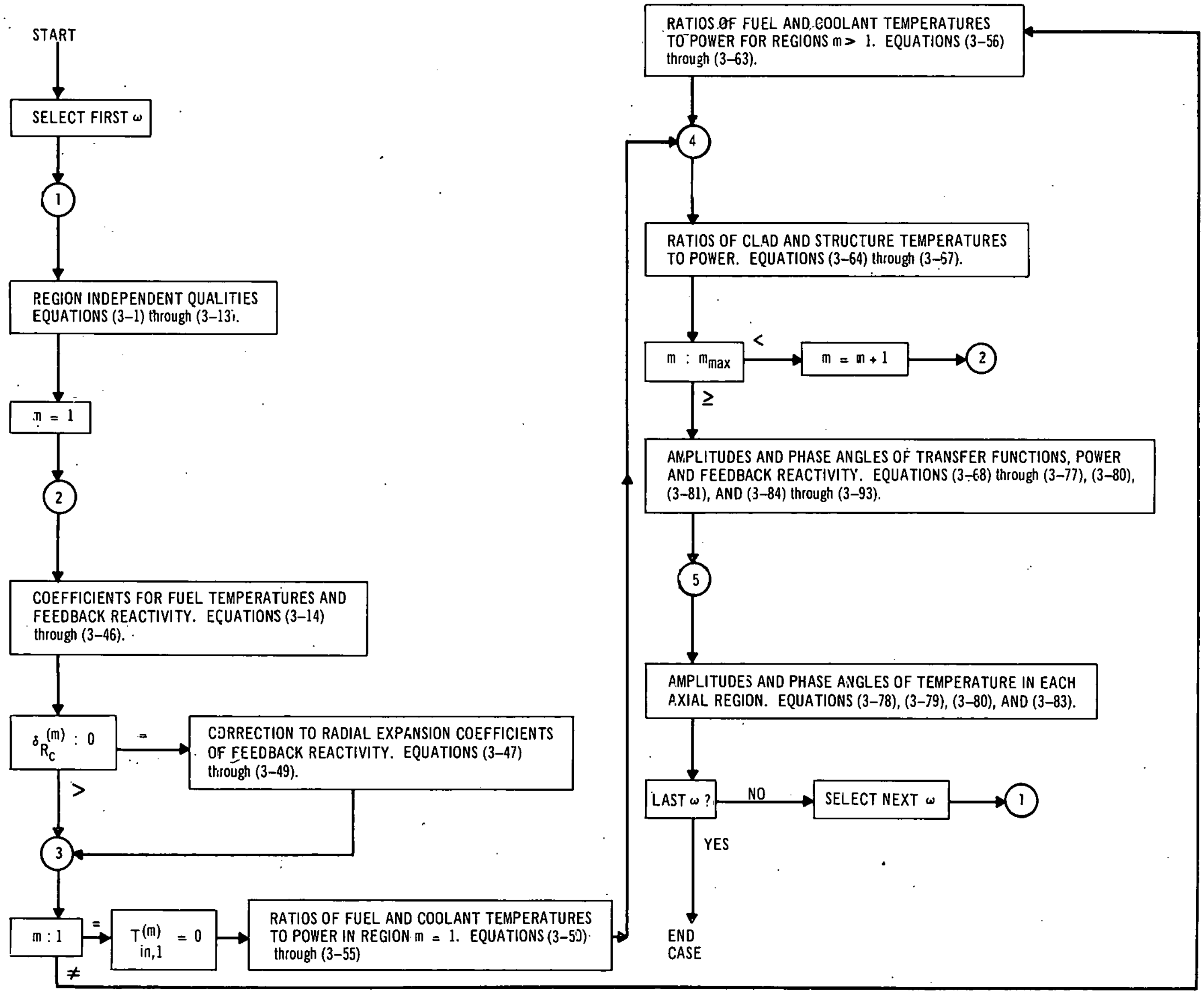
$$k_f = \left({k_f^r}^2 + {k_f^i}^2\right)^{1/2} \tag{3-90}$$

$$\phi_f = \text{arc tan}\left[\frac{k_f^i(i\omega)}{k_f^r(i\omega)}\right] \tag{3-91}$$

$$\left| \frac{P_1}{P_0} \right| = \frac{1}{P_0} \left({P_1^r}^2 + {P_1^i}^2\right)^{1/2} \tag{3-92}$$

$$\phi_p = \text{arc tan}\left(\frac{P_1^i}{P_1^r}\right) \tag{3-93}$$

3.4 Block Diagram



3.5 Output

The output to FLUZ is listed below according to the type of quantities printed.

A. Statements of the choice of options used.

B. Input constants pertaining to the reactor: P_0, k_1, ℓ, $\left(R_{c,0}\partial k/\partial R_{c,0}\right)$, $\left(H_{c,0}\partial k/\partial H_{c,0}\right)$, F_γ, v, C_h, β_j, λ_j, $\Delta Z^{(m)}$, $\delta_{R_c}^{(m)}$, $A^{(m)}$, $\left(\partial k/\partial T_f\right)_{Dop}^{(m)}$. (See Section 3.1 for descriptions of these quantities.)

C. Input constants pertaining to fuel: ρ_f, C_f, $V_{f,0}$, k_{fuel}, $\alpha_{f,r}$, r_f, $N'_{f,0}\left(\partial k/dN'_{f,0}\right)$, $\left(\partial k/\partial d_c\right)^{(m)}$, $\left(\partial d_c/\partial T_f\right)^{(m)}$. (See Section 3.1 for descriptions of these quantities.)

D. Input constants pertaining to cladding: ρ_{cl}, $V_{cl,0}$, k_{clad}, $\alpha_{cl,ax}$, $\alpha_{cl,r}$, r_{cl}, $\left(N'_{cl,0}\,\partial k/\partial N'_{cl,0}\right)^{(m)}$, $\left(\partial d_c/\partial T_{cl}\right)^{(m)}$. (See Section 3.1 for descriptions of these quantities.

E. Input constants pertaining to coolant: ρ_{co}, $V_{co,0}$, C_{co}, $\left(N'_{co,0}\,\partial k/\partial N'_{co,0}\right)$ (See Section 3.1 for descriptions of these quantities.)

F. Input constants pertaining to structure: ρ_s, $V_{s,0}$, $\alpha_{s,ax}$, δ_s, $\left(N'_{s,0}\,\partial k/\partial N'_{s,0}\right)$ (See Section 3.1 for descriptions of these quantities.)

G. Amplitude and phase angle of the following output quantities for each angular velocity ω:

1. $G_0(i\omega)$, zero power transfer function;
2. $G(i\omega)$, power transfer function;
3. $H(i\omega)$, feedback transfer function;
4. $k_f(i\omega)$, feedback reactivity;
5. $P_i(i\omega)/P_0$, fractional oscillating component of power;
6. $T_{f,1}^{(m)}(i\omega)$, $T_{surf,1}^{(m)}(i\omega)$, $T_{cl,1}^{(m)}(i\omega)$, $T_{co,1}^{(m)}(i\omega)$, $T_{co,1}^{(m)}(i\omega)$ and $T_{s,1}^{(m)}(i\omega)$, oscillating components of fuel average, fuel surface, cladding, coolant, and structure temperatures for each of m_{max} axial regions.

The phase angles are all measured with respect to the oscillating impressed reactivity, k_1.

APPENDIX A

EVALUATION OF BESSEL FUNCTIONS

The quantities $ber_0 X_f$, $bei_0 X_f$, $ber_1 X_f$, and $bei_1 X_f$ used in Equations (3-9), (3-10), (3-30), (3-31), (3-32), (3-33), and (3-34) are defined in terms of the Bessel functions

$$J_0\left(\sqrt{-1}\, X_f\right) = ber_0 X_f + i\, bei_0 X_f$$

$$J_1\left(\sqrt{-1}\, X_f\right) = ber_1 X_f + i\, bei_1 X_f$$

and must be provided either from file tables or calculated for $X_f > 0$.

Jahnke and Emde* list tables of the values of the real and imaginary parts of $J_0\left(\sqrt{i}\, x\right)$ and $\sqrt{i}\; J_1\left(\sqrt{i}\, x\right)$ for $0 \geq X \leq 10$. Values for $ber_0 X_f$, $bei_0 X_f$, $ber_1 X_f$, and $bei_1 X_f$ for $0 \leq X_f \leq 10$ may be obtained from these tables thru the use of the following expressions:

$$ber_0 X_f = \left[J_0\left(\sqrt{i}\, X_f\right)\right]^r \tag{A-1}$$

$$bei_0 X_f = -\left[J_0\left(\sqrt{i}\, X_f\right)\right]^i \tag{A-2}$$

$$ber_1 X_f = -\frac{\sqrt{2}}{2}\left\{\left[\sqrt{i}\, J_1\left(\sqrt{i}\, X_f\right)\right]^r + \left[\sqrt{i}\, J_1\left(\sqrt{i}\, X_f\right)\right]^i\right\} \tag{A-3}$$

$$bei_1 X_f = -\frac{\sqrt{2}}{2}\left\{-\left[\sqrt{i}\, J_1\left(\sqrt{i}\, X_f\right)\right]^i + \left[\sqrt{i}\, J_1\left(\sqrt{i}\, X_f\right)\right]^r\right\} \tag{A-4}$$

Tables of the real and imaginary parts of $J_0\left(\sqrt{i}\, X\right)$ and $\sqrt{i}\; J_1\left(\sqrt{i}\, X\right)$ for $0 \leq X \leq 10$ are included as an integral part of the coding. Values for any given $0 \leq X_f \leq 10$ are obtained from the

*Eugene Jahnke and Fritz Emde, Tables of Functions, pp. 246-251 and 258, Dover Publications, New York (1945).

tables by four-point Lagrangian interpolation. Equations (A-1) through (A-4) are then used to convert these to ber and bei values.

When $X_f > 10$, the Bessel functions may be approximated by damped cosine functions, given by

$$J_p(x) \sim \sqrt{\frac{2}{\pi x}} \cos\left[x - (2p+1)\frac{\pi}{4}\right]$$

The values for $ber_0 X_f$, $bei_0 X_f$, $ber_1 X_f$, and $bei_1 X_f$ are then given by

$$ber_0 X_f = \frac{\sqrt{2}}{4}\sqrt{\frac{\sqrt{2}}{\pi X_f}}\, e^{X_f}\left[\frac{1}{2\sqrt{\frac{\sqrt{2}-1}{2}}}\left(\cos\frac{X_f}{\sqrt{2}} + \sin\frac{X_f}{\sqrt{2}}\right)\right.$$

$$\left. - \sqrt{\frac{\sqrt{2}-1}{2}}\left(\sin\frac{X_f}{\sqrt{2}} - \cos\frac{X_f}{\sqrt{2}}\right)\right] \qquad \text{(A-5)}$$

$$bei_0 X_f = \frac{\sqrt{2}}{4}\sqrt{\frac{\sqrt{2}}{\pi X_f}}\, e^{X_f}\left[\frac{1}{2\sqrt{\frac{\sqrt{2}-1}{2}}}\left(\sin\frac{X_f}{\sqrt{2}} - \cos\frac{X_f}{\sqrt{2}}\right)\right.$$

$$\left. + \sqrt{\frac{\sqrt{2}-1}{2}}\left(\cos\frac{X_f}{\sqrt{2}} + \sin\frac{X_f}{\sqrt{2}}\right)\right] \qquad \text{(A-6)}$$

$$ber_1 X_f = + \frac{\sqrt{2}}{4}\sqrt{\frac{\sqrt{2}}{\pi X_f}}\, e^{X_f}\left[\frac{1}{2\sqrt{\frac{\sqrt{2}-1}{2}}}\left(\sin\frac{X_f}{\sqrt{2}} - \cos\frac{X_f}{\sqrt{2}}\right)\right.$$

$$+ \sqrt{\frac{\sqrt{2}-1}{2}}\left(\sin\frac{X_f}{\sqrt{2}} + \cos\frac{X_f}{\sqrt{2}}\right) \qquad \text{(A-7)}$$

$$\mathrm{bei}_1 X_f = +\frac{\sqrt{2}}{4}\sqrt{\frac{\sqrt{2}}{\pi X_f}}\, e^{X_f}\left[\frac{1}{2\sqrt{\frac{\sqrt{2}-1}{2}}}\left(\sin\frac{X_f}{\sqrt{2}} + \cos\frac{X_f}{\sqrt{2}}\right)\right.$$

$$\left. + \sqrt{\frac{\sqrt{2}-1}{2}}\left(\sin\frac{X_f}{\sqrt{2}} - \cos\frac{X_f}{\sqrt{2}}\right)\right] . \qquad \text{(A-8)}$$

Equations (A-5) through (A-8) are used when X_f is greater than 10 in place of the tables and Equations (A-1) through (A-4).

DISTRIBUTION LIST

	Copies
Dr. James Gilbreath Assistant Director for Argonne National Laboratory Argonne, Illinois	4
Dr. S. Lawroski Argonne National Laboratory Argonne, Illinois	1
Mr. A. P. Donnell Atomic Power Development Associates 1911 First Street Detroit 26, Michigan	1
Nuclear Development Corp. of America 5 New Street White Plains, New York Attn: Library (for Dr. K. Goldsmith)	1
H. A. Wagner The Detroit Edison Company 2000 Second Avenue Detroit 26, Michigan	1
United Nuclear Corp. 5 New Street White Plains, New York Attn: Library (for Dr. C. Graves)	1
F. A. Thalgott Argonne National Laboratory Idaho Division P. O. Box 1096 Idaho Falls, Idaho	1
Mr. Arthur S. Griswold Power Reactor Development Co. 1911 First Street Detroit 26, Michigan	1
D. C. DeHuff Westinghouse Electric Corporation Atomic Power Department P. O. Box 355 Pittsburgh 30, Pa.	1
Dr. Karl H. Puechl Nuclear Materials and Equipment Corp. Apollo, Pa.	1
R. C. Werner Mine Safety Appliances Research Corp. Callery, Pa.	1

	Copies
Director, Reactor Engineering Division Attention: C. A. Pursel Chicago Operations Office 9800 South Cass Avenue Argonne, Illinois	2
Mr. W. H. Brummett, Jr. Director, Contracts Division U. S. Atomic Energy Commission San Francisco Operations Office 2111 Bancroft Way Berkeley 4, California	2
Col. J. B. Radcliffe, Jr. Director, Reactor Division U. S. Atomic Energy Commission San Francisco Operations Office 2111 Bancroft Way Berkeley 4, California	2
Dr. G. W. Wensch Director, Sodium Reactors Branch Division of Reactor Development U. S. Atomic Energy Commission Washington 25, D. C.	7
Mr. F. A. Robertson Chief, California Patent Group U. S. Atomic Energy Commission San Francisco Operations Office P. O. Box 808 Livermore, California	1
J. K. Davidson Allis-Chalmers Company 508 Kennedy Street, NW Washington 11, D. C.	2
S. Visner Contract No. AT(11-1)-1317 Combustion Engineering Corporation Nuclear Division Prospect Hill Road Windsor, Connecticut	2
Dr. John C. Woodhouse 1 Guest Lane Wilmington, Delaware	1
H. W. Savage Oak Ridge National Laboratory P. O. Box X Oak Ridge, Tennessee	1
W. R. Simmons Argonne National Laboratory Argonne, Illinois	1

	Copies
W. R. Wykoff Los Alamos Scientific Laboratory Los Alamos, New Mexico	1
O. E. Dwyer Brookhaven National Laboratory Upton, New York	1
U. S. Atomic Energy Commission New York Operations Office 376 Hudson Street New York, New York Attn: Jules Wise	1
U. S. Atomic Energy Commission Canoga Park Area Office P. O. Box 591 Canoga Park, California Attn: C. W. Richards	1
General Directorate Research and Training Euratom Rue Belliard 51-53 Brussels, Belgium Attn: A. Stordeur	20
Mr. D. T. Eggen Atomics International P. O. Box 309 Canoga Park, California	1
M. C. Edlund The Babcock & Wilcox Company Atomic Energy Division Lynchburg, Virginia	1
Mr. Samuel C. Nordlinger United States Embassy London, England	1
Dr. Peter A. Morris United States Embassy Tokyo, Japan	1
Dr. Abraham S. Freidman United States Embassy Paris, France	1
Dr. Martin Biles U. S. Mission to the European Communitites c/o United States Embassy Brussels, Belgium	1
Director Research and Development Division U. S. Atomic Energy Commission Richland Operations Office Post Office Box 550 Richland, Washington 99352	1