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ANALYSIS OF THE INITIAL CRITICAL EXPERIMENTS OF THE EBWR PLUTONIUM RECYCLE PROGRAM

by

P. H. Kier

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ANALYSIS OF THE INITIAL CRITICAL EXPERIMENTS OF THE EBWR PLUTONIUM RECYCLE PROGRAM

by

P. H. Kier

Reactor Physics Division

August 1967

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ABSTRACT

As part of the EBWR Plutonium Recycle Program, the purpose of which is to obtain information useful for the utilization of plutonium in light-water-moderated power reactors, critical experiments were performed prior to power operation. These measurements of critical configurations and reactivity coefficients yield information useful for the modification and refinement of calculational models.

This report describes the experimental results and the analysis of the experimental results for measurements of two plutonium-fuel critical configurations, temperature coefficients of reactivity, boric acid worths and control-rod bank worths. The temperature coefficients and boric acid worths were measured for the 36-plutonium-assembly loading and for the full fuel loadings. These critical experiments led to a revised computational model, which is also described.

I. INTRODUCTION

Irradiation of plutonium-bearing fuel in the EBWR is part of a joint Argonne-Pacific Northwest program to obtain information useful for the utilization of plutonium in light-water-moderated power reactors. Such utilization could be made in either an all-thermal recycle system or as the thermal part of a mixed fast-thermal reactor complex.

Although the primary objective of the program is to irradiate plutonium fuel in an environment representative of a boiling-water reactor to obtain changes in isotopic composition with exposure, much information useful for the construction and testing of calculational models is obtainable from zero-power measurements of critical configurations and reactivity coefficients. Critical experiments[1] with uniform arrays of the plutonium fuel pins that are used in this experiment were performed at Battelle's Pacific Northwest Laboratory before they were placed in assemblies for use in the EBWR. Comparison of the analysis of the preirradiation critical experiments in the EBWR with the analysis of the critical experiments

performed at Pacific Northwest Laboratory indicates refinements in theory that are necessary to account for the effects of nonuniformities found in power reactors, such as control rods and their guide structure, and the assembly can itself.

This report describes the initial, preirradiation measurements of critical configurations and reactivity coefficients and analyzes the results of these experiments. Section II is concerned with the experiments performed with the partial loading of plutonium fuel, and Section III is concerned with the experiments performed with the full loading of plutonium fuel, and enriched and natural uranium fuel. Appendix A describes the calculational model used in the analysis of the measurement of critical configurations, temperature coefficients, and shutdown margins. Appendix B gives the procedure used to analyze the errors in the boric acid worth and temperature-coefficient experiments.

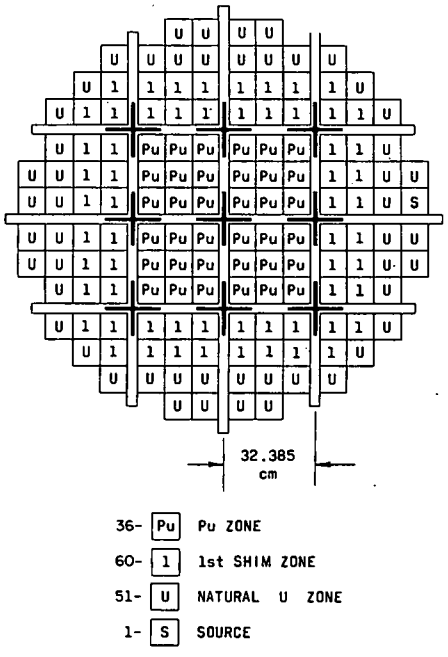

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Fig. 1. Schematic Layout of the Full Loading in EBWR during the Plutonium Recycle Experiment

The 148-assembly core (Fig. 1) is divided into four zones: the central 6 x 6 square, the immediately adjacent 28-assembly row, the next 32-assembly row (excluding the fuel assemblies on the 45° symmetry lines), and the remaining 52 outer assemblies. One of the outside elements is replaced with antimony-beryllium neutron source.

The center 36 assemblies represent the plutonium test zone. These fuel pins contain 1.5 w/o plutonium-uranium oxide. The two rows surrounding the plutonium are referred to as the enriched-uranium shim zones. These zones initially contained only assemblies with 6 w/o enriched uranium oxide fuel, poisoned with europium and samarium oxides to reduce the change of reactivity with burnup. During the experiments, eight shim assemblies were replaced by highly enriched uranium spike assemblies (which were used during the 100-MW operations[2]) to increase the maximum operating power. The outer 52-assembly zone contains assemblies with natural uranium oxide fuel.

Each of these types of fuel assemblies contains fuel-bearing Zircaloy-2 tubes of 0.372-in. ID with 0.025-in. walls (0.945-cm ID with 0.064-cm walls). All assemblies but the spikes contain a 6 x 6 array of fuel pins with a molecular oxide density of 0.02146/barn-cm. Each spike assembly contains a 7 x 7 array of fuel pins. Table I gives the compositions of the four types of fuel. The active length of the fuel pins is 4 ft.

TABLE I. Compositions of the Oxide Fuels Used during the Initial Critical Experiments

Type	Composition	Material	Atom Density, $(barn\text{-}cm)^{-1}$
Plutonium	Depleted UO_2-PuO_2 1.5 w/o PuO_2	Pu^{239}	0.0002931
		Pu^{240}	0.0000250
		Pu^{241}	0.0000023
		U^{238}	0.021091
		U^{235}	0.0000365
		O	0.04292
Shim	UO_2 (6 w/o U^{235}, 0.158 w/o Eu_2O_3, 0.0288 w/o Sm_2O_3)	U^{238}	0.020172
		U^{235}	0.001288
		Sm	0.0000100
		Eu	0.0000517
		O	0.04292
Natural	Natural UO_2	U^{238}	0.021305
		U^{235}	0.000155
		O	0.04292
Spike		U^{238}	0.00005726
		U^{235}	0.00076074
		Ca	0.003924
		Zr	0.016233
		O	0.038048

There are nine cruciform control rods: The center rod is made of hafnium, to be light in weight for rod oscillator experiments, and the other eight rods are made of Type 304 stainless steel containing 2 w/o boron, which is enriched to approximately 90 w/o in the isotope B^{10}.

The hafnium control rod has blades 0.317 cm (0.125 in.) thick x 25.4 cm (10 in.) wide with a 117-cm (46-in.)-long absorber section. The hafnium is welded to a 98-cm (38.5-in.)-long Zircaloy-2 follower section. Below the Zircaloy-2 follower is a 3-in.-wide stainless steel follower. The hafnium rod with followers is pictured in Fig. 7 of Ref. 3.

The remaining eight control rods have boron stainless steel absorber sections, which are 0.635 cm (0.25 in.) thick x 25.4 cm (10 in.) wide. The absorber sections are 152 cm (60 in.) long and are attached to 87.6-cm (34.5-in.)-long follower sections made of Zircaloy-2. Below the Zircaloy-2 follower is a 93-in.-long, 3-in.-wide, stainless steel follower. A boron-stainless steel rod with followers is pictured in Fig. 6 of Ref. 3.

The center, hafnium rod is identified as control rod No. 9; the four corner rods are control rods No. 2, 4, 6, and 8; the four noncorner, off-center or "flat" rods are control rods No. 1, 3, 5, and 7.

Interspersed among the critical experiments described in this report were foil activations of two types: flux traverses and activation ratios.

The former type yields information on the spatial-flux distribution; the latter yields information on the effect of plutonium isotopes on the thermal-flux spectrum. Pile oscillator measurements of the reactor transfer function of the full loading were made at zero power and during the approach to operating power. The results of these foil activation and transfer-function measurements will be described in subsequent reports.

II. EXPERIMENTS WITH PLUTONIUM LOADING

A. Unpoisoned, Unrodded Critical Loading

The purpose of this experiment was to obtain a critical loading of plutonium fuel at room temperature with the reactor water not poisoned with boric acid. After each assembly was loaded the count rate on two fission counters was recorded for two control-rod configurations: (1) control rods No. 1 through 8 at 15 in. with control rod No. 9 fully inserted, and (2) control rods No. 1 through 8 at 15 in. with control rod No. 9 fully withdrawn to 48 in. Initially after every fourth assembly was loaded and then after every second assembly was loaded, the count rates were also obtained with all control rods fully withdrawn. Curves of the reciprocal count rates versus the number of assemblies loaded yielded estimates of the critical loading.

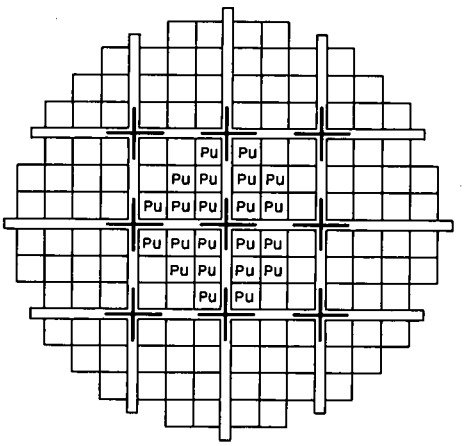

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Fig. 2. Twenty-two-assembly, Unrodded, Unpoisoned Critical Loading

Criticality was attained with the 22-assembly loading shown in Fig. 2 with control rods No. 1 through 8 fully withdrawn to 48 in. and with control rod No. 9 withdrawn to 40.86 in.

Fortuitously, the critical configuration was nearly fully unrodded, a configuration that can be easily represented in calculations. To obtain the multiplication constant of the unrodded assembly, a calculable quantity, the reactivity* worth of withdrawing control rod No. 9 from 41 to 48 in., was determined from period measurements.

The doubling time was obtained from the counting rates of two BF_3 chambers located within the core. The counts for a 20-sec interval of a 30-sec cycle were plotted on semilogarithmic

*Because the prompt-neutron lifetime rather that the generation time is used in the inhour equation, k_{ex} rather than reactivity (k_{ex}/k) is obtained. However, for simplicity of expression, the k_{ex} introduced by the rod withdrawal will sometimes be referred to as "reactivity." Since the period measurements were made with k near unity and the rod movements in the period measurements introduced $k_{ex} < 0.001$, the difference between reactivity and k_{ex} is small.

paper against time after rod movement. In relating doubling time to k_{ex} by use of the inhour equation, we took the prompt-neutron lifetime, ℓ, as 41.7 μsec and the effective delayed-neutron fraction, β_{eff}, as 0.00307.

Control rod No. 9 could not be withdrawn to 48 in. in a single step because an excessively short period would have resulted. Therefore, the rod was withdrawn to such a height that a doubling time of approximately 60 sec resulted. Then a small amount of boric acid was added to the reactor water to raise the critical height of the control rod. Then the control rod was withdrawn to 48 in. Table II gives the result of these period measurements.

TABLE II. Results of Period Measurements for 22-plutonium-assembly Critical Loading

H_3BO_3 Concentration, g/gal	Height of Rod No. 9, in.		Doubling Time, sec	Δk (10^{-4})	$\Delta k/\Delta h$ (10^{-4}/in.)
	Critical	Period			
0	40.86	41.80	60 ± 3	3.50 ± 0.34	3.70 ± 0.58
0	40.86	41.80	63 ± 3	3.35 ± 0.34	3.55 ± 0.56
0.08 ± 0.03	43.8	48.0	150 ± 4	1.65 ± 0.17	0.39 ± 0.05
0.08 ± 0.03	43.6	48.0	123 ± 4	1.93 ± 0.20	0.44 ± 0.05

Table II indicates that the reproducibility of the period measurements without boric acid was good, but the reproducibility of period measurements with boric acid was not so good. The shift in the critical height of the rod is indicative of incomplete mixing of boric acid with the reactor water.

The experiment yielded the differential rod worth between 40.86 and 41.80 in. and between 43.8 and 48 in. The differential worth between 41.80 and 43.8 in. was taken to be the average of the two measured worths, $(2.6 \pm 0.6) \times 10^{-4}$/in. By multiplying the differential rod worths by their associated intervals, we found the worth of withdrawing control rod No. 9 from 40.86 to 48 in. to be approximately $(10.4 \pm 1.3) \times 10^{-4}$. The measured multiplication constant of the unrodded, unpoisoned, 22-assembly loading was therefore 1.00104 ± 0.00013.

The multiplication constant of this loading was calculated with the two-dimensional, diffusion-theory code CANDID2D[4] with four groups of neutrons. Group cross sections were obtained for the three fast groups by use of the GAM[5] code, and for the thermal group by use of the Argonne-revised THERMOS[6] code. To describe adequately flux peaking between fuel assemblies, and between an assembly and the control-rod guide structure, we used a three-stage homogenization procedure to obtain the cross sections for the thermal group. Appendix A gives the details of this calculational model.

This CANDID2D calculation yielded a multiplication constant, k, of 1.007, which is in good agreement with the measured value of 1.001. The computed value of k could be expected to be slightly greater than the measured value because of the construction of control-rod units. Because the Zircaloy-2 portion of a control-rod follower is only about 3 ft long, there is a 1-ft section of stainless steel follower in the core when a control rod is fully withdrawn to 48 in. In the calculation, we assumed that when a control rod is fully withdrawn there is 4 ft of Zircaloy-2 follower in the core. Since stainless steel is more highly absorbing than zirconium, the calculation should be expected to overestimate k slightly.

This calculational model yielded k in far better agreement with experiment than an earlier model,[3,7,8] which is based on the assumption of the space-energy separability of the intracell thermal-neutron flux. This earlier model predicted an unrodded, unpoisoned critical loading of 17.8 assemblies when used with a one-dimensional, diffusion-theory code, REX.[9] The worth of an additional assembly is at least 1% in Δk.

The assumption of the space-energy separability of the thermal flux is quite good when the nuclides present have nearly $1/v$ absorption cross sections, a condition that is satisfied for U^{235}-enriched uranium systems but not for plutonium-enriched systems. The isotopes of plutonium have resonances in the thermal and the near-thermal energy ranges: Pu^{239} has a large resonance at 0.3 eV, Pu^{240} has a giant resonance at 1 eV, and Pu^{240} has a large resonance at 0.26 eV. The depressions in the flux from absorption in these resonances causes the thermal flux to be highly nonseparable in space and energy. For this plutonium loading, the assumption of flux separability caused k to be overestimated by between 1 and 2%, in agreement with calculations of Liikala[10] and of Bindler and Poncelet.[11]

To predict the multiplication constant of the plutonium loading in EBWR accurately, spatial as well as spectral effects must be treated accurately. Calculations performed[12] elsewhere predicted that the unrodded, unpoisoned critical loading would be approximately 15 assemblies. The calculational model used THERMOS to compute the thermal-group cross section and therefore is similar to our model (Appendix A) with regard to spectral effects. However, the treatment of core irregularities (assembly cans, control-rod followers, control-rod guide structure, interassembly water, etc.) was highly approximate. For the thermal group, the water and zirconium in a pin cell were augmented to conserve core material in a single-stage homogenization. Since this model and that given in Appendix A are similar in other respects, the large overestimate of k can be attributed to the approximate treatment of core irregularities.

B. Critical Boric Acid Concentration for 36-assembly Loading

After the unrodded, unpoisoned critical loading was determined, enough boric acid was added to the reactor water so that the reactor would

be subcritical during the loading of the 14 plutonium assemblies required to complete the plutonium test zone. During this loading procedure, to insure that the reactor would be subcritical, the reciprocal count rate for control-rod configurations were recorded and plotted against the number of assemblies loaded. After the full loading of 36 plutonium assemblies was reached, the boric acid concentration was adjusted so that the reactor would be critical with all control rods fully withdrawn to 48 in.

The boric acid concentration for criticality of the unrodded loading of 36 plutonium assemblies was 5.09 g/gal. As this clean, unrodded system can be well represented in a calculation, a four-group CANDID2D computation was made of the multiplication constant for the system when poisoned with 5 g/gal boric acid. The calculation yielded $k = 0.9988$, in excellent agreement with experiment. This calculation, which used the model given in Appendix A, is far better than an earlier calculation, based on the assumption of space-energy separability of the thermal flux and the use of one-dimensional diffusion theory, which predicted a critical boric acid concentration of 6.8 g/gal. Another calculation,[12] based on a highly approximate spatial representation of the core, yielded a critical boric acid concentration of approximately 10 g/gal. For this loading, the boric acid worth is between -1.5 and -1.2% $\Delta k/g/gal$.

C. Boric Acid Worth for 36-assembly Loading

Boric acid worth as a function of boric acid concentration was measured by an experimental procedure that yielded partial differential worths of the central control rod, determined the minimum boric acid concentration for eight-rod shutdown, and verified nine-rod shutdown without boric acid. The eight-rod shutdown and nine-rod shutdown aspects of this experiment are discussed in Section III-C.

The procedure used to obtain boric acid worth is as follows: With the eight noncenter control rods banked at a specified height, the differential worth of the center rod is obtained. Then by use of the water-purification system, boric acid is removed from the reactor water. During this removal process the center rod is inserted to maintain criticality. When the rod has been inserted several inches below its previous critical setting, the removal of boric acid is halted and the differential worth of the center rod is measured. Also at this time, the critical height of the nine-rod bank and of the eight-rod bank with the center rod withdrawn to its maximum allowable height are obtained. This procedure is repeated until the boric acid concentration approached the minimum concentration required for eight-rod shutdown.

The boric acid worth ($\Delta k/\Delta C$) is obtained from the differential rod worths, the change in concentration, ΔC, and the change in the critical height of the center rod, ΔH, as follows:

$$\frac{\Delta k}{\Delta C} = \frac{\Delta k}{\Delta h}\frac{\Delta H}{\Delta C}, \qquad (1)$$

where $\Delta k/\Delta h$ is the average of the preremoval and postremoval differential worths of the center rod.

Before discussing the results of these measurements of rod worth and of boric acid worth, we should mention that during the experiment, inverted rod-worth effects were detected for certain control-rod configurations. When the center rod was partly inserted and the other eight rods were fully withdrawn to 48 in., a slight insertion of the eight-rod bank caused the reactor to go on a positive period. The same effect was observed when the center rod was inserted from 48 in. with the other eight rods partly inserted. This inverted rod-worth effect is caused by the lower portion of the control-rod followers being stainless steel, a moderately strong neutron absorber, instead of Zircaloy, a weak neutron absorber. With the center rod partly inserted, the neutron-flux distribution is distorted so that its peak is in the lower half of the core. As a result, the stainless steel portion of the followers of the fully withdrawn control rods are in a region of great importance. When these fully withdrawn control rods are then inserted, the positive reactivity effect of replacing borated water and steel with Zircaloy in a region of high importance apparently is greater than the negative reactivity effect of replacing Zircaloy with the control-rod absorber in a region of lower importance.

The rod heights at which the inverted rod-worth effect appears have been ascertained for several boric concentrations. The height at which the effect appears is constant at 37 in. for insertion of the eight-rod bank and varies from 40 in., with 5 g/gal H_3BO_3, to 46 in., with 1.5 g/gal H_3BO_3 for insertion of rod No. 9. To preclude insertion of a control rod causing an increase in reactivity, the mechanical stops on the rods were lowered to prevent the center rod from being withdrawn above 40 in. and the other eight rods from being withdrawn above 37 in. Also, switches were installed to cause a rod to be dropped if somehow it were raised 1 in. beyond the level of the mechanical stop.

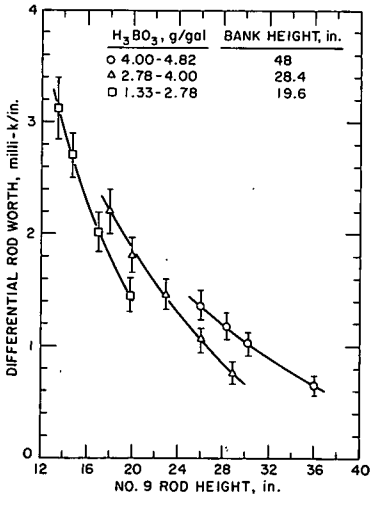

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Fig. 3. Differential Worth of the Central Control Rod, 36-assembly Critical Loading

The results of the measurements of differential rod worth are given in Fig. 3, and the resulting boric acid worths are given in Table III. The rod-worth measurements were performed with the eight-rod bank at three different heights, as shown by the three curves in the figure. It is seen that the differential worth of a rod at a particular height is strongly influenced by the position of the other rods and by the boric acid concentration.

TABLE III. Boric Acid Worths for the 36-plutonium-assembly Loading

H_3BO_3 Concentration, g/gal	Eight-rod Bank Height, in.	Height of Rod No. 9, in.	$\Delta k/\Delta C$, % $\Delta k/g/gal$
4.82 - 4.42	48	35.38 - 29.78	1.19 ± 0.14
4.42 - 4.24	48	30.02 - 28.10	1.17 ± 0.20
4.28 - 3.99	48	28.10 - 25.72	1.25 ± 0.19
4.00 - 3.77	28.35	28.68 - 25.32	1.29 ± 0.19
3.77 - 3.52	28.35	25.32 - 22.84	1.42 ± 0.22
3.52 - 3.12	28.35	22.84 - 19.90	1.32 ± 0.18
3.12 - 2.78	28.35	19.90 - 17.83	1.25 ± 0.19
2.38 - 1.98	19.62	19.66 - 17.04	1.14 ± 0.17
1.98 - 1.54	19.62	17.04 - 14.55	1.33 ± 0.15
1.54 - 1.33	19.62	14.55 - 13.56	1.39 ± 0.25

Boric acid worth would be expected to decrease with increasing concentration because spectrum hardening would reduce the effective cross section of boron. Calculations made with REX[9] predicted such an effect. They predicted[3] about -1.3% in k for a 1-g/gal addition to the unpoisoned system and about -1% in k for a 1-g/gal addition to the system that contains 9 g/gal. A trend toward increasing worth with decreasing boric acid concentration is discernible from the results given in Table III. However, because of the relatively large experimental error, a measured dependence of boric acid worth upon concentration cannot be inferred. From these results, we can only conclude that the boric acid worth for boric acid concentrations between 1.3 and 4.8 g/gal is (-1.25 ± 0.20) % $\Delta k/g/gal$, which is in reasonable agreement with calculation. Errors in measurements of differential rod worth and boric acid concentration precluded more precise determinations of boric acid worth.

D. Uniform Temperature Coefficient of the 36-assembly Loading

The uniform temperature coefficient of reactivity of the 36-assembly loading was obtained from an experimental procedure that yielded the multiplication constant of the unrodded system as a function of temperature for boric acid contents of approximately 5 g/gal. These measurements were made between room temperature and 135°F; they were not extended to higher temperatures because of malfunctioning of the in-core thermocouples and inability to maintain uniform temperature distributions and boric acid concentration at higher temperatures.

The experimental procedure was to heat the reactor to a specified temperature by use of the startup heater. At this temperature, the boric acid concentration was adjusted so that the reactor was critical with a corner rod (rod No. 4) partly inserted and the other eight rods withdrawn to their maximum allowable heights (40 in. for rod No. 9, 37.5 in. for the other seven rods). The worth of withdrawing rod No. 4 to 37.5 in. was

then obtained from period measurements. If too short a period would have been obtained if rod No. 4 were withdrawn to 37.5 in. in one step, rod No. 8 was inserted so that rod No. 4 could be withdrawn in several steps. Because the noncenter rods have so little worth for this loading and because rods No. 4 and 8 are so far apart (approximately 50 cm), we assumed that the reactivity worth of rod No. 4 was independent of the position of rod No. 8. This procedure was repeated at several temperatures.

The experimental procedure yields the multiplication constant as a function of temperature and boric acid concentration. To obtain the temperature coefficient from this information, k must be normalized to a single boric acid concentration C_0, by use of the measured boric acid worth, (-0.0125 ± 0.0020) $\Delta k/g/gal$:

$$k(C_0) = k(C) + \frac{\Delta k}{\Delta C} (C - C_0). \qquad (2)$$

The results of the experiment are summarized in Table IV, where the critical rod positions and the multiplication constants of the system for four temperatures are given. From the critical rod positions, we see that a rod movement of more than 10 in. is required to produce a period of about a minute; this substantiates the assumption that the offcenter rods are noninteracting. For the low temperature range studied, reactivity increases with temperature. From these data, we obtain an average uniform temperature coefficient between 67 and 135°F of $(6.2 \pm 1.4) \times 10^{-5}/°F$.

TABLE IV. Experimental Multiplication Constants of the Unrodded, 36-plutonium-assembly Loading at Several Temperatures

Temp, °F	H_3BO_3 Concentration, g/gal at 70°F	Critical Heights, in.		k (unnormalized)	k, 4.8 g/gal
		Rod No. 4	Rod No. 8		
67.3 ± 0.3	4.80 ± 0.02	13.41 25.36	37.50 22.96	1.00065 ± 0.00003	1.00065 ± 0.00003
94.1 ± 0.2	4.86 ± 0.02	3.32 22.22	37.50 21.97	1.00091 ± 0.0005	1.00166 ± 0.00032
122.5 ± 0.2	5.09 ± 0.02	18.88	37.50	1.00056 ± 0.00003	1.00418 ± 0.00042
135.5 ± 0.2	5.19 ± 0.02	37.50	37.50	1.00000 ± 0.00001	1.00487 ± 0.00050

Two-dimensional, four-group, diffusion-theory computations of the multiplication constant of the unrodded assembly at 70, 120, 170, 220, 270, and 325°F were made with the PDQ-3 code.[13] The boric acid concentration, which was taken as 5.0 g/gal at 70°F, was proportional to the temperature-dependent water density. Spectrum-dependent group cross sections for each temperature was obtained by use of the GAM and the Argonne-revised THERMOS codes, in accordance with the model described in Appendix A.

Table V gives the results of these calculations, with 0.0025 added to the computed k to normalize the calculations to a room-temperature boric acid concentration of 4.8 g/gal. The computed temperature coefficient is positive below and negative above 220°F. Although the computed temperature coefficient is positive between 70 and 170°F, it is much smaller (1.7×10^{-5}/°F) than the measured value [$(6.2 \pm 1.4) \times 10^{-5}$/°F].

TABLE V. Computed Multiplication Constants of the Unrodded, 36-plutonium-assembly Loading at Several Temperatures with a Boric Acid Concentration of 4.8 g/gal

Temp, °F	k	Temp, °F	k
70	0.99427	220	0.99570
120	0.99413	270	0.99528
170	0.99591	325	0.99408

Although this computed temperature coefficient agrees with experiment much better than that obtained with the early model (which predicted a negative temperature coefficient), it significantly underestimates the magnitude of the coefficient. Isolating the causes of this discrepancy between theory and experiment is difficult.

One source of error could be that an inadequate representation of the core structure is used in the calculational model. In Appendix A, Fig. 5 shows a nine-assembly quadrant of the plutonium test zone with its associated auxiliary structure and Fig. 6 shows how this material is represented in the two-dimensional diffusion-theory computations of the multiplication constant. A comparison of these figures shows that representing the EBWR core structure more precisely in the two-dimensional computations would be extremely difficult.

Inadequate representation of the core structure could potentially cause errors in the group cross sections for the quadrant of nine fuel assemblies. There is thermal-flux peaking in the interstitial water between assemblies and between an assembly and the control-rod guide structure. However, this thermal-flux peaking is well represented in the three-stage homogenization procedure used to obtain thermal-group cross sections for the quadrant of fuel. For the fast cross sections, where flux peaking effects are small, the quadrant is homogenized in a single step. If inadequate representation of the core in the procedure for determining group cross sections is the source of the error in the calculation of temperature coefficients, which is doubtful, then correcting the error would be very difficult because existing computer codes do not permit a more precise representation.

In the calculations, the size of fuel pins and control rods did not change with temperature. This neglect of thermal-expansion effects should cause the magnitude of a positive temperature coefficient to be

underestimated. Axial expansion of the fuel pins reduces leakage and thereby increases k. Expansion of the Zircaloy-2 rod followers in the X - Y plane causes borated water to be displaced from the core and produces a positive reactivity effect. Another thermal-expansion effect that increases reactivity arises from the axial expansion of the control rod-control rod follower system. This system is driven upward from below the core and is free, therefore, to expand upward. On the other hand, the reactor vessel is suspended from above the core, so thermal expansion causes the core to be lowered. (At 489°F the core is 13/16 in. lower than at room temperature.) The relative movement of the control rod-control rod followers and the core causes the control rods to be withdrawn some fraction of an inch, since at room temperature they were inserted approximately a foot into the core. For this loading and control-rod configuration, a 1-in. withdrawal of the rods would introduce no more than 0.1% in Δk. For an estimated elongation of the control rods of 0.001 in./°F and a lowering of the core of 0.002 in./°F, neglect of this relative increment of core and control rods results in the magnitude of a positive temperature coefficient being overestimated by 0.3×10^{-5}/°F.

Another potential source of error is the use of inaccurate cross-section data. Errors in the thermal cross sections of Pu^{239} can give the energy dependence of α inaccurately (the ratio of captures to fission) and can cause significant errors in the temperature coefficient because the thermal-neutron spectrum is temperature-dependent (from changes in the density and the differential inelastic scattering cross section of water). The cross-section data for Pu^{239} were obtained from resonance parameters based on the compilation of Hughes et al.[14] To investigate the effect of uncertainties in the thermal cross-section data of Pu^{239} on effective thermal-group cross sections, we made THERMOS computations for plutonium zone fuel at 70 and 170°F with Pu^{239} data based on resonance parameters given by Schmidt.[15] The change in the effective group average $\nu\Sigma_f/\Sigma_a$ with temperature increase from 0.01156 to 0.01907 when Schmidt's parameters were used. This change in the temperature dependence of $\nu\Sigma_f/\Sigma_a$ corresponds to a 0.7×10^{5}/°F increase in the computed temperature coefficient.

III. EXPERIMENTS WITH THE FULL FUEL LOADING

A. Boric Acid Worth

Boric acid worth for the full loading, shown in Fig. 1, was measured by use of an experimental procedure that yielded also the differential worth of the nine-rod bank as a function of bank height, the shutdown margin, and verification of eight-rod and nine-rod shutdown. The aspects of this experiment that are concerned with the shutdown margin and with the eight-rod and the nine-rod shutdown criteria is discussed in Section III-C.

The experimental procedure used to make these measurements is given below. At a specified boric acid concentration, the height of the nine-rod bank for criticality is obtained. Then the differential worth at the center rod is obtained from period measurements. The reactor is then shut down, and after a specified volume of borated water is drained from the reactor vessel, a specified volume of unpoisoned water is added to lower the boric acid concentration to a desired level. Control rods No. 1 through 8 are then withdrawn to their previous setting, and the center rod is withdrawn to its new, lower, critical setting. The differential worth of the center rod at its new setting is obtained from period measurements. To verify eight-rod shutdown, the critical height of the eight-rod bank is obtained with the center rod withdrawn to its maximum allowable height. Finally, the critical nine-rod bank height for the new boric acid concentration is obtained. This procedure is repeated until further removal of boric acid would result in violation of the eight-rod shutdown criterion.

The boric acid worth is obtained from the reactivity worth of the reduction in height of the center rod required to compensate for the reduction in boric acid concentration and is computed from Eq. 1. This reactivity is identical with that associated with the insertion of the nine-rod bank. Thus, the nine-rod-bank worth is the boric acid worth, $(\Delta k/\Delta C)$ multiplied by $\Delta C/\Delta H_B$, where ΔH_B is the change in the critical nine-rod-bank height.

Tables VI and VII summarize the results of the experiment. Table VI gives the results of the period measurements of the differential worth of the center rod, and Table VII gives boric acid worths and the nine-rod bank worths. The differential worth of the nine-rod bank definitely increases as the bank is lowered from 21.3 to 11 in. The explanation for this effect is as follows: With the nine rods banked at a low height, the flux is pushed toward the bottom of the core, where it has a large curvature. For such a flux distribution, a small movement of the bank more effectively causes a loss of neutrons by means of absorption in the rods and increased leakage than for the less compressed flux distribution that would be present if the rods were at a higher setting.

TABLE VI. Results of Period Measurements Associated with Boric Acid Worth Measurement for the Initial Full Loading

H_3BO_3 Concentration, g/gal	Critical Height, Eight-rod Bank, in.	Critical Height, Rod No. 9, in.	Δh, Rod No. 9, in.	Doubling Time, sec	Δk (10^{-4})	Δk/Δh (10^{-4}/in.)
7.66 ± 0.02	21.30 ± 0.02	21.365 ± 0.007	0.884 ± 0.008	48 ± 3	6.82 ± 0.30	7.68 ± 0.41
6.68 ± 0.02	21.30 ± 0.02	12.800 ± 0.007	0.356 ± 0.008	52 ± 3	6.45 ± 0.25	18.1 ± 1.1
6.68 ± 0.02	18.65 ± 0.02	18.654 ± 0.007	0.674 ± 0.008	46 ± 3	7.00 ± 0.35	10.39 ± 0.69
5.84 ± 0.02	18.65 ± 0.02	11.598 ± 0.007	0.414 ± 0.008	33 ± 3	8.65 ± 0.45	20.9 ± 1.5
5.84 ± 0.02	16.60 ± 0.02	16.598 ± 0.007	0.498 ± 0.008	42 ± 3	7.50 ± 0.38	15.0 ± 1.0
5.20 ± 0.02	16.60 ± 0.02	11.846 ± 0.007	0.385 ± 0.008	37 ± 3	8.12 ± 0.41	21.1 ± 1.5
5.20 ± 0.02	15.12 ± 0.02	15.206 ± 0.007	0.371 ± 0.008	59 ± 3	5.86 ± 0.20	15.80 ± 0.89
4.67 ± 0.02	15.12 ± 0.02	11.366 ± 0.007	0.306 ± 0.008	46 ± 3	7.05 ± 0.35	23.0 ± 1.8
4.67 ± 0.02	14.10 ± 0.02	14.159 ± 0.007	0.327 ± 0.008	56 ± 3	6.19 ± 0.22	18.6 ± 1.1
4.18 ± 0.02	14.10 ± 0.02	10.981 ± 0.007	0.255 ± 0.008	55 ± 3	6.12 ± 0.22	24.0 ± 1.6
4.18 ± 0.02	13.22 ± 0.02	13.286 ± 0.007	0.284 ± 0.008	63 ± 3	5.57 ± 0.18	19.6 ± 1.2
3.70 ± 0.02	13.22 ± 0.02	10.334 ± 0.007	0.270 ± 0.008	48 ± 3	6.80 ± 0.30	25.2 ± 1.9
3.70 ± 0.02	12.45 ± 0.02	12.446 ± 0.007	0.338 ± 0.008	44 ± 3	7.27 ± 0.37	21.6 ± 1.6
3.32 ± 0.02	12.45 ± 0.02	10.057 ± 0.007	0.240 ± 0.008	54 ± 3	6.30 ± 0.23	26.2 ± 1.8
3.32 ± 0.02	11.80 ± 0.02	11.824 ± 0.007	0.284 ± 0.008	49 ± 3	6.73 ± 0.28	23.7 ± 1.6
3.00 ± 0.02	11.80 ± 0.02	10.097 ± 0.007	0.240 ± 0.008	55 ± 3	6.16 ± 0.23	25.7 ± 1.8
3.00 ± 0.02	11.31 ± 0.02	11.413 ± 0.007	0.258 ± 0.008	51 ± 3	6.58 ± 0.25	25.4 ± 1.8
2.80 ± 0.02	11.31 ± 0.02	10.197 ± 0.007	0.235 ± 0.008	53 ± 3	6.31 ± 0.23	26.9 ± 1.9
2.80 ± 0.02	11.00 ± 0.02	11.010 ± 0.007	0.247 ± 0.008	50 ± 3	6.63 ± 0.26	26.8 ± 1.9

TABLE VII. Boric Acid Worths and Nine-rod Bank Worths for the Initial Full Loading

Change in H_3BO_3 Concentration, g/gal	Change in Critical Nine-rod-bank Height, in.	Change in Reactivity, % Δk	Δk/ΔC, % Δk/g/gal	$\Delta k/\Delta H_B$, % Δk/in.
7.56 - 6.68	21.30 - 18.65	1.10 ± 0.10	-1.25 ± 0.16	0.42 ± 0.04
6.68 - 5.84	18.65 - 16.60	1.09 ± 0.12	-1.29 ± 0.16	0.53 ± 0.07
5.84 - 5.20	16.60 - 15.12	0.929 ± 0.095	-1.45 ± 0.22	0.63 ± 0.08
5.20 - 4.67	15.12 - 14.10	0.746 ± 0.076	-1.41 ± 0.22	0.73 ± 0.10
4.67 - 4.18	14.10 - 13.22	0.678 ± 0.064	-1.38 ± 0.21	0.77 ± 0.11
4.18 - 3.70	13.22 - 12.45	0.662 ± 0.068	-1.38 ± 0.23	0.86 ± 0.13
3.70 - 3.32	12.45 - 11.80	0.571 ± 0.060	-1.50 ± 0.28	0.88 ± 0.15
3.32 - 3.00	11.80 - 11.31	0.431 ± 0.045	-1.34 ± 0.26	0.88 ± 0.16
3.00 - 2.80	11.31 - 11.00	0.319 ± 0.034	-1.59 ± 0.41	1.03 ± 0.24

A trend toward increasing boric acid worth with decreasing boric acid concentration is discernible. However, because of the relatively large experimental errors of between ±15 and ±25%, the change in boric acid worth with concentration is within the experimental uncertainty. Therefore, we can only report that for boric acid concentrations between 3.0 and 7.6 g/gal, the boric acid worth for the full loading is (-1.40 ± 0.25) % Δk/g/gal.

Comparison between theory and experiment is unsatisfactory. REX calculations predicted boric acid worths of about -1.1% in Δk for an addition of 1 g/gal to the unpoisoned system and about -0.9% in Δk for an addition of 1 g/gal to the system that contains 9 g/gal. Part of this discrepancy could arise in the interpretation of the experiment from the use of an inaccurate β_{eff} in the inhour equation that relates doubling time to k_{ex}.

We obtained β_{eff} from an eight-group, perturbation-theory calculation made with the B1188RP[16] code that used real and adjoint flux distributions determined by a REX calculation for an unrodded system. Thermal-fission, delayed-neutron decay constants and group yields were used for Pu^{239} and U^{235}, and fast-fission values used for U^{238} and Pu^{240}.[17] The detailed spectra for the delayed-neutron groups were taken from the work of Batchelor and Hyder.[18] This model yielded ℓ = 41.7 μsec and β_{eff} = 0.00307 for the 36-assembly loading, and ℓ = 25.5 μsec and β_{eff} = 0.00534 for the full loading.

Although β_{eff} was computed for an unrodded system, the experiment was performed with the nine control rods inserted below the midplane of the core. This partly rodded system has real and adjoint flux distributions significantly different from those for the unrodded system. If the experimental flux distributions are shifted toward shim zones, where fission is predominantly in U^{235}, the computed β_{eff} is low, and the reactivity (in absolute units) associated with a given asymptotic period is underestimated; if the experimental flux distributions are shifted toward the plutonium zone, where fission is predominantly in Pu^{239}, the computed β_{eff} is high, and the reactivity associated with a given period is overestimated. Unfortunately, experimental and computational information on the flux distribution for the partly rodded system is not available so we

cannot compute the magnitude and sign of the error in differential rod worth arising from the use of an inaccurate β_{eff} in the inhour equation.

The order of magnitude of the effect of β_{eff} estimated was by computing the effect that varying the effective delayed-neutron yields of U^{235}, Pu^{239}, and Pu^{240}, which are dependent on the real and adjoint flux distribution, had on the k_{ex} associated with a given period. For the cold, full loading and a period of 60 sec, increasing the effective delayed-neutron yields of Pu^{239} and Pu^{240} by 10% and decreasing the effective neutron yields of U^{235} by 10%, relative to the result of the perturbation-theory calculation, changed k_{ex} from 7.54×10^{-4} to 7.10×10^{-4}. When the effective delayed-neutron yield of U^{235} was increased by 10% and those of Pu^{239} and Pu^{240} were reduced by 10%, k_{ex} changed from 7.54×10^{-4} to 7.97×10^{-4}. The latter change in effective delayed-neutron yields is about the same as the change resulting from a burnup of 0.15% of the plutonium zone fuel. The position of the control rod can possibly have as much an effect on β_{eff} as this burnup does. An error in experimental k_{ex} of 5-10% may therefore be introduced by the use of an inaccurate β_{eff} in the inhour equation.

B. Uniform Temperature Coefficient

The uniform temperature coefficient of reactivity for the full fuel loading was obtained from an experimental procedure that yielded the multiplication constant of the system, with five rods withdrawn to their maximum allowable heights and four rods fully inserted, as a function of temperature for boric acid concentrations, of approximately 4.5 g/gal. These measurements were not made with an unrodded system, in contrast to those for the plutonium zone loading, because we would have required a boric acid concentration in excess of that at which the void coefficient would have become positive.

The experimental procedure was to heat the reactor to a specified temperature by use of the startup heater. At this temperature, the boric acid concentration was adjusted so that the reactor was critical with the offcenter, noncorner rods (No. 1, 3, 5, and 7) fully inserted, the center rod and three corner rods (No. 2, 6, and 8) withdrawn to their maximum allowable heights; and the other corner rod (No. 4) partly inserted. The worth of the part of rod No. 4 in the core was obtained from the period resulting when the rod was withdrawn to 36 in. If too short a period would have resulted if the rod were withdrawn to 36 in. in one step, rod No. 8 was inserted as required, so that rod No. 4 could be withdrawn in several steps. After the reactor water was cooled to other specified temperatures, the procedure was repeated.

With the assumptions that the control-rod worths are not strongly temperature-dependent and that the worth of rod No. 4 is not strongly dependent upon the position of rod No. 8, the experimental procedure yields

the multiplication constant of the system with four rods inserted, as a function of temperature and boric acid concentration. To obtain a temperature coefficient from this information, we must normalize k to a reference boric acid concentration by use of the measured boric acid worth (-0.0140 ± 0.0025) Δk/g/gal, and Eq. 2.

Table VIII gives the results of the period measurements. The integral worth of the inserted portion of rod No. 4 was obtained at 78°F from the measurements of 5/19/66, at 121°F from the measurements of 6/2/66, and at 153°F from the measurements of 5/20/66 and 5/31/66. Table IX gives the resulting multiplication constant, before and after normalization to a boric acid concentration of 4.39 g/gal. From this information, a uniform temperature coefficient of (3.65 ± 1.73) x 10^{-5} Δk/°F between 78 and 153°F is obtained. Because of the influence of the shim and the natural uranium zones, this temperature coefficient, although still positive, is less than that for the partial plutonium loading.

TABLE VIII. Results of Period Measurements Associated with the Measurement of the Temperature Coefficient of the Initial Full Loading

Date	H_3BO_3 Concentration, g/gal	Height of Rod No. 8, in.	Height of Rod No. 4, in.		Δk (10^{-4})	Temp, °F
			Level	Period		
5/19/66	4.39 ± 0.02	36.5	23.516	26.616	6.70 ± 0.89	78.8 ± 0.5
	4.39 ± 0.02	32.005	26.616	30.092	7.09 ± 0.90	78.9 ± 0.3
	4.39 ± 0.02	28.521	30.092	36.106	9.08 ± 1.20	78.4 ± 0.4
5/20/66	4.37 ± 0.02	36.5	7.566	12.443	5.75 ± 0.73	153.5 ± 0.8
	4.38 ± 0.02	33.336	12.443	16.232	6.74 ± 0.87	152.0 ± 0.9
	4.39 ± 0.02	30.550	16.232	19.329	6.88 ± 0.90	151.2 ± 0.8
	4.38 ± 0.02	27.947	19.329	23.516	10.09 ± 1.30	150.5 ± 0.8
5/31/66	4.42 ± 0.02	24.173	23.516	26.056	6.45 ± 0.84	153.7 ± 0.5
	4.42 ± 0.02	22.209	26.056	28.820	6.55 ± 0.85	153.4 ± 0.5
	4.42 ± 0.02	20.515	28.820	32.048	6.84 ± 0.87	152.6 ± 0.8
	4.42 ± 0.02	17.213	32.048	36.106	6.33 ± 0.81	152.2 ± 0.7
6/1/66	4.57 ± 0.02	36.5	23.516	26.631	6.95 ± 0.90	154.7 ± 0.7
	4.58 ± 0.02	32.019	26.631	30.536	8.12 ± 1.05	153.4 ± 0.8
	4.58 ± 0.02	28.470	30.536	36.106	8.58 ± 1.11	152.7 ± 0.8
6/2/66	4.39 ± 0.02	36.5	16.138	19.133	6.14 ± 0.81	121.5 ± 0.6
	4.39 ± 0.02	32.580	19.133	22.333	7.20 ± 0.93	121.6 ± 0.7
	4.39 ± 0.02	29.045	22.333	25.351	6.97 ± 0.90	121.7 ± 0.6
	4.39 ± 0.02	26.209	25.351	28.529	7.21 ± 0.95	122.0 ± 0.6
	4.39 ± 0.02	23.329	28.529	31.809	6.59 ± 0.85	122.1 ± 0.6
	4.39 ± 0.02	20.769	31.809	36.106	6.49 ± 0.86	122.1 ± 0.6

TABLE IX. Experimental Multiplication Constants of the Initial Full Loading with the Noncorner, Offcenter Control Rods Fully Inserted

Temp, °F	H_3BO_3 Concentration, g/gal	Critical Height, Rod No. 4, in.[a]	k with Rod No. 4 Withdrawn	k Normalized to H_3BO_3 Concentration of 4.39 g/gal
78.7 ± 0.6	4.39 ± 0.02	23.52 ± 0.03	1.00229 ± 0.00030	1.00229 ± 0.00030
121.5 ± 0.8	4.39 ± 0.02	16.14 ± 0.03	1.00406 ± 0.00053	1.00406 ± 0.00053
153.5 ± 0.9	4.37 ± 0.02	7.50 ± 0.03	1.00531 ± 0.00072	1.00503 ± 0.00100

[a]The center rod and the other three corner rods are withdrawn to their maximum permissible heights.

The assumption that the rod worths are not strongly dependent upon temperature was tested by determining the reactivity worth of withdrawing rod No. 4 from 23.5 in. (with rod No. 8 initially at 36.5 in.) at two

temperatures. From the measurements of 5/19/66, the worth of withdrawing the rod at 78°F was (0.229 ± 0.030) % Δk; from the measurements of 6/1/66, the worth of withdrawing the rod at 153°F was (0.236 ± 0.031) % Δk. Since the difference between these two worths is less than the experimental error, a dependence of rod worth upon temperature cannot be inferred.

The assumption that the worth of rod No. 4 is not strongly influenced by the position of rod No. 8 was tested by determining the reactivity worth of withdrawing rod No. 4 from 23.5 in. at 153°F with rod No. 8 at different heights. From the measurements of 5/31/66, with rod No. 8 initially at 24.2 in., the reactivity worth of withdrawing rod No. 4 was (0.262 ± 0.034) % Δk; from the measurements of 6/1/66, with rod No. 8 initially at 36.5 in., the reactivity worth of withdrawing rod No. 4 was (0.236 ± 0.031) % Δk. Since the difference in rod worth is nearly equal to the experimental uncertainties, the magnitude of the effect of the position of rod No. 8 on the worth of withdrawing rod No. 4 cannot be inferred. However, if the worth of withdrawing rod No. 4 is increased by positioning rod No. 8 more deeply in the core, the magnitude of the positive temperature is overestimated since, as can be seen from Table VIII, rod No. 8 was deeper in the core during the measurements at 120 and 153°F than it was during the measurements at 78°F. If the worth of withdrawing rod No. 4 was overestimated by approximately 0.0003 Δk during the measurements at 153°F, because of the position of rod No. 8, the temperature coefficient was overestimated by approximately 0.6×10^{-5} Δk/°F.

Two-dimensional, four-group, diffusion-theory computations of the multiplication constant of the assembly at 70, 120, 170, 220, 270, and 325°F were made with the PDQ-3 Code. The boric acid concentration, which was taken as 5.0 g/gal at 70°F, was proportional to the temperature-dependent water density. Spectrum-dependent group cross sections for each temperature and fuel mixture were obtained by use of the GAM and the Argonne-revised THERMOS codes, in accordance with the model described in Appendix A. The calculations were performed with three control-rod configurations (only the center rod out, the corner and the center rods out, and all rods out) so that rod worths could be estimated.

Table X gives the results of these calculations, with 0.0084 added to the computed multiplication constants to normalize them to 4.4 g/gal.

TABLE X. Computed Temperature-dependent Multiplication Constants and Rod Worths for the Initial Full Loading with a Boric Acid Concentration of 4.4 g/gal

Temp, °F	k, Rods out			Rod Worths	
	Center	Center-corner	All	Corners (flats in)	Flats (others out)
70	0.9625	1.0198	1.0968	0.0573	0.0770
120	0.9597	1.0166	1.0946	0.0569	0.0780
170	0.9584	1.0140	1.0929	0.0555	0.0789
220	0.9555	1.0100	1.0910	0.0495	0.0810
270	0.9519	1.0050	1.0886	0.0531	0.0836
325	0.9481	1.0011	1.0865	0.0530	0.0854

Table X indicates that the calculations yield a negative temperature coefficient of $-5 \times 10^{-5}/°F$ between 70 and 170°F. Here the magnitude of the discrepancy between measurement and calculation is 60% larger than for the plutonium loading.

It is difficult to determine the effects that account for the larger discrepancy between measurement and theory for the temperature coefficient of the full system. Thermal-group cross sections for fuel regions were derived with the assumption that the control rods were withdrawn; however, during the measurements, four control rods were inserted. The spectral hardening introduced by the additional absorber could affect the temperature dependence of the fuel cross sections. However, when thermal cross sections for the plutonium fuel were derived with the control rods inserted, the change in the thermal $\nu\Sigma_f/\Sigma_a$ with temperature was the same as before.

It should be remembered that the experimental configuration during the measurement of the temperature coefficient of the full loading was much more complicated than the configuration for the corresponding measurement with the partial plutonium loading. For the partial plutonium loading, the system was virtually unrodded and there was only one type of fuel assembly; for the full loading, four control rods were inserted and three types of fuel assemblies were present. The four control rods that were inserted distort the neutron-flux distribution which adds uncertainty to the k_{ex} obtained from a period measurement, as was discussed in the preceding section. In general, it is just to be expected that additional complications in the experimental configuration introduce uncertainties in the interpretation of the measurements and approximations in the representation of the configuration in calculations.

C. Eight-rod Shutdown and Nine-rod Shutdown for Several Loadings

The reactor must be operated so that it can be shut down by the insertion of all nine rods when the reactor water is unpoisoned (nine-rod shutdown criterion) and so that it can be shut down by the insertion of any eight rods when the reactor water is borated (eight-rod shutdown criterion). For several loadings, nine-rod shutdown was verified and the minimum boric acid concentration for eight-rod shutdown was determined.

For these loadings, we obtained the critical nine-rod bank height and the critical eight-rod bank height for several boric acid concentrations with rod No. 9 (the control rod with the greatest worth) withdrawn to its maximum allowable height. For loadings for which boric acid worth was measured, these bank heights were obtained as part of the experiment in which boric acid worth was measured. Figure 4 is a typical plot of these bank heights as a function of boric acid concentration. To obtain the minimum boric acid concentration for satisfaction of the eight-rod shutdown criterion, we extrapolate to zero bank height the curve of critical eight-rod bank height versus boric acid concentration.

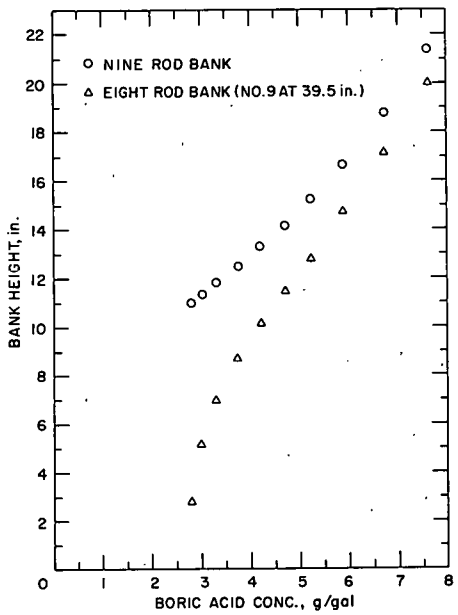

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Fig. 4. Critical Eight-rod Bank Heights and Nine-rod Bank Heights as a Function of Boric Acid Concentration for the Full Loading

The nine-rod shutdown criterion is satisfied if the critical nine-rod bank height is positive when the reactor water is not borated. However, for the loadings considered, operation without boric acid would violate the eight-rod shutdown criterion. Figure 4 and Table VII indicate that the nine-rod bank worth becomes reasonably constant at low bank heights. Therefore, nine-rod shutdown was tested by linearly extrapolating the plot of critical nine-rod bank height versus boric acid concentration to zero boric acid concentration.

Table XI gives the minimum boric acid concentration for eight-rod shutdown, the critical nine-rod bank height at this minimum allowable boric acid concentration, and the extrapolated nine-rod bank height for the unpoisoned system. From the extrapolated bank heights, we see that the nine-rod shutdown criterion is undoubtedly satisfied for all loadings.

TABLE XI. Minimum Boric Acid Concentration for Eight-rod Shutdown and Critical Nine-rod Bank Heights for Several Loadings

Loading	Minimum H_3BO_3 Concentration, g/gal	Critical Nine-rod Bank Heights, in.	
		Minimum H_3BO_3	Unborated
Pu	1.25	14.6	11.8
Pu + 1 shim	1.70	12.6	9.3
Pu + 2 shim	2.45	10.9	6.9
Full	2.70	10.7	6.4
Full (eight spikes)	3.40	10.2	5.5

It is convenient now to introduce the shutdown margin, which is defined as the compliment of the multiplication constant, 1 - k, of the unpoisoned system with all control rods fully inserted. The shutdown margin should be as small as possible, commensurate with safety; otherwise the loading will not have sufficient reactivity to reach an adequately high operating power. During power operation, because of burnup of the europium and samarium poisons in the shim fuel, reactivity will probably increase by less than 1% initially.[3] Therefore, the shutdown margin should be between 1 and 2%. The eight-rod shutdown criterion precludes direct measurement of this quantity. Therefore, the shutdown margin of the full loading was estimated by use of the assumptions of constant bank

worth, $\Delta k/\Delta H$, and constant boric acid worth, $\Delta k/\Delta C$. These assumptions are consistent with the results of the measurements given in Section III-B. The reactor was critical with the nine rods banked at 10.7 in. when the reactor was poisoned with 2.7 g/gal boric acid. Extrapolating from this point to the unpoisoned, fully rodded configuration results in the following shutdown margin:

$$SM = 10.7\ (\Delta k/\Delta H) + 2.7\ (\Delta k/\Delta C). \qquad (3)$$

With $\Delta k/\Delta H = 0.009 \pm 0.002$ Δk/in. and $\Delta k/\Delta C = -0.0140 \pm 0.0025$ $\Delta k/g/gal$, the estimate of the shutdown margin is $(6 \pm 3)\%$. Because of the large uncertainty in this estimate, we made a two-dimensional, four-group computation of the multiplication constant of the unpoisoned, fully rodded reactor using the 20-GRAND code.[19] This computation yielded a shutdown margin of 3.4%, which is just within the uncertainty of our estimate. A shutdown margin of 4-6% limits operating power and the maximum exposure that can be attained during the limited duration of the program. A computation made with 20-GRAND predicts an operating power of about 33 MW for this loading.

Since EBWR has been safely operated at powers up to 70 MW without carryover of steam, we changed the fuel loading to increase reactivity and permit operation at powers approaching 70 MW. Computations made with 20-GRAND predicted that if eight 6%-enriched uranium assemblies in the first shim zone were replaced by highly enriched uranium spike assemblies, which were used during the earlier operation of EBWR at 100 MW, the shutdown margin would be reduced by 2% and the operating power would be raised to 55 MW. Therefore, this fuel substitution was made. The eight spike assemblies were put in the center locations of the rows of three assemblies bounded by control rods (see Fig. 1). The results of the measurements of boric acid worth and of temperature coefficients for the revised loading are given in Sections III-D and III-E, respectively.

D. Boric Acid Worth (Eight Spike Assemblies)

After the fuel substitution, in which eight enriched uranium shim assemblies were replaced with spike assemblies, boric acid worths and nine-rod bank worths were remeasured using the same procedure as before (see Section III-B). Table XII gives the results of the period measurements of differential rod worth, and Table XIII gives the boric acid worths and nine-rod bank worths.

As before, the nine-rod bank worth increases as the bank is lowered. The boric acid worths exhibit a trend toward increasing worth with decreasing concentration, but because the change is within the uncertainty of experiment, only an average worth of -0.0125 ± 0.0025 $\Delta k/g/gal$ between boric acid concentrations of 3.6 and 8.2 g/gal is reported. These boric acid

worths are in better agreement with calculated predictions (-1.1% in Δk for an addition of 1 g/gal to the unpoisoned system, and -0.9% in Δk for an addition of 1 g/gal to the system that contains 9 g/gal) than the measurements performed before the fuel substitution.

TABLE XII. Results of Period Measurements Associated with Boric Acid Worth Measurements for Revised Full Loading

H_3BO_3 Concentration, g/gal	Eight-rod Bank Height, in.	Critical Height, Rod No. 9, in.	Δh, Rod No. 9, in.	Doubling Time, sec	Δk (10^{-4})	$\Delta k/\Delta h$, % Δk/in.
8.22 ± 0.02	18.95 ± 0.03	19.264 ± 0.007	0.945 ± 0.008	53 ± 3	6.37 ± 0.64	0.0674 ± 0.0074
7.41 ± 0.02	18.95 ± 0.03	10.912 ± 0.007	0.433 ± 0.008	52 ± 3	6.42 ± 0.64	0.148 ± 0.018
7.41 ± 0.02	17.00 ± 0.03	17.093 ± 0.007	0.557 ± 0.008	68 ± 3	5.50 ± 0.55	0.098 ± 0.011
6.69 ± 0.02	17.00 ± 0.03	10.417 ± 0.007	0.426 ± 0.008	50 ± 3	6.58 ± 0.66	0.155 ± 0.018
6.69 ± 0.02	15.50 ± 0.03	15.594 ± 0.007	0.484 ± 0.008	68 ± 3	5.45 ± 0.54	0.112 ± 0.013
6.15 ± 0.02	15.50 ± 0.03	10.512 ± 0.007	0.463 ± 0.008	39 ± 3	7.80 ± 0.78	0.169 ± 0.020
6.15 ± 0.02	14.40 ± 0.03	14.490 ± 0.007	0.520 ± 0.008	51 ± 3	6.53 ± 0.65	0.125 ± 0.014
5.61 ± 0.02	14.40 ± 0.03	10.060 ± 0.007	0.367 ± 0.008	52 ± 3	6.47 ± 0.65	0.176 ± 0.022
5.61 ± 0.02	13.40 ± 0.03	13.603 ± 0.007	0.468 ± 0.008	50 ± 3	6.65 ± 0.66	0.142 ± 0.017
5.14 ± 0.02	13.40 ± 0.03	10.079 ± 0.007	0.349 ± 0.008	52 ± 3	6.45 ± 0.64	0.185 ± 0.023
5.14 ± 0.02	12.68 ± 0.03	12.777 ± 0.007	0.466 ± 0.008	45 ± 3	7.12 ± 0.71	0.153 ± 0.018
4.71 ± 0.02	12.68 ± 0.03	9.643 ± 0.007	0.327 ± 0.008	51 ± 3	6.58 ± 0.66	0.201 ± 0.025
4.71 ± 0.02	12.00 ± 0.03	12.086 ± 0.007	0.437 ± 0.008	43 ± 3	7.38 ± 0.74	0.169 ± 0.020
4.33 ± 0.02	12.00 ± 0.03	9.519 ± 0.007	0.320 ± 0.008	52 ± 3	6.46 ± 0.65	0.202 ± 0.025
4.33 ± 0.02	11.40 ± 0.03	11.584 ± 0.007	0.437 ± 0.008	38 ± 3	7.98 ± 0.80	0.182 ± 0.022
3.95 ± 0.02	11.40 ± 0.03	9.097 ± 0.007	0.284 ± 0.008	53 ± 3	6.37 ± 0.64	0.224 ± 0.029
3.95 ± 0.02	10.90 ± 0.03	11.057 ± 0.007	0.375 ± 0.008	42 ± 3	7.47 ± 0.75	0.200 ± 0.024
3.60 ± 0.02	10.90 ± 0.03	8.836 ± 0.007	0.283 ± 0.008	52 ± 3	6.45 ± 0.64	0.228 ± 0.029
3.60 ± 0.02	10.40 ± 0.03	10.518 ± 0.007	0.325 ± 0.008	51 ± 3	6.50 ± 0.65	0.200 ± 0.025

TABLE XIII. Boric Acid Worths and Nine-rod Bank Worths for the Revised Full Loading

Change in H_3BO_3 Concentration, g/gal	Change in Critical Nine-rod Bank Height, in.	Change in Reactivity, % Δk	$\Delta k/\Delta C$, % Δk/g/gal	$\Delta k/\Delta H_B$, % Δk/in.
8.22 - 7.41	18.9 - 17.0	0.90 ± 0.12	-1.11 ± 0.18	0.42 ± 0.06
7.41 - 6.69	17.0 - 15.5	0.84 ± 0.11	-1.17 ± 0.20	0.56 ± 0.09
6.69 - 6.15	15.5 - 14.4	0.714 ± 0.093	-1.32 ± 0.24	0.65 ± 0.11
6.15 - 5.61	14.4 - 13.4	0.667 ± 0.087	-1.24 ± 0.23	0.67 ± 0.12
5.61 - 5.14	13.4 - 12.7	0.575 ± 0.075	-1.22 ± 0.25	0.80 ± 0.16
5.14 - 4.71	12.7 - 12.0	0.555 ± 0.072	-1.29 ± 0.27	0.82 ± 0.17
4.71 - 4.33	12.0 - 11.4	0.476 ± 0.062	-1.25 ± 0.27	0.79 ± 0.17
4.33 - 3.95	11.4 - 10.9	0.506 ± 0.066	-1.33 ± 0.29	1.01 ± 0.23
3.95 - 3.60	10.9 - 10.5	0.475 ± 0.062	-1.35 ± 0.30	1.18 ± 0.30

The reactivity effect of the fuel substitution can be estimated from the change in the critical height of the nine-rod bank for a given boric acid concentration. With a boric acid concentration of 6.68 g/gal, the replacement of eight 6%-enriched shim assemblies with highly enriched spike assemblies, lowered the critical height of the nine-rod bank from 18.65 to 15.5 in. For this range of bank heights, the bank worth was (0.55 ± 0.12) % Δk/in. By multiplying the bank worth by the distance the bank was moved, we found the fuel substitution to increase reactivity by 0.0173 ± 0.002, which is in reasonable agreement with the calculated prediction of 0.02.

E. Uniform Temperature Coefficient (Eight Spike Assemblies)

To obtain the uniform temperature coefficient of reactivity for the full loading with eight spike assemblies in the first shim zone, we used the same procedure utilized before the fuel substitution (see Section III-B). These measurements were carried out between room temperature and 362°F, the maximum temperature attainable with heat supplied by the startup heater.

The results of the period measurements of differential k_{ex} are given in Table XIV, in which the boric acid concentrations are the equivalent room-temperature concentrations. The multiplication constant of the reactor with five rods withdrawn is obtained from the sum of the differential k_{ex} obtained when rod No. 4 is withdrawn to 36 in. in steps while rod No. 8 is inserted to keep the reactor from becoming supercritical. Table XV gives the resulting k as a function of temperature and boric acid concentration. To calculate temperature coefficients, we normalized these multiplication constants to a room-temperature boric acid concentration of 6.25 g/gal by using the boric acid worth previously obtained for this loading, -0.0125 ± 0.0025 g/gal (see Section III-D). From the normalized k, the average uniform temperature coefficient for four temperature ranges were obtained.

TABLE XIV. Results of Period Measurements Associated with the Measurement of the Temperature Coefficient of the Revised Full Loading

Date	H_3BO_3 Concentration, g/gal	Temp, °F	Height of Rod No. 8, in.	Height of Rod No. 4, in.		Δk, 10^{-4}
				Level	Period	
9/16/66	6.02 ± 0.02	152.5 ± 1.0	36.5	29.910	32.588	5.67 ± 0.96
	6.02 ± 0.02	152.5 ± 1.0	33.3	32.588	35.866	5 80 ± 0.98
9/16/66	5.71 ± 0.03	64.5 ± 1.0	36.5	28.994	32.150	6.91 ± 1.17
	5.71 ± 0.03	64.5 ± 1.1	33.3	32.150	35.866	6.20 ± 1.05
	6.35 ± 0.04	362.0 ± 2	36.5	27.591	29.863	6.36 ± 1.08
	6.35 ± 0.04	362.0 ± 2	34.0	29.863	32.107	6.66 ± 1.13
	6.35 ± 0.04	362.0 ± 2	32.5	32.107	35.997	7.97 ± 1.35
10/5/66	6.14 ± 0.04	321.0 ± 2	36.5	15.533	17.733	5.39 ± 0.91
	6.14 ± 0.04	321.0 ± 2	34.2	17.733	19.982	6.30 ± 1.07
	6.14 ± 0.04	321.0 ± 2	31.9	19.982	22.623	8.27 ± 1.40
	6.14 ± 0.04	321.0 ± 2	29.7	22.623	25.108	7.55 ± 1.28
	6.14 ± 0.04	321.0 ± 2	27.6	25.108	27.591	7.35 ± 1.25
10/6/66	6.16 ± 0.04	247.0 ± 2	36.5	21.882	23.853	5.60 ± 0.95
	6.16 ± 0.04	247.0 ± 2	33.5	23.853	26.202	6.34 ± 1.08
	6.16 ± 0.04	247.0 ± 2	30.9	26.202	28.667	6.70 ± 1.14
	6.16 ± 0.04	247.0 ± 2	28.5	28.667	31.154	6.42 ± 1.09
	6.16 ± 0.04	247.0 ± 2	26.3	31.154	33.539	5.72 ± 0.97
	6.16 ± 0.04	247.0 ± 2	24.6	33.539	35.844	4.95 ± 0.84

TABLE XV. Experimental Multiplication Constants and Temperature Coefficients of the Revised Full Loading with the Noncorner, Offcenter Control Rods Fully Inserted

Temp, °F	H_3BO_3 Concentration, g/gal	Critical Height, Rod No. 4, in.	k, Rod No. 4 Withdrawn[a]	k, Normalized to H_3BO_3 Concentration of 6.25 g/gal	$\Delta k/\Delta T$, $\%\Delta k/°F$
64.5 ± 1	5.71 ± 0.03	28.994	1.00131 ± 0.00016	0.99456 ± 0.00052	
					0.00435 ± 0.00083
152.5 ± 1	6.02 ± 0.02	29.910	1.00115 ± 0.00014	0.99843 ± 0.00039	
					0.00430 ± 0.00105
246.0 ± 1	6.16 ± 0.04	21.882	1.00357 ± 0.00025	1.00245 ± 0.00075	
					0.00236 ± 0.00148
321.0 ± 2	6.14 ± 0.04	15.533	1.00559 ± 0.00032	1.00422 ± 0.00082	
					-0.00212 ± 0.00320
362.0 ± 2	6.35 ± 0.04	27.591	1.00210 ± 0.00021	1.00335 ± 0.00071	

[a]The other three corner rods and the center rod are withdrawn to their maximum permissible heights.

The results given in Table XV indicate a positive temperature coefficient at low temperatures, which becomes smaller as temperature is raised to become negative apparently about 300°F. Much of the uncertainty in the temperature coefficients arises from uncertainties in boric acid concentration. Boric acid concentration can be obtained by titration to no better than ±0.02 g/gal. Additional uncertainties were present at elevated temperatures from observed drifts in boric acid concentration of about 0.04 g/gal. These uncertainties in boric acid concentration may seem small, but they introduce uncertainties in reactivity of the order of the change resulting from a change in temperature of about 20°F.

The temperature coefficient of the revised full loading were not calculated. The change in the fuel loading should not increase the accuracy of the calculations. Since the calculations for the initial full loading were inaccurate, we felt that little would be learned from similar calculations for the revised full loading.

IV. CONCLUSIONS

1. It is possible to predict within 1% the cold multiplication constant of loadings of PuO_2-UO_2 assemblies and of loadings of PuO_2-UO_2 assemblies plus enriched UO_2 assemblies plus natural UO_2 assemblies in EBWR if care is taken to represent accurately the intracell, space-energy, thermal-flux distribution and the thermal-flux peaking in extra-fuel-assembly core structure and water. The assumption of separability of the intracell thermal flux in energy and space in the derivation of thermal-group cross sections caused k to be overestimated by 1-2%. Inadequate representation of thermal-flux peaking in extra-fuel-assembly water and core structure caused additional overestimates of k. Also, for small

systems, two-dimensional diffusion-theory codes rather than one-dimensional codes should be used to compute k, because the latter, by underestimating leakage, overestimated k.

2. For the loadings considered in this experiment, it was difficult to measure reactivity coefficients precisely and to calculate them accurately. Computations consistently predicted temperature coefficients significantly more negative than the experimental results. As was discussed in Sections II-D and III-B, the discrepancy between theory and experiment in temperature-coefficient measurements would be reduced if the effects of axial thermal expansion could be included in the calculation and if control-rod interaction effects could be assessed in the interpretation of the measurements. For partly rodded systems, for which the flux distributions were significantly different from those assumed in the computation of β_{eff}, uncertainties in the k_{ex} associated with a period introduce additional uncertainties in the experimental results.

There was fair agreement between measured and computed boric acid worths. However, because of uncertainties in changes in boric acid concentration, an expected dependence of boric acid worth upon boric acid concentration could not be inferred from the experimental results.

APPENDIX A

Revised Calculational Model

Before the critical experiments discussed in this report had been performed, the calculational model[3,7] used GAM,[5] DSN,[20] and SOFOCATE[21] to obtain group-averaged cross sections for one-dimensional, four-group, diffusion-theory calculations of critical configurations and reactivity coefficients. Flaws in this model became apparent when it yielded inaccurate predictions of the unrodded, unpoisoned critical loading and of the critical boric acid concentration for the unrodded, 36-plutonium-assembly loading (Sections II-A and II-B). The major revisions in the model were to use a THERMOS-type computation to obtain thermal-group cross sections and to use two-dimensional, four-group, diffusion-theory computations of reactivity. The reasons for these changes will now be discussed.

Because DSN plus SOFOCATE was used to derive the thermal-group cross sections, the early model assumed that the intracell thermal flux is separable in space and energy; i.e., it is the product of a function of space and a function of energy. Because of depressions in the thermal-neutron flux in the fuel pin above 0.2 eV from absorption by the isotopes of plutonium, the intracell thermal flux is not separable in space and energy. Since the predictions underestimated the critical loading by more than four assemblies, and since Liikala[10] had showed that calculations using thermal cross sections based on separability of the thermal flux overestimated reactivity for mixed U-Pu oxide lattices, DSN plus SOFOCATE was dropped in favor of the Argonne-revised THERMOS[6] code, which computes space- and energy-dependent fluxes.

When THERMOS-derived thermal cross sections were used in a one-dimensional calculation of the unpoisoned, unrodded critical loading, the critical loading increased from 17.8 to 20.1 assemblies. Since each assembly is worth approximately 1% in Δk, the multiplication constant is still overestimated by 2%. The one-dimensional code, by assuming a circularized fuel configuration, could be expected to underestimate leakage and thereby overestimate reactivity. Therefore, the model was changed further by using two-dimensional, diffusion-theory codes to compute multiplication constants, instead of one-dimensional codes.

1. Geometrical and Compositional Considerations

Figure 5 shows a quadrant of nine plutonium assemblies and its associated EBWR core structure. Two cells were used to obtain group cross sections. One cell, with dimensions of 12.75 x 12.75 in., is bounded by the center lines of the control-rod channels and included all reactor materials (control-rod guides, spacers, control-rod followers, and water). This "large cell" was used in one-dimensional calculations. The second cell, with dimensions of 12.25 x 12.25 in., is bounded by the control-rod guide structure.

This "small cell" is used to obtain fuel cross sections for two-dimensional calculations, in which the control rods (or Zircaloy followers), the Zircaloy spacers, and the interstitial water between the control rods and the guide structure, are separate regions. Figure 6 shows the representation of the core configuration with quadrant symmetry for the two-dimensional, diffusion-theory codes.

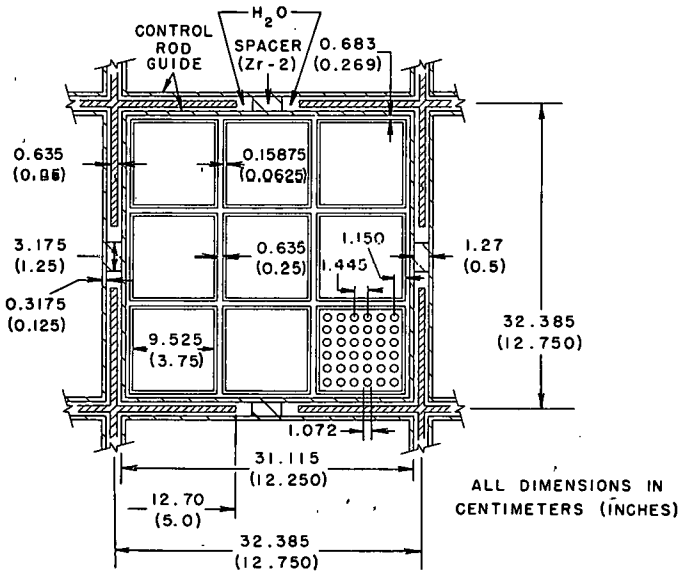

Fig. 5
Nine Fuel Assemblies and Associated Core Structure

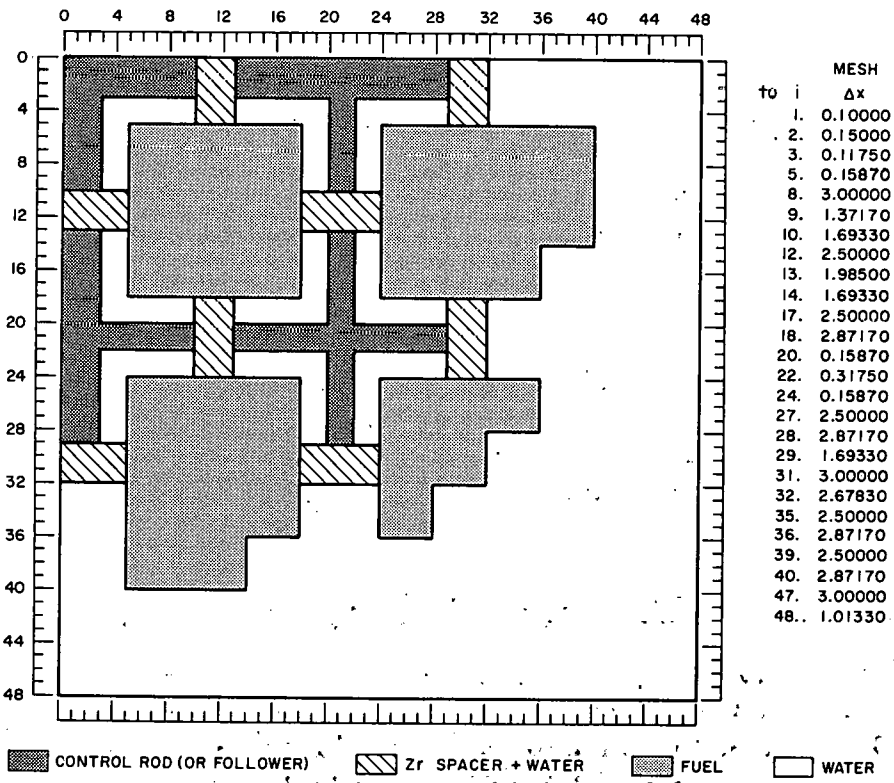

Fig. 6. Mockup of a Quadrant of EBWR for Two-dimensional Computations

Thermal-group cross sections were obtained from the Argonne-revised THERMOS code with a three-stage homogenization procedure so that thermal-flux peaking effects are accounted for more accurately. The first stage was to homogenize a cell centered about a fuel pin and containing the fuel pin, its cladding, and its associated water. This composition was then used in a cell associated with a fuel assembly. In this "assembly" cell, the inner region is 36 homogenized pin cells plus extra water in the interior of an assembly, the middle region is the Zircaloy can, and the outer region is the interassembly water. This homogenized assembly material is then used in the large-cell calculation.

The large cell consists of four regions: (1) the nine homogenized fuel assemblies, (2) the water between the outer assemblies and the guide structure, (3) the zirconium control-rod guide, and (4) the homogenized water, zirconium spacer, and zirconium rod follower inside the control-rod channel. The inner three regions are homogenized to obtain small-cell cross sections. Table XVI gives the volume fractions of fuel, water, and Zircaloy-2 in the various cells.

TABLE XVI. Cell Volume Fractions

Fuel	Material	Pin	Assembly	Small	Large
Plutonium, Shim, Natural	Fuel	0.33569	0.25507	0.23468	0.21663
	Zr	0.09630	0.13326	0.16301	0.188541
	H_2O	0.56801	0.61167	0.60231	0.59483
Spike	Fuel	0.44887	0.34724	0.31948	0.29492
	Zr	0.12982	0.16052	0.18809	0.21246
	H_2O	0.42131	0.49223	0.49242	0.49262

Because flux peaking in interstitial water at above thermal energies is small, group cross sections for the fuel are obtained from a GAM problem in which the homogenized small-cell composition is used.

Two types of water were used as compositions in the two-dimensional, diffusion-theory computations: "reflector water," which is outside the fuel assemblies, and "core water," which is water between the control-rod guide structure and the fuel assemblies. Group cross sections for reflector water were obtained from a neutron spectrum associated with pure (or borated) water. For the thermal group, cross sections for core water were obtained for the neutron spectrum associated with the water next to the control-rod guides (Region 2 of the large cell). For the fast groups, cross sections of core water were obtained for a neutron spectrum characteristic of the small-cell plutonium fuel composition.

In the temperature-coefficient measurements for the full fuel loadings, several control rods were fully inserted. In the associated calculations, for the thermal group, a control rod was represented by a logarithmic

boundary condition $(Dd\phi/dx)/\phi = -C$, where D is the thermal-diffusion coefficient of the medium bordering the rod. As in the physics analysis of the EBWR core 1A,[22] C was taken as 0.46948, which corresponds to a black slab bordering core water.

2. Cross-section Considerations

Group cross sections for the three fast groups of four-group calculations were obtained by use of the GAM[5] code. The normal GAM library[23] was employed (P-1 option), except that resonance parameters were included in the library for Pu^{240} (material No. 132) and U^{235} (material No. 133). Also, the Hanford RBU Library[24] values were employed for U^{238} (material No. 139) because Moon[8] found that GAM's U^{238} (material No. 12) had a deficiency in the capture cross section in the unresolved resonance regions, which caused reactivity to be overestimated in his analysis of the Yankee critical experiments.

The GAM library Pu^{239} (material No. 14), which does not have resonance parameters, was used, except that ν was taken as 2.89 in the second and third groups. The resonance integral code, RIC,[25] was used to obtain self-shielding factors for Pu^{239} for input to GAM. RIC evaluates Doppler-broadened resonance integrals in a manner similar to that used in GAM. Self-shielding factors were obtained from comparison of RIC-derived cross sections and the infinite-dilution cross sections of Pu^{239}.

TABLE XVII. Group Structure of the Four-group Cross-section Set

Group	Lower Energy, eV
1	5.53×10^3
2	1.44
3	0.532
4	0

Table XVII gives the energy structure of the four-group set. All neutrons are born in Group 1. Group 2 covers the fast-resonance energy range, and Group 3 covers the epithermal energy range in which absorption by the 1-eV resonance of Pu^{240} predominates. Group 4 is the thermal group.

Thermal cross sections were obtained by using the Argonne-revised THERMOS[6] code with computations having 30 velocity groups. The velocity mesh, which is given in Table XVIII, yielded the neutron flux in great detail near the 0.3-eV resonance of Pu^{239}. For hydrogen in water, the GAKER code, based on the Nelkin scattering model, was used to obtain temperature-dependent scattering cross sections; for other material, the heavy-gas model was used to obtain scattering cross sections.

Thermal cross sections for the isotopes of plutonium and uranium were obtained from the set developed by Moon.[8] Doppler-broadened absorption and fission cross sections for Eu^{151} and Sm^{149} were computed from

single-level resonance parameters given in BNL-325.[14] The W-subroutine[26] of O'Shea and Thacher was used to compute the complex probability integral, from which the symmetric (Ψ) and antisymmetric (X) Doppler-broadened line-shape functions are obtained. Table XIX gives these resonance parameters. For all calculations, absorption and fission cross sections derived from resonance parameters were based on a temperature of 293°K. For the temperatures encountered during the temperature-coefficient measurements, in which heat was supplied by nonnuclear means, the velocity-dependent absorption and fission cross sections were virtually independent of temperature.

TABLE XVIII. Velocity Mesh of 30-group THERMOS Calculations

Group	v_i	Δv_i	E_i, eV	Group	v_i	Δv_i	E_i, eV
1	0.1	0.16	0.00253	16	2.43	0.14	0.149394
2	0.26	0.16	0.001710	17	2.57	0.14	0.167104
3	0.42	0.16	0.004463	18	2.71	0.14	0.185806
4	0.58	0.16	0.008511	19	2.85	0.14	0.205499
5	0.74	0.16	0.013854	20	2.99	0.14	0.226184
6	0.90	0.16	0.020493	21	3.13	0.14	0.247862
7	1.06	0.16	0.028427	22	3.27	0.14	0.270530
8	1.22	0.16	0.037656	23	3.42047	0.16094	0.296000
9	1.38	0.16	0.048181	24	3.57094	0.14	0.322616
10	1.54	0.16	0.06001	25	3.71966	0.15744	0.350048
11	1.70	0.16	0.073117	26	3.87710	0.15744	0.380307
12	1.86	0.16	0.087528	27	4.03454	0.15744	0.411821
13	2.01	0.14	0.102214	28	4.19198	0.15744	0.444589
14	2.15	0.14	0.116949	29	4.34943	0.15744	0.478614
15	2.29	0.14	0.132676	30	4.50687	0.15744	0.513890

TABLE XIX. Resonance Parameters Used to Obtain Cross Sections for Use by THERMOS

Nuclide	E_r, eV	Γ_n, eV	Γ_γ, eV	g
Eu^{151}	0.327	0.000065	0.070	0.5
	0.461	0.000756	0.093	0.5
	1.055	0.000235	0.090	0.5
	1.830	0.000035	0.090	0.5
	2.712	0.00023	0.092	0.5
	3.361	0.00185	0.092	0.5
	3.710	0.00074	0.088	0.5
Sm^{149}	0.0976	0.00050	0.063	0.5625
	0.873	0.00082	0.060	0.5
	4.98	0.00226	0.067	0.5
	6.48	0.00080	0.062	0.5
	9.00	0.00900	0.062	0.5

APPENDIX B

Error Analysis for Boric Acid Worth and Temperature-coefficient Measurements

1. Differential Rod Worth

Differential rod worth, $\Delta k/\Delta h$, is obtained from the asymptotic period, or doubling time, that resulted when a control rod was withdrawn a distance Δh from the position for which the reactor was critical. The relative error in the differential rod worth is

$$\frac{\delta(\Delta k/\Delta h)}{\Delta k/\Delta h} = \frac{\delta(\Delta k)}{\Delta k} + \frac{\delta(\Delta h)}{\Delta h}, \qquad \text{(B-1)}$$

where $\delta(\Delta k)$ is the uncertainty in reactivity and $\delta(\Delta h)$ is the uncertainty in the change in rod position.

The position of the rod that was moved, with the reactor on a period, could be read precisely. A disk, which makes one revolution for every 2.618 in. of rod movement, has been attached to the control-rod drive shaft. The disk setting can be read with an uncertainty of $\pm 0.5°$ on the closed-circuit television in the control room. Use of this disk, which is virtually free of backlash, permits rod position to be determined to within ± 0.005 in. the uncertainty $\delta(\Delta h)$ arises mainly from drifts in the critical height of the rod from drifts in temperature and boric acid concentration. We estimate that $\delta(\Delta h)$, was ± 0.025 in. during these experiments.

There are three sources of error in the determination of reactivity from period measurements:

1. Fitting the experimental points to a straight line on semi-logarithmic graph paper.
2. The difference between the apparent period and the true asymptotic period.
3. Errors in the inhour equation arising from the use of inaccurate parameters.

In fitting the experimental points to straight lines, we found uncertainties in period of 1 to 2 sec for periods of roughly a minute. This relative uncertainty in period of $\pm 3\%$ corresponds to a relative uncertainty in reactivity of about 3%.

The doubling time was obtained from the counting rates of two BF_3 chambers located outside the core. The counts for a 20-sec interval of a

30-sec cycle were plotted against time on semilogarithmic paper as the counts rose from approximately 10^3 to 5×10^4. There was an interval of 1.5 to 2 sec between the time when we stopped the rod and the time we began to record counts. For these waiting times with periods of approximately a minute, the difference between the apparent period and the true asymptotic period is about 5%,[27] which corresponds to an error in reactivity of about 5%. When the contributions from these two factors are combined, a relative experimental error in differential rod worth of 10% is obtained.

In this analysis, we assumed that the experimental errors are random. The error in differential rod worth arising from the use of an inaccurate β_{eff} in the inhour equation is systematic and hence contributes to the discrepancy between theory and experiment. Therefore, the effects of the use of inaccurate β_{eff} have been given in the discussion of the discrepancy between theory and experiment for the boric acid worth measurements for the full loading (Section III-A).

2. Boric Acid Worth

The reactivity worth of a change in boric acid concentration, $\Delta k/\Delta C$, was obtained from the change in the critical height of the center rod, ΔH, when the boric acid concentration is changed by an amount ΔC. Differential rod-worth measurements were taken at the critical height before dilution, H_1, and the critical height after dilution, H_2. With the assumption that the differential worth varies linearly with height (see Fig. 3), boric acid worth is obtained from Eq. 1. The relative experimental uncertainty in boric acid worth is taken as

$$\frac{\delta(\Delta k/\Delta C)}{\Delta k/\Delta C} = \frac{\delta(\Delta k/\Delta h)}{\Delta k/\Delta h} + \frac{\delta(\Delta H)}{\Delta H} + \frac{\delta(\Delta C)}{\Delta C}, \qquad \text{(B-2)}$$

where

$$\Delta k/\Delta h = \frac{\Delta k/\Delta h|_{H_1} + \Delta k/\Delta h|_{H_2}}{2}, \qquad \text{(B-3)}$$

and

$$\delta(\Delta k/\Delta h) = \left[\delta\left(\Delta k/\Delta h|_{H_1}\right)^2 + \delta\left(\Delta k/\Delta h|_{H_2}\right)^2\right]^{1/2} \qquad \text{(B-4)}$$

The errors in the differential rod worths are obtained as described in the first section of this appendix. The uncertainty in boric acid concentration as obtained from titration is about 0.03 g/gal. This estimate of uncertainty in boric acid concentration is based on determination of boric acid concentration in samples prepared by the Chemistry Division and analyzed at the EBWR facility. If the error in the determination of one boric acid concentration is δC, the error in the difference of two boric acid concentrations

$\delta(\Delta C) = \sqrt{2}\,\delta C$; therefore for our measurements, $\delta(\Delta C) \cong 0.04$ g/gal. Similarly, the uncertainty in the height of a control rod for criticality is 0.025 in.; therefore, for our measurements, $\delta(\Delta H) \cong 0.035$ in.

3. Temperature Coefficient of Reactivity

The temperature coefficient of reactivity was obtained from the temperature dependence of the multiplication constant, normalized to a reference boric acid concentration, for specified control-rod configurations. For the measurements with the 36-plutonium-assembly loading, this configuration was all control rods withdrawn to their maximum allowable heights; for the full loading, the specified configuration was that the center rod and the four corner rods be withdrawn to their maximum allowable heights and the four flat rods be fully inserted.

The multiplication constant, k, for the specified control-rod configuration was obtained from the sum of the k_{ex} when rod No. 4 was withdrawn in steps to its maximum allowable height; i.e.,

$$k = 1 + \sum_i k_{ex_i}. \tag{B-5}$$

The uncertainty in k; δk, was obtained from

$$\delta k = \left[\Sigma \left(\delta k_{ex_i}\right)^2\right]^{1/2}, \tag{B-6}$$

where δk_{ex_i} is the uncertainty in a k_{ex}. To obtain the temperature coefficient, it was necessary to normalize k as a function of temperature to a reference boric acid concentration as given by Eq. 2. This introduces a relatively large uncertainty in the normalized multiplication constant, k_0, from the uncertainty in boric acid concentration, δC, and the uncertainty in boric acid worth, $\delta(\Delta k/\Delta C)$. Since these uncertainties are assumed to be random, the uncertainty in the normalized multiplication constant was taken as

$$\delta k_0 = \{(\delta k)^2 + [\delta C \times (\Delta k/\Delta C)]^2 + [(C - C_0) \times \delta(\Delta k/\Delta C)]^2\}^{1/2}. \tag{B-7}$$

The average temperature coefficient between T_1 and T_2 is simply

$$\frac{\Delta k}{\Delta T} = \frac{k_{0_2} - k_{0_1}}{T_2 - T_1}. \tag{B-8}$$

The relative error in the temperature coefficient is then

$$\frac{\delta(\Delta k/\Delta T)}{\Delta k/\Delta T} = \frac{\delta(T_2 - T_1)}{T_2 - T_1} + \frac{\delta k_{0_1}}{k_{0_1}} + \frac{\delta k_{0_2}}{k_{0_2}}. \tag{B-9}$$

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