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**Neutron Capture Cross Sections**  
of Some Rare Earth Nuclei

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by

George I. Bell

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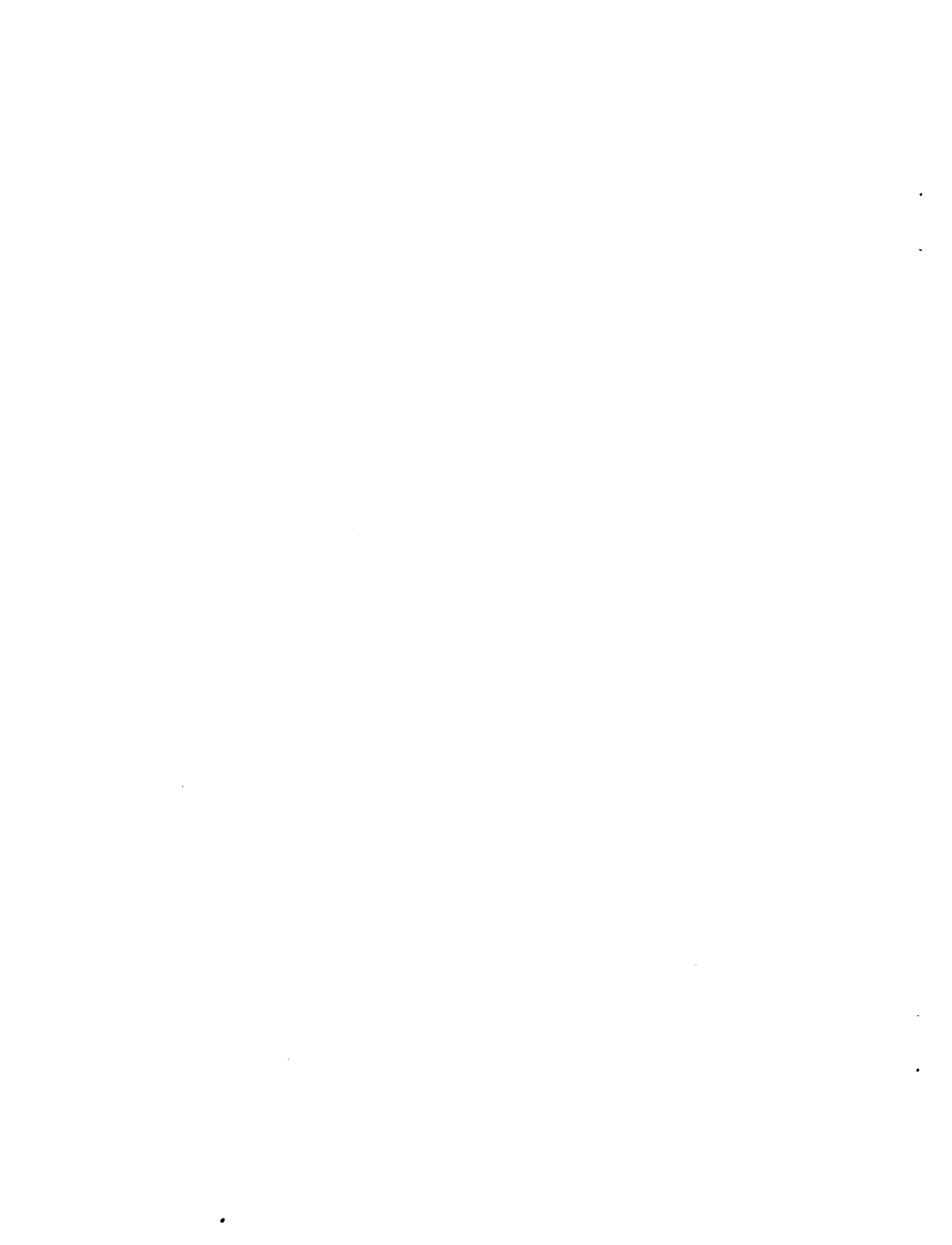
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## ~~ABSTRACT~~

Statistical theory has been used to calculate neutron capture cross sections of some rare earth nuclei for neutron energies of 1 and 10 keV. For nuclei with measured low energy resonance parameters, these parameters are used in the theory for s-wave capture; for other nuclei, the level density is computed from a semi-empirical expression, and the strength function and  $\Gamma_\gamma$  are determined by interpolation - or extrapolation. Qualitative agreement was found with the experimental values of  $\sigma(n,\gamma)$ . Particular attention is given to the isotopes of Eu ( $Z = 63$ ) which probably have larger  $\sigma(n,\gamma)$  than any other rare earths. Capture cross sections for the stable Eu nuclei ( $A = 151, 153$ ) are large because of large strength functions and small competition from compound elastic scattering (i.e. small  $\langle \Gamma_n \rangle$ ). The odd-odd Eu isotopes (especially Eu<sup>152</sup>) should have slightly larger  $\sigma(n,\gamma)$  (because of even smaller  $\langle \Gamma_n \rangle$ ) but their capture cross sections are unlikely to be more than about 20% larger than that for Eu<sup>151</sup>. Results for other nuclei are given.

## ACKNOWLEDGEMENTS

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## INTRODUCTION

In this note we consider the capture cross sections of rare earth nuclides for neutrons in roughly the energy range from 1 to 10 keV. We are interested in the cross sections averaged over many resonances, and in this energy range the effects of inelastic scattering and non-zero neutron angular momenta are unimportant.

Two somewhat different reasons stimulated the present exercise. The first was an interest in which rare earth nuclei may be expected to have the largest capture cross sections. It is well known that  $\sigma(n,\gamma)$  is large for many of the rare earths, and an examination of the data<sup>1</sup> indicates that Europium ( $Z = 93$ ,  $A = 151, 153$ ) has a larger capture cross section than any other rare earth element, larger by about a factor two over the energy range of interest. We wish to consider whether this notably large Eu cross section can be easily understood and whether other nuclides, for example  $\text{Eu}^{152}$ ,  $\text{Pm}^{146}$ , or  $\text{Lu}^{176}$ , might have substantially larger  $\sigma(n,\gamma)$ .

Our second reason arises from the similarity between the nuclear properties of the rare earth nuclei and heavy nuclei ( $A \gtrsim 238$ ). Both classes of nuclei are in general deformed and well removed from major shells. In a recent study,<sup>2</sup> we calculated  $\sigma(n,\gamma)$  for a variety of heavy nuclei, and it is of interest to see how well the same techniques may be applied to the rare earth nuclei. For the rare earths there are somewhat more data on  $\sigma(n,\gamma)$  than for the heavy nuclei.

In the next section statistical theory is described for the calculation of s-wave neutron capture cross sections. The results of the calculations are summarized in tables 2 and 3 where the available experimental data are also presented. Conclusions are drawn in the final section.

## METHOD OF CALCULATION

For neutron energies below 10 keV  $\ell = 0$  neutrons provide nearly all the capture cross section of the rare earths. This is because, for the rare earths, s-wave strength functions are large while the p-wave strength functions are small.<sup>3</sup> Thus we will now consider only s-wave capture, though an estimate of the p-wave capture will be made later on.

When an  $\ell = 0$  neutron encounters a target nucleus of spin  $I$ , compound nuclei of spin  $J = I \pm 1/2$  result (unless  $I = 0$ , in which case  $J = 1/2$ ). For odd  $A$  rare earths  $I$  is typically rather large ( $\sim 7/2$ ) so that the states  $J = I + 1/2$  and  $J = I - 1/2$  are present in about equal numbers and we do not distinguish between them. From statistical theory we then have<sup>4</sup>

$$\sigma_{n,\gamma} \approx 2\pi^2 \lambda^2 \frac{1}{D_J} \left\langle \frac{\Gamma_n \Gamma_\gamma}{\Gamma_n + \Gamma_\gamma} \right\rangle \quad (1)$$

where  $\lambda$  is the reduced neutron DeBroglie wavelength,  $D_J$  is the spacing of levels of spin  $J$ ,  $\Gamma_\gamma$  is the capture width of a resonance,  $\Gamma_n$  is the s-wave neutron width of a resonance, and the average  $\langle \rangle$  is performed over many resonances. Introducing the transmission for  $\ell = 0$  neutrons,  $T_0(E)$ ,

$$T_0(E) = 2\pi \frac{\langle \Gamma_n \rangle}{D_J} \quad (2)$$

and the function

$$S_1 \left( \frac{\Gamma_\gamma}{\langle \Gamma_n \rangle} \right) = \frac{\left\langle \frac{\Gamma_n \Gamma_\gamma}{\Gamma_n + \Gamma_\gamma} \right\rangle}{\frac{\langle \Gamma_n \rangle \Gamma_\gamma}{\langle \Gamma_n \rangle + \Gamma_\gamma}} \quad (3)$$

which has been graphed by Lane and Lynn<sup>4</sup> for constant  $\Gamma_\gamma$  and a Porter-Thomas distribution of  $\Gamma_n$ , we may rewrite eq. (1),

$$\sigma_{n,\gamma} = \pi\lambda^2 \frac{S_1 \left( \frac{\Gamma_\gamma}{\langle \Gamma_n \rangle} \right)}{1 + \frac{\langle \Gamma_n \rangle}{\Gamma_\gamma}} T_0(E) \quad (4)$$

(In practice  $0.68 \leq S_1 \leq 1.0$ , so that  $S_1$  is not a very important factor.) In addition the s-wave strength function  $S_0 = \langle \Gamma_n^0 \rangle / D_J$ , where  $\langle \Gamma_n^0 \rangle$  is the average neutron width reduced to 1 eV according to

$$\langle \Gamma_n^0 \rangle = \frac{\langle \Gamma_n(E) \rangle}{\sqrt{E}}$$

is related to  $T_0$  for small  $T_0$

$$T_0(E) = 2\pi\sqrt{E} S_0 \quad (5)$$

Thus we have

$$\sigma_{n,\gamma}(E) = 2\pi^2\lambda^2\sqrt{E} S_0 \frac{S_1 \left( \frac{\Gamma_\gamma}{\langle \Gamma_n \rangle} \right)}{1 + \frac{\langle \Gamma_n \rangle}{\Gamma_\gamma}} \quad (6)$$

This is the expression which we shall use for  $\sigma(n,\gamma)$ . It may be noted that  $2\pi\lambda^2\sqrt{E} S_0$  is simply the cross section for compound nuclear formation. The factor

$$\left[ 1 + \frac{\langle \Gamma_n \rangle}{\Gamma_\gamma} \right]^{-1}$$

would be the fraction of the decays of the compound nucleus which proceed by  $\gamma$  emission if  $\Gamma_n$  were a constant. The function

$$S_1 \left( \frac{\Gamma_\gamma}{\langle \Gamma_n \rangle} \right)$$

arises from the fluctuations of  $\Gamma_n$  corresponding to a one channel process. We assume that  $\Gamma_\gamma$  is constant among resonances of a given nuclide.

For many of the rare earths  $\langle \Gamma_n^0 \rangle$ ,  $D_{\text{obs}}$ , and  $\Gamma_\gamma$  are known from experiments with low energy ( $0 < E_n \lesssim 100$  eV) neutrons.  $D_{\text{obs}}$  is the observed level spacing, which for even-even targets ( $I = 0, J = 1/2$ ) equals  $D_J$ , while for other targets ( $J = I \pm 1/2$ )  $D_J \approx 2 D_{\text{obs}}$ . Because the strength function,  $S_0$ , varies by about a factor three over the rare earths, experimental values should be used where available. A tabulation of experimental parameters<sup>5</sup> is given in table 1. Many of the parameters, including most of the neutron widths, are uncertain to  $\sim \pm 20\%$ .

It will be noted that parameters are given only for one even-even target nucleus ( $\text{Dy}^{162}$ ). The reason is that the level spacings for the even-even nuclei are so large (see following) that few resonances will be detected. Note further that only one odd-odd target nuclide ( $\text{Lu}^{176}$ ) has resonance parameters in table 1. This and  $\text{La}^{138}$  are the only naturally occurring odd-odd rare earths. It will be seen that the similarity of the resonance parameters for  $\text{Lu}^{175}$  and  $\text{Lu}^{176}$  is probably somewhat atypical inasmuch as the difference between neutron binding energies for  $\text{Lu}^{176}$  and  $\text{Lu}^{177}$  is smaller than usual.

For nuclides not having measured resonance parameters, we will interpolate between measured values of  $\Gamma_\gamma$  and  $S_0$  which are relatively slowly varying. We will calculate  $D_J$  from a level density formula of the form<sup>6,7,2</sup> (we have replaced  $U^{5/4J}$  of ref. 7 by  $U'$ )

$$D_J = C U' e^{-a\sqrt{U'}} \frac{\exp(J + \frac{1}{2})^2 / 2\sigma^2}{2J + 1} \quad (7)$$

where according to Gilbert and Cameron<sup>7</sup>  $a \approx 9.0$  and  $\sigma \approx 5$  for the rare earths. Here  $U'$  is the excitation energy in the compound nucleus,  $U$ ,

Table 1  
Average Resonance Parameters

Target			No. of Res.	I	$\langle \Gamma_n^0 \rangle$ (mV)	$\Gamma_\gamma$ (mV)	$D_{\text{obs}}$ (eV)	$S_0 \times 10^4$	$U^a$
Element	Z	A							
Pm	61	147	9	5/2	2.6	80	3	4.3	5.92
Sm	62	147	13	7/2	6.0	59	7	4.3	8.14
		149	29	7/2	2.3	62	2.7	4.3	7.98
		151	5	5/2	0.57	62	0.95	3.0	8.22
Eu	63	151	21	5/2	0.42	91	0.70	3.0	6.29
		153	18	5/2	0.51	97	1.1	2.3	6.39
Gd	64	155	24	3/2	0.76	109	1.9	2.0	8.53
		157	5	7/2	2.4	96	7.5	1.6	7.93
Tb	65	159	16	3/2	0.72	100	2.3	1.6	6.40
Dy	66	161	27	5/2	0.68	120	2	1.7	8.20
		162	4	0	21.6	140	120	1.8	6.25
		163	9	5/2	3.6	103	9	2.0	7.66
Ho	67	165	15	7/2	2.3	66	5.8	2.0	6.33
Er	68	167	4	7/2	1.4	70	4	1.8	7.77
Tm	69	169	10	1/2	2.0	70	6.8	1.5	6.38
Lu	71	175	16	7/2	1.2	60	3.3	1.8	6.19
		176	21	(7?)	0.82	60	2.3	1.8	6.89
Hf	72	177	12	7/2	1.08	61	3	1.8	7.62
		179	23	9/2	1.08	60	3	1.8	7.33

<sup>a</sup>Binding energies, U, in compound nucleus from ref. 8

corrected for pairing according to Newton<sup>6</sup>

$$\begin{aligned}
 U' &= U - 2\Delta && \text{even-even compound nuclei} \\
 &U - \Delta && \text{odd A compound nuclei} \\
 &U && \text{odd-odd compound nuclei} \\
 \Delta &= 0.41(4 - A/100)
 \end{aligned}$$

For the low neutron energies of interest  $U$  may be set equal to the neutron binding energy.  $C$  is determined from the observed level densities of table 1 to be about  $10 e^{9 \times 2.4} \text{ mV/MeV}$ . Thus eq. (7) may be written

$$D_J \approx 10 U' e^{-9(\sqrt{U'} - 2.4)} \frac{\exp\left(0.02\left(J + \frac{1}{2}\right)^2\right)}{2J + 1} \frac{\text{mV}}{\text{MeV}} \quad (8)$$

This expression gives all the  $D_{\text{obs}}$  values of table 1 to within about a factor three and most to within better than a factor two.

Let us now consider qualitatively which nuclides will have large capture cross sections. From eq. (6), we see that the cross section is determined by the strength function,  $S_0$ , and the ratio  $\Gamma_\gamma / \langle \Gamma_n \rangle$ . From table 1 we see that strength functions are largest at the lower  $A$  end of the rare earths. It should be noted that the next few elements lighter than Pm (Nd, Pr, Ce, and La) all have neutron magic nuclides and hence rather small  $\sigma_{n,\gamma}$ . In addition, for large  $\sigma_{n,\gamma}$  we want a large ratio  $\Gamma_\gamma / \langle \Gamma_n \rangle$ ; or since  $\Gamma_\gamma$  varies much less than  $\Gamma_n$ , we want a small value of  $\langle \Gamma_n \rangle$ . Since  $\langle \Gamma_n \rangle / D_J$  is proportional to the strength function it is clear that small values of  $D_J$  will favor large  $\sigma_{n,\gamma}$ . According to eq. (8)  $D_J$  decreases rapidly with  $U'$ , the effective excitation energy of the compound nucleus.

Let us now consider how  $U'$  will be affected by pairing effects for the various kinds of nuclei. Suppose  $U_0$  is the binding energy of an unpaired neutron, and  $2\Delta$  is the added binding energy from pairing. Then the following table summarizes the various energies.

Target Z	Nucleus N	Binding Energy of Added Neutron = U	Pairing Energy Correction	Effective Excitation Energy = U'
even	even	$U_0$	$-\Delta$	$U_0 - \Delta$
even	odd	$U_0 + 2\Delta$	$-2\Delta$	$U_0$
odd	even	$U_0$	0	$U_0$
odd	odd	$U_0 + 2\Delta$	$-\Delta$	$U_0 + \Delta$

From the table we see that  $U'$  should be largest for odd-odd targets, smallest for even-even targets, and intermediate for odd A targets. In addition it is seen from eq. (7) that even-even targets will have large values of  $D_J$  because they have  $J = 1/2$  while the other targets will usually have larger  $J$ . Hence, we anticipate that odd-odd nuclides will have the largest values of  $\sigma_{n,\gamma}$ , that even-even nuclides will have the smallest, while odd A nuclides will be intermediate.

## RESULTS

We are now in a position to compute  $\sigma_{n,\gamma}$  for nuclides with measured resonance parameters. These parameters are used (from table 1) in eq. (6). For nuclides not having measured resonance parameters, we interpolate from table 1 to obtain  $\Gamma_\gamma$  and  $S_0$  and use the level density formula eq. (8) together with  $S_0$  to deduce  $\langle \Gamma_n \rangle$ . Cross sections were computed for neutron energies of 1 and 10 keV. At 1 keV  $2\pi^2 \lambda^2 \sqrt{E} \approx 1.30 \times 10^5$  b, while at 10 keV  $2\pi^2 \lambda^2 \sqrt{E} \approx 4.10 \times 10^4$  b.

Results for nuclides with measured resonance parameters are given in table 2. Upon comparing the results with experimental values of  $\sigma_{n,\gamma}$  in table 2 we see that there is fairly good agreement, with, however, the calculated values being low by about 10% to 50%. Assuming that the measured  $\sigma_{n,\gamma}$  are correct (although an accuracy of only  $\pm 25\%$  is indicated in ref. 1), this discrepancy is probably due to using too small strength functions and/or too large values of  $\langle \Gamma_n \rangle$ . We should, however, verify that neglect

Table 2  
 Capture Cross Sections for Nuclides of Table 1

<u>Target</u>	<u>Nuclide</u>	<u>Capture Cross Sections</u>				<u>Exp</u>	<u>Ref.</u>
		<u>1 keV</u>		<u>10 keV</u>			
		<u>Calc</u>	<u>Exp</u>	<u>Calc</u>	<u>Exp</u>		
Pm	147	19		2.9			
Sm	147	9.2		1.2	2.4	9	
	149	17		2.6	3.4	9	
	151	23		4.4			
Eu	151	28		6.2			
	153	20	35	4.5	7	1,9	
Gd	155	17		3.4			
	157	8.1		1.3			
Tb	159	13	15	2.6	4	1	
Dy	161	15		3.3			
	162	2.9		0.36			
	163	8.4		1.3			
Ho	165	8.4	14	1.3	3	1	
Er	167	10		1.7			
Tm	169	7.3	10	1.1	1.5	1	
Lu	175	10	14	1.7	3	1	
	176	12		2.1			
Hf	177	11		1.8			
	179	11		1.8			

of p-wave capture is not an important source of error. As an upper limit to the p-wave contribution we take  $3\pi\lambda^2 T_1(E)$  where the p-wave strength function<sup>3</sup> ( $S_1$ )  $\lesssim 10^{-4}$  and

$$T_1 = 2\pi \sqrt{E} \frac{(kR)^2}{1 + (kR)^2} S_1$$

(with  $k = 1/\lambda$  and  $R$  the nuclear radius,  $R \approx 1.3 A^{1/3}$  f). We find the contribution from p-wave capture to be less than 0.1 b at 1 keV and 0.3 b at 10 keV. Thus the p-wave contribution is negligible at 1 keV and nearly so at 10 keV.

Clearly one can obtain better agreement between calculated and experimental values of  $\sigma_{n,\gamma}$  by making moderate adjustments of the resonance parameters for use in eq. (6). This, however, we will not bother to do.

Qualitatively we see that the calculated cross sections (and the few experimental ones) are in accord with our expectations. Most of the target nuclides of table 2 have odd A. The cross sections are largest near the low A end of the table because the strength function is larger there. The odd Z and odd N nuclides have similar cross sections. The one even-even nucleus ( $\text{Dy}^{162}$ ) has clearly the smallest  $\sigma_{n,\gamma}$ . The one odd-odd nucleus ( $\text{Lu}^{176}$ ) appears undistinguished from its odd A neighbors in the table. This somewhat unexpected result arises because as seen from table 1  $\text{Lu}^{176}$  has resonance parameters similar to its odd A neighbors. This in turn is because the neutron binding energy in  $\text{Lu}^{177}$  is unexpectedly low (6.89 MeV), clearly lower than for any other paired neutron in table 1.

Let us now compute some capture cross sections for nuclides not having measured resonance parameters. In particular we are interested in low A rare earths as having probably the largest values of  $\sigma_{n,\gamma}$ . A word of caution may be appropriate here. It is well known<sup>10</sup> that nuclides having  $145 \lesssim A \lesssim 151$  are on the border between spherical near-neutron magic nuclei ( $N \approx 82$ ) and clearly deformed rare earth nuclei ( $A \gtrsim 153$ ). For examples the deformations of  $\text{Eu}^{151}$  and  $\text{Eu}^{153}$  differ by about a factor

two.<sup>10</sup> Since the strength function peak near  $A = 150$  may be associated with deformation<sup>3</sup> one might expect large strength function changes near  $A = 150$ . However, from table 1, no rapid changes are observed; and we therefore proceed by interpolating on  $S_0$ .

Some results are given in table 3. Included at the end of the table are some nuclides with measured  $\sigma_{n,\gamma}$  and known resonance parameters, where however instead of using the measured  $\langle \Gamma_n \rangle$  we have calculated  $D$  from eq. (8) and thence  $\langle \Gamma_n \rangle$ . The experimental values of  $\Gamma_\gamma$  and  $S_0$  were used so that for these nuclides the only difference from table 2 is that calculated rather than experimental values of  $D_J$  (and  $\langle \Gamma_n \rangle$ ) were used.

From table 3 we see that our qualitative expectations are confirmed. The even-even nuclei have clearly the smallest values of  $\sigma_{n,\gamma}$ , and the odd-odd nuclei are predicted to have the largest values. The odd  $A$  capture cross sections are not much smaller than those for odd-odd nuclei because already for many of the odd  $A$  nuclei,  $\sigma_{n,\gamma}$  is not far from the compound nucleus cross section.

We observe by comparing tables 2 and 3 that use of the calculated  $D_J$  rather than the observed  $D_J$  actually improves the agreement between calculated  $\sigma_{n,\gamma}$  and experimental  $\sigma_{n,\gamma}$  - except in the case of Europium. For  $\text{Eu}^{151}$  and  $\text{Eu}^{153}$  the calculated values of  $D_J$  and  $\langle \Gamma_n \rangle$  are larger than the experimental values, and thus we overestimate the competition from elastic scattering. It thus appears that capture in  $\text{Eu}$  is favored not only by a large strength function but also by very small  $\langle \Gamma_n \rangle$ .

It is illuminating to consider how the cross sections of various nuclides in table 3 depend on the parameters involved in eq. (6). The behavior is quite different in the limits  $\langle \Gamma_n \rangle / \Gamma_\gamma \gg 1$  and  $\langle \Gamma_n \rangle / \Gamma_\gamma \ll 1$ . If  $\langle \Gamma_n \rangle / \Gamma_\gamma \gg 1$ , elastic scattering predominates (as is the case for even-even targets) and neglecting the almost constant function  $S_1$  in eq. (6),  $\sigma_{n,\gamma} \sim \Gamma_\gamma / D_J$ , for constant  $E$ . Thus for nuclides with small  $\sigma_{n,\gamma}$ , the cross sections are sensitive to  $D_J$  and insensitive to  $S_0$ . We believe that for the  $\text{Sm}$  even-even isotopes our calculated values of  $D_J$  are probably too large and we thus underestimate  $\sigma_{n,\gamma}$ . If, on the other

Table 3

Calculated Capture Cross Sections for Nuclides without Measured Resonance Parameters  
 Plus Some Nuclides from Table 1 where Calculated Rather Than Measured  $D_J$  Are Used

Target	Nuclide	I	U(MeV) <sup>a</sup>	$D_J$ (eV) <sup>b</sup>	$S_0 \times 10^4$	$\Gamma_\gamma$ mV	$\sigma_{n,\gamma}$ (barns)			
							1 keV calc	1 keV exp <sup>d</sup>	10 keV calc <sup>c</sup>	10 keV exp <sup>d</sup>
Pm	146	7	7.63	3.0	4.3	80	27		4.7	
Sm	144	0	6.76	31	4	60	5.2		0.6	0.3
	148	0	5.85	150	4.3	60	1.1		0.2	0.5
	150	0	5.61	300	3.5	60	0.7		0.1	0.7
	152	0	5.89	140	3.0	60	1.4		0.2	0.8
	154	0	5.82	150	2.5	60	1.2		0.2	0.6
Eu	152	3	8.54	0.61	3.0	90	34		8.3	
	154	3	8.19	1.0	2.2	90	24		5.7	
	155	5/2	6.33	4.4	2.0	90	15		2.8	
	156	3	7.52	3.0	2.0	90	17		3.4	
Dy	164	0	5.64	200	2.0	120	1.7		0.2	
	165	7/2	7.15	25	2.0	120	7.7		1.1	
	166	0	(5.3)	330	2.0	120	1.2		0.13	
Sm	147	7/2	8.14	5.8	4.3	60	16		2.4	2.4
	149	7/2	7.98	7.5	4.3	60	14		2.0	3.4
Eu	151	5/2	6.29	4.8	3.0	90	19		3.2	7
	153	5/2	6.39	4.0	2.3	97	17	35	3.3	
Tb	159	3/2	6.4	5.4	1.6	100	13	15	2.4	4
Ho	165	7/2	6.33	3.7	2.0	66	14	14	2.7	3
Tm	169	1/2	6.38	10	1.5	70	8	10	1.4	1.5
Lu	175	7/2	6.19	4.8	1.8	60	12	14	2.1	3

<sup>a</sup> U data from ref. 9

<sup>b</sup>  $D_J$  calculated from eq. (8)

<sup>c</sup> p-wave contribution of  $\lesssim 0.3$  neglected

<sup>d</sup> data from refs. 1 and 9

hand  $\langle \Gamma_n \rangle / \Gamma_\gamma \ll 1$  (as is roughly the case for the largest 1 keV cross sections of table 3), then for constant energy  $\sigma_{n,\gamma} \sim S_0 \sim \langle \Gamma_n \rangle / D_J$ , and the cross section is completely determined by the strength function. Thus for the odd-odd targets  $D_J$  per se need not be known accurately so long as  $S_0$  is known.

### CONCLUSIONS

1. The large capture cross sections of  $\text{Eu}^{151}$  and  $\text{Eu}^{153}$  arise from their large strength functions combined with small values of neutron width,  $\langle \Gamma_n \rangle$ . The adjacent even Z nuclei (Sm and Gd) have comparable strength functions; their odd A isotopes have somewhat larger  $\langle \Gamma_n \rangle$  values than Eu and thus somewhat smaller  $\sigma_{n,\gamma}$  while the even A isotopes have much larger  $\langle \Gamma_n \rangle$  values, and thus because of the increased competition from compound elastic scattering, they have much smaller  $\sigma(n,\gamma)$ . The odd Z heavier rare earths (Tb, Z = 65, A = 159; Ho, Z = 67, A = 165; etc.) have smaller strength functions than Eu and hence smaller  $\sigma(n,\gamma)$ .

2. Some of the odd-odd nuclei near the strength function maximum (probably  $\text{Eu}^{152}$  and perhaps  $\text{Eu}^{154}$  and  $\text{Pm}^{146}$ ) are predicted to have larger capture cross sections than  $\text{Eu}^{151}$  or  $\text{Eu}^{153}$ . This is because they should have even smaller values of  $\langle \Gamma_n^0 \rangle$  and thus reduced competition from compound elastic scattering. However the competition is not large for odd A Eu isotopes ( $\langle \Gamma_n \rangle / \Gamma_\gamma < 1$ ) and hence we do not expect the  $\text{Eu}^{152}$  cross section to be larger than about 1.20 times the  $\text{Eu}^{151}$  cross section. (This figure comes from comparing the  $\text{Eu}^{152}$  entries of table 3 with the  $\text{Eu}^{151}$  entries of table 2.) Cross sections for the heavier Eu isotopes are expected to decrease as in table 3. For convenience, we tabulate below the calculated 1 keV Eu cross sections, using experimental  $D_J$  for  $\text{Eu}^{151}$  and  $\text{Eu}^{153}$ :

A	151	152	153	154	155	156
$\sigma(n,\gamma)$ , barns	28	34	20	24	15	17

3. The calculated and experimental capture cross sections are in qualitative agreement as can be seen from table 3. Quantitatively there are two areas of disagreement worth noting. First, the calculated  $\sigma_{n,\gamma}$  for Eu is smaller than observed. The disagreement is largest when calculated  $\langle \Gamma_n \rangle$  are used (table 3) and is roughly halved when the smaller experimental  $\langle \Gamma_n \rangle$  are used (in table 2). It appears, though, that somewhat smaller  $\langle \Gamma_n \rangle$  and/or larger  $S_0$  are required to reproduce the experimental  $\sigma(n,\gamma)$  for Eu. Second, for the even-even Sm isotopes calculated  $\sigma(n,\gamma)$  are generally smaller than observed. Part of this discrepancy is caused by our neglect of p-wave contributions to  $\sigma(n,\gamma)$  ( $\lesssim 0.3$  b at 10 keV), but we feel that in addition our level density formula has probably overestimated  $D_J$  for the nuclides concerned. We have found  $D_J$  data for only one even-even rare earth (Dy<sup>162</sup>) and this is quite uncertain. Sm<sup>144</sup> should not be expected to agree with our calculation since it is neutron magic. It is also possible that  $S_0$  is not very regular for the Sm nuclei due to deformations changing with A.

4. There are a number of differences in detail between the present treatment of neutron capture by rare earths and our previous study of the heavy elements.<sup>2</sup> In particular, for the rare earths we are near an s-wave strength function maximum and a p-wave strength function minimum.<sup>3</sup> Therefore, for the rare earths we neglected p-wave capture and used the available information on variations with A of the s-wave strength function. For the heavy nuclei, the s-wave strength function is moderate and probably slowly varying<sup>3</sup> while little is known concerning the p-wave strength function. Therefore, we treated both strength functions as constant for the heavy nuclei, and p-wave capture was not negligible for many of the targets at 20 keV.

For the heavy nuclei there is more information on  $D_J$  for even-even targets which is helpful for deriving level density expressions.

From the present exercise we conclude that the qualitative features of capture cross sections of the rare earths (and heavy nuclei) may be calculated using simple statistical theory without shell corrections.

Quantitative errors will arise from use of incorrect parameters in the theory. As the calculation is carried out, the largest  $\sigma(n,\gamma)$  values will be uncertain mostly because of strength function uncertainties, and the smallest  $\sigma(n,\gamma)$  values will be uncertain mostly because of  $D_J$  uncertainties.

Of course for the heavy nuclei one has added complications if fission is possible.

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