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RCS and Antenna Modeling with MOM Using Hybrid Meshes

J.M. Putnam
McDonnell Douglas Corporation
J.D. Kotulski
Sandia National Laboratories

Abstract

During the last decade, the method of moments(MOM) has become a robust technique for solving electromagnetic problems for arbitrary three-dimensional geometries. There are several reasons why the MOM technique has become so widely used. First, modeling fully three-dimensional geometries has been facilitated by the development of robust basis functions, such as the roof-top functions introduced by Rao-Wilton-Glisson(RWG) for triangular meshes. Secondly, complex boundary conditions can be readily incorporated into the formulation. These boundary conditions, for example, can include conducting, dielectric, resistive, magnetically conducting, and the impedance boundary condition. Finally, the advent of modern fast parallel and vector computer architectures has permitted the solutions of larger and more complex problems.

In this presentation, we will investigate the use of hybrid meshes for modeling RCS and antenna problems in three dimensions. We will consider two classes of hybrid basis functions. These include combinations of quadrilateral and triangular meshes for arbitrary 3D geometries, and combinations of axisymmetric body-of-revolution (BOR) basis functions and triangular facets. In particular, we will focus on the problem of enforcing current continuity between two surfaces which are represented by different types of surface discretizations and unknown basis function representations. We will illustrate the use of an operator-based code architecture for the implementation of these formulations, and how it facilitates the incorporation of the various types of boundary conditions in the code. Both serial and parallel code implementation issues for the formulations will be discussed.

Results will be presented for both scattering and antenna problems. The emphasis will be on accuracy, and robustness of the techniques. Comparisons of accuracy between triangularly meshed and quadrilateral meshed geometries will be shown. The use of hybrid meshes for modeling BORs with attached appendages will also be presented.

Introduction

The method of moments has been used to solve many electromagnetic problems over the years since its inception. With the advent of massively parallel architectures, large complex problems have been solved. To further extend the class of problems that can be solved hybrid techniques can be incorporated. These hybrid techniques can be other MOM formulations coupled together, or MOM combined with high-frequency asymptotic techniques. In this paper alternate MOM formulations will be presented and incorporated within the CARLOS-3D code which has been ported to massively parallel architectures and uses the MOM technique to solve Stratton-Chu integral equations. The different hybrid formulations will be presented, discussed, and tested.

Quadrilateral Patch Formulation

Basis functions can be defined on a surface which is arbitrarily meshed with quadrilateral patches in a manner which is analogous to the RWG basis functions [1] on a triangularly meshed surface. Use of these linear quad roof-top basis functions on large smooth surfaces has been shown to reduce the number of unknowns required. In addition, wire structures can be simply represented as a thin quadrilateral patched surface, with the current being axially directed along the wire. These quad basis functions are edge based and possess all of the important properties of the edge based RWG triangular basis functions, which make them well suited for modeling surfaces of arbitrary shape. Namely, the component of current normal to each interior edge is continuous across the edge,

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line charges do not exist along the boundary of the two quad patches which define the edge, and the symmetric form of the scalar potential term in the MOM implementation can be used. Also, since the linear quad basis functions are edge based, the modifications to a RWG based code are straightforward [2,3].

On a surface which is represented by a quadrilateral mesh, linear roof-top basis functions are defined on pairs of adjacent quads which define each interior edge. Junction edges between surfaces are handled using half basis functions which are equated to enforce current continuity between surfaces. An edge formed by a pair of quad patches is illustrated in Figure 1. Each patch is represented as a local parametric bi-linear surface formed by four vertex points, in terms of the parametric coordinates u and v.

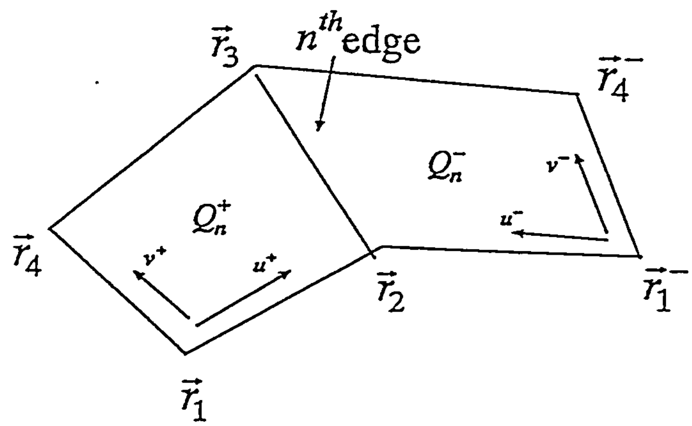


Figure 1. Quad-patch geometry.

The basis function for the n-th edge is defined in terms of the parametric variables u, v $(0 \le u \le 1, 0 \le v \le 1)$ as

$$\bar{f}_n(u,v) = \begin{cases} \dfrac{l_n u \bar{u}}{\sqrt{g(u,v)}} , & \bar{r} \in Q_n^+ \\[2ex] \dfrac{-l_n u \bar{u}}{\sqrt{g(u,v)}} , & \bar{r} \in Q_n^- \end{cases}$$

where l_n is the length of the n-th edge, and for a point (u,v) on the positive side(Q_n^+) of the edge, we have

$$\bar{r}(u,v) = \bar{r}_1 + u(\bar{r}_2 - \bar{r}_1) + v(\bar{r}_4 - \bar{r}_1) + uv(\bar{r}_1 - \bar{r}_2 + \bar{r}_3 - \bar{r}_4) \quad ,$$

$$\bar{u} = \frac{\partial \bar{r}}{\partial u} = (\bar{r}_2 - \bar{r}_1) + v(\bar{r}_1 - \bar{r}_2 + \bar{r}_3 - \bar{r}_4) \quad ,$$

$$\sqrt{g(u,v)} = \left\| \bar{u} \times \bar{v} \right\| \quad \text{and} \quad \iint ds = \iint \sqrt{g(u,v)}\, du dv \quad .$$

Similar expressions are obtained for the negative side (Q_n^-).

In addition, the surface divergence of the basis function is given by

$$\nabla \cdot \bar{f}_n(u,v) = \begin{cases} \dfrac{l_n}{\sqrt{g(u,v)}} , & \bar{r} \in Q_n^+ \\[2ex] \dfrac{-l_n}{\sqrt{g(u,v)}} , & \bar{r} \in Q_n^- \end{cases} \quad .$$

The current is expanded in terms of these basis functions, and then substituted into a surface integral equation formulation and solved using the Galerkin method of moments technique. The surface integral equation formulation used depends on the boundary conditions that need to be satisfied. Each of the formulations is implemented in terms of an operator structure which is independent of the form of the basis function. The resulting matrix elements can be computed in a manner analogous to the RWG case using a combination of analytic and numerical procedures to compute the self and non-self terms.

Hybrid Quad/Triangle Patch Formulation

When modeling a complex geometry, it is often advantageous to have the freedom to generate hybrid meshes containing both triangular and quadrilateral patches, where quads are used for the large smooth parts and triangles are used for the fine detailed parts of the geometry. The forms of the edge based linear quad and RWG roof-top basis functions allow them to be combined at edges formed by adjacent quads and triangles as illustrated in Figure 2.

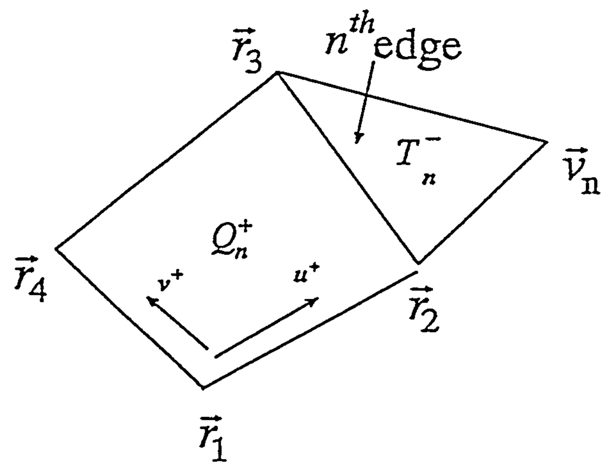


Figure 2. Hybrid Quad/Triangle Geometry.

Continuity of the normal component of current across hybrid edges is preserved, where these hybrid basis functions are defined as

$$\bar{f}_n(u,v) = \begin{cases} \dfrac{l_n u \bar{u}}{\sqrt{g(u,v)}} , & \bar{r} \in Q_n^+ \\[2ex] \dfrac{-l_n(\bar{r} - \bar{v}_n^-)}{2A_n^-} , & \bar{r} \in T_n^- \end{cases}$$

where A_n^- is the area of the triangle.

Hybrid BOR/Patch Formulation

For a BOR geometry, the surface is parameterized in terms of (t, φ), where t is the distance along the generating curve defining the BOR, and $\varphi(0 \le \varphi \le 2\pi)$ is the circumferential variable. A point on the surface is given by $(x,y,z) = (\rho(t)\cos\varphi, \rho(t)\sin\varphi, z(t))$, where $\rho(t)$ is the distance from the z axis to a point on the generating curve. For the BOR/Patch formulation, the currents on the 3D meshed part of the geometry are represented by the basis functions defined above, and the currents on the BOR part are represented using overlapping triangle functions for the t variation of the current and an entire-domain Fourier series representation for the φ variation

[4]. The BOR basis functions and current expansion in terms of the orthogonal tangent vectors $\hat{t}$ and $\hat{\varphi}$ on the surface are given by

$$\bar{J}(t,\varphi) = \sum_{n,k} (a^{t}_{nk}\bar{j}^{t}_{nk} - a^{\varphi}_{nk}\bar{j}^{\varphi}_{nk}) \quad ,$$

where

$$\bar{j}^{\alpha}_{nk}(t,\varphi) = \hat{\alpha}\frac{T_k(t)}{\rho(t)}e^{jn\varphi}$$

and $T_k(t)$ is the k-th overlapping triangle function of the surface.

When combinations of BOR and patch basis functions are substituted into a surface integral equation formulation and solved using the MOM technique, the Fourier mode interactions between test and source functions on the BOR surfaces decouple and the resulting matrix equation has the form

$$\begin{bmatrix} Z^{pp} & Z_0^{ps} & Z_{-1}^{ps} & Z_1^{ps} & \cdots & Z_n^{ps} \\ Z_0^{sp} & Z_0^{ss} & 0 & 0 & 0 & 0 \\ Z_{-1}^{sp} & 0 & Z_{-1}^{ss} & 0 & 0 & 0 \\ Z_1^{sp} & 0 & 0 & Z_1^{ss} & 0 & 0 \\ \vdots & 0 & 0 & 0 & \ddots & 0 \\ Z_n^{sp} & 0 & 0 & 0 & 0 & Z_n^{ss} \end{bmatrix} \begin{bmatrix} I^p \\ I_0^s \\ I_{-1}^s \\ I_1^s \\ \vdots \\ I_n^s \end{bmatrix} = \begin{bmatrix} V^p \\ V_0^s \\ V_{-1}^s \\ V_1^s \\ \vdots \\ V_n^s \end{bmatrix}$$

where the superscripts s (BOR) and p (patch) specify the surface on which the test (1st superscript) and source (2nd superscript) functions reside, and the subscript gives the Fourier mode number. The submatrices are given by Z, and the column vectors I and V represent the unknown coefficients and known source voltages, respectively. The variable n specifies the largest positive and negative Fourier mode numbers used in the current expansion. The sparse form of this matrix equation can be exploited to more efficiently solve the system of equations. In addition, the matrix sub-blocks possess certain symmetries which can be taken advantage of during the matrix fill process.

Current Continuity Between Hybrid Surface Representations

When dealing with hybrid surface representations which intersect, surface current continuity between the different representations must be maintained. For the combination of triangular patched and quadrilateral patched surfaces, this is a simple matter since both representations are edge based, and the functional form of the two types of basis functions allows them to be connected at each hybrid edge. For the case of the BOR/Patch formulation, it is not a simple procedure to match the unknowns in order to explicitly enforce current continuity. In fact, for certain classes of intersecting surfaces, such as the intersection between a circular cylinder and a plate, strict enforcement of the junction condition would result in coupling of the Fourier modes on the BOR surface. This would destroy the primary advantage of a BOR formulation.

The simplest procedure for allowing current continuity between intersecting BOR and patched surfaces involves overlapping of the intersecting surfaces. This amounts to extending the patched surface so that it overlaps onto the rotationally symmetric BOR surface. Typically, the overlap should be on the order of a half basis function, although it can be larger. If the overlap region is too small, then the current variation in the intersecting region will be overly constrained, and result in a poor representation for the actual current. This procedure is analogous to overlapping wires in order to form a junction without explicitly implementing the Kirchhoff junction condition. This procedure of overlapping intersecting surfaces simply results in regions of the surface which are represented by two different types of current expansions. From the theoretical point, this procedure is perfectly valid, however, the numerical implementation requires care due to the singularity in the Green's function in the overlapping region. A robust implementation should use a singularity extraction procedure to handle this case, however, we have found that the equivalent distance approximation used by Mautz and Harrington[5] in their BOR formulation is adequate. Results will be presented which validate this procedure for both 3D patched surfaces and intersecting BOR/patch surfaces.

Parallel Implementation

The parallel implementation of the quad patch version and the quad/triangle patch version uses the previous parallelization effort on CARLOS 3D v2.0 [6]. The code was originally structured to run on either a workstation or Intel Paragon. By incorporating the parallel message passing protocol MPI (Message Passing Interface) it can now be run on a workstation, workstation cluster, or a massively parallel machine that supports MPI.

In the parallel version of the code all the input that specifies the type of problem to be solved is read by one node. This node processes the information then sends it to the rest of the nodes. The next step requires the matrix fill procedure and is partitioned among the different nodes. The solution technique to solve the matrix equation depends on the matrix description. For the quad/triangle hybrid formulation the resulting matrix is dense and LU decomposition optimized for parallel platforms can be used. For this case the matrix solve consists of n^3 operations which dominates the total solution time. For this reason the block matrix fill algorithm, that realizes optimal load balancing, is prescribed by the solver. The two parallel dense solvers used are the Intel Prosolver-DES package for out-of core solutions and the Sandia in-core solver. The Sandia in-core solver has also been configured to run under the MPI protocol. The BOR/Patch hybrid formulation yields a sparse matrix so using an LU decomposition optimized for a dense matrix would be inefficient in time and memory management. Two alternatives would be a sparse LU solver or an iterative solver. These possibilities have not been exercised on parallel platforms for this problem. Once the solution is obtained the scattering cross section or scattered fields are computed on separate nodes and then accumulated to one node to write to an output file.

Results

The RCS for the one meter NASA almond was computed for three different patch representations: quad, triangular, and quad/triangular patch descriptions. The spatial resolution of the different grids are 106 facets/λ^2 and 215 facets/λ^2 for the quad and triangular patches, respectively. The hybrid grid is shown in Figure 3 where triangular facets are used from the tip to 0.2m from the tip. The RCS was calculated at 2 GHz and is shown in Figure 4.

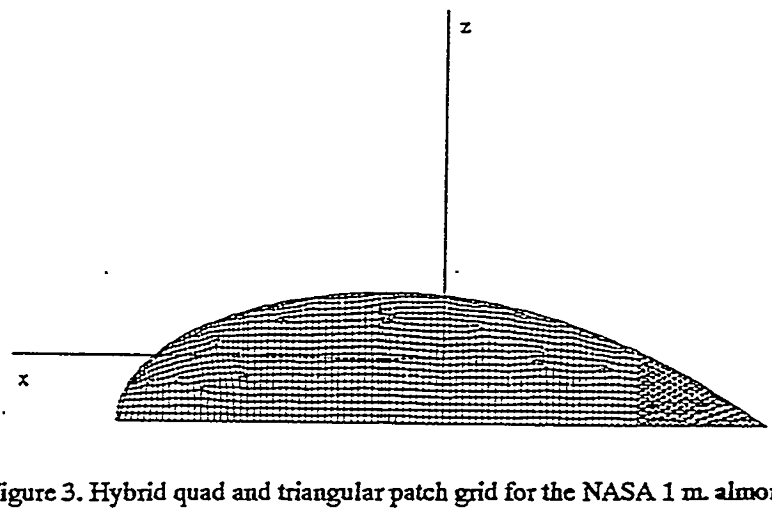


Figure 3. Hybrid quad and triangular patch grid for the NASA 1 m. almond.

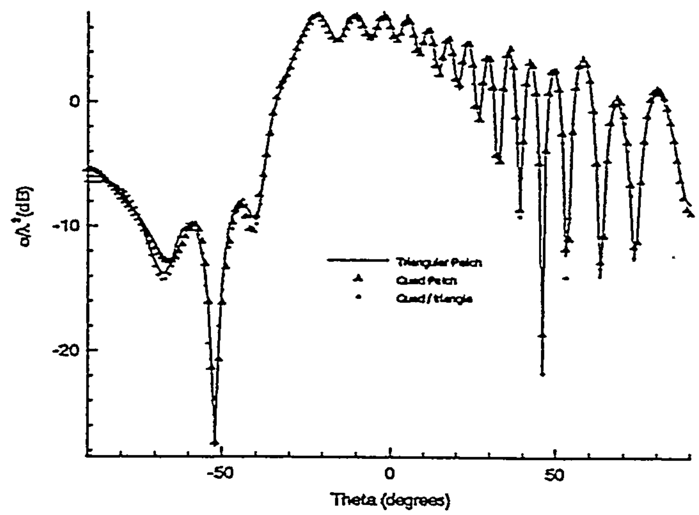
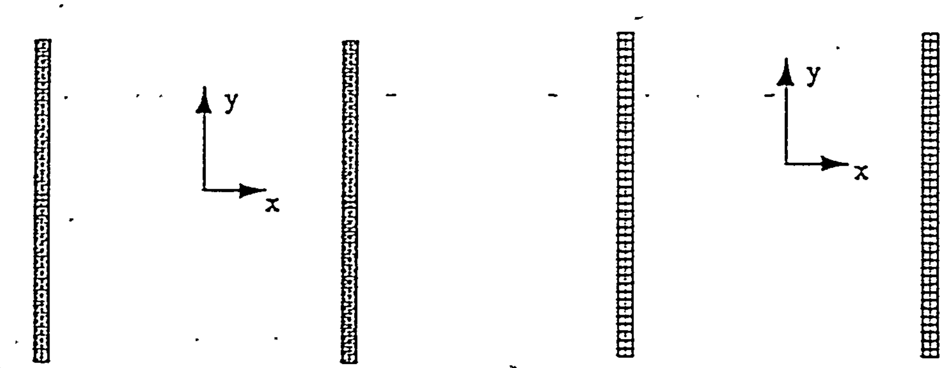


Figure 4. Monostatic RCS of a one meter almond at 2 GHz (ϑ-pol, $\varphi = 0$).

The results show good agreement with minor variations in the tip region.

Another problem to consider is the modeling of a wire by using the quad-patch representation. Two wires each 0.75m long separated by 0.75m were modeled using a dense triangular patch mesh and a coarse quad mesh. The triangular patch mesh models the thickness of the antenna (0.0375m) while the quad mesh does not. The quad and triangular patch models for this configuration are shown in Figure 5. The monostatic RCS of this configuration is shown in Figure 6.



a) Triangular patch b)Quad patch

Figure 5. Quad and triangular patch models for the two antenna configuration.

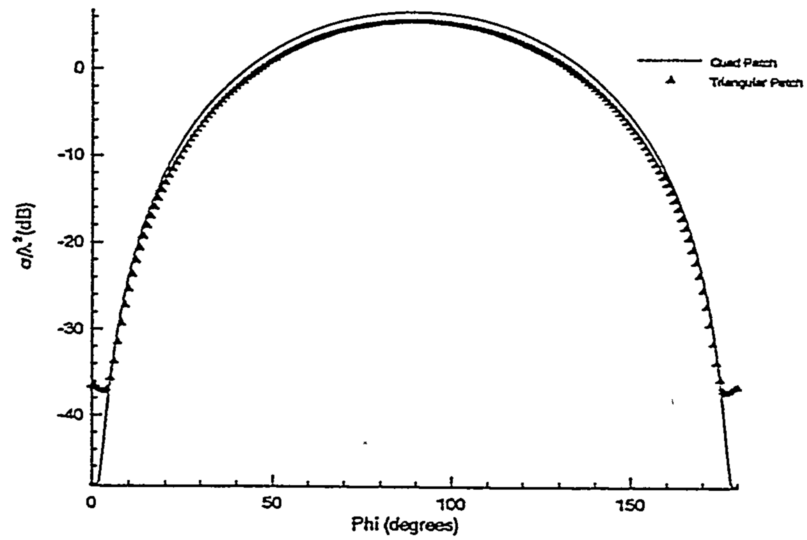


Figure 6. Monostatic RCS for wire configuration at 200MHz (ϑ - pol, $\vartheta = 0$).

Again excellent agreement is seen except in the region where the thickness of the antenna is important. For modeling of thin wires the quad basis function is more natural since the current is decomposed into axially and circumferentially directed currents.

The final example is the BOR/Patch hybrid model for a cone which is 2λ long with a base radius of $.5\lambda$. The monostatic RCS is shown in Figure 7 for a BOR model and a BOR/Patch model with overlap. The overlap region was chosen to be a half triangle function on the BOR surface.

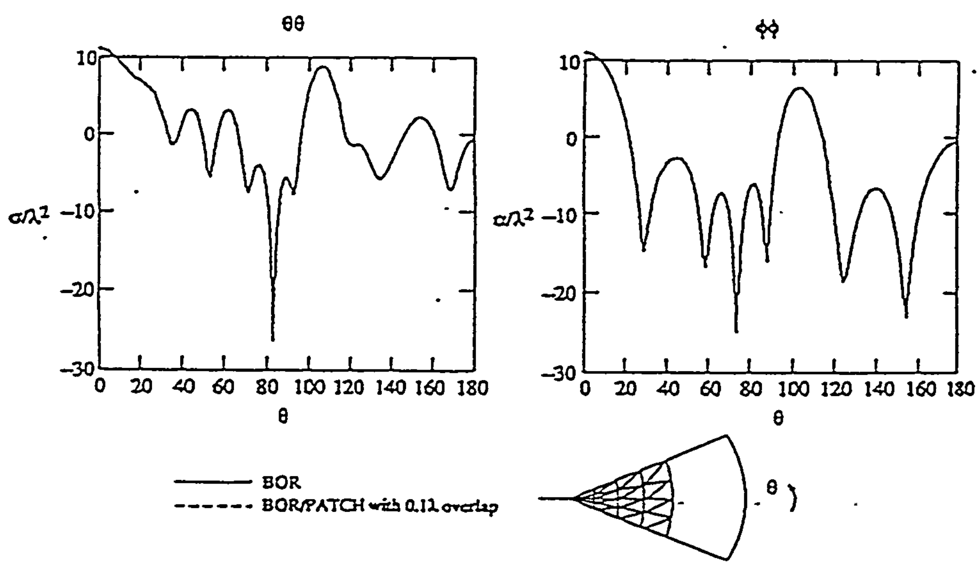


Figure 7. Monostatic RCS comparisons for a 2λ cone - BOR and hybrid BOR/Patch.

Conclusions

A number of different hybrid schemes have been presented and tested. The methods considered in this paper were incorporated into the CARLOS-3D code which has been ported to massively parallel systems and uses a modular operator formulation so that only a few routines needed to be added. The use of these hybrid schemes extends the range of problems that can be solved using the method of moments by modeling the geometry of interest more efficiently.

References

[1] S.M. Rao, D.R. Wilton, and A.W. Glisson, "Electromagnetic Scattering by Surfaces of Arbitrary Shape", IEEE Trans. Ant. Prop.,AP-30,3,pp.409-418,1982.

[2] J.M. Putnam and M.B. Gedera, "CARLOS-3D: A General-Purpose Three-Dimensional Method-of-Moments Scattering Code", IEEE Antennas & Propagation Magazine, April, 1993.

[3] J.M. Putnam and J.D. Kotulski, "Parallel CARLOS-3D Code Development", Proceedings of the 12th Annual Review of Progress in Applied Computational Electromagnetics, Monterey,CA, March 1996.

[4] L.N. Medgyesi-Mitschang and J.M. Putnam, "Electromagnetic Scattering from Axially Inhomogeneous Bodies of Revolution",IEEE Trans. Ant. Prop. AP-32,8,pp.797-806,1984.

[5]J.R. Mautz and R.F. Harrington, "H-field, E-field, and Combined Field Solutions for Bodies of Revolution", Tech. Rep. RADC-TR-77-109, Rome Air Development Center, Griffiss Air Force Base, NY, March 1977.

[6] J.M. Putnam, D.D. Car, and J.D. Kotulski, "Parallel CARLOS-3D - An Electromagnetic Boundary Integral Method for Parallel Platforms", Submitted to Engineering Analysis with Boundary Elements, June 1996.