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MASTER

**TWIGL-A PROGRAM TO SOLVE THE
TWO-DIMENSIONAL, TWO-GROUP,
SPACE-TIME NEUTRON DIFFUSION
EQUATIONS WITH TEMPERATURE
FEEDBACK**

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**BETTIS ATOMIC POWER LABORATORY, PITTSBURGH, PA.,
OPERATED FOR THE U. S. ATOMIC ENERGY COMMISSION
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TWIGL - A PROGRAM TO SOLVE THE TWO-DIMENSIONAL, TWO-GROUP, SPACE-TIME
NEUTRON DIFFUSION EQUATIONS WITH TEMPERATURE FEEDBACK

J. B. Yasinsky, M. Natelson, L. A. Hageman

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TABLE OF CONTENTS

| | <u>Page</u> |
|---|-------------|
| I. INTRODUCTION | 1 |
| II. THE GROUP DIFFUSION EQUATIONS | 2 |
| III. SOLUTION OF THE SPATIAL DIFFERENCE EQUATIONS | 7 |
| IV. THE TEMPERATURE EQUATIONS | 9 |
| V. TREATMENT OF INPUT PARAMETERS | 13 |
| VI. THE STEADY STATE PROBLEM | 17 |
| VII. EDIT FEATURES | 20 |
| VIII. INPUT FORMAT | 21 |
| IX. SAMPLE PROBLEM | 31 |
| X. COMMENTS | 47 |
| REFERENCES | 50 |

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The maximum number of spatial mesh points that may be used in the TWIGL (Ref 1) program has been reduced by one in each dimension. The following changes should appear on page 21 of Ref 1.

NPX = number of x (or r) mesh points (1 and NPX are boundary points).

$NPX \leq 36$ for rectangular geometry. $NPX \leq 35$ for cylindrical geometry.

NPZ = number of z mesh points (1 and NPZ are boundary points). $NPZ \leq 36$.

The central memory field length (CFL on the jobcard) must be set at 137 for TWIGL problems.

Ref 1: J.B. Yasinsky, M. Natelson, L.A. Hageman, "TWIGL - A Program to Solve the Two-Dimensional, Two-Group, Space-Time Neutron Diffusion Equations with Temperature Feedback," WAPD-TM-743 (February 1968).

TWIGL is a two-dimensional, two-group, space-time diffusion theory program for rectangular or cylindrical geometries. From zero to six groups of delayed neutron precursors may be used. Temperature feedback effects are accounted for via a one-pass, no-boiling thermal-hydraulic model. The relevant differential equations are represented by space-time difference equations which are solved at each time step by an accelerated iterative procedure. This report describes the governing equations, solution technique and input information.

TWIGL - A PROGRAM TO SOLVE THE TWO-DIMENSIONAL, TWO-GROUP, SPACE-TIME NEUTRON DIFFUSION EQUATIONS WITH TEMPERATURE FEEDBACK

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I. INTRODUCTION

TWIGL is a program for the solution of the two-dimensional, two-group, space-time diffusion and delayed precursor equations, accounting for temperature feedback, in x-z or r-z geometry. This program solves transient neutron diffusion problems that are initiated by specified time variations of the reactor material parameters or by specified changes in the core inlet coolant temperature or flow rate. All transient problems are assumed to start from equilibrium, and TWIGL will, on option, generate the steady-state flux and average temperatures. Zero flux or zero current boundary conditions can be applied on any of the external boundaries.

The thermal-hydraulic model in TWIGL assumes that no boiling occurs and that the coolant makes only one pass through the core (with flow in the z direction).

TWIGL was written in FORTRAN IV and compiled on the CDC-6600 computer. If desired, the initial neutron flux distribution may be generated by the PDQ-7 (Ref 1) program. The PDQ-7 generated fluxes can be saved on a disc file and read into TWIGL via the file manager subroutines (Ref 2).

In the next section the governing neutron diffusion equations are presented and reduced to relevant space-time difference equations. Section III describes the mechanism for the solution of the spatial difference equations that must be solved at each time step. This is followed, in section IV, by a description of the temperature equations. Section V discusses the treatment of the input parameters, while the steady-state solution technique is presented in section VI. This is followed in the remaining sections by a description of the edit features, the input format, and a sample problem.

II. THE GROUP DIFFUSION EQUATIONS

The two-group diffusion and delayed precursor equations solved by TWIGL are

$$\nabla \cdot D_1(\vec{r}, t) \nabla \phi_1(\vec{r}, t) - \Sigma_1(\vec{r}, t) \phi_1(\vec{r}, t) + (1-\beta) \left[v \Sigma_{f_1}(\vec{r}, t) \phi_1(\vec{r}, t) + v \Sigma_{f_2}(\vec{r}, t) \phi_2(\vec{r}, t) \right] + \sum_{i=1}^I \lambda_i c_i(\vec{r}, t) = \frac{1}{V_1} \frac{\partial}{\partial t} \phi_1(\vec{r}, t) \quad (1a)$$

$$\nabla \cdot D_2(\vec{r}, t) \nabla \phi_2(\vec{r}, t) - \Sigma_2(\vec{r}, t) \phi_2(\vec{r}, t) + \Sigma_{r_1}(\vec{r}, t) \phi_1(\vec{r}, t) = \frac{1}{V_2} \frac{\partial}{\partial t} \phi_2(\vec{r}, t) \quad (1b)$$

$$\beta^i \left[v \Sigma_{f_1}(\vec{r}, t) \phi_1(\vec{r}, t) + v \Sigma_{f_2}(\vec{r}, t) \phi_2(\vec{r}, t) \right] - \lambda_i c_i(\vec{r}, t) = \frac{\partial}{\partial t} c_i(\vec{r}, t) , \quad (1c)$$

$$i = 1, \dots, I ,$$

where \vec{r} represents x, z or r, z . These equations are to be solved subject to zero flux boundary conditions on external surfaces and equilibrium initial conditions.

The time-differenced form of Eqs (1) is obtained relative to a sequence of time points t_1, t_2, \dots, t_J , which are separated by time intervals $\Delta t_j \equiv t_{j+1} - t_j$. In the interval between t_j and t_{j+1} we approximate the time derivatives as

$$\frac{\partial}{\partial t} \eta \approx (\eta^{j+1} - \eta^j) / \Delta t_j \quad (2)$$

where η represents ϕ_1 , ϕ_2 , C_i . The delayed neutron precursor terms are central differenced over this time interval as

$$C_i \approx \frac{1}{2} (C_i^{j+1} + C_i^j) , \quad (3)$$

while the flux terms are differenced in a variable manner as

$$\sigma\phi \approx \theta\sigma^{j+1}\phi^{j+1} + (1-\theta)\sigma^j\phi^j \quad (4)$$

where σ represents D_1 , D_2 , Σ_1 , Σ_2 , Σ_{r1} , $\nu\Sigma_{f1}$, $\nu\Sigma_{f2}$. This is similar to the time differencing employed in the WIGL-2 (Ref 3) program. θ is specified to accommodate the desired differencing ($0 < \theta \leq 1$); for example, $\theta = 1/2$ gives central differencing or $\theta = 1$ gives backward differencing. Forward differencing, $\theta = 0$, is not permitted. If we use (2), (3) and (4) in Eqs (1) and eliminate the C_i^{j+1} term in the fast group equation by means of the precursor relations we obtain

$$\begin{aligned} & \theta \left\{ -\nabla \cdot D_1^{j+1} \nabla \phi_1^{j+1} + \Sigma_1^{j+1} \phi_1^{j+1} - (1-B^j) [\nu\Sigma_{f1}^{j+1} \phi_1^{j+1} + \nu\Sigma_{f2}^{j+1} \phi_2^{j+1}] \right\} + \frac{1}{V_1 \Delta t_j} \phi_1^{j+1} \\ &= (1-\theta) \left\{ \nabla \cdot D_1^j \nabla \phi_1^j - \Sigma_1^j \phi_1^j + (1-B^j) [\nu\Sigma_{f1}^j \phi_1^j + \nu\Sigma_{f2}^j \phi_2^j] \right\} + \frac{1}{V_1 \Delta t_j} \phi_1^j + \sum_{i=1}^I \frac{\lambda_i}{(1 + \frac{\lambda_i}{2} \Delta t_j)} C_i^j \end{aligned} \quad (5a)$$

$$\begin{aligned} & \theta \left\{ -\nabla \cdot D_2^{j+1} \nabla \phi_2^{j+1} + \Sigma_2^{j+1} \phi_2^{j+1} - \Sigma_{r1}^{j+1} \phi_1^{j+1} \right\} + \frac{1}{V_2 \Delta t_j} \phi_2^{j+1} \\ &= (1-\theta) \left\{ \nabla \cdot D_2^j \nabla \phi_2^j - \Sigma_2^j \phi_2^j + \Sigma_{r1}^j \phi_1^j \right\} + \frac{1}{V_2 \Delta t_j} \phi_2^j \end{aligned} \quad (5b)$$

$$C_i^{j+1} = \left(\frac{1 - \frac{\lambda_i}{2} \Delta t_j}{1 + \frac{\lambda_i}{2} \Delta t_j} \right) C_i^j + \theta B_i^j [\nu\Sigma_{f1}^{j+1} \phi_1^{j+1} + \nu\Sigma_{f2}^{j+1} \phi_2^{j+1}] \Delta t_j + (1-\theta) B_i^j [\nu\Sigma_{f1}^j \phi_1^j + \nu\Sigma_{f2}^j \phi_2^j] \Delta t_j \quad (5c)$$

where $j = 1, 2, \dots, J$ and $I = 1, 2, \dots, I$ and

$$B_i^j = \frac{\beta^i}{\lambda \left(1 + \frac{1}{2} \Delta t_j \right)} \quad (6)$$

$$B^j = \sum_{i=1}^I B_i^j .$$

Equations (5) are then spatially differenced relative to a two-dimensional mesh grid, where the mesh points x_1, x_2, \dots, x_{NPX} span the x (or r) dimension and the points z_1, z_2, \dots, z_{NPZ} span the z dimension. $h^x(n)$ is the mesh spacing between points x_n and x_{n+1} and $h^z(m)$ is the spacing between points z_m and z_{m+1} . The material parameters are assumed constant within each mesh rectangle and are represented as

$$\sigma = \sigma(n, m) ; \quad x_n < x < x_{n+1} , \quad z_m < z < z_{m+1} .$$

The relevant difference equations are obtained from Eqs (5) as described in Ref 4. These equations take the form

$$\theta \left\{ -\mathcal{A}_{1z}^{j+1}(n, m-1) \varphi_1^{j+1}(n, m-1) - \mathcal{A}_{1x}^{j+1}(n-1, m) \varphi_1^{j+1}(n-1, m) + \mathcal{A}_1^{j+1}(n, m) \varphi_1^{j+1}(n, m) \right. \\ \left. - \mathcal{A}_{1x}^{j+1}(n, m) \varphi_1^{j+1}(n+1, m) - \mathcal{A}_{1z}^{j+1}(n, m) \varphi_1^{j+1}(n, m+1) - (1-B^j) \nu \bar{\Sigma}_{f_2}^{j+1}(n, m) \varphi_2^{j+1}(n, m) \right\} \quad (7a)$$

$$+ \bar{v}_1^{-1}(n, m) \varphi_1^{j+1}(n, m) / \Delta t_j = \mathcal{J}_1^j(n, m)$$

$$\theta \left\{ -\mathcal{A}_{2z}^{j+1}(n, m-1) \varphi_2^{j+1}(n, m-1) - \mathcal{A}_{2x}^{j+1}(n-1, m) \varphi_2^{j+1}(n-1, m) + \mathcal{A}_2^{j+1}(n, m) \varphi_2^{j+1}(n, m) \right. \\ \left. - \mathcal{A}_{2x}^{j+1}(n, m) \varphi_2^{j+1}(n+1, m) - \mathcal{A}_{2z}^{j+1}(n, m) \varphi_2^{j+1}(n, m+1) \right\} - \bar{\Sigma}_{r_1}^{j+1}(n, m) \varphi_1^{j+1}(n, m) \quad (7b)$$

$$+ \bar{v}_2^{-1}(n, m) \varphi_2^{j+1}(n, m) / \Delta t_j = \mathcal{J}_2^j(n, m)$$

$$\begin{aligned}
\bar{C}_i^{j+1}(n, m) = & \left(\frac{1 - \frac{\lambda_i}{2} \Delta t_j}{1 + \frac{\lambda_i}{2} \Delta t_j} \right) \bar{C}_i^j(n, m) + B^j \theta \left[v \bar{\Sigma}_{f_1}^{j+1}(n, m) \varphi_1^{j+1}(n, m) + v \bar{\Sigma}_{f_2}^{j+1}(n, m) \varphi_2^{j+1}(n, m) \right] \\
& + B^j (1-\theta) \left[v \bar{\Sigma}_{f_1}^j(n, m) \varphi_1^j(n, m) + v \bar{\Sigma}_{f_2}^j(n, m) \varphi_2^j(n, m) \right]
\end{aligned} \tag{7c}$$

where (for $g=1, 2$, $k = j, j+1$)

$$\mathcal{A}_{gx}^k(n, m) = \tilde{A}_4(n) [h^z(m) D_g^k(n, m) + h^z(m-1) D_g^k(n, m-1)] / 2h^x(n)$$

$$\mathcal{A}_{gx}^k(n-1, m) = \tilde{A}_3(n) [h^z(m) D_g^k(n-1, m) + h^z(m-1) D_g^k(n-1, m-1)] / 2h^x(n-1)$$

$$\mathcal{A}_{gz}^k(n, m) = [h^x(n) \tilde{A}_1(n) D_g^k(n, m) + h^x(n-1) \tilde{A}_0(n) D_g^k(n-1, m)] / 2h^z(m) \tag{8a}$$

$$\mathcal{A}_{gz}^k(n, m-1) = [h^x(n) \tilde{A}_1(n) D_g^k(n, m-1) + h^x(n-1) \tilde{A}_0(n) D_g^k(n-1, m-1)] / 2h^z(m-1)$$

$$\mathcal{A}_1^k(n, m) = \mathcal{A}_{1x}^k(n, m) + \mathcal{A}_{1x}^k(n-1, m) + \mathcal{A}_{1z}^k(n, m) + \mathcal{A}_{1z}^k(n, m-1) + \bar{\Sigma}_1^k(n, m) - (1-B^j)v \bar{\Sigma}_{f_1}^k(n, m)$$

$$\mathcal{A}_2^k(n, m) = \mathcal{A}_{2x}^k(n, m) + \mathcal{A}_{2x}^k(n-1, m) + \mathcal{A}_{2z}^k(n, m) + \mathcal{A}_{2z}^k(n, m-1) + \bar{\Sigma}_2^k(n, m)$$

and (for $\sigma = \Sigma_1, \Sigma_2, \Sigma_{r_1}, v \Sigma_{f_1}, v \Sigma_{f_2}, 1/v_1, 1/v_2$)

$$\begin{aligned}
\bar{\sigma}(n, m) = & \frac{1}{4} [\tilde{A}_1(n) h^x(n) h^z(m) \sigma(n, m) + \tilde{A}_1(n) h^x(n) h^z(m-1) \sigma(n, m-1) \\
& + \tilde{A}_0(n) h^x(n-1) h^z(m-1) \sigma(n-1, m-1) + \tilde{A}_0(n) h^x(n-1) h^z(m) \sigma(n-1, m)]
\end{aligned} \tag{8b}$$

The source terms in Eqs (7a) and (7b) are defined as

$$\begin{aligned}
\mathcal{A}_1^j(n, m) = & (1-\theta) [\mathcal{A}_{1z}^j(n, m) \varphi_1^j(n, m-1) + \mathcal{A}_{1x}^j(n-1, m) \varphi_1^j(n-1, m) - \mathcal{A}_1^j(n, m) \varphi_1^j(n, m) \\
& + \mathcal{A}_{1x}^j(n, m) \varphi_1^j(n+1, m) + \mathcal{A}_{1z}^j(n, m) \varphi_1^j(n, m+1)] + \bar{v}_1^{-1}(n, m) \varphi_1^j(n, m) / \Delta t_j \\
& + \sum_{i=1}^I \lambda_i \bar{C}_i^j(n, m) / (1 + \frac{\lambda_i}{2} \Delta t_j)
\end{aligned} \tag{8c}$$

$$\begin{aligned}
\mathcal{A}_2^j(n,m) = & (1-\theta) \left[\mathcal{A}_{2z}^j(n,m-1) \varphi_2^j(n,m-1) + \mathcal{A}_{2x}^j(n-1,m) \varphi_2^j(n-1,m) - \alpha_2^j(n,m) \varphi_2^j(n,m) \right. \\
& \left. + \mathcal{A}_{2x}^j(n,m) \varphi_2^j(n+1,m) + \mathcal{A}_{2z}^j(n,m) \varphi_2^j(n,m+1) \right] + \bar{v}_2^{-1}(n,m) \varphi_2^j(n,m) / \Delta t_j .
\end{aligned} \tag{8d}$$

In addition, for x-z (rectangular) geometry we have

$$\tilde{A}_0(n) = \tilde{A}_1(n) = \tilde{A}_3(n) = \tilde{A}_4(n) = 1 \quad \text{for all } n$$

and for r-z (cylindrical) geometry we have

$$\begin{aligned}
\tilde{A}_0(n) &= R_n - \frac{1}{4} h^x(n-1) \\
\tilde{A}_1(n) &= R_n + \frac{1}{4} h^x(n) \\
\tilde{A}_3(n) &= R_n - \frac{1}{2} h^x(n-1) \\
\tilde{A}_4(n) &= R_n + \frac{1}{2} h^x(n)
\end{aligned} \tag{8e}$$

where R_n is the radial distance from the origin to mesh point x_n .

Since all problems are assumed to start from an equilibrium state, the initial ($j=1$) precursor terms at points (n,m) are calculated by the equilibrium relation

$$\bar{c}_1^1(n,m) = \frac{\beta^1}{\lambda_1} [\nu \bar{\Sigma}_f^1(n,m) \varphi_1^1(n,m) + \nu \Sigma_{f2}^1(n,m) \varphi_2^1(n,m)] . \tag{9}$$

At each time point j , Eqs (7a) and (7b) each constitute a set of inhomogeneous algebraic equations. These equations may be written conveniently as two coupled matrix equations

$$A_1 \Phi_1 = S_1 + B_2 \Phi_2 \tag{10a}$$

$$A_2 \Phi_2 = S_2 + B_1 \Phi_1 \tag{10b}$$

where both $\underline{\Phi}_1$ and $\underline{\Phi}_2$ are column vectors composed of a number of components equal to the number of spatial solution points.

III. SOLUTION OF THE SPATIAL DIFFERENCE EQUATIONS

The TWIGL program uses the cyclic Chebyshev polynomial method (Ref 5) to solve matrix equation (10). Given an initial guess vector $\underline{\Phi}_2(0)$, the cyclic Chebyshev method generates successive estimates for the solution vectors $\underline{\Phi}_1$ and $\underline{\Phi}_2$ by the process

$$\left. \begin{aligned} \underline{\Phi}_1(k+1) &= \alpha_{k+1} \left[\underline{s}_1(k+1) - \underline{\Phi}_1(k) \right] + \underline{\Phi}_1(k), \quad \text{where } A_1 \underline{s}_1(k+1) = B_2 \underline{\Phi}_2(k) + \underline{s}_1, \\ \underline{\Phi}_2(k+1) &= \beta_{k+1} \left[\underline{s}_2(k+1) - \underline{\Phi}_2(k) \right] + \underline{\Phi}_2(k), \quad \text{where } A_2 \underline{s}_2(k+1) = B_1 \underline{\Phi}_1(k) + \underline{s}_2. \end{aligned} \right\} \quad (11)$$

If $\alpha_{k+1} = \beta_{k+1} = 1$, (11) is the Gauss-Seidel method of iteration.

For the cyclic Chebyshev method, the iteration parameters α_{k+1} and β_{k+1} are functions of the eigenvalue domain of the Jacobi iteration matrix

$$J \equiv \begin{pmatrix} 0 & A_1^{-1} B_2 \\ A_2^{-1} B_1 & 0 \end{pmatrix}.$$

It is known that the eigenvalues of J are bounded by the ellipse $x^2/\sigma^2 + y^2/\epsilon^2 = 1$, where σ is the spectral radius of J and $\epsilon^2 < \sigma^2$. For this case, the Chebyshev parameters α and β are given (Ref 6) by $\alpha_1 = 1$, $\beta_1 = 2/(2-\rho^2)$ and for $k \geq 1$

$$\alpha_{k+1} = \frac{1}{1 - \frac{\rho}{4} \beta_k}, \quad \beta_{k+1} = \frac{1}{1 - \frac{\rho}{4} \alpha_{k+1}}, \quad (12)$$

where $\rho^2 = \sigma^2 - \epsilon^2$. In what follows we assume that $\epsilon^2 \ll \sigma^2$ and that an estimate $\tilde{\epsilon}$ for ϵ may be picked to satisfy $\epsilon \leq \tilde{\epsilon} < \sigma^*$. An accurate estimate for σ is now

* It is thought (Ref 7) that the eigenvalues of J are real but that the inner iterations (defined on p. 8) introduce complex eigenvalues whose moduli depend on the degree of convergence of the inner iterations. In the TWIGL program, the value chosen for $\tilde{\epsilon}$ is based on the error reduction achieved by the inner iterations.

needed for the efficient use of the Chebyshev method. The strategy given below for estimating σ is discussed in more detail in Ref 6, Appendix B.

In the TWIGL program, numerical estimates for σ are obtained by observing the decay rate of the residual vector $\underline{y}(k+1) \equiv \underline{s}_2(k+1) - \underline{\Phi}_2(k)$, where $\underline{s}_2(k+1)$ and $\underline{\Phi}_2(k)$ are given by (11). The residual vector quotient is defined as

$$Q(k+1) \equiv \frac{\|\underline{y}(k+1)\|}{\|\underline{y}(k)\|} , \quad (13)$$

where $\|\cdot\|$ denotes some suitable vector norm. Since $\lim_{k \rightarrow \infty} Q(k) = \sigma^2$ for the Gauss-Seidel method (Ref 5), an initial estimate for σ^2 may be obtained by doing a few Gauss-Seidel iterations and computing $Q(k)$ before starting the Chebyshev process. Once the Chebyshev process has started, new estimates for σ can be obtained by comparing the convergence rate actually being obtained with the theoretical convergence rate one would obtain if the estimate being used for σ were correct. In order to compute a new estimate for σ , however, one needs (Ref 6, Appendix B) to stop the Chebyshev process, do one Gauss-Seidel iteration, estimate a new σ , and then start the Chebyshev process over again, i.e., the sequence of Chebyshev parameters, α_k and β_k , are started over from $k=1$ using this new estimate for σ .

In order to carry out the iterative process (11), group equations of the form

$$A \underline{s}_g = \underline{k}_g , \quad g = 1, 2, \quad (14)$$

must be solved for \underline{s}_g . In (14), \underline{k}_g is a known vector. The solutions \underline{s}_g to the group equations (14) are approximated by the one-line successive overrelaxation method (Ref 5). The iterations used to obtain these approximations are called inner iterations while the iterations of (11) are called outer iterations. A fixed number, m_g , of inner iterations are performed in group g every outer iteration. The program tries to choose m_g such that the inner iteration error vector

after performing m_g iterations is about 0.05 of the initial error vector. This prediction is made in what is called the "omega routine", which is carried out prior to the start of the outer iterations. The relaxation factor, ω_g , required by the overrelaxation method is also determined in the "omega routine".

The acceleration parameters σ and ω_g computed for the first time step are used for succeeding time steps. When it is determined by the program that these constants are no longer adequate, they will be recomputed.

The iterative process (11) is considered converged when

$$\frac{\| \underline{y}(k+1) \|}{\| \Phi_2(k) \|} < \delta(1-Q(k+1)), \quad (15)$$

where $\underline{y}(k+1)$ is the residual vector, $Q(k+1)$ is defined by (13), and δ is an input number. The ℓ_2 vector norm is used in (15), i.e., $\| \underline{x} \|^2 = \sum_{i=1}^n x_i^2$, where the x_i are components of \underline{x} .

IV. THE TEMPERATURE EQUATIONS

The heat transfer description in TWIGL is essentially that used in WIGL2 (Ref 3), restricted to one-pass, radial type (flow along the z axis) cores. The reactor volume is subdivided into thermal-hydraulic (t.h.) regions within which we desire to know the average coolant temperature \bar{T}_c and the average fuel temperature \bar{T}_f . The equations describing the time behavior of \bar{T}_c and \bar{T}_f in a given t.h. region (neglecting boiling effects) are

$$\left. \begin{aligned} v_c \left(\frac{\partial \rho_c H}{\partial T_c} \right) \frac{d}{dt_h} \bar{T}_c &= v_c \left[\frac{1}{A_H U} + \frac{1}{A_H h_o \left(\frac{W}{W_o} \right)^{.8}} \right]^{-1} (\bar{T}_f - \bar{T}_c) + 2W_r C_c (T_b - \bar{T}_c) + rqV_f \\ \rho_f v_f C_f \frac{d}{dt_h} \bar{T}_f &= (1-r)qV_f - v_c \left[\frac{1}{A_H U} + \frac{1}{A_H h_o \left(\frac{W}{W_o} \right)^{.8}} \right]^{-1} (\bar{T}_f - \bar{T}_c) \end{aligned} \right\} \quad (16)$$

$$W = \begin{cases} W_0 & ; t \leq t_w \\ W_0 \left[\frac{1}{1 + \alpha_w (t - t_w)} \right] & ; t > t_w \end{cases} . \quad (17)$$

The symbolism used in Eqs (16) and (17) is identical with that of WIGL2. For the sake of completeness we include their definitions.

V = volume of coolant in the t.h. region in question (ft^3)
 $\left(\frac{\partial \rho_c H}{\partial T_c} \right)^F$ = heat required to raise temperature of one cubic foot of coolant one degree F. ($\text{btu}/\text{ft}^3/\text{^o F}$)
 $(\rho_c = \text{coolant density in lb}/\text{ft}^3; H = \text{coolant enthalpy in btu/lb})$
 \bar{T}_c = average coolant temperature in t.h. region in question (^o F)
 t_h = time measured in hours
 A_H = total heat transfer area for the composition in question \pm total volume coolant in that composition (ft^{-1})
 U = conductivity/conduction length of the fuel cladding ($\text{btu}/\text{ft}^2/\text{^o F}/\text{hr}$)
 h_0 = film coefficient at initial flow rate ($\text{btu}/\text{ft}^2/\text{^o F}/\text{hr}$)
 W = flow rate into core (lb/hr)
 W_0 = flow rate into core at time, $t_h = 0$ (lb/hr)
 \bar{T}_f = average fuel temperature in the t.h. region in question (^o F)
 W_r = flow rate through the t.h. region in question (lb/hr)
 C_c = specific heat of coolant ($\text{btu}/\text{lb}/\text{^o F}$)
 T_b = inlet temperature of the t.h. region in question (^o F)
 r = fraction of fission power appearing in coolant
 q = average fission power in region ($\text{btu}/\text{hr}/\text{ft}^3$ of fuel)
 V_f = volume of fuel in region (ft^3) ($V_c + V_f$ = total volume of the region in question)
 ρ_f = density of fuel (lb/ft^3)
 C_f = specific heat of fuel in region in question ($\text{btu}/\text{lb}/\text{^o F}$)
 α_w = coefficient specifying flow coast down (sec^{-1})

The t.h. regions are rectangles on the two-dimensional spatial mesh structure constructed by defining t.h. channels ($N=1, 2, \dots, NTHC$) along the x axis and the t.h. planes ($M=1, 2, \dots, NTHP$) along the z axis. Coolant flow is along the z axis, in the direction of increasing z mesh points. The inlet temperatures of the t.h. channels in the first t.h. plane ($M=1$) are known, and are given by

$$T_b(N,1) = \begin{cases} T_{in} & ; \quad t \leq t_{in} \\ T_{in} \left[\frac{1}{1 + \alpha_{in}(t-t_{in})} \right] & ; \quad t > t_{in} \end{cases} \quad (18)$$

where T_{in} is the initial core inlet coolant temperature and $N=1, 2, \dots, NTHC$.

For each of the t.h. channels in the t.h. planes $M=2, 3, \dots, NTHP$ we have

$$T_b(N,M) = 2\bar{T}_c(N,M-1) - T_b(N,M-1) \quad (19)$$

which follows from the fact that \bar{T}_c is the average coolant temperature.

The flow rate through t.h. channel N is given by

$$W_r(N) = f(N)W \quad (20)$$

where $f(N)$ is the specified flow fraction for t.h. channel N .

The fission power in t.h. region (N,M) is (in BTU/HR)

$$(qV_f) = Q(N,M) = Y(1.08133 \times 10^{-10}) \iint (\sum_{f_1} \phi_1 + \sum_{f_2} \phi_2) dA \quad (21)$$

where, for rectangular geometry,

$$dA = dx dz$$

$$Y = \text{core depth (cm)}$$

or, for cylindrical geometry,

$$dA = x dx dz$$

$$Y = 2\pi .$$

For t.h. region (N,M), we may write Eqs (16) as

$$A(N,M) \frac{d}{dt_h} \bar{T}_c(N,M) = B(N,M)[\bar{T}_f(N,M) - \bar{T}_c(N,M)] + C(N)[T_b(N,M) - \bar{T}_c(N,M)] + r(N,M)Q(N,M) \quad (22)$$

$$\alpha(N,M) \frac{d}{dt_h} \bar{T}_f = [1 - r(N,M)]Q(N,M) - B(N,M)[\bar{T}_f(N,M) - \bar{T}_c(N,M)]$$

where (for the material composition in t.h. region (N,M))

$$A(N,M) = V_c \left(\frac{\partial \rho_c^H}{\partial T_c} \right)$$

$$B(N,M) = V_c \left[\frac{1}{A_H^U} + \frac{1}{A_H^H \cdot \left(\frac{W}{W_o} \right)^8} \right]^{-1} \quad (23)$$

$$C(N) = 2W_r(N)C_c$$

$$\alpha(N,M) = V_c \left(\rho_f \frac{V_f}{V_c} C_f \right).$$

Equations (22) are backward differenced in time to yield

$$\begin{aligned} \bar{T}_c^{j+1}(N,M) = & \left\{ \frac{A(N,M)}{\Delta t_{h,j}} + B^{j+1}(N,M) + C^{j+1}(N) - \frac{B^{j+1}(N,M)^2}{\left[\frac{\alpha(N,M)}{\Delta t_{h,j}} + B^{j+1}(N,M) \right]} \right\}^{-1} \\ & \left\{ \frac{A(N,M)}{\Delta t_{h,j}} \bar{T}_c^j(N,M) + \frac{B^{j+1}(N,M)\alpha(N,M)}{\Delta t_{h,j} \left[\frac{\alpha(N,M)}{\Delta t_{h,j}} + B^{j+1}(N,M) \right]} \bar{T}_f^j(N,M) \right. \\ & \left. + C^{j+1}(N)T_b^{j+1}(N,M) + \left[r(N,M) + \left[\frac{1-r(N,M)}{\frac{\alpha(N,M)}{\Delta t_{h,j}} + B^{j+1}(N,M)} \right] B^{j+1}(N,M) \right] Q^{j+1}(N,M) \right\} \end{aligned} \quad (24a)$$

$$\bar{T}_f^{j+1}(N, M) = \frac{1}{\left[\frac{\alpha(N, M)}{\Delta t_{h_j}} + B^{j+1}(N, M) \right]} \left\{ \frac{\alpha(N, M)}{\Delta t_{h_j}} \bar{T}_f^j(N, M) + B^{j+1}(N, M) \bar{T}_c^{j+1}(N, M) + [1 - r(N, M)] Q^{j+1}(N, M) \right\} \quad (24b)$$

where superscripts, j , refer to time points, and Δt_{h_j} is the appropriate time interval in hours ($\Delta t_{h_j} = \frac{\Delta t_j}{3600}$).

Equations (24) are solved for each t.h. region ($N=1, 2, \dots, NTHC$; $M=1, 2, \dots, NTHP$) only after the fluxes ϕ_1^{j+1} , ϕ_2^{j+1} have been obtained and used to form Q^{j+1} . Then \bar{T}_c^{j+1} and \bar{T}_f^{j+1} are used to up-date the material parameters (see Section V.) to be used when calculating ϕ_1^{j+2} , ϕ_2^{j+2} .

V. TREATMENT OF INPUT PARAMETERS

In TWIGL the continuous x (or r) and z dimensions are replaced by a sequence of mesh points $1, 2, \dots, NPX$ and $1, 2, \dots, NPZ$, respectively. If x - y geometry is specified, zero flux conditions are assumed at the mesh points (x, z) , $x=1, NPX$. If r - z geometry is specified, then zero flux conditions are assumed at (NPX, z) , and symmetry ($\frac{\partial \phi}{\partial r} = 0$) conditions are assumed at mesh points $(1, z)$. Zero flux conditions are always assumed at the top and bottom of the reactor (mesh points $(x, 1)$ and (x, NPZ)). (See Section X for a discussion of how symmetry boundary conditions can be applied on external surfaces.) Variable mesh spacing is permitted in both the x (or r) and z directions. TWIGL input requires NNN values of x (or r) mesh intervals, $h^x(n)$, and break points $Nl(n)$. The mesh spacing $h^x(1)$ applies from x (or r) mesh point 1 to $Nl(1)$, $h^x(2)$ applies to point $Nl(2), \dots, h^x(NNN)$ applies to mesh point $Nl(NNN) = NPX$. Input of the z mesh spacings consists of specifying MMM values of mesh spacings $h^z(m)$ and break points $Ml(m)$. The continuous time domain is replaced by a sequence of time points $1, 2, \dots, NTS$, where the initial conditions apply at time point 1. The time step size, Δt_j , is permitted to be variable. Input requires NPT values of $\Delta t(j)$ and break points $Tl(j)$, where time step $\Delta t(1)$ applies to time point $Tl(1)$, time step $\Delta t(2)$ applies to time point $Tl(2), \dots, \Delta t(NPT)$ applies to time point

$T1(NPT) = NTS$. The neutron diffusion and t.h. material parameters are to be input by material compositions $n=1, 2, \dots, NC$. The compositions are assigned to regions of the spatial mesh structure by supplying a number, NOVL, of overlay regions which assign a given material composition to a specific spatial region. This is accomplished by specifying sets of integers $NR(n)$, $NA(n)$, $NB(n)$, $MA(n)$, $MB(n)$ for $n=1, \dots, NOVL$, where $NR(n)$ is the composition that is to be placed in the spatial region bounded in the x (or r) direction by mesh points $NA(n)$ and $NB(n)$, and bounded in the z direction by mesh points $MA(n)$ and $MB(n)$.

An input parameter NCT indicates that material compositions $n=1, 2, \dots, NCT$ ($NCT \leq NC$) are to have specified linear time variations assigned to their neutron diffusion parameters. (Note that this requires one to order the composition numbers such that the time varying compositions have the lowest composition numbers.) For each of the time varying compositions one can specify a time variation of any or all neutron diffusion parameters by means of one, two or three linear ramps. The input parameter $NTAU (=0, 1, 2, 3)$ indicates 0, 1, 2 or 3 linear ramps are to be used. Let σ represent $D_1, D_2, \Sigma_1, \Sigma_2, \Sigma_{r_1}, v\Sigma_f, v\Sigma_{f_2}$. Then the specified time variation (independent of temperature) of the material parameter σ in composition k is calculated as (for $NTAU=3$)

$$\begin{aligned}
 \sigma_k(t) &= \sigma_k^* + \Delta\sigma_k^{(1)} \left(\frac{t}{\tau_1} \right) & , \quad 0 \leq t \leq \tau_1 \\
 \sigma_k(t) &= \sigma_k(\tau_1) + \Delta\sigma_k^{(2)} \left(\frac{t - \tau_1}{\tau_2 - \tau_1} \right) & , \quad \tau_1 \leq t \leq \tau_2 \\
 \sigma_k(t) &= \sigma_k(\tau_2) + \Delta\sigma_k^{(3)} \left(\frac{t - \tau_2}{\tau_3 - \tau_2} \right) & , \quad \tau_2 \leq t \leq \tau_3 \\
 \sigma_k(t) &= \sigma_k(\tau_3) & , \quad t \geq \tau_3
 \end{aligned} \tag{25}$$

where σ^* is the reference value and $\Delta\sigma^{(1)}, \Delta\sigma^{(2)}, \Delta\sigma^{(3)}$ are input changes in σ over the time intervals $\tau_1, \tau_2 - \tau_1$, and $\tau_3 - \tau_2$, respectively. τ_1, τ_2 , and τ_3 are input.

In addition to the specified time variations, the neutron diffusion parameters, σ , are functions of the local coolant temperature and fuel metal temperature. Hence to complete the time dependence of the neutron parameters we must specify, by composition, coolant and fuel temperature coefficients.

The required input for each composition ($n=1, 2, \dots, NC$) is then as follows.

$D_1^*(n)$, $D_2^*(n)$, $\Sigma_1^*(n)$, $\Sigma_2^*(n)$, $\Sigma_{r1}^*(n)$, $\nu\Sigma_{f1}^*(n)$, $\nu\Sigma_{f2}^*(n)$ are the base material parameters corresponding to the reference temperatures $\bar{T}_f^*(n)$ and $\bar{T}_c^*(n)$.

$$\left(\frac{\partial \Sigma_1}{\partial T_f^{1/2}} \right)_n, \quad \left(\frac{\partial \nu\Sigma_{f1}}{\partial T_f^{1/2}} \right)_n, \quad \left(\frac{\partial \Sigma_{r1}}{\partial T_f^{1/2}} \right)_n, \quad \left(\frac{\partial D_1}{\partial T_c} \right)_n, \quad \left(\frac{\partial D_2}{\partial T_c} \right)_n$$

$$\left(\frac{\partial \nu\Sigma_{f1}}{\partial T_c} \right)_n, \quad \left(\frac{\partial \nu\Sigma_{f2}}{\partial T_c} \right)_n, \quad \left(\frac{\partial \Sigma_1}{\partial T_c} \right)_n, \quad \left(\frac{\partial \Sigma_2}{\partial T_c} \right)_n, \quad \left(\frac{\partial \Sigma_{r1}}{\partial T_c} \right)_n$$

are the necessary temperature coefficients. The required t.h. input for composition n is

$$\left(\frac{\partial \rho_c H}{\partial T_c} \right)_n, \quad A_{H_n}, \quad U_n, \quad h_{o_n}, \quad \left(\rho_f \frac{V_f}{V_c} C_f \right)_n, \quad \left(\frac{V_c}{V_c + V_f} \right)_n.$$

If linear time variations are specified then for compositions $n=1, \dots, NCT$ we require $\Delta D_1^{(1)}(n)$, $\Delta D_2^{(1)}(n)$, $\Delta \Sigma_1^{(1)}(n)$, $\Delta \Sigma_2^{(1)}(n)$, $\Delta \Sigma_{r1}^{(1)}(n)$, $\Delta \nu\Sigma_{f1}^{(1)}(n)$, $\Delta \nu\Sigma_{f2}^{(1)}(n)$ which are the changes desired during the time interval 0 to τ_1 . If NTAU=1, no further changes are required. If NTAU = 2 or 3 we need $\Delta D_1^{(2)}(n)$, $\Delta D_2^{(2)}(n)$, $\Delta \Sigma_1^{(2)}(n)$, $\Delta \Sigma_2^{(2)}(n)$, $\Delta \Sigma_{r1}^{(2)}(n)$, $\Delta \nu\Sigma_{f1}^{(2)}(n)$, $\Delta \nu\Sigma_{f2}^{(2)}(n)$ which are the changes desired during the time interval τ_1 to τ_2 . If NTAU=2, no further changes are required. If NTAU=3 we need $\Delta D_1^{(3)}(n)$, $\Delta D_2^{(3)}(n)$, $\Delta \Sigma_1^{(3)}(n)$, $\Delta \Sigma_2^{(3)}(n)$, $\Delta \Sigma_{r1}^{(3)}(n)$, $\Delta \nu\Sigma_{f1}^{(3)}(n)$, $\Delta \nu\Sigma_{f2}^{(3)}(n)$ which are the changes desired during the time interval τ_2 to τ_3 .

At time t_j , in a t.h. region (N, M) containing composition n , the neutron diffusion parameters are constructed as

$$\begin{aligned}
D_1^j(n) &= D_1^*(n) + \left(\frac{\partial D_1}{\partial T_c} \right)_n [\bar{T}_c^j(N, M) - T_c^*(n)] + [\Delta D_1(n)]^j \\
D_2^j(n) &= D_2^*(n) + \left(\frac{\partial D_2}{\partial T_c} \right)_n [\bar{T}_c^j(N, M) - T_c^*(n)] + [\Delta D_2(n)]^j \\
\Sigma_1^j(n) &= \Sigma_1^*(n) + \left(\frac{\partial \Sigma_1}{\partial T_c} \right)_n [\bar{T}_c^j(N, M) - T_c^*(n)] \\
&+ \left(\frac{\partial \Sigma_1}{\partial T_f^{1/2}} \right)_n \left[\sqrt{\bar{T}_f^j(N, M) + 459.72} - \sqrt{T_f^*(n) + 459.72} \right] + [\Delta \Sigma_1(n)]^j \\
\Sigma_2^j(n) &= \Sigma_2^*(n) + \left(\frac{\partial \Sigma_2}{\partial T_c} \right)_n [\bar{T}_c^j(N, M) - T_c^*(n)] + [\Delta \Sigma_2(n)]^j \\
\Sigma_{r_1}^j(n) &= \Sigma_{r_1}^*(n) + \left(\frac{\partial \Sigma_{r_1}}{\partial T_c} \right)_n [\bar{T}_c^j(N, M) - T_c^*(n)] \\
&+ \left(\frac{\partial \Sigma_{r_1}}{\partial T_f^{1/2}} \right)_n \left[\sqrt{\bar{T}_f^j(N, M) + 459.72} - \sqrt{T_f^*(n) + 459.72} \right] + [\Delta \Sigma_{r_1}(n)]^j \\
\nu \Sigma_{f_1}^j(n) &= \nu \Sigma_{f_1}^* + \left(\frac{\partial \nu \Sigma_{f_1}}{\partial T_c} \right)_n [\bar{T}_c^j(N, M) - T_c^*(n)] \\
&+ \left(\frac{\partial \nu \Sigma_{f_1}}{\partial T_f^{1/2}} \right)_n \left[\sqrt{\bar{T}_f^j(N, M) + 459.72} - \sqrt{T_f^*(n) + 459.72} \right] + [\Delta \nu \Sigma_{f_1}(n)]^j \\
\nu \Sigma_{f_2}^j(n) &= \nu \Sigma_{f_2}^* + \left(\frac{\partial \nu \Sigma_{f_2}}{\partial T_c} \right)_n [\bar{T}_c^j(N, M) - T_c^*(n)] + [\Delta \nu \Sigma_{f_2}(n)]^j
\end{aligned} \tag{26}$$

where the last term on the right side of each expression is that change due to the specified linear variations of the neutron parameters.

Specifically, we have (letting $\sigma = D_1, D_2, \Sigma_1, \Sigma_2, \Sigma_{r_1}, \nu \Sigma_{f_1}, \nu \Sigma_{f_2}$) for compositions $n=1, \dots, NCT$

$$[\Delta\sigma(n)]^j = \begin{cases} (\Delta\sigma_n^{(1)}/\tau_1) \sum_{k=1}^{j-1} \Delta t_k & ; \quad 0 \leq t \leq \tau_1 \\ \Delta\sigma_n^{(1)} + \left(\Delta\sigma_n^{(2)}/(\tau_2 - \tau_1) \right) \left(\sum_{k=1}^{j-1} \Delta t_k - \tau_1 \right) & ; \quad \tau_1 \leq t \leq \tau_2 \\ \Delta\sigma_n^{(1)} + \Delta\sigma_n^{(2)} + \left(\Delta\sigma_n^{(3)}/(\tau_3 - \tau_2) \right) \left(\sum_{k=1}^{j-1} \Delta t_k - \tau_2 \right) & ; \quad \tau_2 \leq t \leq \tau_3 \\ \Delta\sigma_n^{(1)} + \Delta\sigma_n^{(2)} + \Delta\sigma_n^{(3)} & ; \quad t \geq \tau_3 \end{cases} \quad (27)$$

and for compositions $n = NCT+1, \dots, NC$, where no changes have been specified,
 $[\Delta\sigma(n)]^j = 0$.

The inverse group velocities $1/v_1$ and $1/v_2$ are input constants that are independent of space and time. The input integer $IP = 0, 1, \dots, 5$ or 6 indicates 0, 1, ..., 5 or 6 groups of delayed neutron precursors. If $IP \neq 0$, then delayed neutron fractions β_i and decay constants λ_i ($i=1, \dots, IP$) must be input.

VI. THE STEADY STATE PROBLEM

TWIGL solves problems with or without temperature feedback. Let us first consider the steady state when the no feedback option is exercised. For such problems TWIGL does not generate the steady-state flux distribution. These fluxes, $\phi_1^1(n, m)$ and $\phi_2^1(n, m)$, must be generated by a static group diffusion program which uses the same spatial differencing as that employed in TWIGL. PDQ-7 (Ref 1) is such a program, and TWIGL is equipped to read, from a disc file, flux distributions generated by PDQ-7. Additional input for such problems is a normalization factor which multiplies the steady-state flux distributions before time dependent calculations are begun, and the PDQ-7 eigenvalue, λ . The eigenvalue is used to divide $v\Sigma_{f_1}$ and $v\Sigma_{f_2}$, thus guaranteeing criticality. It is important to note that $v\Sigma_{f_1}$ and $v\Sigma_{f_2}$ in Eqs (26) are divided by λ . On option, the initial fluxes may be input

by cards rather than read from a disc file. If the initial fluxes are to be read from a PDQ-7 disc file, then the job-ID (IDJOB) and file-ID (IDFILE) identifying the disc and location of the desired fluxes must be input.

When thermal feedback is to be considered two options exist in TWIGL for generating the steady-state solutions. The first option, which we refer to as "approximate steady state" permits a fast calculation of a set of steady-state fluxes and average temperatures. Here the assumption is made that the input fluxes (either by disc file or cards) are the steady-state fluxes. These fluxes are then used to generate the t.h. region fission source term $Q^1(N,M)$ (Eq 21), and the steady-state version of Eqs (22) are solved for the initial values, $\bar{T}_c^1(N,M)$ and $\bar{T}_f^1(N,M)$. Then, the approximation is made that the reference temperatures, used in Eqs (26), are equal to the initial temperatures, $\bar{T}_c^1(N,M)$ and $\bar{T}_f^1(N,M)$. That is (for $N=1, \dots, NTHC$; $M=1, \dots, NTHP$)

$$\begin{aligned}\bar{T}_c^1(N,M) &= T_c^*(N,M) = T_b^1(N,M) + Q^1(N,M)/C^1(N) \\ \bar{T}_f^1(N,M) &= T_f^*(N,M) = \bar{T}_c^1(N,M) + (1-r(N))Q^1(N,M)/B^1(N,M)\end{aligned}\quad (28)$$

This procedure does yield an equilibrium state; however, the reference temperatures T_c^* and T_f^* do not correspond to the reference material parameters D_g^* , Σ_g^* , etc., appearing in Eqs (26). Note that for this situation the reference temperatures to be used are not composition dependent, but are different for every different t.h. region. This "approximate steady state" is inexpensive to generate since it involves no special iterations or time calculations.

The second steady-state option we refer to simply as "steady state". The strategy used here is precisely that used in WIGL-3 (Ref 8), with the exception that no rod searches are possible. The method used is as follows. The user inputs an initial flux guess and indicates that no delayed neutrons are to be considered. In addition, a desired power level, P_o , is input. The neutron equations (10) are

then solved as transient equations. At each time step the power, P^j , is calculated and used to change either the reference coolant or reference fuel temperatures such as to bring the reactor closer to critical.

In addition, at each time step, the temperature equations are solved as steady-state equations using normalized values of the fission power source term, $\tilde{Q}^j(N, M)$, corresponding to the desired power level, P_o . This procedure is repeated until the calculated power, P^j , satisfies a specified convergence criteria for three successive time steps.

The specific calculations performed are

(1) At each time step ϕ_1^j and ϕ_2^j are obtained from the no delayed neutron, transient neutron equations.

(2) The power, P^j , is computed and normalized values of the fission power source term are obtained as

$$\tilde{Q}^j(N, M) = Q^j(N, M)P_o/P^j. \quad (29)$$

(3) The steady-state temperature equations

$$\begin{aligned} \tilde{T}_c^j(N, M) &= T_b^j(N, M) + \tilde{Q}^j(N, M)/C^j(N) \\ \tilde{T}_c^j(N, M) &= \tilde{T}_c^j(N, M) + (1-r(N))\tilde{Q}^j(N, M)/B^j(N, M) \end{aligned} \quad (30)$$

are solved for all t.h. regions.

(4) Next the check for convergence is made as

$$\frac{|P^j - P_o|}{P_o} < \epsilon \quad (31)$$

where P^j is the unnormalized power, P_o the desired power, and ϵ an input parameter. If Eq (31) is satisfied for three successive time steps, convergence is achieved.

(5) If convergence is not achieved at time t_j , then either the reference coolant or the reference fuel temperature is altered as (for $n=1, \dots, NC$)

$$T^{*j+1}(n) = T^{*j}(n) + \alpha_{SS}[P^j/P_o - 1]\Delta t_j. \quad (32)$$

(6) The fluxes ϕ_1^j and ϕ_2^j are then normalized to the desired power P_o , the altered reference temperatures (Eq 32) are used in Eqs (26), and new fluxes ϕ_1^{j+1} , ϕ_2^{j+1} are obtained.

When convergence is achieved the steady-state fluxes are punched on cards (to be used as input for the appropriate transient problem) and a complete edit is performed.

VII. EDIT FEATURES

All of the input information is edited for every TWIGL problem. In addition, at specified time points, TWIGL will edit any or all of the following: ϕ_1^j , ϕ_2^j , \bar{T}_c^j , \bar{T}_f^j , Q^j . Zero boundary values of the fluxes are not edited.

At every time point the time is printed along with the total fission power. In addition, for the first time step, the following iteration information is edited by the program during outer iteration ℓ .

- (1) ITER $\equiv \ell$
- (2) DELTA \equiv the left side of inequality (15).
- (3) ERROR REDUCT $\equiv Q(\ell)$, where $Q(\ell)$ is defined by Eq (13).
- (4) MIN GROUP REDUCT \equiv the error reduction achieved by inner iterations.
- (5) DEG \equiv the degree of the Chebyshev polynomial being generated by iteration ℓ . If DEG $\equiv 0$, then a Gauss-Seidel iteration is being carried out.
- (6) RATE $\equiv \frac{\text{Actual convergence rate of outer iteration } (\ell-1)}{\text{Theoretical convergence rate of outer iteration } (\ell)}$
If outer iteration $(\ell-1)$ was a Gauss-Seidel iteration, RATE is set equal to unity.
- (7) ALPHA \equiv the α_ℓ of (11).
- (8) BETA \equiv the β_ℓ of (11).

If a new Chebyshev polynomial is started on iteration k , then the following additional information is printed.

- (a) SIGMA = the present estimate for the spectral radius σ of J .
- (b) PRED ERROR RED = the theoretical error reduction one should achieve per iteration if the estimate for σ were correct.
- (c) ELLIPSE MINOR AXIS = the estimate $\tilde{\epsilon}$ for ϵ being used.

For other time steps, the above information is printed only for the last iteration.

VIII. INPUT FORMAT

The following cards must appear in their given order and format.

Card 1: Title - alphanumeric characters, columns 1-80

Card 2: δ, θ

Format (2E10.0)

δ = convergence parameter, Eq (15)

θ = time differencing parameter ($0 < \theta \leq 1$), Eq (4)

Card 3: NPX, NPZ, NTS, NNN, MMM, NPT, NC, NCT, NOVL, IP, NPRT, NTAU, NGO, IFEED

Format (14I5)

NPX = number of x (or r) mesh points (1 and NPX are boundary points).
 $NPX \leq 37$ for rectangular geometry. $NPX = 36$ for cylindrical geometry.

NPZ = number of z mesh points (1 and NPZ are boundary points).
 $NPZ \leq 37$.

NTS = number of time points (time point 1 represents $t=0$)

NNN = x (or r) mesh interval breakpoint parameter ($NNN = 1 +$ number of changes in x (or r) mesh spacing). $NNN \leq 24$.

MMM = z mesh interval breakpoint parameter ($MMM = 1 +$ number of changes in z mesh spacing). $MMM \leq 24$.

NPT = time step breakpoint parameter ($NPT = 1 +$ number of changes in time step size). $NPT \leq 12$.

NC = number of material compositions. NC \leq 20

NCT = number of material compositions with specified linear time variations. NCT \leq 20

NOVL = number of overlay regions. NOVL \leq 40

IP = number of delayed neutron groups. 0 \leq IP \leq 6

NPRT = number of times flux, temperatures and regionwise power is to be printed. NPRT \leq 50

NTAU = number of linear ramps for time varying compositions
(0 \leq NTAU \leq 3)

NGO = 1 for cylindrical geometry; = 0 for rectangular geometry.

IFEED = 1 if temperature feedback is desired; = 0 if not.

Card 4: λ , $1/v_1$, $1/v_2$, NSS, INPTF

Format (3E10.0,2I5)

λ = eigenvalue which divides fission cross sections.

$1/v_1$ = fast group inverse neutron speed (sec-cm $^{-1}$), Eq (1)

$1/v_2$ = thermal group inverse neutron speed (sec-cm $^{-1}$), Eq (1)

NSS = 1 for "steady state" option; = 0 for "approximate steady state"; = -1 for transient problem using initial conditions generated from a "steady state" problem or for a no feedback transient problem.

INPTF = 1 if ϕ^1 and ϕ^2 are input by cards; = 0 if ϕ^1 and ϕ^2 are to be read from disc file.

Card(s) 5: $h^x(1)$, $h^x(2)$, ..., $h^x(NNN)$

Format (12E6.0)

$h^x(1)$ = x mesh spacing (cm) that applies from mesh point 1 to N1(1). (see CARDS 6)

$h^x(2)$ = x mesh spacing that applies for mesh point N1(1) to N1(2).

⋮

$h^x(NNN)$ = x mesh spacing that applies for mesh point N1(NNN-1) to N1(NNN) = NPX.

Card(s) 6: Nl(1), Nl(2),...,Nl(NNN)

Format (12I5)

These are the x mesh interval breakpoints (see Card(s) 5).

Card(s) 7: h^z(1), h^z(2),...,h^z(MMM)

Format (12E6.0)

h^z(1) = z mesh spacing (cm) that applies from mesh point 1 to Ml(1). (see Card(s) 8)

h^z(2) = z mesh spacing that applies from mesh point Ml(1) to Ml(2).

⋮

h^z(MMM) = z mesh spacing that applies from mesh point Ml(MMM-1) to Ml(MMM) = NPY.

Card(s) 8: Ml(1), Ml(2),...,Ml(MMM)

Format (12I5)

These are the y mesh interval breakpoints (see Card(s) 7)

Card 9: Δt(1), Δt(2),...,Δt(NPT)

Format (12E6.0)

Δt(1) = time step size (sec) that applies from time point 1 to time point Tl(1) (see Card 10).

Δt(2) = time step size that applies from time point Tl(1) to time point Tl(2).

⋮

Δt(NPT) = time step size that applies from time point Tl(NPT-1) to time point Tl(NPT) = NTS.

Card 10: Tl(1), Tl(2),...,Tl(NPT)

Format (12I5)

These are the time step size breakpoints (see Card 9).

Card(s)11: This series consists of NOVL cards, each containing 5 integers.

NR(1), NA(1), NB(1), MA(1), MB(1)

NR(2), NA(2), NB(2), MA(2), MB(2)

⋮

⋮

NR(NOVL), NA(NOVL), NB(NOVL), MA(NOVL), MB(NOVL)

Format (5I5)

NR(1) = composition number identifying the material that is to be placed in the spatial region bounded by x mesh points NA(1) and NB(1), and y mesh points MA(1) and MB(1). Note that NA(1) < NB(1), MA(1) < MB(1).

NR(2) = composition number identifying the material that is to be placed in the spatial region bounded by x mesh points NA(2) and NB(2), and y mesh points MA(2) and MB(2).

⋮

Cards 12: This series consists of NC cards, each containing 7 numbers describing the reference reactor material (Eqs 26)

$D_1^*(1)$, $D_2^*(1)$, $\Sigma_1^*(1)$, $\Sigma_2^*(1)$, $\Sigma_{r1}^*(1)$, $v\Sigma_{f1}^*(1)$, $v\Sigma_{f2}^*(1)$

$D_1^*(2)$, $D_2^*(2)$, $\Sigma_1^*(2)$, $\Sigma_2^*(2)$, $\Sigma_{r1}^*(2)$, $v\Sigma_{f1}^*(2)$, $v\Sigma_{f2}^*(2)$

⋮

$D_1^*(NC)$, $D_2^*(NC)$, $\Sigma_1^*(NC)$, $\Sigma_2^*(NC)$, $\Sigma_{r1}^*(NC)$, $v\Sigma_{f1}^*(NC)$, $v\Sigma_{f2}^*(NC)$

Format (7E10.0)

$D_1^*(N)$ = fast group diffusion coefficient (cm) for composition N.

$D_2^*(N)$ = thermal diffusion coefficient (cm) for composition N.

$\Sigma_1^*(N)$ = fast group total cross section (cm^{-1}) for composition N.

$\Sigma_2^*(N)$ = thermal group absorption cross section (cm^{-1}) for composition N.

$\Sigma_{r1}^*(N)$ = fast to thermal-group removal cross section (cm^{-1}) for composition N.

$v\Sigma_{f1}^*(N)$ = fast group fission cross section (cm^{-1}) times v (neutrons/fission) for composition N.

$v\Sigma_{f2}^*(N)$ = thermal group fission cross section (cm^{-1}) times v (neutrons/fission) for composition N.

Card(s) 13: This series consists of IP cards, each containing two numbers. If IP = 0 omit these cards.

β_1, λ_1

β_2, λ_2

⋮

β_{IP}, λ_{IP}

Format (2E10.0)

β_i = delayed neutron fraction for the i^{th} group of delayed precursors (Eqs 1)

λ_i = decay constant (sec^{-1}) for the i^{th} group of delayed precursors (Eqs 1)

Card 14: If NTAU = 0 omit this card. If NTAU = 1, 2 or 3, then this card contains 1, 2 or 3 numbers, respectively (Eqs 25, 27)

τ_i , $i = 1, \dots, \text{NTAU}$

Format (3E10.0)

τ_1 = time (sec) at which linear ramp 1 ends for time varying materials

τ_2 = time (sec) at which linear ramp 2 ends for time varying material

τ_3 = time (sec) at which linear ramp 3 ends for time varying material

Cards 15: This series of cards describes the changes in the macroscopic reactor parameters for the time varying compositions. If NTAU = 0 omit all these cards. (Eqs 25, 27)

The changes for linear ramp 1 ($t=0$ to τ_1 sec) are input on NCT cards as

$$\begin{aligned} \Delta D_1^{(1)}(1), \Delta D_2^{(1)}(1), \Delta \Sigma_1^{(1)}(1), \Delta \Sigma_2^{(1)}(1), \Delta \Sigma_{r_1}^{(1)}(1), \nu \Sigma_{f_1}^{(1)}(1), \nu \Sigma_{f_2}^{(1)}(1) \\ \vdots \\ \Delta D_1^{(1)}(\text{NCT}), \Delta D_2^{(1)}(\text{NCT}), \Delta \Sigma_1^{(1)}(\text{NCT}), \Delta \Sigma_2^{(1)}(\text{NCT}), \Delta \Sigma_{r_1}^{(1)}(\text{NCT}), \nu \Sigma_{f_1}^{(1)}(\text{NCT}), \nu \Sigma_{f_2}^{(1)}(\text{NCT}) \end{aligned}$$

The changes for linear ramp 2 ($t=\tau_1$ to τ_2 sec) are input on NCT cards as (If NTAU = 1 omit these cards)

$$\begin{aligned} \Delta D_1^{(2)}(1), \Delta D_2^{(2)}(1), \Delta \Sigma_1^{(2)}(1), \Delta \Sigma_2^{(2)}(1), \Delta \Sigma_{r_1}^{(2)}(1), \nu \Sigma_{f_1}^{(2)}(1), \nu \Sigma_{f_2}^{(2)}(1) \\ \vdots \\ \Delta D_1^{(2)}(\text{NCT}), \Delta D_2^{(2)}(\text{NCT}), \Delta \Sigma_1^{(2)}(\text{NCT}), \Delta \Sigma_2^{(2)}(\text{NCT}), \Delta \Sigma_{r_1}^{(2)}(\text{NCT}), \nu \Sigma_{f_1}^{(2)}(\text{NCT}), \nu \Sigma_{f_2}^{(2)}(\text{NCT}) \end{aligned}$$

The input changes for linear ramp 3 ($t=\tau_2$ to τ_3 sec) are input as NCT cards as (If NTAU = 1 or 2 omit these cards)

$$\begin{aligned} \Delta D_1^{(3)}(1), \Delta D_2^{(3)}(1), \Delta \Sigma_1^{(3)}(1), \Delta \Sigma_2^{(3)}(1), \Delta \Sigma_{r_1}^{(3)}(1), \Delta \nu \Sigma_{f_1}^{(3)}(1), \Delta \nu \Sigma_{f_2}^{(3)}(1) \\ \vdots \\ \Delta D_1^{(3)}(\text{NCT}), \Delta D_2^{(3)}(\text{NCT}), \Delta \Sigma_1^{(3)}(\text{NCT}), \Delta \Sigma_2^{(3)}(\text{NCT}), \Delta \Sigma_{r_1}^{(3)}(\text{NCT}), \Delta \nu \Sigma_{f_1}^{(3)}(\text{NCT}), \Delta \nu \Sigma_{f_2}^{(3)}(\text{NCT}) \end{aligned}$$

Format (7E10.0)

$\Delta D_1^{(k)}(N)$ = the change in the fast group diffusion coefficient for composition N that occurs during the linear ramp k time interval.

$\Delta D_2^{(k)}(N)$ = same as above for thermal group.

$\Delta \Sigma_1^{(k)}(N)$ = the change in the fast group total cross section for composition N that occurs during the linear ramp k time interval.

$\Delta \Sigma_2^{(k)}(N)$ = same as above for thermal group.

$\Delta \Sigma_{r1}^{(k)}(N)$ = the change in the fast to thermal-group removal cross section for composition N that occurs during the linear ramp k time interval.

$\Delta v\Sigma_{f1}^{(k)}(N)$ = the change in the fast group fission cross section times v for composition N that occurs during the linear ramp k time interval.

$\Delta v\Sigma_{f2}^{(k)}(N)$ = same as above for thermal group.

Card 16: KTS(1), KTS(2),...,KTS(NPRT)

Format (18I4)

KTS(1) = the first time point for which the flux is to be printed.
Note that KTS(1) ≥ 2 since the initial flux (time point 1) is automatically printed.

⋮

KTS(NPRT) = the last time point for which the flux is to be printed.

Card 17: IDJOB, IDFLE, XNORM (skip this card if INPTF = 1)

Format (A8, A10, E1C.0)

IDJOB = the PDQ-7 job ID for the initial flux.

IDFLE = the PDQ-7 file ID for the initial flux.

XNORM = normalization factor which multiplies initial flux after it is read from the disc file.

Card 18: ed1, ed2, ed3, ed4, ed5

Format (5I5)

ed1 = 1 if ϕ_1 is to be printed at edit times
= 0 otherwise

ed2 = 1 if ϕ_2 is to be printed at edit times
= 0 otherwise

ed3 = 1 if \bar{T}_c is to be printed at edit times
= 0 otherwise

ed4 = 1 if \bar{T}_f is to be printed at edit times
= 0 otherwise

ed5 = 1 if Q is to be printed at edit times
= 0 otherwise

If IFEED = 0 (no feedback) skip Card(s) 19-29

Card 19: $v, C_c, Y, W_o, t_w, \alpha_w, T_{in}, t_{in}, \alpha_{in}$

Format (9E8.0), see Eqs (16)

v = number of neutrons/fission

C_c = specific heat of coolant (btu/lb/ $^{\circ}$ F)

Y = y dimension for x-z problems (cm)

W_o = initial flow rate into core (lb/hr)

t_w = time at which flow rate begins to change (sec)

α_w = parameter affecting change of flow rate (sec $^{-1}$)

T_{in} = core inlet temperature ($^{\circ}$ F)

t_{in} = time at which core inlet temperature begins to change (sec)

α_{in} = parameter affecting change of core inlet temperature (sec $^{-1}$)

Card 20: NTHC, NTHP

Format (2I5)

NTHC = number of thermal-hydraulic channels (≤ 10)

NTHP = number of thermal-hydraulic planes (≤ 10)

Card 21: NCH(1), ..., NCH(NTHC)

Format (10I5)

NCH(I) = the number of the x (or r) mesh point that forms the right boundary of t.h. channel I.

Card 22: NPL(1), ..., NPL(NTHP)

Format (10I5)

NPL(I) = the number of the z mesh point that forms the top boundary of t.h. plane I.

Card(s) 23: This series of cards is composed of NTHP cards each with NTHC numbers with Format (LOI5)

COMP(1,1), COMP(2,1), ..., COMP(NTHC,1)

COMP(1,2), COMP(2,2), ..., COMP(NTHC,2)

⋮

COMP(1,NTHP), COMP(2,NTHP), ..., COMP(NTHC,NTHP)

COMP(I,J) = composition no. of material in t.h. region lying in t.h. channel I and t.h. plane J. (Note that each t.h. region may contain only one material composition)

Card(s) 24: This series of cards consists of NC cards each containing 7 numbers with Format (7E10.0) Eqs (16 and 23)

$\left(\frac{V_c}{V_c + V_f}\right)_1, \left(\frac{\partial \rho_c^H}{\partial T_c}\right)_1, A_{H_1}, U_1, h_{o_1}, \left(\rho_f \frac{V_f}{V_c} C_f\right)_1, r_1$

⋮

$\left(\frac{V_c}{V_c + V_f}\right)_{NC}, \left(\frac{\partial \rho_c^H}{\partial T_c}\right)_{NC}, A_{H_{NC}}, U_{NC}, h_{o_{NC}}, \left(\rho_f \frac{V_f}{V_c} C_f\right)_{NC}, r_{NC}$

$\left(\frac{V_c}{V_c + V_f}\right)_n$ = coolant volume fraction for composition n

$\left(\frac{\partial \rho_c^H}{\partial T_c}\right)_n$ = heat required to raise the temperature of one cubic foot of coolant, in composition n, one degree F (btu/ft³/°F)

A_{H_n} = total heat transfer area for composition n divided by the total volume of coolant for that composition (ft⁻¹)

U_n = conductivity/conduction length of the fuel cladding in composition n (btu/ft²/°F/hr)

h_{o_n} = film coefficient at initial flow rate for composition n (btu/ft²/°F/hr)

$\left(\rho_f \frac{V_f}{V_c} C_f\right)_n$ = heat capacity of fuel in composition n (btu/ft³/°F)

r_n = fraction of fission power appearing in coolant for composition n.

Card(s) 25: This series of cards is composed of NC cards each containing 10 numbers with Format (10E7.0) (Eqs 16)

$$\left(\frac{\partial D_1}{\partial T_c} \right)_1, \left(\frac{\partial D_2}{\partial T_c} \right)_1, \left(\frac{\partial \Sigma_1}{\partial T_c} \right)_1, \left(\frac{\partial \Sigma_1}{\partial T_f^{1/2}} \right)_1, \left(\frac{\partial \Sigma_2}{\partial T_c} \right)_1, \left(\frac{\partial \Sigma_{r1}}{\partial T_c} \right)_1, \left(\frac{\partial \Sigma_{r1}}{\partial T_f^{1/2}} \right)_1, \left(\frac{\partial v\Sigma_{f1}}{\partial T_c} \right)_1, \left(\frac{\partial v\Sigma_{f1}}{\partial T_f^{1/2}} \right)_1, \left(\frac{\partial v\Sigma_{f2}}{\partial T_c} \right)_1$$

⋮

$$\left(\frac{\partial D_1}{\partial T_c} \right)_{NC}, \left(\frac{\partial D_2}{\partial T_c} \right)_{NC}, \left(\frac{\partial \Sigma_1}{\partial T_c} \right)_{NC}, \left(\frac{\partial \Sigma_1}{\partial T_f^{1/2}} \right)_{NC}, \left(\frac{\partial \Sigma_2}{\partial T_c} \right)_{NC}, \left(\frac{\partial \Sigma_{r1}}{\partial T_c} \right)_{NC}, \left(\frac{\partial \Sigma_{r1}}{\partial T_f^{1/2}} \right)_{NC}, \left(\frac{\partial v\Sigma_{f1}}{\partial T_c} \right)_{NC}, \left(\frac{\partial v\Sigma_{f1}}{\partial T_f^{1/2}} \right)_{NC}, \left(\frac{\partial v\Sigma_{f2}}{\partial T_c} \right)_{NC}$$

$\left(\frac{\partial D_1}{\partial T_c} \right)_n$ = change in D_1 with coolant temperature in composition n ($\text{cm}^{-1}/^\circ\text{F}$)

$\left(\frac{\partial D_2}{\partial T_c} \right)_n$ = change in D_2 with coolant temperature in composition n ($\text{cm}^{-1}/^\circ\text{F}$)

$\left(\frac{\partial \Sigma_1}{\partial T_c} \right)_n$ = change in fast total cross section with coolant temperature for composition n ($\text{cm}^{-1}/^\circ\text{F}$)

$\left(\frac{\partial \Sigma_1}{\partial T_f^{1/2}} \right)_n$ = change in fast total cross section with square root of fuel temperature for composition n ($\text{cm}^{-1}/\sqrt{^\circ\text{R}}$)

$\left(\frac{\partial \Sigma_2}{\partial T_c} \right)_n$ = change in thermal absorption cross section with coolant temperature for composition n ($\text{cm}^{-1}/^\circ\text{F}$)

$\left(\frac{\partial \Sigma_{r1}}{\partial T_c} \right)_n$ = change in fast removal cross section with coolant temperature for composition n ($\text{cm}^{-1}/^\circ\text{F}$)

$\left(\frac{\partial \Sigma_{r1}}{\partial T_f^{1/2}} \right)_n$ = change in fast removal cross section with coolant temperature for composition n ($\text{cm}^{-1}/\sqrt{^\circ\text{R}}$)

$\left(\frac{\partial v\Sigma_{f1}}{\partial T_c} \right)_n$ = change in $v\Sigma_{f1}$ with the coolant temperature for composition n ($\text{cm}^{-1}/^\circ\text{F}$)

$\left(\frac{\partial v\Sigma_{f1}}{\partial T_f^{1/2}} \right)_n$ = change in $v\Sigma_{f1}$ with the square root fuel temperature in composition n ($\text{cm}^{-1}/\sqrt{^\circ\text{R}}$)

$\left(\frac{\partial v\Sigma_{f2}}{\partial T_c} \right)_n$ = change in $v\Sigma_{f2}$ with the coolant temperature for composition n ($\text{cm}^{-1}/^\circ\text{F}$)

Card 26: $f(1), f(2), \dots, f(NTHC)$

Format (10E7.0)

$f(N)$ = flow rate fraction for t.h. channel N. (Eq 20)

Card 27: P_o , ϵ , α_{SS} , NSRCH (If NSS $\neq 1$ skip this card)

Format (3E10.0, I5)

P_o = desired power (megawatts) $P_o > 0$ (Eqs 29, 31)

ϵ = steady-state convergence parameter (Eq 31)

α_{SS} = parameter specifying response of reference temperature to change in power. (Eq 32)

NSRCH = 1 if T_f^* is to be varied; = 0 if T_c^* is to be varied. (Eq 32)

If NSS = 0 skip cards 28 and 29

Card(s) 28: $T_c^*(1), \dots, T_c^*(NC)$

Format (7E10.0)

$T_c^*(n)$ = reference coolant temperature for composition n. (Eqs 26)

Card(s) 29: $T_f^*(1), \dots, T_f^*(NC)$

Format (7E10.0)

$T_f^*(n)$ = reference fuel temperature for composition n. (Eqs 26)

If INPTF = 0 (initial fluxes read from disc file) skip Card(s) 30 and 31

Card(s) 30: This series contains the initial fast group fluxes, $\phi_1^1(n,m)$. They are to be input 6 to a card in format (6E12.6). Zero boundary values of the fluxes are not to be input. Recall that TWIGL assumes zero flux conditions on all external surfaces. Thus fluxes are not input for z mesh points $m=1, NPZ$.

..., $\phi_1^1(n,2), \dots$

..., $\phi_1^1(n,3), \dots$

⋮

..., $\phi_1^1(n,NPZ-1), \dots$

$\phi_1^1(n,m)$ = initial flux for group 1 at x (or r) mesh point n and z mesh point m. (neutrons/cm²/sec)

Card(s) 31: Same as card series 30, but for $\phi_2^1(n,m)$, the initial thermal flux.

If a "steady state" problem is run (NSS=1), then the steady-state fluxes ϕ_1^1 and ϕ_2^1 are automatically punched on cards in the required format of Card(s) 30 and 31.

IX. SAMPLE PROBLEM

In order to illustrate the input-edit characteristics of the TWIGL program, a sample problem is presented. This is an x-z transient problem using initial fluxes and temperatures generated by a previously run TWIGL "steady state" (NSS=1) problem. The reactor geometry is shown below (Fig. 1).

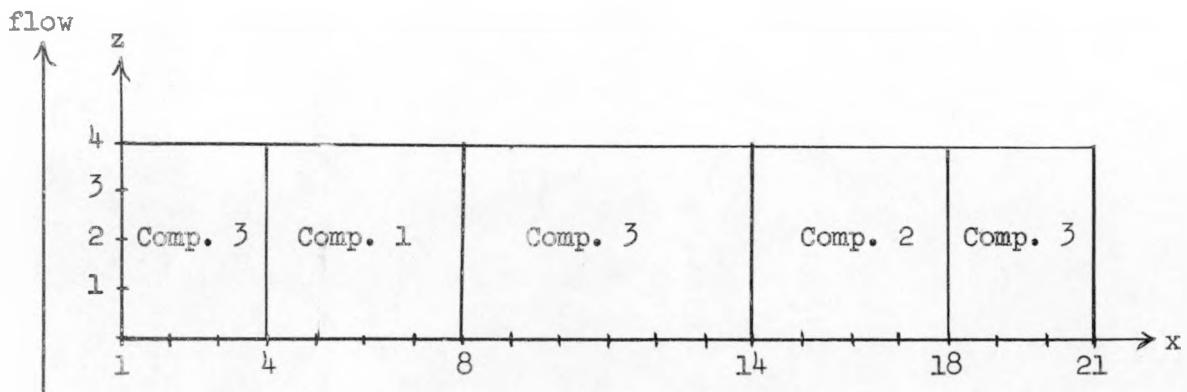


Fig. 1. Test Problem Geometry

Composition 1 differs from composition 2 only in that the thermal group absorption in composition 1 has the specified time behavior shown in Fig. 2.

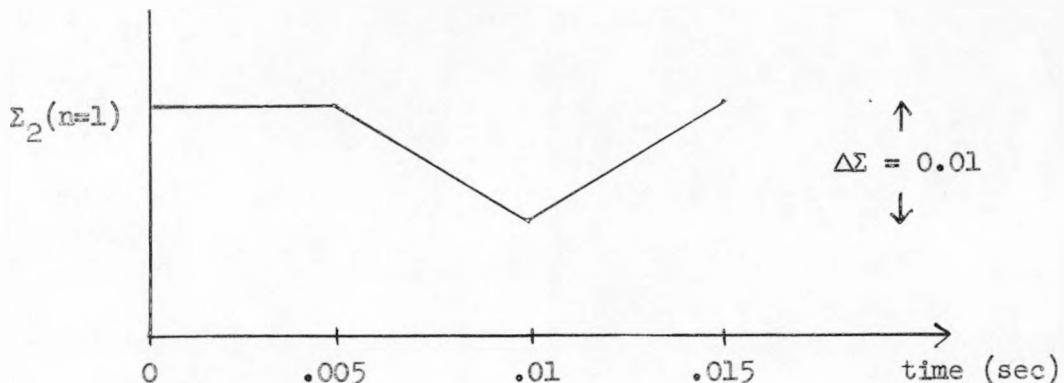


Fig. 2. Time Dependent Thermal Absorption

SAMPLE PROBLEM FOR WAPD-TM-743

| | | | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|
| 500568+03 | 513318+03 | 500640+03 | 513318+03 | 500568+03 | |
| 584678+03 | 142651+04 | 595314+03 | 142651+04 | 584678+03 | |
| 626847+14 | 192851+15 | 534671+15 | 937259+15 | 108474+16 | 939969+15 |
| 539333+15 | 200865+15 | 828086+14 | 538778+14 | 828086+14 | 200865+15 |
| 539333+15 | 939969+15 | 108474+16 | 937259+15 | 534671+15 | 192851+15 |
| 626847+14 | | | | | |
| 626847+14 | 192851+15 | 534671+15 | 937259+15 | 108474+16 | 939969+15 |
| 539333+15 | 200865+15 | 828086+14 | 538778+14 | 828086+14 | 200865+15 |
| 539333+15 | 939969+15 | 108474+16 | 937259+15 | 534671+15 | 192851+15 |
| 626847+14 | | | | | |
| 300763+13 | 868805+13 | 119957+14 | 140090+14 | 161562+14 | 140499+14 |
| 121065+14 | 907133+13 | 398295+13 | 260908+13 | 398295+13 | 907133+13 |
| 121065+14 | 140499+14 | 161562+14 | 140090+14 | 119957+14 | 868805+13 |
| 300763+13 | | | | | |
| 300763+13 | 868805+13 | 119957+14 | 140090+14 | 161562+14 | 140499+14 |
| 121065+14 | 907133+13 | 398295+13 | 260908+13 | 398295+13 | 907133+13 |
| 121065+14 | 140499+14 | 161562+14 | 140090+14 | 119957+14 | 868805+13 |
| 300763+13 | | | | | |

INITIAL FAST FLUX

Z= 1
 6268470+14 1928510+15 2346710+15 9372590+15 1084740+16 9399690+15 5393330+15 2008650+15 8280860+14 5387780+14
 8280860+14 2008650+15 2393330+15 9399690+15 1084740+16 9372590+15 5346710+15 1928510+15 6268470+14

Z= 2
 6268470+14 1928510+15 2346710+15 9372590+15 1084740+16 9399690+15 5393330+15 2008650+15 8280860+14 5387780+14
 8280860+14 2008650+15 2393330+15 9399690+15 1084740+16 9372590+15 5346710+15 1928510+15 6268470+14

INITIAL THERMAL FLUX

Z= 1
 3007630+13 8688050+13 1199570+14 1400900+14 1615620+14 1404990+14 1210650+14 9071330+13 3982950+13 2609080+13
 3982950+13 9071330+13 1210650+14 1404990+14 1615620+14 1400900+14 1199570+14 8688050+13 3007630+13

Z= 2
 3007630+13 8688050+13 1199570+14 1400900+14 1615620+14 1404990+14 1210650+14 9071330+13 3982950+13 2609080+13
 3982950+13 9071330+13 1210650+14 1404990+14 1615620+14 1400900+14 1199570+14 8688050+13 3007630+13

RECTANGULAR GEOMETRY

NUMBER OF X(R) MESH POINTS=21

NUMBER OF Z MESH POINTS= 4

NUMBER OF TIME POINTS= 31

NUMBER OF PRECURSOR GROUPS= 6

BETA=2470000-03 LAMDA=1270000-01 I= 1

BETA=1384500-02 LAMDA=3170000-01 I= 2

BETA=1222000-02 LAMDA=1150000+00 I= 3

BETA=2645500-02 LAMDA=3110000+00 I= 4

BETA=8320000-03 LAMDA=1400000+01 I= 5

BETA=1690000-03 LAMDA=3870000+01 I= 6

1/V1=2000000-06 1/V2=5000000-05

TIME DIFFERENCING THETA=1000+01

MATERIAL PARAMETERS VARY UNTIL TIME=5000000-02

MATERIAL PARAMETERS VARY UNTIL TIME=1000000-01

MATERIAL PARAMETERS VARY UNTIL TIME=1500000-01

GROUP 1 MATERIAL PARAMETERS

SAMPLE PROBLEM FOR WARD-TM-743

TWIGL U YASIN D 11/20/67.IBAI001 PAGE 2

| COMPOSITION | DIFFUSION | SIGMA TOTAL | SIG REMOVAL | NU FISSION |
|-------------|------------|-------------|-------------|------------|
| 1 | 1390000+01 | 2500000-01 | 6000000-02 | 2555562-01 |
| 2 | 1390000+01 | 2500000-01 | 6000000-02 | 2555562-01 |
| 3 | 1190000+01 | 1600000-01 | 6000000-02 | 5111120-02 |

| COMPOSITION | DELTADIFFUS | DELTASIGMAT | DELTASIGMAR | DELTANUFISS | NRAMP |
|-------------|-------------|-------------|-------------|-------------|-------|
| 1 | 0000000+00 | 0000000+00 | 0000000+00 | 0000000+00 | 1 |
| 1 | 0000000+00 | 0000000+00 | 0000000+00 | 0000000+00 | 2 |
| 1 | 0000000+00 | 0000000+00 | 0000000+00 | 0000000+00 | 3 |

GROUP 2 MATERIAL PARAMETERS

| COMPOSITION | DIFFUSION | SIGMA TOTAL | SIG REMOVAL | NU FISSION |
|-------------|------------|-------------|-------------|------------|
| 1 | 5200000+00 | 4300000+00 | | 7325943+00 |
| 2 | 5200000+00 | 4300000+00 | | 7325943+00 |
| 3 | 7000000+00 | 1300000+00 | | 8518539-01 |

| COMPOSITION | DELTADIFFUS | DELTASIGMAT | DELTASIGMAR | DELTANUFISS | NRAMP |
|-------------|-------------|-------------|-------------|-------------|-------|
| 1 | 0000000+00 | 0000000+00 | | 0000000+00 | 1 |
| 1 | 0000000+00 | -1000000-01 | | 0000000+00 | 2 |
| 1 | 0000000+00 | 1000000-01 | | 0000000+00 | 3 |

COMPOSITION OVERLAY

| COMPOSITION | X(R)PT TO | X(R)PT | ZPOINT TO | ZPOINT |
|-------------|-----------|--------|-----------|--------|
| 3 | 1 | 4 | 1 | 4 |
| 1 | 4 | 8 | 1 | 4 |
| 3 | 8 | 14 | 1 | 4 |
| 2 | 14 | 18 | 1 | 4 |
| 3 | 18 | 21 | 1 | 4 |

TIME STEP TO TIME POINT

1000-02 31

X(R) MESH INTERVAL TO POINT,

10000+02 21

Z MESH INTERVAL TO POINT,

14142+02 4

SPATIAL CONVERGENCE PARAMETER=1000000-03

INITIAL FLOW RATE INTO CORE= 20000+06

FLOW RATE INTO CORE BEGINS TO CHANGE AT TIME(SEC)= 10000+09

FLOW RATE CHANGE PARAMETER= 00000+00

INITIAL INLET TEMPERATURE= 50000+03

INLET TEMPERATURE BEGINS TO CHANGE AT TIME(SEC)= 10000+09

INLET TEMPERATURE CHANGE PARAMETER= 00000+00

SPECIFIC HEAT OF COOLANT= 15000+01

THIRD DIMENSION(CM)= 10000+01

NUMBER OF NEUTRONS PER FISSION= 25000+01

NUMBER OF TH CHANNELS= 5

TH CHANNEL INTERFACES= 4 8 14 18 21

NUMBER OF TH PLANES= 1

TH PLANE INTERFACES= 4

TH CHANNEL FLOW FRACTIONS= 15000+00 20000+00 30000+00 20000+00 15000+00

FUEL TEMPERATURE COEFFICIENTS

| COMP. | SIG1 | SIGR | FIS1 |
|-------|----------|----------|----------|
| 1 | 62000-04 | 00000+00 | 00000+00 |
| 2 | 62000-04 | 00000+00 | 00000+00 |
| 3 | 44000-04 | 00000+00 | 00000+00 |

COOLANT TEMPERATURE COEFFICIENTS

| COMP. | D1 | D2 | SIG1 | SIG2 | SIGR | FIS1 | FIS2 |
|-------|----|----|------|------|------|------|------|
|-------|----|----|------|------|------|------|------|

SAMPLE PROBLEM FOR WARD-TM-743

TWIGL U YASIN D 11/20/67.IBAI001 PAGE 4

| | | | | | | | |
|---|----------|----------|----------|----------|----------|----------|----------|
| 1 | 00000+00 | 00000+00 | 00000+00 | 18000-03 | 00000+00 | 00000+00 | 00000+00 |
| 2 | 00000+00 | 00000+00 | 00000+00 | 18000-03 | 00000+00 | 00000+00 | 00000+00 |
| 3 | 00000+00 | 00000+00 | 00000+00 | 12000-04 | 00000+00 | 00000+00 | 00000+00 |

| COMP | V.RATIO | AH | U | H0 | R | RHOVC | RHOH/TC |
|------|----------|----------|----------|----------|----------|----------|----------|
| 1 | 57000+00 | 20000+03 | 27000+03 | 50000+04 | 00000+00 | 30000+02 | 58000+02 |
| 2 | 57000+00 | 20000+03 | 27000+03 | 50000+04 | 00000+00 | 30000+02 | 58000+02 |
| 3 | 33000+00 | 16000+03 | 27000+03 | 50000+04 | 00000+00 | 94000+02 | 58000+02 |

TOTAL FISSION POWER(MEGA WATTS)=100000+01

COOLANT TEMPERATURE

TH PLANE NUMBER 1
5005680+03 5133180+03 5006400+03 5133180+03 5005680+03

37

FUEL TEMPERATURE

TH PLANE NUMBER 1
5846780+03 1426510+04 5953140+03 1426510+04 5846780+03

TH REGION FISSION POWER(BTU/HR)

TH PLANE NUMBER 1
5113319+05 1598199+07 1151118+06 1598199+07 5113319+05

TIME= 100000-02

NUMBER OF INNER ITERATIONS FOR GROUP 1 = 3

NUMBER OF INNER ITERATIONS FOR GROUP 2 = 3

OMEGA FOR GROUP 1 = 10409055+01

OMEGA FOR GROUP 2 = 10001508+01

| | | | |
|----------|--------------------|----------------------------|--------------------------------|
| ITER = 1 | DELTA = 1329736-05 | ERROR REDUCT. = 8944777+08 | MIN GROUP REDUCT. = 1411132-01 |
| DEG. = 0 | RATE = 1000000+01 | ALPHA = 1000000+01 | BETA = 1000000+01 |
| ITER = 2 | DELTA = 4776127-06 | ERROR REDUCT. = 3591788+00 | MIN GROUP REDUCT. = 1850244-01 |
| DEG. = 0 | RATE = 1000000+01 | ALPHA = 1000000+01 | BETA = 1000000+01 |
| ITER = 3 | DELTA = 3313552-06 | ERROR REDUCT. = 6937741+00 | MIN GROUP REDUCT. = 2365003-01 |
| DEG. = 0 | RATE = 1000000+01 | ALPHA = 1000000+01 | BETA = 1100000+01 |
| ITER = 4 | DELTA = 2834217-06 | ERROR REDUCT. = 8553412+00 | MIN GROUP REDUCT. = 2454351-01 |
| DEG. = 0 | RATE = 1000000+01 | ALPHA = 1000000+01 | BETA = 1000000+01 |

| | | | | | |
|----------|----------|--------------------|-------------------|----------------------------|--------------------------------|
| ITER = 5 | DEG. = 0 | DELTA = 2608158-06 | RATE = 1000000+01 | ERROR REDUCT. = 9202398+00 | MIN GROUP REDUCT. = 2471729-01 |
| | | | | ALPHA = 1000000+01 | BETA = 1000000+01 |
| | | SIGMA = 9000000+00 | | PRED ERROR RED= 3928645+00 | ELL1PSE MINOR AXIS= 0000000+00 |
| ITER = 6 | DEG. = 2 | DELTA = 2453089-06 | RATE = 1000000+01 | ERROR REDUCT. = 9405447+00 | MIN GROUP REDUCT. = 2475859-01 |
| | | | | ALPHA = 1000000+01 | BETA = 1680672+01 |

TOTAL FISSION POWER(MEGAWATTS)=100000+01

TIME= 200000-02

| | | | | | |
|----------|----------|--------------------|-------------------|----------------------------|--------------------------------|
| ITER = 6 | DEG. = 6 | DELTA = 3000106-06 | RATE = 1000000+01 | ERROR REDUCT. = 8203991+00 | MIN GROUP REDUCT. = 2478037-01 |
| | | | | ALPHA = 1412823+01 | BETA = 1400750+01 |
| | | SIGMA = 9000000+00 | | PRED ERROR RED= 3928645+00 | ELL1PSE MINOR AXIS= 0000000+00 |

TOTAL FISSION POWER(MEGAWATTS)=100000+01

TIME= 300000-02

| | | | | | |
|----------|----------|--------------------|-------------------|----------------------------|--------------------------------|
| ITER = 6 | DEG. = 6 | DELTA = 2500828-06 | RATE = 1000000+01 | ERROR REDUCT. = 8206473+00 | MIN GROUP REDUCT. = 2478058-01 |
| | | | | ALPHA = 1412823+01 | BETA = 1400750+01 |
| | | SIGMA = 9000000+00 | | PRED ERROR RED= 3928645+00 | ELL1PSE MINOR AXIS= 0000000+00 |

38

TOTAL FISSION POWER(MEGAWATTS)=100001+01

TIME= 400000-02

| | | | | | |
|----------|----------|--------------------|-------------------|----------------------------|--------------------------------|
| ITER = 6 | DEG. = 6 | DELTA = 2074364-06 | RATE = 1000000+01 | ERROR REDUCT. = 8206536+00 | MIN GROUP REDUCT. = 2478061-01 |
| | | | | ALPHA = 1412823+01 | BETA = 1400750+01 |
| | | SIGMA = 9000000+00 | | PRED ERROR RED= 3928645+00 | ELL1PSE MINOR AXIS= 0000000+00 |

TOTAL FISSION POWER(MEGAWATTS)=100001+01

TIME= 500000-02

| | | | | | |
|----------|----------|--------------------|-------------------|----------------------------|--------------------------------|
| ITER = 6 | DEG. = 6 | DELTA = 1716758-06 | RATE = 1000000+01 | ERROR REDUCT. = 8206518+00 | MIN GROUP REDUCT. = 2478060-01 |
| | | | | ALPHA = 1412823+01 | BETA = 1400750+01 |
| | | SIGMA = 9000000+00 | | PRED ERROR RED= 3928645+00 | ELL1PSE MINOR AXIS= 0000000+00 |

TOTAL FISSION POWER(MEGAWATTS)=100001+01

```
*****
TIME= 600000-02
ITER = 55    DELTA = 1076144-04    ERROR REDUCT. = 8881980+00    MIN GROUP REDUCT. = 2466261-01
DEG. = 104    RATE = 1268990+00    ALPHA = 1392864+01    BETA = 1392864+01
SIGMA = 9000000+00    PRED ERROR RED= 3928645+00    ELL1PSE MINOR AXIS= 0000000+00
TOTAL FISSION POWER(MEGAWATTS)=104822+01
```

```
*****
TIME= 700000-02
NUMBER OF INNER ITERATIONS FOR GROUP 1 = 3
NUMBER OF INNER ITERATIONS FOR GROUP 2 = 3
OMEGA FOR GROUP 1 = 10409055+01
OMEGA FOR GROUP 2 = 10001508+01
ITER = 27    DELTA = 3176403-04    ERROR REDUCT = 6628199+00    MIN GROUP REDUCT. = 2465305-01
DEG. = 32    RATE = 9749756+00    ALPHA = 1655859+01    BETA = 1655859+01
SIGMA = 9781642+00    PRED ERROR RED= 6558575+00    ELL1PSE MINOR AXIS= 0000000+00
TOTAL FISSION POWER(MEGAWATTS)=114282+01
```

63

```
*****
TIME= 800000-02
ITER = 29    DELTA = 1883736-04    ERROR REDUCT. = 7540168+00    MIN GROUP REDUCT. = 2465472-01
DEG. = 52    RATE = 6693522+00    ALPHA = 1655858+01    BETA = 1655858+01
SIGMA = 9781642+00    PRED ERROR RED= 6558575+00    ELL1PSE MINOR AXIS= 0000000+00
TOTAL FISSION POWER(MEGAWATTS)=129867+01
```

```
*****
TIME= 900000-02
NUMBER OF INNER ITERATIONS FOR GROUP 1 = 3
NUMBER OF INNER ITERATIONS FOR GROUP 2 = 3
OMEGA FOR GROUP 1 = 10409049+01
OMEGA FOR GROUP 2 = 10001508+01
ITER = 32    DELTA = 2261101-04    ERROR REDUCT. = 6972727+00    MIN GROUP REDUCT. = 2464890-01
DEG. = 42    RATE = 9651851+00    ALPHA = 1688262+01    BETA = 1688262+01
SIGMA = 9828043+00    PRED ERROR RED= 6882622+00    ELL1PSE MINOR AXIS= 0000000+00
TOTAL FISSION POWER(MEGAWATTS)=155479+01
```

TIME= 100000-01

| | | | |
|--------------------|----------------------------|--------------------------------|--------------------------------|
| ITER = 36 | DELTA = 1797068-04 | ERROR REDUCT. = 7917545+00 | MIN GROUP REDUCT. = 2464862-01 |
| DEG. = 66 | RATE = 6250349+00 | ALPHA = 1688262+01 | BETA = 1688262+01 |
| SIGMA = 9828043+00 | PRED ERROR RED= 6882622+00 | ELL1PSE MINOR AXIS= 0000000+00 | |

TOTAL FISSION POWER(MEGAWATTS)=200128+01

FAST FLUX

Z= 1

| | | | | | | | | | |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 1835550+15 | 5668937+15 | 1578674+16 | 2774679+16 | 3211949+16 | 2779915+16 | 1587650+16 | 5824532+15 | 2229511+15 | 1059940+15 |
| 1038213+15 | 2137265+15 | 5582470+15 | 9682363+15 | 1115436+16 | 9629778+15 | 5491288+15 | 1980289+15 | 6435757+14 | |

Z= 2

| | | | | | | | | | |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 1835551+15 | 5668937+15 | 1578674+16 | 2774679+16 | 3211949+16 | 2779915+16 | 1587651+16 | 5824532+15 | 2229511+15 | 1059940+15 |
| 1038213+15 | 2137265+15 | 5582470+15 | 9682363+15 | 1115436+16 | 9629778+15 | 5491288+15 | 1980289+15 | 6435757+14 | |

THERMAL FLUX

OF

Z= 1

| | | | | | | | | | |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 8713087+13 | 2530770+14 | 3588219+14 | 4236613+14 | 4887131+14 | 4244689+14 | 3609860+14 | 2604640+14 | 1060694+14 | 5090832+13 |
| 4983436+13 | 9662064+13 | 1253309+14 | 1447045+14 | 1661103+14 | 1439133+14 | 1231737+14 | 8917534+13 | 3086520+13 | |

Z= 2

| | | | | | | | | | |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 8713087+13 | 2530770+14 | 3588220+14 | 4236613+14 | 4887131+14 | 4244690+14 | 3609860+14 | 2604641+14 | 1060694+14 | 5090833+13 |
| 4983436+13 | 9662064+13 | 1253309+14 | 1447045+14 | 1661103+14 | 1439133+14 | 1231737+14 | 8917534+13 | 3086520+13 | |

COOLANT TEMPERATURE

TH PLANE NUMBER 1

5005681+03 5133188+03 5006399+03 5133181+03 5005680+03

FUEL TEMPERATURE

TH PLANE NUMBER 1

5847184+03 1428259+04 5953369+03 1426535+04 5846785+03

TH REGION FISSION POWER(BTU/HR)

TH PLANE NUMBER 1

1504312+06 4756426+07 2280392+06 1644516+07 5250255+05

TIME= 110000-01

SAMPLE PROBLEM FOR WARD-TM-743

TWIGL U YASIN D 11/20/67.IBAI001 PAGE 8

NUMBER OF INNER ITERATIONS FOR GROUP 1 = 3
NUMBER OF INNER ITERATIONS FOR GROUP 2 = 3
OMEGA FOR GROUP 1 = 10409027+01
OMEGA FOR GROUP 2 = 10001508+01

ITER = 30 DELTA = 2419382-04 ERROR REDUCT. = 6970260+00 MIN GROUP REDUCT. = 2464608-01
DEG. = 38 RATE = 9645927+00 ALPHA = 1687852+01 BETA = 1687852+01
SIGMA = 9827501+00 PRED ERROR RED= 6878511+00 ELL1PSE MINOR AXIS= 0000000+00

TOTAL FISSION POWER(MEGA WATTS)=221093+01

TIME= 120000-01

ITER = 16 DELTA = 2506278-04 ERROR REDUCT. = 4756707+00 MIN GROUP REDUCT. = 2471346-01
DEG. = 26 RATE = 1986483+01 ALPHA = 1687917+01 BETA = 1687897+01
SIGMA = 9827501+00 PRED ERROR RED= 6878511+00 ELL1PSE MINOR AXIS= 0000000+00

TOTAL FISSION POWER(MEGA WATTS)=217470+01

TIME= 130000-01

ITER = 14 DELTA = 5000150-04 ERROR REDUCT. = 1371916+00 MIN GROUP REDUCT. = 2469720-01
DEG. = 22 RATE = 5317460+01 ALPHA = 1688147+01 BETA = 1688055+01
SIGMA = 9827501+00 PRED ERROR RED= 6878511+00 ELL1PSE MINOR AXIS= 0000000+00

TOTAL FISSION POWER(MEGA WATTS)=196419+01

TIME= 140000-01

ITER = 23 DELTA = 3984771-04 ERROR REDUCT. = 5379117+00 MIN GROUP REDUCT. = 2464854-01
DEG. = 40 RATE = 1657110+01 ALPHA = 1687851+01 BETA = 1687851+01
SIGMA = 9827501+00 PRED ERROR RED= 6878511+00 ELL1PSE MINOR AXIS= 0000000+00

TOTAL FISSION POWER(MEGA WATTS)=168580+01

TIME= 150000-01

ITER = 22 DELTA = 3284389-04 ERROR REDUCT. = 3657811+00 MIN GROUP REDUCT. = 2465172-01
DEG. = 38 RATE = 2687789+01 ALPHA = 1687852+01 BETA = 1687852+01
SIGMA = 9827501+00 PRED ERROR RED= 6878511+00 ELL1PSE MINOR AXIS= 0000000+00

TOTAL FISSION POWER(MEGAWATTS)=141398+01

TIME= 160000-01

| | | | |
|-----------|--------------------|----------------------------|--------------------------------|
| ITER = 22 | DELTA = 2256470-04 | ERROR REDUCT. = 3654605+00 | MIN GROUP REDUCT. = 2465325-01 |
| DEG. = 38 | RATE = 2690131+01 | ALPHA = 1687852+01 | BETA = 1687852+01 |
| | SIGMA = 9827501+00 | PRED ERROR RED= 6878511+00 | ELLIPSE MINOR AXIS= 0000000+00 |

TOTAL FISSION POWER(MEGAWATTS)=124876+01

TIME= 170000-01

| | | | |
|-----------|--------------------|----------------------------|--------------------------------|
| ITER = 21 | DELTA = 4071327-04 | ERROR REDUCT. = 4912998+00 | MIN GROUP REDUCT. = 2462473-01 |
| DEG. = 36 | RATE = 1899358+01 | ALPHA = 1687853+01 | BETA = 1687852+01 |
| | SIGMA = 9827501+00 | PRED ERROR RED= 6878511+00 | ELLIPSE MINOR AXIS= 0000000+00 |

TOTAL FISSION POWER(MEGAWATTS)=114849+01

42

TIME= 180000-01

| | | | |
|-----------|--------------------|----------------------------|--------------------------------|
| ITER = 21 | DELTA = 2594807-04 | ERROR REDUCT. = 4914998+00 | MIN GROUP REDUCT. = 2462696-01 |
| DEG. = 36 | RATE = 1898270+01 | ALPHA = 1687853+01 | BETA = 1687852+01 |
| | SIGMA = 9827501+00 | PRED ERROR RED= 6878511+00 | ELLIPSE MINOR AXIS= 0000000+00 |

TOTAL FISSION POWER(MEGAWATTS)=108770+01

TIME= 190000-01

| | | | |
|-----------|--------------------|----------------------------|--------------------------------|
| ITER = 20 | DELTA = 3289227-04 | ERROR REDUCT. = 5663976+00 | MIN GROUP REDUCT. = 2463461-01 |
| DEG. = 34 | RATE = 1519229+01 | ALPHA = 1687854+01 | BETA = 1687853+01 |
| | SIGMA = 9827501+00 | PRED ERROR RED= 6878511+00 | ELLIPSE MINOR AXIS= 0000000+00 |

TOTAL FISSION POWER(MEGAWATTS)=105082+01

TIME= 200000-01

| | | | |
|-----------|--------------------|----------------------------|--------------------------------|
| ITER = 19 | DELTA = 3566263-04 | ERROR REDUCT. = 6244400+00 | MIN GROUP REDUCT. = 2464469-01 |
| DEG. = 32 | RATE = 1258525+01 | ALPHA = 1687858+01 | BETA = 1687856+01 |

SAMPLE PROBLEM FOR WAPD-FM-743

TWIGL U YASIN D 11/20/67.IBAI001 PAGE 10

SIGMA = 9827501+00 PRED ERROR RED= 6878511+00 ELL1PSE MINOR AXIS= 0000000+00

TOTAL FISSION POWER(MEGAWATTS)=102844+01

FAST FLUX

Z= 1

6594880+14 2028282+15 5621241+15 9850857+15 1139997+16 9878743+15 5669227+15 2110781+15 8666358+14 5544859+14
8375202+14 2021805+15 5424454+15 9452472+15 1090770+16 9424546+15 5376407+15 1939267+15 6303560+14

Z= 2

6594881+14 2028282+15 5621242+15 9850859+15 1139997+16 9878745+15 5669228+15 2110782+15 8666359+14 5544860+14
8375203+14 2021805+15 5424455+15 9452472+15 1090770+16 9424546+15 5376407+15 1939267+15 6303560+14

THERMAL FLUX

Z= 1

3169347+13 9151245+13 1262166+14 1473170+14 1698822+14 1477377+14 1273575+14 9546162+13 4174217+13 2697570+13
4029774+13 9132560+13 1217731+14 1412937+14 1624674+14 1408723+14 1206308+14 8737467+13 3024824+13

Z= 2

3169348+13 9151248+13 1262166+14 1473170+14 1698822+14 1477378+14 1273575+14 9546164+13 4174218+13 2697571+13
4029775+13 9132560+13 1217731+14 1412937+14 1624674+14 1408724+14 1206308+14 8737468+13 3024824+13

COOLANT TEMPERATURE

TH PLANE NUMBER 1

5005683+03 5133303+03 5006399+03 5133183+03 5005681+03

FUEL TEMPERATURE

TH PLANE NUMBER 1

5848175+03 1432454+04 5953943+03 1426659+04 5846814+03

TH REGION FISSION POWER(BTU/HR)

TH PLANE NUMBER 1

5379686+05 1680055+07 1184441+06 1607153+07 5141977+05

TIME= 210000-01

ITER = 12 DELTA = 4107126-04 ERROR REDUCT. = 5346520+00 MIN GROUP REDUCT. = 2448750-01

DEG. = 18 RATE = 1685901+01 ALPHA = 1689172+01 BETA = 1688760+01

SIGMA = 9827501+00 PRED ERROR RED= 6878511+00 ELL1PSE MINOR AXIS= 0000000+00

TOTAL FISSION POWER(MEGAWATTS)=101431+01

TIME= 220000-01

| | | | |
|--------------------|----------------------------|--------------------------------|--------------------------------|
| ITER = 11 | DELTA = 4530700-04 | ERROR REDUCT. = 2608136+00 | MIN GROUP REDUCT. = 2462389-01 |
| DEG. = 16 | RATE = 3648856+01 | ALPHA = 1690637+01 | BETA = 1689770+01 |
| SIGMA = 9827501+00 | PRED ERROR RED= 6878511+00 | ELL1PSE MINOR AXIS= 0000000+00 | |

TOTAL FISSION POWER(MEGAWATTS)=1005999+01

TIME= 230000-01

| | | | |
|--------------------|----------------------------|--------------------------------|--------------------------------|
| ITER = 11 | DELTA = 2646843-04 | ERROR REDUCT. = 2615214+00 | MIN GROUP REDUCT. = 2462752-01 |
| DEG. = 16 | RATE = 3641497+01 | ALPHA = 1690637+01 | BETA = 1689770+01 |
| SIGMA = 9827501+00 | PRED ERROR RED= 6878511+00 | ELL1PSE MINOR AXIS= 0000000+00 | |

TOTAL FISSION POWER(MEGAWATTS)=1001110+01

TIME= 240000-01

| | | | |
|--------------------|----------------------------|--------------------------------|--------------------------------|
| ITER = 11 | DELTA = 1551692-04 | ERROR REDUCT. = 2622348+00 | MIN GROUP REDUCT. = 2463094-01 |
| DEG. = 16 | RATE = 3634102+01 | ALPHA = 1690637+01 | BETA = 1689770+01 |
| SIGMA = 9827501+00 | PRED ERROR RED= 6878511+00 | ELL1PSE MINOR AXIS= 0000000+00 | |

TOTAL FISSION POWER(MEGAWATTS)=998215+00

47

TIME= 250000-01

| | | | |
|--------------------|----------------------------|--------------------------------|--------------------------------|
| ITER = 10 | DELTA = 3459506-04 | ERROR REDUCT. = 4528183+00 | MIN GROUP REDUCT. = 2465874-01 |
| DEG. = 14 | RATE = 2189151+01 | ALPHA = 1693716+01 | BETA = 1691895+01 |
| SIGMA = 9827501+00 | PRED ERROR RED= 6878511+00 | ELL1PSE MINOR AXIS= 0000000+00 | |

TOTAL FISSION POWER(MEGAWATTS)=996541+00

TIME= 260000-01

| | | | |
|--------------------|----------------------------|--------------------------------|--------------------------------|
| ITER = 10 | DELTA = 2024470-04 | ERROR REDUCT. = 4531110+00 | MIN GROUP REDUCT. = 2466286-01 |
| DEG. = 14 | RATE = 2187365+01 | ALPHA = 1693716+01 | BETA = 1691895+01 |
| SIGMA = 9827501+00 | PRED ERROR RED= 6878511+00 | ELL1PSE MINOR AXIS= 0000000+00 | |

TOTAL FISSION POWER(MEGAWATTS)=995543+00

TIME= 270000-01

| | | | |
|--------------------|--------------------|----------------------------|--------------------------------|
| ITER = 9 | DELTA = 2642422-04 | ERROR REDUCT. = 5601425+00 | MIN GROUP REDUCT. = 2465482-01 |
| DEG. = 12 | RATE = 1661982+01 | ALPHA = 1700143+01 | BETA = 1696349+01 |
| SIGMA = 9827501+00 | | PRED ERROR RED= 6878511+00 | ELLIPSE MINOR AXIS= 0100000+00 |

TOTAL FISSION POWER(MEGAWATTS)=994971+00

TIME= 280000-01

| | | | |
|--------------------|--------------------|----------------------------|--------------------------------|
| ITER = 8 | DELTA = 2828998-04 | ERROR REDUCT. = 6618822+00 | MIN GROUP REDUCT. = 2465068-01 |
| DEG. = 10 | RATE = 1279962+01 | ALPHA = 1713374+01 | BETA = 1705592+01 |
| SIGMA = 9827501+00 | | PRED ERROR RED= 6878511+00 | ELLIPSE MINOR AXIS= 0000000+00 |

TOTAL FISSION POWER(MEGAWATTS)=994646+00

TIME= 290000-01

| | | | |
|--------------------|--------------------|----------------------------|--------------------------------|
| ITER = 8 | DELTA = 1780874-04 | ERROR REDUCT. = 6619962+00 | MIN GROUP REDUCT. = 2465312-01 |
| DEG. = 10 | RATE = 1279428+01 | ALPHA = 1713374+01 | BETA = 1705592+01 |
| SIGMA = 9827501+00 | | PRED ERROR RED= 6878511+00 | ELLIPSE MINOR AXIS= 0000000+00 |

TOTAL FISSION POWER(MEGAWATTS)=994435+00

TIME= 300000-01

| | | | |
|--------------------|--------------------|----------------------------|--------------------------------|
| ITER = 7 | DELTA = 1738325-04 | ERROR REDUCT. = 8104122+00 | MIN GROUP REDUCT. = 2467079-01 |
| DEG. = 8 | RATE = 1000000+01 | ALPHA = 1739865+01 | BETA = 1724404+01 |
| SIGMA = 9827501+00 | | PRED ERROR RED= 6878511+00 | ELLIPSE MINOR AXIS= 0000000+00 |

TOTAL FISSION POWER(MEGAWATTS)=994318+00

FAST FLUX

Z= 1

| | | | | | | | | | |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 6200791+14 | 1907637+15 | 5288669+15 | 9270582+15 | 1072929+16 | 9297533+15 | 5335045+15 | 1987341+15 | 8201963+14 | 5337542+14 |
| 8266583+14 | 2007278+15 | 5390510+15 | 9395035+15 | 1084210+16 | 9368094+15 | 5344159+15 | 1927594+15 | 6265503+14 | |

Z= 2

| | | | | | | | | | |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 6200790+14 | 1907637+15 | 5288668+15 | 9270581+15 | 1072929+16 | 9297532+15 | 5335045+15 | 1987340+15 | 8201962+14 | 5337542+14 |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|

SAMPLE PROBLEM FOR WAPD-TM-743

TWIGL U YASIN D 11/20/67.1BA1001 PAGE 13

8266583+14 2007278+15 5390510+15 9395035+15 1084210+16 9368093+15 5344159+15 1927594+15 6265503+14

THERMAL FLUX

Z= 1

2975118+13 8593904+13 1186527+14 1385619+14 1597995+14 1389684+14 1197547+14 8975082+13 3944991+13 2594414+13
3976032+13 9064993+13 1210013+14 1404288+14 1614832+14 1400224+14 1198998+14 8683902+13 3006201+13

Z= 2

2975117+13 8593904+13 1186526+14 1385619+14 1597995+14 1389684+14 1197547+14 8975081+13 3944991+13 2594414+13
3976032+13 9064992+13 1210013+14 1404288+14 1614832+14 1400224+14 1198998+14 8683901+13 3006201+13

COOLANT TEMPERATURE

TH PLANE NUMBER 1

5005685+03 5133417+03 5006399+03 5133187+03 5005681+03

FUEL TEMPERATURE

TH PLANE NUMBER 1

5848170+03 1432414+04 5953941+03 1426661+04 5846514+03

TH REGION FISSION POWER(BTU/HR)

94

TH PLANE NUMBER 1

5057894+05 1580812+07 1144612+06 1597414+07 5110984+05

X. COMMENTS

Since the TWIGL spatial difference equations are solved, at each time step, by an iterative technique, the user must be aware of the possibility of either slow convergence or no convergence at all. (If at any time step, the spatial solution is not converged within 500 outer iterations the problem is terminated.) The rate of convergence is dependent upon the magnitude of the largest eigenvalue of the Jacobi iteration matrix (Section III). The magnitude of this eigenvalue must be less than unity for convergence to occur. The $1/\Delta t$ terms on the left side of Eqs (5 or 10) act as effective absorption terms; hence it follows that the magnitude of the largest eigenvalue of the Jacobi matrix decreases as Δt decreases. This says that the smaller the value of Δt used, the faster the spatial solutions converge. For very fast transients (near the prompt critical range) a small value of Δt is usually employed in order to follow the very rapid changes in the system, thus insuring, at the same time, fast convergence of the TWIGL spatial equations. However, for problems in the slow delayed critical range, one wants to use larger values of Δt since the state of the reactor is changing very slowly. Such values of Δt , although adequate for accuracy as to the time differencing approximation, can result in very slow convergence. Hence it may be wise to run a problem for the first time, using only one or two time steps to insure that the Δt being used is adequate for convergence.

The convergence criteria (Eq 15) is an integral criteria. Hence a given value of δ does not guarantee that the pointwise error be less than δ . Initial experience seems to indicate that values of $\delta \sim 10^{-3}$ yield pointwise accuracy to about three decimal places. $\delta \sim 10^{-2}$ would probably be adequate for most applications.

The choice of the value of θ , the time differencing parameter (Eq 4) can also affect the convergence of the spatial equations, as well as the accuracy and stability of the time differencing approximation. If we divide both sides of

Eqs (5 or 10) by θ we find that effective absorption terms, $1/v\Delta t\theta$ appear. In light of the above discussion, we see that the use of $\theta < 1$ will increase the convergence rate over the $\theta=1$ situation. However, for $\theta < 1$, negative fluxes can occur (inaccurate oscillating solutions in time). This results in the dilemma of wanting to decrease θ (to increase the rate of convergence), and at the same time increase θ (to avoid oscillations). Initial experience with TWIGL indicates that when Δt is sufficient to insure a reasonable rate of convergence for supercritical problems, it is at the same time much smaller than necessary for good accuracy in time, and small enough to give stable solutions with θ near $1/2$. For subcritical problems $\theta=1$ should always be used. Also it is wise to use $\theta=1$ for "steady state" problems since we have no a priori idea of the time behavior that will result for such problems. In fact, for any TWIGL problem with feedback, $\theta=1$ is certainly the safest approach.

When "steady state" ($\text{NSS}=1$) problems are run on TWIGL the user must be especially careful of the Δt 's used for the first few time steps. It is quite possible that the initial flux distribution coupled with the initial reactor parameters for such problems can result in a very reactive situation (since no delayed neutrons are present, any supercriticality is superprompt-criticality). Hence unless very small Δt 's are used for the first few time steps, the spatial solutions will not converge. It appears wise to use initial values of Δt that are the same order of magnitude of the $1/v$ values used. Then, after a few time steps, Δt can be increased to a more reasonable value.

The TWIGL user, when running a "steady state" problem must also be careful to choose a value of α_{SS} (SS) with the proper sign to insure that the control response will push the reactor power towards the desired power. If α_{SS} is too small in magnitude the convergence to the desired steady state will be very slow, but if the magnitude of α_{SS} is too large the reactor power will oscillate about the desired power level. Experience with one-dimensional WIGL-3 steady state problems should educate

the user as to the proper choice of α_{SS} , Δt , and $1/v_1$, $1/v_2$ for TWIGL "steady state" problems. (Note that the physical values of $1/v_1$ and $1/v_2$ need not be used for steady state problems.)

If rectangular geometry is specified in a TWIGL problem, then zero flux conditions are automatically assumed at mesh points on the boundaries (i.e. mesh points $(1,z)$, (NPX,z) and mesh points $(x,1)$, $(xNPZ)$). If cylindrical geometry is specified, then zero flux conditions are assumed on the z surfaces (mesh points $(x,1)$ and (x,NPZ)) and on the outer radial surface (mesh points (NPX,z)). Symmetry conditions apply at r mesh points $(1,z)$. However, for either rectangular or cylindrical geometry, problems with symmetry conditions on the external surfaces can be run on TWIGL. This is accomplished by specifying a fictitious material composition with D_1 , D_2 , Σ_1 , Σ_2 , Σ_r^1 , $v\Sigma_{f1}$ and $v\Sigma_{f2}$ equal to zero. Then this fictitious composition is overlaid between the last two mesh points on the surface where symmetry boundary conditions are desired. For example, consider an x - y problem for which we desire symmetry conditions on both x boundaries. In order to accomplish this we overlay the fictitious composition between x mesh points 1 and 2, and $NPX-1$ and NPX , respectively. Note, though, that this results in the physical x boundaries of the problem being located at x mesh points 2 and $NPX-1$. If for such problems the initial flux (or flux guess) is generated using the PDQ-7 program, then the symmetry conditions for the PDQ-7 problem should be achieved by the use of the fictitious composition. That is, one would run the PDQ problem with zero flux conditions on all external surfaces, and by overlapping the fictitious composition on the desired external surfaces symmetry conditions would be obtained. This is necessary to insure consistency between the PDQ-7 and TWIGL mesh structures.

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2. C. J. Pfeifer, "CDC-6600 FORTRAN Programming - Bettis Environmental Report", WAPD-TM-668 (January 1967).
3. A. F. Henry and A. V. Vota, "WIGL2 - A Program for the Solution of the One-Dimensional, Two-Group, Space-Time Diffusion Equations Accounting for Temperature, Xenon, and Control Feedback", WAPD-TM-532 (October 1965).
4. L. A. Hageman, "Numerical Methods and Techniques Used in the Two-Dimensional Neutron-Diffusion Program PDQ-5", WAPD-TM-364 (February 1963).
5. R. S. Varga, Matrix Iterative Analysis, Prentice-Hall, Inc., Englewood Cliffs, New Jersey (1962).
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Computer Code Abstract

1. Name of Code: TWIGL (Ref 1)
2. Computer for which Program is Designed: CDC 6600
Programming Language: FORTRAN IV
3. Nature of Physical Problem Solved: TWIGL solves the two-dimensional, two-group, space-time neutron diffusion equations in rectangular or cylindrical geometry in the presence of temperature feedback. The neutron diffusion and delayed precursor equations are differenced in both space and time. The thermal-hydraulic description is based on a no-boiling, one-pass model formulated in terms of regionwise averaged coolant and fuel metal temperatures.
4. Method of Solution: The program uses the cyclic Chebyshev polynomial method (Ref 2) to solve the spatial difference equations for each time step.
5. Restrictions on the Complexity of the Problem: The maximum number of allowable spatial mesh points is 37 in the x (or r) direction and 37 in the z direction. All problems use two energy groups. Up to six groups of delayed neutrons may be employed. The program accepts a maximum of 20 different material compositions and 100 thermal-hydraulic regions for which the average temperatures are defined.
6. Related and Auxiliary Programs: This program solves two-dimensional problems similar to the one-dimensional problems solved by the WIGL2 (Ref 3) program. However, the methods of solution differ. The PDQ-7 (Ref 4) program may be used to generate initial flux distributions that can be saved on a disc file and read into the TWIGL program.
7. Running Time: Absolute running times per mesh point, per time step are not quotable since the convergence of the spatial difference equations at each

time step is a function of the type of transient (subcritical, supercritical, etc.) being analyzed, the time step size being employed and the desired degree of accuracy. A cylindrical reactor transient problem with 15 radial and 21 axial mesh points took 223 sec for 150 time steps. One group of delayed neutrons was employed and the time step size was 0.001 sec for all time steps.

8. Unusual Features: This program solves transient neutron diffusion problems that are initiated by specified time variations of the reactor material parameters or by specified changes in the core inlet coolant temperature or flow rate. All transient problems are assumed to start from equilibrium, and TWIGL will, by either of two options, generate the steady-state fluxes and average temperatures. The initial flux distributions may be input by punched cards or read from a PDQ-7 (Ref 4) disc file via the file manager subroutines (Ref 5).
9. Status: The program is in the process of being put into production.
10. Machine Requirements: The program was written for a CDC-6600 with a central memory of at least 64K.
11. Operating System: The appropriate software and hardware associated with the system for which this program was written is contained in Ref 5. This particular version of the program was constructed within the SCOPE 2.0 operating system.
12. Other Programming or Operating Information or Restrictions: None.

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