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A SIMPLIFIED MONTE CARLO APPROACH TO DEEP PENETRATION PROBLEMS*

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ABSTRACT

A method of increasing the sampling efficiency in Monte Carlo calculations of thick shield penetration has been developed. The procedure alters the effective mean free path in such a way as to maximize the rate of convergence of the transmission probability. The approach is semi-empirical in nature and has been shown to be remarkably insensitive to geometry. The primary dependence appears to be on the non-absorption probability at each collision, with secondary dependence on the distance to escape. The procedure is simple enough to permit its incorporation into existing Monte Carlo codes with a minimum of programming effort.

In many general Monte Carlo codes in use today, several techniques are available for reducing the variance in the calculated transmission of particles through thick regions. Some of these utilize particle splitting and Russian roulette at arbitrarily chosen boundaries, as well as other forms of importance sampling. Unfortunately, the success of this class of biasing scheme rests to a large degree on the skill of the user of the code, and quite often on his "a priori" knowledge of the very problem he is trying to solve.

An investigation was made of some of the existing alternate approaches, since it was felt that the intuition required to effectively utilize position or direction dependent importance sampling would be hard to come by in complicated geometric configurations. The hope was that a technique could be found which would rely less heavily on the code users' experience.

To investigate the problem of thick shield penetration we assumed an infinite plane source of mono-energetic neutrons, incident normally on the left face of a slab of thickness $m\lambda$, where λ is the mean free path. This is depicted in figure 1. λ , as well as the scattering probability, P_s , was taken to be constant. The scattering was assumed isotropic in the laboratory system since this presents the most difficult convergence problem in calculating the transmission.

Energy dependence was eliminated in order to isolate the problem of spatial penetration from any extraneous factors.

We used slab geometry so that we could compare our numerical results with those from analytic treatments. It also served to considerably reduce the computation costs of this investigation.

Since we were seeking techniques which would apply in any geometry or coordinate system, we tried those which would be functions only of the distance to escape at each collision point. One such approach, available in some codes, relies on the adjustment in some way of the effective mean free path which the particle sees in traversing the medium.

The most direct approach was to replace the actual mean free path, λ, by

$$\lambda' = \lambda/a \qquad (1)$$

where "a" is a parameter set usually between 0 and 1.

The straightforward Monte Carlo approach to selecting a path length is to randomly select it from the actual distribution of such distances:

$$f(s)\,ds = e^{-s/\lambda}\frac{ds}{\lambda} \tag{2}$$

where s is the distance to the next collision.

This is done by picking a random number, R, from a uniform distribution of such numbers between 0 and 1 and setting it equal to the cumulative distribution function for s:

$$R = \int_0^s e^{-s'/\lambda}\frac{ds'}{\lambda} \tag{3}$$

from which

$$s = -\lambda \ln(1-R), \text{ or simply } s = -\lambda \ln R \tag{4}$$

since (1-R) has the same distribution as R.

If we select instead from

$$g(s)\,ds = e^{-s/\lambda'}\frac{ds}{\lambda'} \tag{5}$$

then

$$s = -\lambda' \ln R, \text{ or, in terms of the parameter "a", } s = -\frac{\lambda}{a}\ln R \tag{6}$$

To preserve the proper expectation value for the number of particles traveling a distance, s, we assign a weighting factor

$$W_f = \frac{\lambda'}{\lambda} e^{(s/\lambda' - s/\lambda)} = \frac{1}{a}\, e^{-\frac{s}{\lambda}(1-a)} \tag{7}$$

so that the product of $W_f \times g = f$.

We coupled this technique with the familiar one of eliminating absorption and weighting the particle by the scattering probability, P_s , so that the statistical weight of a particle which collides after traveling a distance s is

$$W = W' P_s \left[\frac{1}{a}\, e^{-\frac{s}{\lambda}(1-a)}\right] \tag{8}$$

where W' is the weight resulting from the previous collision.

We used no directional biasing except for the fact that equation (8) was used only when a particle was moving forward. When the particle was moving toward the source, we set

$$W = W' P_s \quad \text{, essentially using "a" = 1.} \tag{9}$$

Note, in equation (8) that a value of "a" exists which will preserve the statistical weight of a particle traveling one mean free path. This is obtained by setting $W = W'$ and $s/\lambda = 1$.

When a comprehensive parameter study was run for a 20 M. F. P. slab, with various values of P_s , it was found that using "a" obtained in this manner gave very satisfactory results.

Figure 2 shows the transmission probability for a 20 M. F. P. slab with a scattering probability of 0.9, obtained for several values of "a". The curves are plotted as a function of $\sqrt{N}$ (where N is the number of particle histories) for ease of interpretation. The constant under the radical merely serves to normalize the first printed output to "1" on the curve. In this way the second output, plotted at "2", has a probable error half that at "1". At "4" it is half that at "2", et cetera.

The analytic value(1), 3.0×10^{-5}, is an excellent agreement with the consensus of the better Monte Carlo results.

Satisfactory results were obtained for "a" between 0.5 and 0.9 with the best results occurring around "a" = 0.7. Variance estimates, as well as comparisons with a straightforward calculation, indicate a reduction in the variance by a factor of nearly 400 for this case. The estimated probable error after 1000 particle histories is about $.06 \times 10^{-5}$ or 20% of the established value. Values of "a" between 0.6 and 0.8 would all give probable errors within 30% at this point. (The value of "a" which would preserve the statistical particle weight for a 1 M. F. P. flight is 0.61, within the above mentioned range.)

These 1000 histories consumed 16 seconds of IBM 7094 time in this simple problem. Should a large code, on a real problem, be even twenty times slower, this calculation would still be quite practical.

Poorly chosen values of "a", however, can actually lead to erroneous results in any reasonable number of particle histories. This occurred most dramatically when "a" was too low. These values over-emphasize long flights at the expense of intermediate path lengths of particles making a large number of collisions. Note that for "a" = .150, the results after 100,000 histories appear reliable within about 50%, but are actually low by more than an order of magnitude. "a" = 0.400 looks quite well converged after 100,000 histories but is still low by around 50%.

A similar set of runs was made for $m = 20$, $P_s = 0.5$. The analytic result[1] is less certain here, but is around 1×10^{-8}. Only two curves are shown in figure 3, but results fall within 20% of 1×10^{-8} for all "a"'s up to .375. The lower curve, "a" = .085, shows the effect of poor sampling. (Note the slow rise and long periods of fall-off.) The curve for "a" = .320 behaves beautifully and was about as good a result as was obtained. ("a" = .230 would preserve a particle's statistical weight for 1 M. F. P. flight. This result was only slightly inferior to the curve for "a" = .320.)

By varying the slab thickness, $m\lambda$, as well as P_s, it was found that setting

$$P_s^{\frac{m}{30}}\left[\frac{1}{a}\, e^{-(1-a)}\right] = 1 \qquad (10)$$

would yield near optimum values of "a" at least to 30 M. F. P. Figure 4 shows "a" vs P_s, obtained from equation (10) for $m = 20$ and $m = 30$. The plot is on Gaussian probability paper since it behaves more linearly on this paper in the normal range of interest. In this range there is not too much difference between the curves. The difference is greatest when P_s is low, but, as our results indicate, the variance is less sensitive to "a" in this range than for high P_s. Effectively, introducing the "m" dependence has a noticeable but relatively minor effect when compared to the effect of preserving the weight of particles traveling about 1 M. F. P.

The application to arbitrary geometry is clear. By interpolation from a table of "a" vs P_s one can obtain a satisfactory value of "a" at every collision in an energy dependent problem. Since, in a flexible geometry Monte Carlo code, the procedure for determining the distance to excape in a given direction is usually needed for other reasons, it would require only minor modification to incorporate this scheme in such a code, along with a tabulation of "a" vs P_s for a few appropriate values of m.

The relative insensitivity of the best choice of "a" to m indicates that in any geometric configuration, the reliability of the results will not be altered, even when the distance to escape is a sharply varying function of direction or energy.

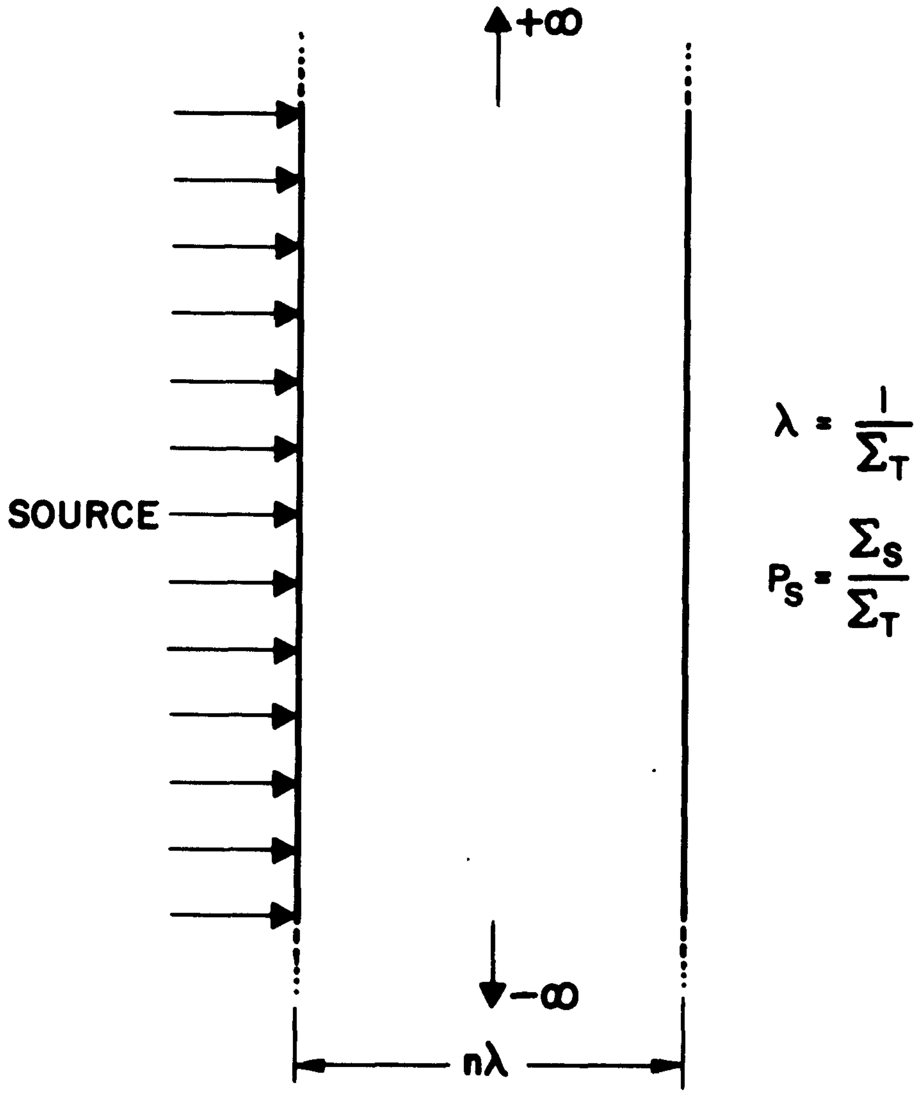


Figure 1

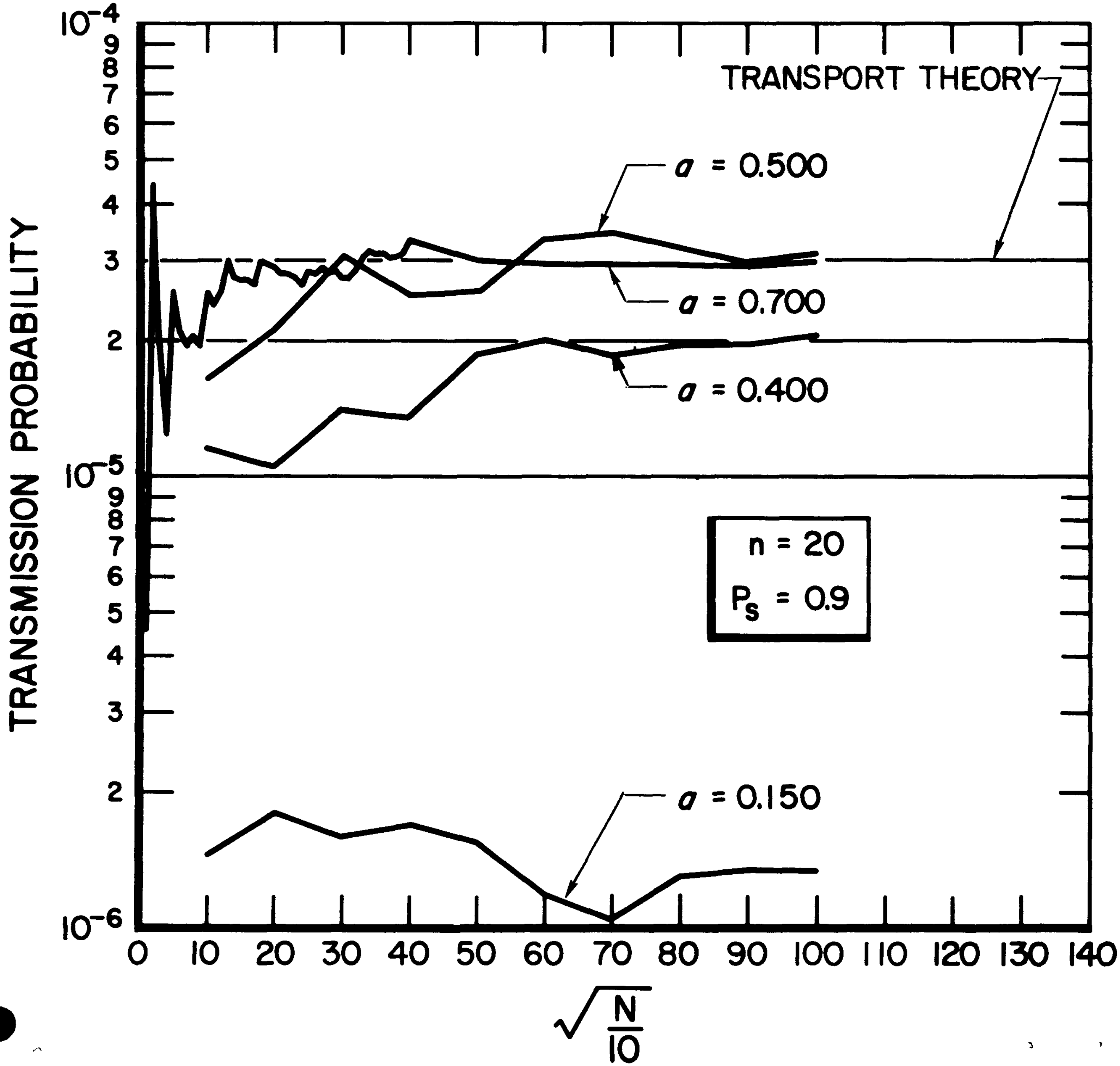
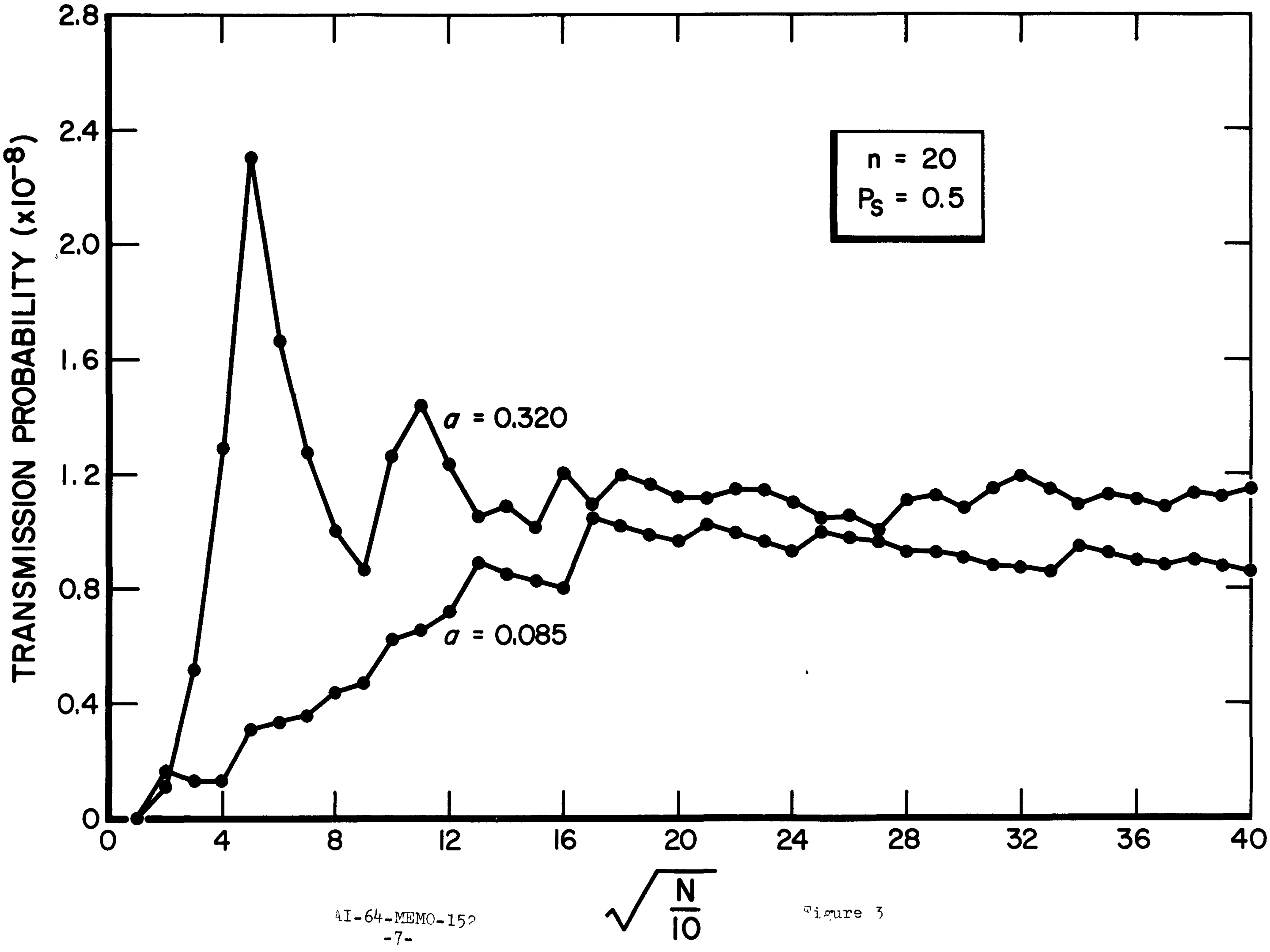
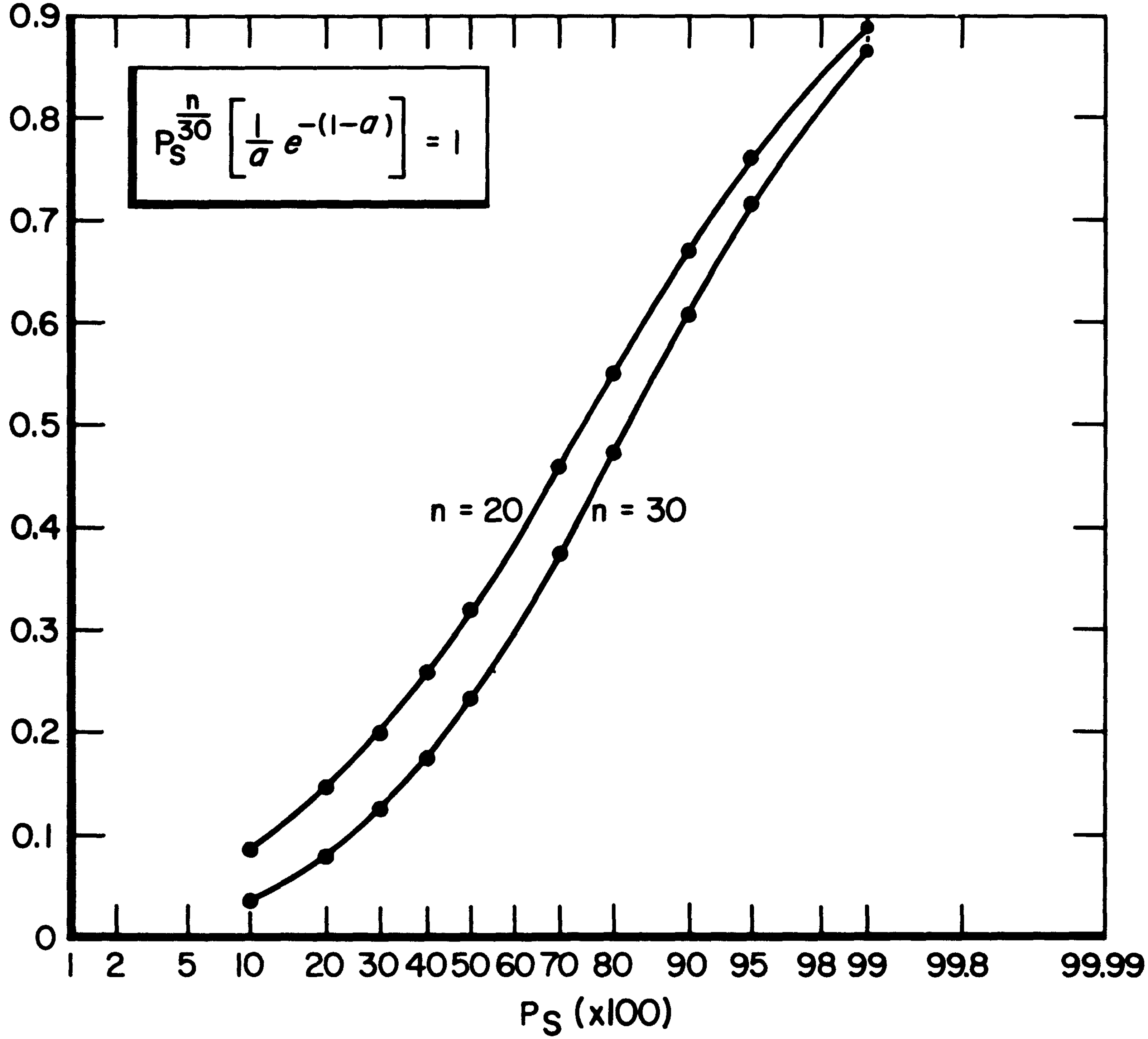

TRANSPORT THEORY
a = 0.500
a = 0.700
a = 0.400
a = 0.150
n = 20
P_s = 0.9
TRANSMISSION PROBABILITY
10^{-4}
10^{-5}
10^{-6}
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140
$\sqrt{\frac{N}{10}}$

Figure 2



TRANSMISSION PROBABILITY (x10^{-8})
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
0
4
8
12
16
20
24
28
32
36
40
$\sqrt{\frac{N}{10}}$
n = 20
P_S = 0.5
a = 0.320
a = 0.085

Figure 3



$P_S^{\frac{n}{30}} \left[\frac{1}{a} e^{-(1-a)} \right] = 1$
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
a
n = 20
n = 30
1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.8 99.99
P_S (x100)

Figure 4

REFERENCES

(1) B. D. O'Reilly and R. C. Lewis, "Solution of the One-Energy Transport Equation in a Finite Slab by Orthogonal Polynomial Expansion", Douglas Report SM-42028, July, 1962.

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