Westinghouse Atomic Power Divisions

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EURAEC-2045
WCAP-3680-7

CONTROL OF XENON INSTABILITIES IN LARGE PWR'S

QUARTERLY PROGRESS REPORT FOR THE PERIOD ENDING JUNE 30, 1968

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Project Engineer

Prepared for the New York Operations Office
U. S. Atomic Energy Commission
Under AEC Contract AT(30-1)-3680

July 1968

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SECTION 1

PROGRAM DESCRIPTION

This program investigates the characteristics and control of spatial instabilities in large pressurized water reactors (large PWR's), with particular emphasis on azimuthal xenon instabilities (x-y plane). The program consists of the following technical tasks:

1. Task EUXE-200 - Effect of Core Parameters on Spatial Oscillations

   The aim of this task is to analyze the effect of variations in core design and operating parameters on the propensity for spatial oscillations, with emphasis on those resulting from xenon redistribution. Parameters to be analyzed include, but are not limited to, core dimensions, fuel and moderator temperature feedbacks, and power distributions.

2. Task EUXE-300 - Remedial Control Procedures

   Under this task, two-dimensional x-y calculations will be performed to establish control methods and detector locations which will prevent divergent oscillations. Calculated results will be used to develop criteria for the application of remedial measures to large pressurized water reactors.

3. Task EUXE-400 - Three-Dimensional Analysis

   Under this task, either direct or synthesized three-dimensional calculations will be performed to further study spatial instabilities in large PWR cores with conditions conducive to oscillations. If a three-dimensional oscillation develops after perturbations have been introduced, selected remedial methods (as developed under Task EUXE-300) will be applied and evaluated. The unique characteristics of three-dimensional oscillations (if observed) will be identified and the criteria for the application of remedial measures will be developed.

SECTION 2

PROGRESS SUMMARY

Major progress for the three-month period ending June 30, 1968 is summarized below:

A comparison was made between a half core model with symmetry about a principal axis and a half-core model of the same core with diagonal symmetry. The latter was found to yield a more divergent oscillation, and will be used in further studies of different concepts for controlling azimuthal xenon-induced oscillations.

The theoretical basis for a flux expansion in time at each time-step for xenon depletion studies was formulated. In digital simulation calculations with finite time steps, the expansion will generate an expression for the time-dependent flux at each spatial point over a time-step interval, and will allow a more accurate estimate of the xenon and iodine number densities than the conventional constant flux approximation in the depletion equations for xenon. It will also allow a considerably improved source-guess for the flux iteration calculations at the beginning of each time-step, thereby reducing required computer time for xenon oscillation studies.

SECTION 3

EUXE-100

PROGRAM MANAGEMENT

M. J. O'Boyle, Project Engineer — Development Projects, PWR Plant Division

A. P. Suda

This is the seventh in a series of technical progress reports on the U.S.-Euratom program titled "Control of Xenon Instabilities in Large PWR's." The preceding reports in this series are:

EURAEC-1721 WCAP-3680-1	"Control of Xenon Instabilities in Large PWR's, Technical Progress Report for the Period Ending September 30, 1966"
EURAEC-1781 WCAP-3680-2	"Control of Xenon Instabilities in Large PWR's, Quarterly Progress Report for the Period Ending December 31, 1966"
EURAEC-1830 WCAP-3680-3	"Control of Xenon Instabilities in Large PWR's, Quarterly Progress Report for the Period Ending March 31, 1967"
EURAEC-1880 WCAP-3680-4	"Control of Xenon Instabilities in Large PWR's Quarterly Progress Report for the Period Ending June 30, 1967"
EURAEC-1925 WCAP-3680-5	"Control of Xenon Instabilities in Large PWR's, Quarterly Progress Report for the Period Ending September 30, 1967"
EURAEC-2008 WCAP-3680-6	"Control of Xenon Instabilities in Large PWR's, Semi-Annual Progress Report for the Period Ending March 31, 1968."

The following topical report has also been prepared under this program:

EURAEC-1974 WCAP-3680-20	"Xenon-Induced Spatial Instabilities in Large Pressurized Water Reactors"

Program management is not discussed in this technical progress report, since separate reports emphasizing the administrative aspects of the program are published monthly for limited distribution.

SECTION 4

EUXE-200

EFFECT OF CORE PARAMETERS ON SPATIAL OSCILLATIONS

G. H. Minton, Manager — Reactor Physics and Mathematics Section

J. E. Olhoeft, Manager — Reactor Physics

C. G. Poncelet

A. M. Christie

The work performed under this task has been completed. The work is described in detail in the following topical report, the abstract of which is also presented:

Poncelet, C. G. and Christie, A. M., "Xenon-Induced Spatial Instabilities in Large Pressurized Water Reactors," WCAP-3680-20 (March 1968).

ABSTRACT:

The characteristics of free-running xenon-induced spatial oscillations in large pressurized water reactors (large PWR's) are investigated both from a phenomenalogical point of view and from the standpoint of the theoretical and calculational methods employed in their analysis. The digital simulation of spatial xenon instabilities with multidimensional, multi-group diffusion theory programs is investigated in detail. An extensive study of the effect of a finite, non-zero time-step length on calculated stability characteristics is presented, including a set of correlations which permits the extrapolation of digital calculations to effectively zero time-step length. The sensitivity of the digital results to the spatial mesh length, the energy mesh, and the treatment of temperature feedback effects is investigated. The effect of a non-zero flux convergence criterion on spatial xenon oscillations is investigated. A comparison is made of modal theory results and digital simulation calculations. The space-dependent transfer function formalism is employed

in a number of analyses, including the effect of delayed neutrons and the effect of finite temperature feedback time lags.

The space-time characteristics of xenon-induced instabilities in the two-dimensional plane perpendicular to the direction of coolant flow are studied for a large variety of core sizes, power distributions and temperature feedback effects. The effect of perturbation size and location is evaluated. Extensive parametric and sensitivity analyses are performed, based on a modal expansion method. Parameters investigated are core size, power distribution, power level, power coefficient, fuel enrichment, Xe-135 absorption cross section, I-135 fission yield and direct Xe-135 fission yield. Parametric calculations with one- and two-dimensional diffusion theory programs are performed to corroborate the modal theory results.

SECTION 5

EUXE-300

REMEDIAL CONTROL PROCEDURES

D. L. Miller, Manager — Nuclear Engineering

F. B. Skogen

1. SUMMARY

This report covers work in progress as well as work accomplished. A comparison has been made between a half-core model with symmetry about a principal axis and a half-core model of the same core with diagonal symmetry. The latter was found to yield a more divergent oscillation.

2. X-Y ANALYSIS

In the two-dimensional analysis of azimuthal xenon instabilities in large PWR's, two different core geometries were considered. Both geometries are half cores: one being symmetric about a principal axis, the other about a diagonal. A comparison study was conducted to determine the model's influence on the calculated results. The core used in the analysis is an 11-foot core with a power density of 104.5 kw/ℓ, with a typical checkerboard loading pattern. The core has burnable poison and a beginning of first cycle critical boron concentration of 1100 ppm. The core has zero moderator feedback. The calculational model incorporates pointwise Doppler corrections.

Figures 1 and 2 illustrate the rod positions and the two geometries considered in the analysis. The systems were perturbed by inserting a local poison in the form of a control rod in the peripheral region of the core. The poison was removed after one hour, at which time the flux distribution was sufficiently distorted to initiate a xenon transient. Subsequent oscillations were examined using two-hour time-steps (Figures 3 and 4). Although both models produced a diverging oscillation, the half core with diagonal symmetry was

Figure 1

Half Core Geometry with Symmetry About a Principal Axis

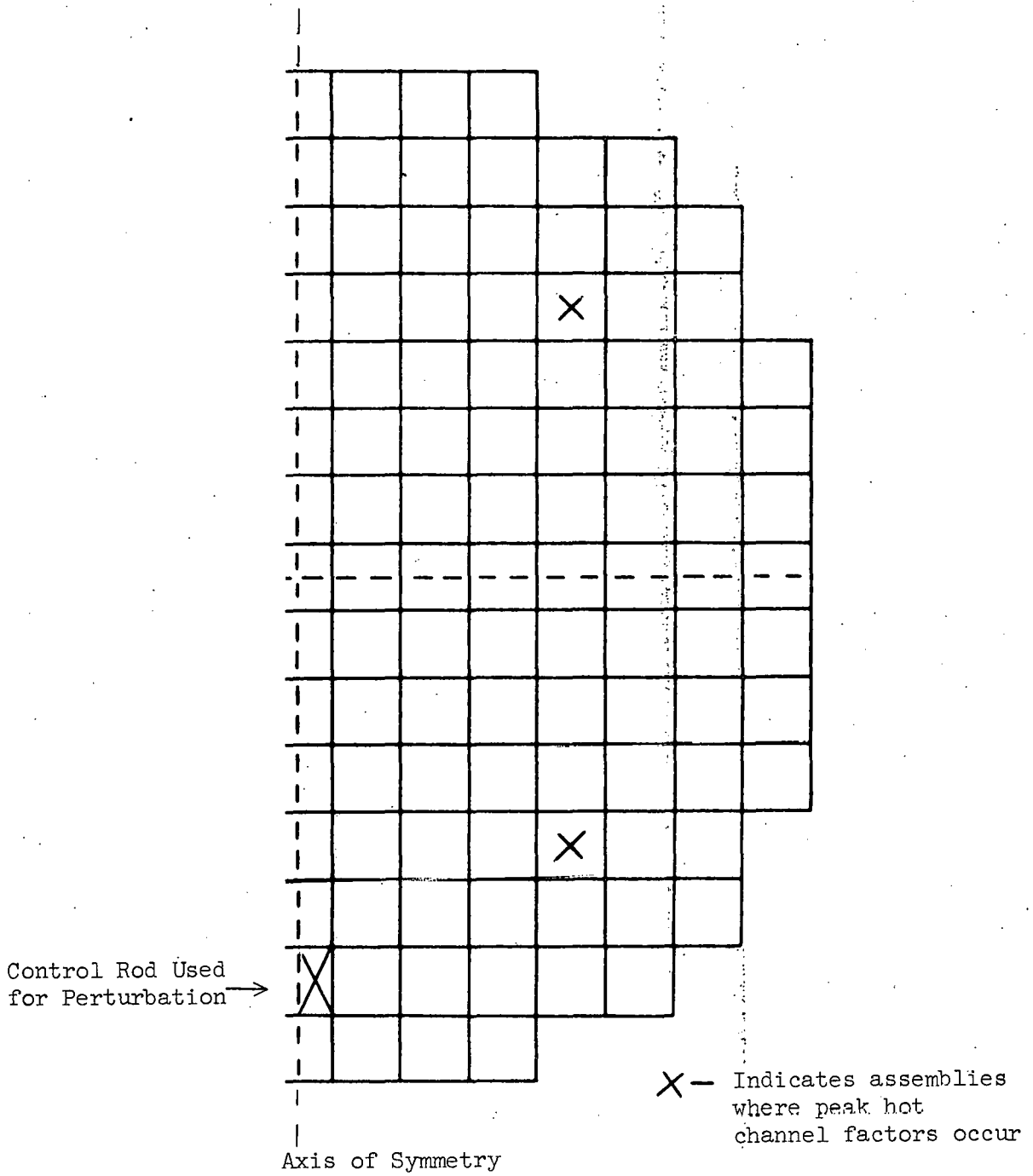
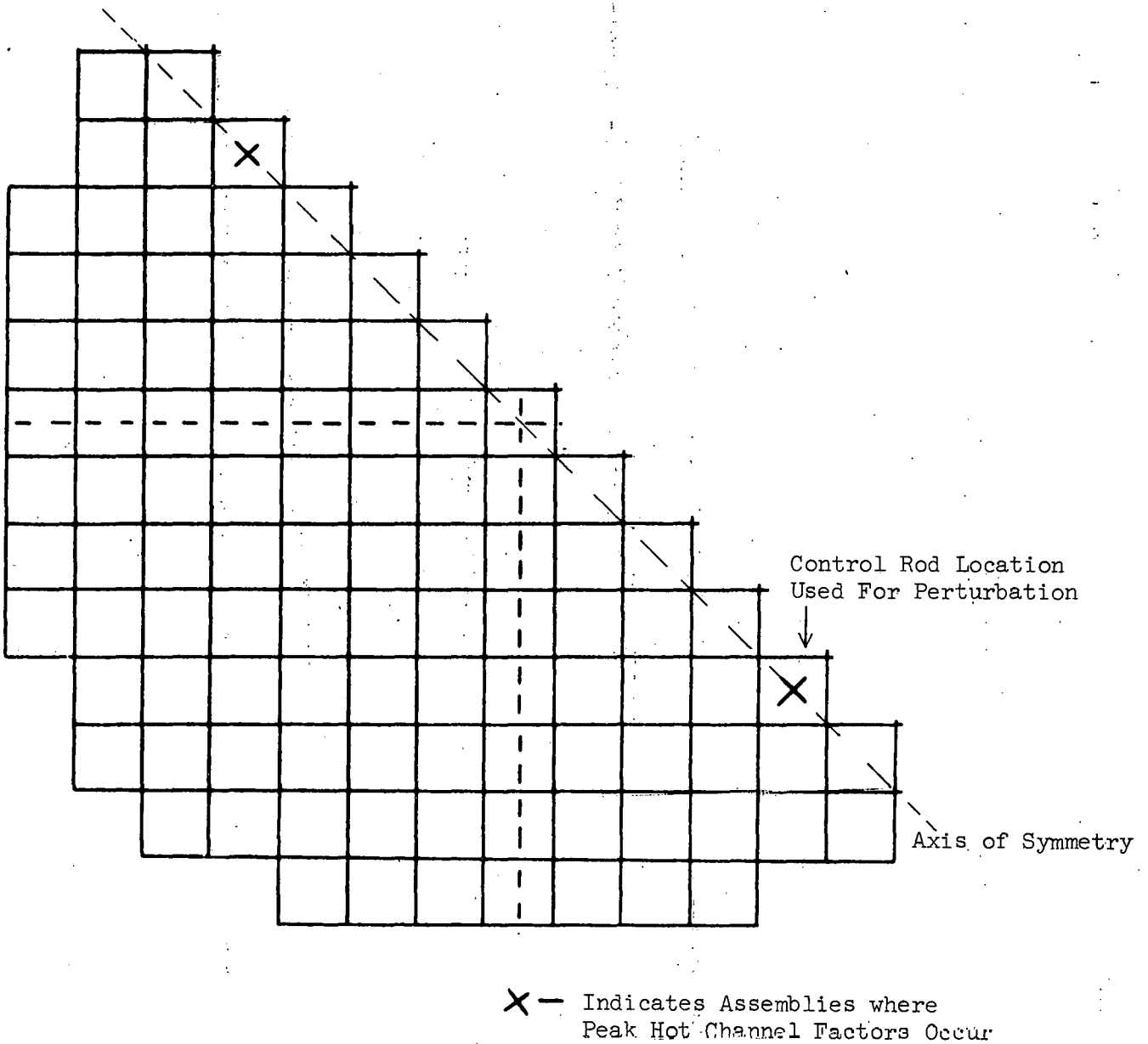


Figure 2

Half Core Geometry with Diagonal Symmetry



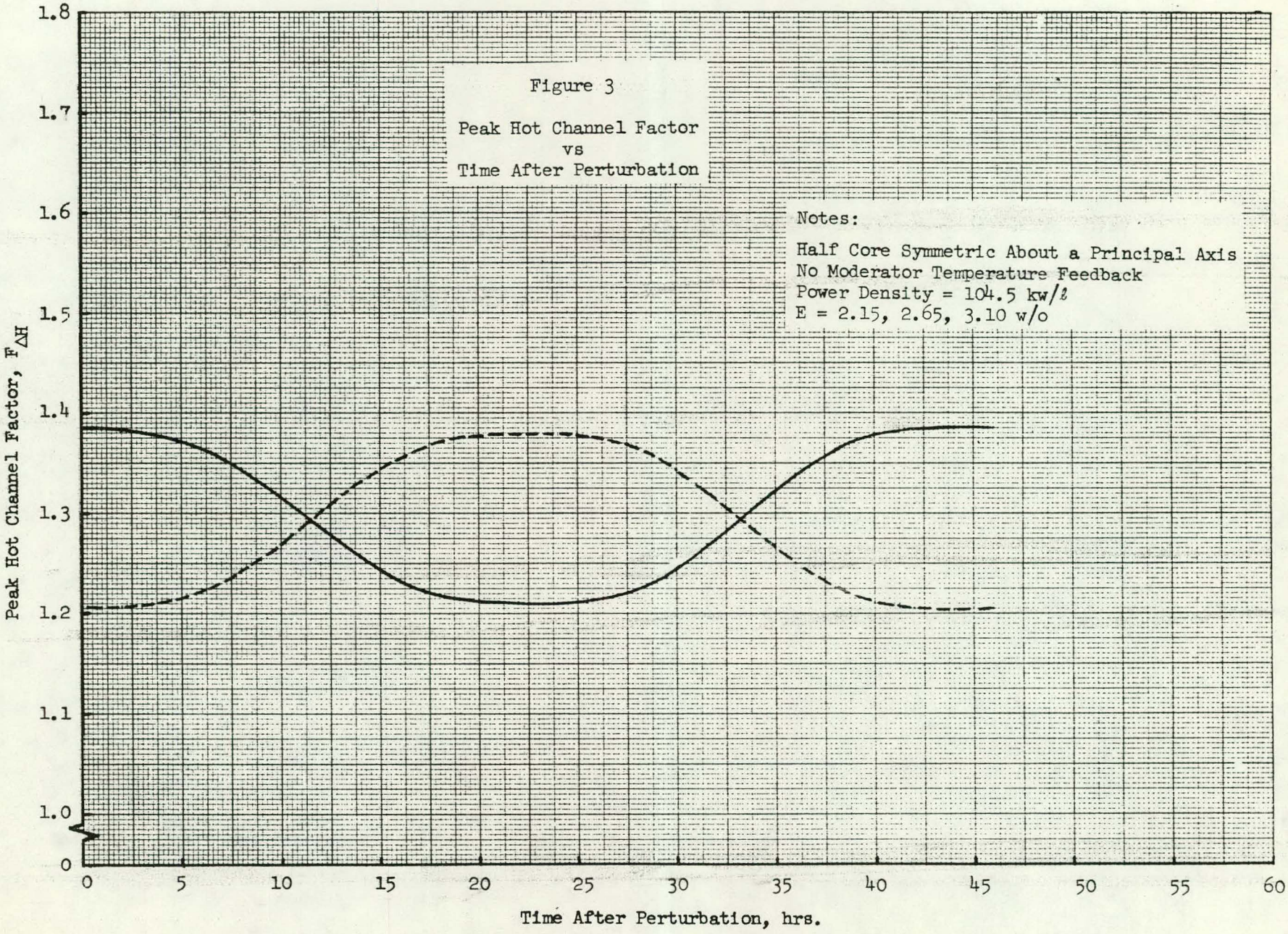


Figure 3
Peak Hot Channel Factor
vs
Time After Perturbation
Notes:
Half Core Symmetric About a Principal Axis
No Moderator Temperature Feedback
Power Density = 104.5 kw/ℓ
E = 2.15, 2.65, 3.10 w/o
Peak Hot Channel Factor, $F_{\Delta H}$
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
0
5
10
15
20
25
30
35
40
45
50
55
60
Time After Perturbation, hrs.

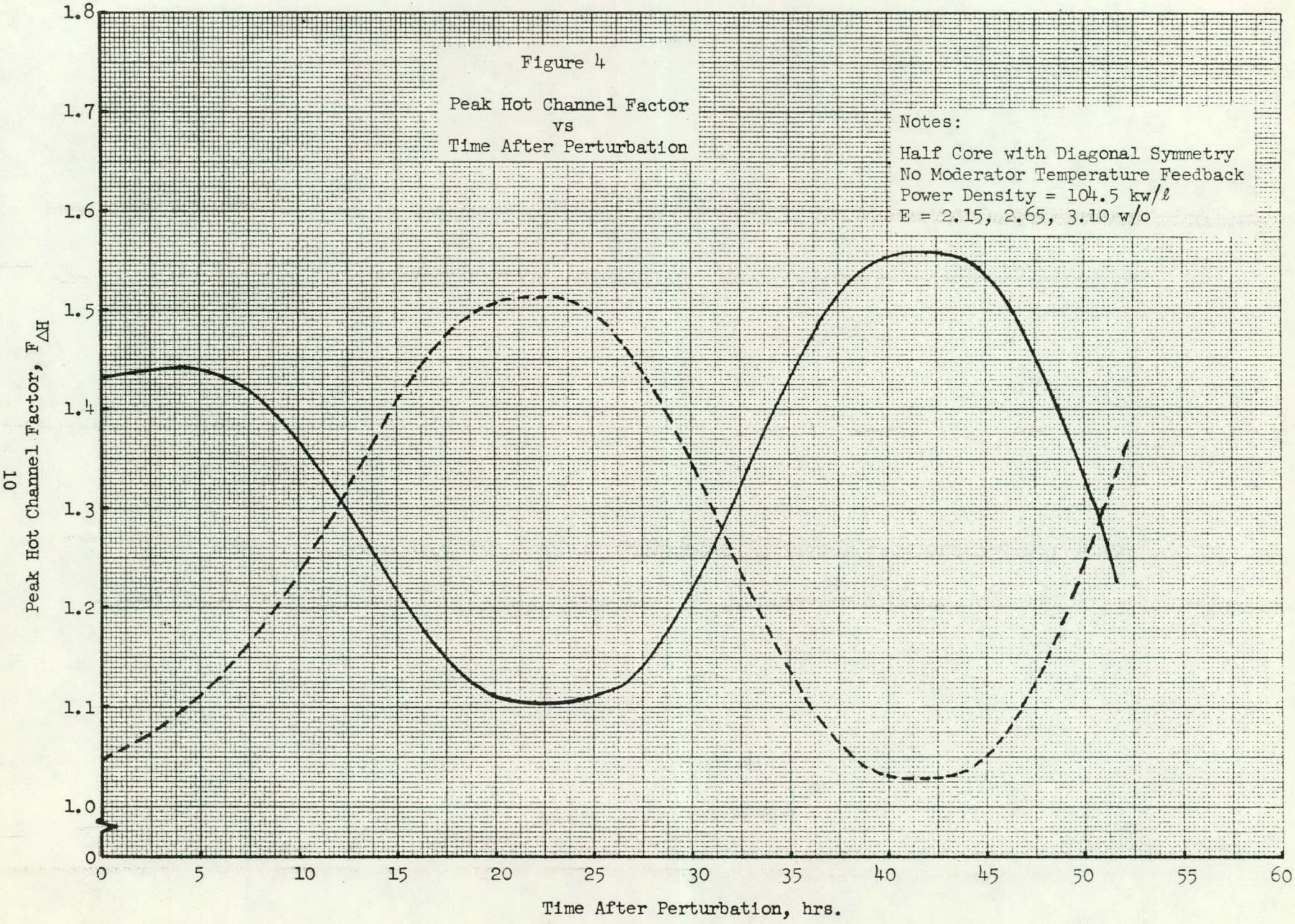


Figure 4
Peak Hot Channel Factor
vs
Time After Perturbation
Notes:
Half Core with Diagonal Symmetry
No Moderator Temperature Feedback
Power Density = 104.5 kw/ℓ
E = 2.15, 2.65, 3.10 w/o
Peak Hot Channel Factor, $F_{\Delta H}$
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
0
5
10
15
20
25
30
35
40
45
50
55
60
Time After Perturbation, hrs.

clearly more divergent. Consequently, future work will concentrate on this core model as a basis for azimuthal xenon control studies.

With reference to WCAP-3680-20 (EURAEC-1974)[1] the stability index is defined as

$$b = \frac{1}{t_2 - t_1} \ln \frac{\partial \phi_2}{\partial \phi_1}$$

where $t_2 - t_1$ = time difference between two successive peaks,

$$\partial \phi_1 = \frac{\phi_{max} - \phi_{min}}{0.5\ (\phi_{max} + \phi_{min})}$$

where ϕ_{max} and ϕ_{min} are the maximum and minimum values of the power at time t_1. Using this definition, the stability indices for the half core with symmetry about the principal axis and half core with diagonal symmetry were calculated to be +0.0039 and +0.014, respectively. The indices have not been corrected to zero time-step length.

Two-dimensional x-y calculations are in progress where different concepts for controlling azimuthal xenon-induced oscillations are being explored.

SECTION 6

EUXE-400

THREE-DIMENSIONAL ANALYSIS

G. H. Minton, Manager — Reactor Physics and Mathematics Section

J. E. Olhoeft, Manager — Reactor Physics

C. G. Poncelet

A. M. Christie

1. INTRODUCTION

In preparation for performing digital simulation calculations in three dimensions for xenon stability studies, work has been initiated to develop a technique for pointwise flux-level extrapolation in time appropriate to diffusion theory calculations. A technique of this nature is necessary to reduce the errors due to finite time-steps that were investigated and reported[1] under Task EUXE-200. Furthermore, it is important for full 3-D analysis because of the anticipated requirement for long execution times on the computer to follow xenon oscillations and, accordingly, the desirability of running with long time-step lengths.

In the numerical solution of the neutron diffusion equation coupled to the xenon and iodine equations, the question of time-step length over which the xenon and iodine chains are depleted must be carefully examined. It has been found that considerable error in the growth rate of the spatial oscillations due to xenon feedback may be generated if no account is taken of the fact that the flux used in the calculation changes substantially between time steps. The magnitude of the error in the growth rate made by assuming that the flux keeps its initial value across a time-step when the depletion of xenon is calculated can been seen in Figure 5.[1] Several approaches to the solution of this problem for uncontrolled oscillations have been studied and some of these are discussed briefly below.

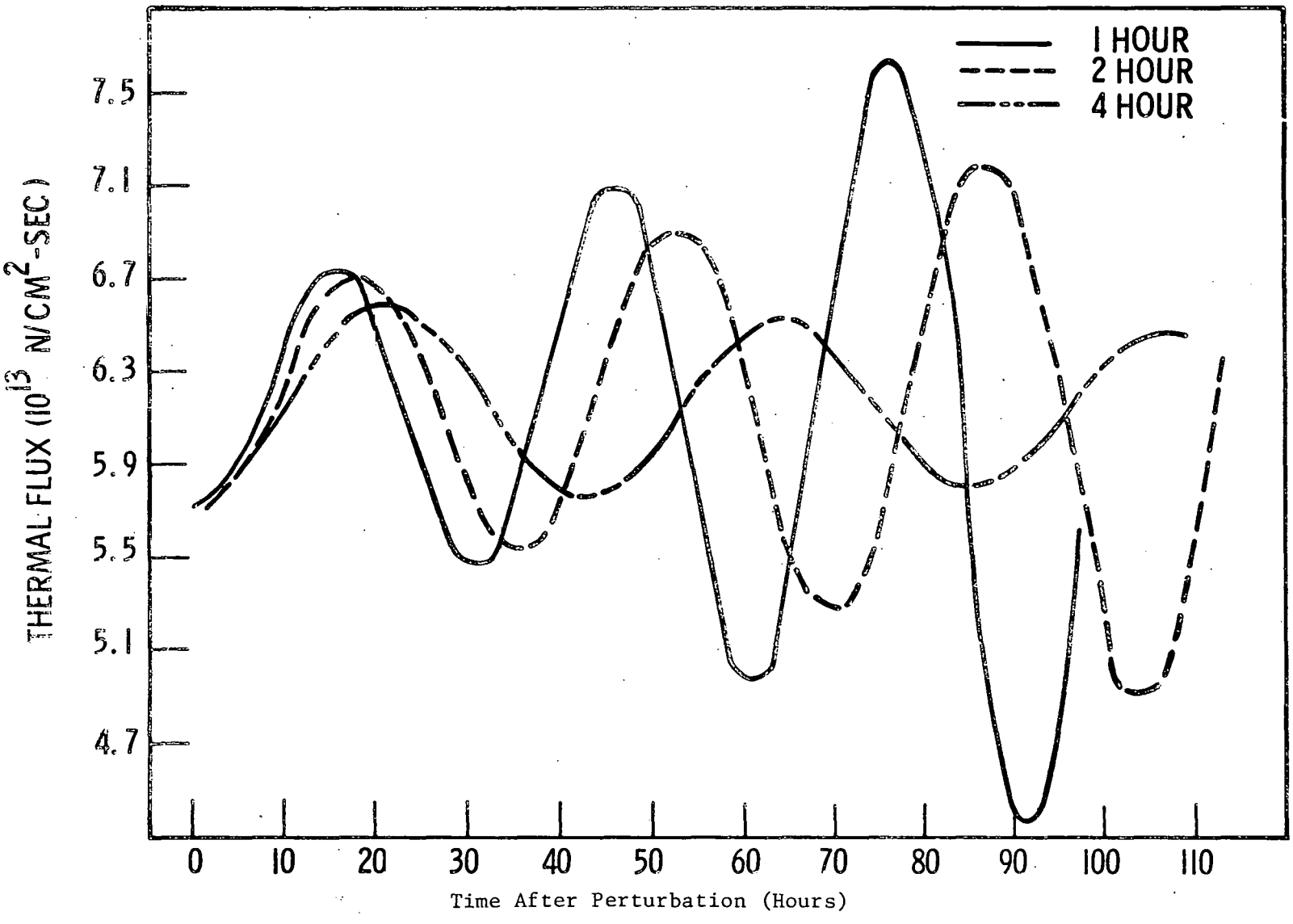


Figure 5. Flux Change with Time Steps

The problem can be solved if the diffusion and xenon equations are first linearized. A correction to the growth rate of the oscillations can then be derived which will modify the apparent growth rate to give a more accurate estimate. This approach is the one taken by B. Davison.[2] It is assumed that the oscillations are of one exponential mode and that the true flux can be written in the form

$$\phi(\underline{r},t) = \exp(\omega t)\ \psi(\underline{r}) \tag{1}$$

A similar expression is assumed for the flux in a calculation with a finite time-step, τ, the only difference being that ω is replaced by ω_τ. To a considerable degree of accuracy the relationship between ω and ω_τ is a function of only the time-step length and the reactor power level.[1] However, this analysis is only relevant to free-running oscillations and not to controlled oscillations, since ω can only be found once ω_τ is known.

A simpler numerical approach can be taken in which the flux at several previous time-steps is used to calculate the fit:

$$\phi(t) = \sum_{i=0}^{N} a_i t^i \tag{2}$$

This time-dependent flux is then substituted into the xenon and iodine equations and the resulting two equations are solved numerically using an integration scheme such as Runge-Kutta. This method was investigated by R. M. Pearch and R. E. Roth[3] and is apparently unstable for a combination of long time step and high flux. It also contains an inherent error in that a fit is not possible for the first few time steps.

Another approach to the problem is to deplete the xenon with a constant flux which is averaged in such a way as to give the same result as the true time-varying flux. The solution of the xenon and iodine equations would then be of the standard form. However, calculation of the effective flux, $\bar{\phi}$, with which to deplete the xenon and iodine is not a trivial problem. It can be approximated by averaging the fitted flux given by equation (2)[1,4]

$$\bar{\phi} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left(\sum_{i=0}^{N} a_i t^i \right) dt \qquad (3)$$

While it is doubtful that this approach will be much faster than that of Pearce and Roth,[3] it will not, however, suffer from numerical instability as does their method.

The final method, and the one to be dealt with in some detail in this report, is one in which the higher time derivatives of the flux are calculated at one time-step and used to deplete the xenon over a time-step to a greater degree of accuracy than by either of the two previous methods.[5] Intuitively, if at one point in time the variables and describing equations are known, it should be possible to determine the higher derivatives of these variables. The advantage of this approach lies in the fact that the method can be used in control-type calculations where the flux essentially exhibits discontinuities in time. In none of the previous methods can the appropriate adjustments be made in this kind of study.

An additional advantage accrued in the above approach is that when using the flux derivatives, a good source guess for the next time-step can be calculated, thus considerably reducing the number of iterations required for convergence. This is also true of the flux extrapolation technique described using equation (2).

2. DERIVATION OF FLUX DERIVATIVES FOR EXPANSION OF FLUX

The neutron diffusion equation in one-group form and the associated xenon and iodine equations can be written as:

$$-\nabla \cdot D\nabla\phi + \Sigma_a \phi + (X\sigma^X\phi) = \frac{\nu\Sigma_f}{\lambda} \phi \qquad (5)$$

$$\frac{\partial X}{\partial t} = - \lambda_X X + \lambda_I I + (y_X \Sigma_f - \sigma^X X)\ \phi \qquad (6)$$

$$\frac{\partial I}{\partial t} = - \lambda_I I + y_I \Sigma_f \phi \tag{7}$$

where

X is the xenon number density
I is the iodine number density
λ_X is the xenon decay constant
λ_I is the iodine decay constant
y_X is the direct xenon yield
y_I is the iodine yield

The rest of the notation is standard.

The variables may be written as

$$\begin{aligned} \phi(\underline{r},t) &= \phi_0(\underline{r}) + \delta\phi(\underline{r},t) \\ X(\underline{r},t) &= X_0(\underline{r}) + \delta X(\underline{r},t) \\ I(\underline{r},t) &= I_0(\underline{r}) + \delta I(\underline{r},t) \end{aligned} \tag{8}$$

It should be noted that this expansion is not about the fundamental values; rather ϕ_0, X_0 and I_0 represent the values of the variables at one particular time-step. Substituting (8) into (5), (6) and (7), eliminating the parts of the resulting equations which represent the solution of equation (5), (6) and (7) for that time-step, and neglecting higher order non-linear terms, results in

$$-\nabla \cdot D \nabla \delta\phi + \Sigma_a \delta\phi + X_0 \sigma^X \delta\phi - \frac{\nu\Sigma_f}{\lambda} \delta\phi = -\sigma^X \phi_0 \delta X \tag{9}$$

$$\frac{\partial}{\partial t} \delta X = -(\lambda_X + \sigma^X \phi_0) \delta X + \lambda_I \delta I + (y_X \Sigma_f - \sigma^X X_0) \delta\phi \tag{10}$$

$$\frac{\partial}{\partial t} \delta I = -\lambda_I \delta I + y_I \Sigma_f \delta\phi \tag{11}$$

Temperature feedbacks have been neglected in these equations, but will be discussed in Sub-Section 3 which follows.

Combining terms, equations (9), (10) and (11) can be written as

$$M\delta\phi = -\sigma^{X}\phi_{0}\delta X \tag{12}$$

$$\dot{\delta X} = a\delta X + b\delta I + c\delta\phi \tag{13}$$

$$\dot{\delta I} = d\delta I + e\delta\phi \tag{14}$$

Since (12) through (14) are all linear, one can differentiate (12) n times and (13), (14) (n-1) times to obtain:

$$M\delta\phi^{(n)} = -\sigma^{X}\phi_{0}\delta X^{(n)} \tag{15}$$

$$\delta X^{(n)} = a\delta X^{(n-1)} + b\delta I^{(n-1)} + c\delta\phi^{(n-1)} \tag{16}$$

$$\delta I^{(n)} = d\delta I^{(n-1)} + e\delta\phi^{(n-1)} \tag{17}$$

Using equations (15) through (17) on a recursive basis, it is possible to build up successive derivatives. The procedure to calculate the derivatives would be as follows:

1. At time-step (i) calculate the flux distribution ϕ_0 from previously computed values of X_0 and I_0.
2. Calculate first xenon and iodine derivatives, using equations (6) and (7).
3. Calculate the first flux derivative, using equation (15).
4. Calculate n^{th} xenon derivative from $(n-1)^{th}$ xenon, iodine and flux derivatives from equation (16) ($n \geq 2$).

5. Calculate n^{th} iodine derivative from $(n-1)^{th}$ iodine and flux derivatives from equation (17) ($n \geq 2$).

6. Calculate n^{th} flux derivative, using n^{th} xenon derivative from equation (16) ($n \geq 2$).

7. If flux expansion with n^{th} derivative is not accurate enough, go to to Step 4 above.

Note that the calculation of each derivative is non-iterative, as the problem being solved is inhomogeneous and not an eigenvalue calculation. Hence, the time required to calculate several derivatives is not excessive.

Explicitly, the flux expansion past time-step k is given by

$$\phi(x,t) = \phi(x,t_k) + \sum_{i=1}^{N} \frac{(t-t_k)^i}{i!} \phi^{(n)}(x,t_k) \qquad (18)$$

This is essentially a Taylor expansion of the flux in time and must be performed for each point in the core. The time-dependent flux derived from equation (18) could be averaged using equation (3). However, it is expected that a pure numerical solution using equation (18) would be more accurate.

The analytic solution of the xenon and iodine equations also involves the computation of exponentials. These require considerable computer time, while the numerical solution does not. It is anticipated, therefore, that the numerical integration of the xenon and iodine equations over the time step being considered will be competitive in computer time to the analytical solution using an average flux. Since the flux at time-step (n+1) will be known to a considerable degree of accuracy, the source at that time can be easily computed. This is then used as the source-guess for the $(n+1)^{th}$ time-step.

It can be seen that in this analysis, no perturbation was made to the fundamental eigenvalue in equation (5). For xenon oscillations, this assumption is valid to a considerable degree of accuracy. The main source of error is likely to lie in the linearization of the equations; therefore, for large oscillations in the non-linear range, this error may not be negligible. A

qualitative estimate of this error can be determined from the number of iterations required to converge on the solution. This is a reflection on the accuracy of the extrapolated source guess.

3. TREATMENT OF FEEDBACK EFFECTS

In the foregoing discussion, temperature feedback effects have been ignored. One may allow some arbitrary cross section Σ (absorption, removal, etc.) to be a function of fuel temperature, T_f, and moderator density, ρ_m. Then, explicitly,

$$\Sigma = \Sigma(T_f, \rho_m) \tag{19}$$

Feedback effects can be included in a manner similar to sub-section 2 above by representing the cross section as a constant value corresponding to the beginning of a time step, Σ_o, and a time-dependent term, $\delta\Sigma$, as was done in equation (8). Thus substitution of these terms in the diffusion equation (5) and linearizing results in an equation similar to equation (9), but with allowance for feedback. Following the procedure described in sub-section 2, derivation of equations involving higher derivatives could be obtained, but because of the explicit dependence of the cross sections on fuel temperature and moderator density, very complex expressions would be expected. More simplified expressions may be obtained by representing the cross sections as polynomials in flux at each spatial point. However, care must be exercised to allow a distinction in directional dependency normal or parallel to flow. This complication follows from the fact that the variation of moderator density colinear with the direction of coolant flow is dependent upon the integral of the power in the coolant channel. Because of these complications, further investigation will be conducted on the treatment of feedback effects.

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