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LASER IMPLOSION OF DT TO DENSITIES > 1000 g/cm^3: OPTIMISM PULSE SHAPE; FUSION YIELD VS LASER ENERGY*

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Introduction: Laser Implosion Scheme[1]

In laser compression of DT to densities greater than 1000 g/cm^3 followed by thermonuclear ignition and efficient burn, the optimum pulse shape is determined by considerations of entropy and Fermi-degeneracy, hydrodynamics and Rayleigh-Taylor instability, and thermonuclear ignition and self-heating. The required implosion symmetry is achieved by irradiating the pellet from all sides by multiple laser beams as well as by electron transport in the atmosphere ablated from the pellet. Taylor instability is suppressed by sufficiently rapid implocion as well as by generating the implosion pressure by subsonic ablation driven by diffusive electron transport. The thermonuclear yield is determined by considerations of inertia, $\overline{\sigma V}$, density, depletion, thermonuclear self-heating and ignition, degeneracy, implosion efficiency, and laser energy.

In a typical implosion a mm size spherical pellet of liquid or solid DT[†] located in a vacuum chamber is irradiated by a laser prepulse to generate a low density atmosphere extending to several pellet radii.

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[1]Nuckolls, J., Wood, L., Thiessen, A., and Zimmerman, G., NATURE, 239, 139, 1972.

[†]The pellet may be hollow and may be seeded (~ 0.1 atom percent of high Z material).

*Work performed under the auspices of the U.S. Atomic Energy Commission.

The applied laser power is optimally increased from about 10^{11} to 10^{15} watts in 10-20 ns. The laser light is absorbed in the outer atmosphere via inverse bremsstrahlung or by plasma instabilities, generating hot electrons. These hot electrons transport throughout the atmosphere heating electrons (via electron-electron collisions) to temperatures which increase from one to 10 Kev. The pellet surface is heated and ablated by the hot atmosphere, generating pressures which optimally increase from 10^6 to 10^{11} atmospheres. This many order of magnitude increase in implosion pressure occurs during transit of the initial shock to the center of the pellet, so that the outer part of the unablated portion of the pellet is near isentropically compressed into a spherical shell with density > 100 g/cm^3, while at the same time this shell is inwardly accelerated to velocities greater than 3×10^7 cm/sec. As the internal pressure becomes larger than the ablation pressure, the rapidly converging shell slows down, and is compressed, still near isentropically at sub-degeneracy temperatures, to densities greater than 1000 g/cm^3. At the same time the non-Fermi degenerate central region is compressed by the shell to densities greater than 1000 g/cm^3 and heated to temperatures greater than 10 Kev, initiating thermonuclear burn. The product of density and radius is greater than 1 g/cm^2, so that the DT alpha particles are absorbed, heating the pellet to higher temperatures. The fusion energy is produced in $\lesssim 10^{-11}$ seconds, and typical burn efficiencies are 10-40%.

We briefly consider here the optimum pulse shape and the thermonuclear yield.

Pulse Shape

The optimum laser pulse shape—which maximizes the fusion yield for a given laser energy—satisfies four conditions.

(1) Densities greater than 1000 g/cm^3 are achieved.

(2) The implosion occurs in less than one sonic transit time so that long wavelength Taylor unstable growth is tolerable.

(3) In most of the compressed pellet the electrons are Fermi degenerate. This minimizes the required implosion pressure and laser energy required for compression.

(4) The central core of the compressed pellet—having a radius comparable to the DT alpha range—reaches temperatures of ~ 10 Kev, initiating rapid thermonuclear burn and self-heating. This minimizes the laser energy required for ignition.

These four conditions are satisfied by a pulse shape having the following three properties.

(1) The initial implosion velocity is ~ 1 cm/μs, several times sound speed. This shock is sufficiently weak so that most of the pellet can be compressed to a Fermi-degenerate state, but with convergence is strong enough so that the center is significantly shock heated.

(2) The implosion pressure is increased with time so that the hydrodynamic characteristics coalesce near the central core. This insures isentropic compression of most of the pellet while heating the central region.

(3) The peak implosion velocity is sufficiently large so that densities of 1000 g/cm^3 are reached, and central ignition occurs.

By means of hundreds of implosion/burn computer calculations it has been found that the optimum Lagrangian implosion pressure history is approximately

$$P = P_o\left(\frac{h}{h_o\tau}\right)^S \quad \text{, where}$$

$h = \int_o^R \rho dR$, $\tau = 1 - \frac{t}{t'}$, t' is the collapse time

$S = \frac{2\gamma}{\gamma+1}$, $\gamma = \frac{5}{3}$ for an ideal Fermi gas

$P_o \sim \frac{1}{2} - 1$ Mb, $P_{max} \gtrsim 2 \times 10^6$ Mb

Similarly the optimal laser power history (which generates approximately this pressure history) is approximately

$$\dot{E} = \dot{E}_o\ \tau^{-3/2S}$$

It is not surprising that the exponent in the power equation is 3/2 of that in the pressure equation since in the ablation process power $\sim$ pressure x velocity and velocity is proportional to the square root of pressure.

If the pellets are hollow or seeded, or if electron decoupling† or preheat†† are dominant, these equations are significantly modified.

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†decoupling: electron-electron collision time is too long to support the required implosion pressure.

††preheat: energetic electrons deposit energy inside the compressing fuel making it more difficult to compress, instead of in the surface layer to generate implosion pressures.

No analytic derivation of these equations is known for implosions which satisfy the required conditions. Similar pressure equations--but which do not depend on h, and with the wrong exponent--result from consideration of subsonic spherical implosions[2]. The above pressure equation results from analytic solution of the problem of the subsonic, isentropic compression of a plane (where $h = h_0 = \text{const}$). Initially the plane and spherical solutions must be similar, and the h variation is the simplest generalization from plane to spherical geometry.

Implosion calculations show that the optimal power history can be satisfactorily approximated by a histogram of 5-10 adjoining pulses. This suggests a practical means of generating the pulse shape. Starting with the final pulse with least duration--but highest power and energy--which need not be accurately shaped, the preceeding pulses can be generated with sufficient accuracy with beam splitters, attenuators/amplifiers, and optical paths of various lengths. For example by sending a pulse through a 50/50 beam splitter, two pulses are generated of the initial duration but each with half the initial intensity. One of these pulses may be reamplified to preserve the original pulse. The other pulse may be split again to 1/4 the intensity of the initial pulse. One quarter intensity pulse may then be delayed by a pulse width relative to the other (via unequal optical paths) and recombined with the other quarter intensity pulse to form a single pulse of twice the initial duration and one quarter the original intensity. With amplification (or attenuation) the intensity may be modified as desired. With suitable optical paths, this new double duration pulse may be shifted one original pulse width in time and combined with

[2]Kidder, R., UCRL 74040 (1972).

the original pulse to form a two pulse histogram. These operations may be used to approximate arbitrary pulse shapes with histograms. Since the same input pulse is always amplified to produce the same output pulse, the amplifier may have any reproducible combination of linearity and non-linearity.

Thermonuclear Energy

The fusion energy is proportional to the product of the burn rate and the inertial confinement time. If ϕ is the fractional burn-up,

$$\frac{\phi}{1-\phi} \sim \rho \, \overline{\sigma v} \, \frac{R}{C}$$

where $\frac{1}{1-\phi}$ corrects for depletion, ρ is density, $\overline{\sigma v}$ is the Maxwell averaged reaction rate, R is radius, and C is sound speed. If ion temperatures of $\sim$ 20–50 Kev are reached $\frac{\overline{\sigma v}}{C}$ depends weakly on temperature. Then computer explosion calculations predict that

$$\phi \simeq \frac{R\rho}{5+R\rho}$$

This corresponds to an inertial confinement time of $\sim \frac{R}{4C}$, which is not surprising since in a sphere half the mass is beyond 80% of the radius. The specific fusion energy is $\phi\varepsilon_{TN}$, where ε_{TN} is the energy released by 100% burn-up, $\sim 3 \times 10^{11}$ joules/gram.

At ignition the specific heat energy in the pellet may be written as

$$C_p \theta_{IGN} \beta^{-1}$$

where C_p is the heat capacity, θ_{IGN} is the ignition temperature, and β^{-1} is a correction for self-heating of the DT by the alpha particles. Optimum designs use $\theta_{IGN} \sim$ 10 Kev in order to avoid the relatively long time to self-heat from 5 Kev, the ideal ignition temperature, to 10 Kev. In the highly compressed pellets considered here, β is a function of the DT alpha particle range, $(\rho R)_\alpha$ ($\sim$ 0.25 g/cm^2 at electron temperatures of 8 Kev), and of the ρR of the pellet. If it is assumed that the temperature is θ_{IGN} from the center to $(\rho R)_\alpha$, and falls off as R^{-2} beyond, then it follows that

$$\beta^{-1} = \frac{(\rho R)_\alpha^2}{(\rho R)^2} \left[3-2\frac{(\rho R)_\alpha}{\rho R}\right]$$

Because of probable implosion symmetry limitations, a minimum of 0.03 is imposed on β^{-1}.

At high densities the specific compressional energy of Fermi-degenerate DT is

$$\varepsilon_F\left[\frac{3}{5} + \frac{\pi^2}{4}\left(\frac{\theta}{\varepsilon_F}\right)^2 + \cdot\cdot\cdot\right],\ \theta < \varepsilon_F$$

where ε_F, the Fermi-energy, is

$$\frac{h^2}{8m}\left(\frac{3}{\pi}n_e\right)^{2/3}$$

n_e is the electron density, h is Planck's constant, and m is the electron mass. The specific compressional energy may be written as

$$3 \times 10^5\ \rho^{2/3}\ \phi_\eta \text{ joules/g}$$

where ϕ_η corrects for deviation from an ideal Fermi-gas (and is usually about 2) due mainly to the 1Mb initial shock, and also because the implosion is not quite isentropic.

For simplicity we make the somewhat pessimistic assumption that the ignition energy and compressional energy should be simply added to obtain the total internal energy.

Of the laser light inputted to the pellet, some may reflect, and 90-95% of the energy absorbed is lost via the blowoff from the ablation process. Assume that a fraction, α, of the laser light energy couples to compress and ignite the dense core. Then the overall gain equation is

$$G = \frac{\alpha\epsilon_{TN}\phi}{C_p\theta_I\beta^{-1} + 3 \times 10^5\rho^{2/3}\phi_\eta}$$

where ϕ and β are both functions of ρR. Over a limited range of ρ and ρR this equation may be approximated by a power law, and solved for the laser light energy

$$E \sim \alpha^{-a}\rho^{-b} \text{ (at constant gain)}$$

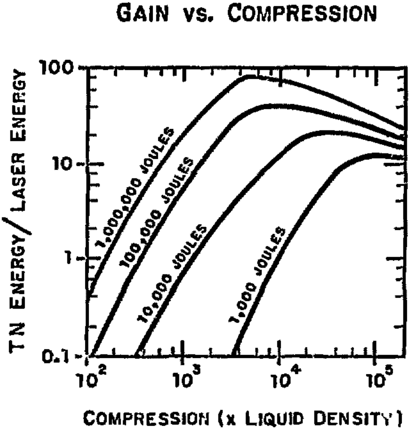
Over a wide range of conditions ($100 < \rho < 1000$, $1/2 < \rho R < 2$) "b" is about 2 and "a" varies from 4/3 to 2, so that the dependence on compression is at least as strong and generally stronger than the dependence on absorption. If depletion, degeneracy, and the dependence of self-heating on compression are ignored, the dependence on absorption is stronger than that on compression[2], i.e. "a" = 4 and "b" = 2.

Figure 1 shows the scaling of this gain equation—modified somewhat to agree with computer calculations—with compression and laser light energy. Gains approaching 100 are predicted for laser energies of 10^6J. At compressions less than $\sim 10^3$, the gain increases strongly with increasing compression because of increasing burn efficiency and thermonuclear self-heating of the fuel. The gain decreases with compressions much greater than 10^4 because of depletion (of the DT) and because ablative energy losses increase and the energy of compression (against degeneracy pressure) becomes dominant.

With a laser efficiency of 10% and a 40% thermal efficiency net electrical energy could be generated with a gain of 25 which occurs for a laser energy less than 10^5 joules (at a compression of 10^4). Less than 1 KJ of laser light may be sufficient to generate an equal thermonuclear energy, if optimally employed.

Hybrid reactors, in which the 14 MeV neutrons which escape from the explosion chamber are used to fission natural uranium or thorium, may generate more energy than used to pump the laser even with low efficiency, low energy lasers, such as 1% efficient, 10 KJ Nd glass lasers.

Fig. 1


GAIN VS. COMPRESSION
TN ENERGY/LASER ENERGY
100
10
1
0.1
10^2
10^3
10^4
10^5
1,000,000 JOULES
100,000 JOULES
10,000 JOULES
1,000 JOULES
COMPRESSION (x LIQUID DENSITY)

1st Slide

LASER FUSION IMPLOSION SCHEME

DT PELLET: Spherical, Hollow?, Seeded? (0.1%)

ATMOSPHERE: Via Laser Prepulse; Symmetry

LASER: Multi-Beam (Symmetry), Pulse Shaped Optimally

ABSORPTION: Inverse Bremsstrahlung, Plasma Instabilities

HOT ELECTRONS: Heat Atmosphere, Symmetry (Transport)

SUBSONIC ABLATION: Implosion Pressure, Taylor Instability

IMPLOSION: > 1000 g/cm^3; Isentropic; Fermi-degenerate

IGNITION: Central, Self-Heating

BURN: $\sim$ 10 ps; 10-40% Efficiency

2nd Slide

OPTIMUM PULSE SHAPE

CONDITIONS

DENSITY > 1000 g/cm^3

SUPERSONIC (Initially) - Taylor Instability

FERMI-DEGENERATE-Most of Mass

IGNITION-Central Region

PROPERTIES

VELOCITY 10^6 cm/s initially, 3 x 10^7 cm/s max

CHARACTERISTICS-Coalesce in Central Region

EQUATIONS

$$P = P_0 \left(\frac{h}{h_0 \tau}\right)^s, \quad h = \int_0^R \rho dR, \quad \tau = 1 - \frac{t}{t^*},$$

t^* collapse time

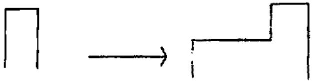
$$s = \frac{2\gamma}{\gamma+1}, \quad \gamma = \frac{5}{3} \text{ (Fermi-gas)}$$

$$P_0 \simeq \frac{1}{2} - 1 \text{ mb}, \quad P_{max} \gtrsim 2 \times 10^4 \text{ mb}$$

$$\dot{E} = \dot{E}_0 \, \tau^{-3/2s}$$

3rd SLIDE

GENERATION OF PULSE SHAPE



$$[I,\ \Delta t,\ t]_1 \xrightarrow{\text{Splitter}} [\tfrac{1}{2} I,\ \Delta t,\ t + a]_{2,3}$$

$$[\qquad]_2 \xrightarrow{\text{Amplifier}} [I,\ \Delta t,\ t + b]_4$$

$$[\qquad]_3 \xrightarrow[\text{Unequal Optical Paths}]{\text{Splitter}} [\tfrac{1}{4} I,\ \Delta t,\ t + c]_5\ ,\ [\tfrac{1}{4} I,\ \Delta t,\ t + c + \Delta t]_6$$

$$[\qquad]_5 + [\qquad]_6 \xrightarrow{\text{Combine}} [\tfrac{1}{4} I,\ 2\Delta t,\ t + d]_7$$

$$[\qquad]_7 \xrightarrow[\text{Optical Path}]{\text{Amplify}} [\tfrac{1}{2} I,\ 2\Delta t,\ t + b - \Delta t]_8$$

$$[\qquad]_8 + [\qquad]_4 \xrightarrow{\text{Combine}} [\tfrac{1}{2} I,\ 2\Delta t,\ t' - 2\Delta t;\ I,\ \Delta t,\ t']_9$$

4th SLIDE

THERMONUCLEAR ENERGY

BURN: $\phi \sim \rho\, \overline{\sigma v}\, \frac{R}{4c} \simeq \frac{R\rho}{6+R\rho}$

IGNITION: $C_p\, \theta_{IGN}\beta^{-1}$; $\beta^{-1} \simeq \frac{(\rho R)_\alpha^2}{(\rho R)^2}\left[3-2\frac{(\rho R)_\alpha}{\rho R}\right]$

COMPRESSION: $\epsilon_F\left[\frac{3}{5} + \frac{\pi^2}{4}\left(\frac{\theta}{\epsilon_F}\right)^2 + \cdots\right] \simeq 3 \times 10^5 \rho^{2/3} \phi_\eta$ J/g

COUPLING: $\alpha \sim 0.05$, Ablation; Reflection?

GAIN: $\text{Gain} \sim \frac{\text{Coupling} \times \text{Burn}}{\text{Ignition} + \text{Compression}}$