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A Tool to Identify Parameter Errors in Finite Element Models

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ABSTRACT

A popular method for updating finite element models with modal test data utilizes optimization of the model based on design sensitivities. The attractive feature of this technique is that it allows some estimate and update of the physical parameters affecting the hardware dynamics. Two difficulties are knowing which physical parameters are important and which of those important parameters are in error. If this is known, the updating process is simply running through the mechanics of the optimization. Most models of real systems have a myriad of parameters. This paper discusses an implementation of a tool which uses the model and test data together to discover which parameters are most important and most in error. Some insight about the validity of the model form may also be obtained. Experience gained from applications to complex models will be shared.

NOMENCLATURE

FEM	Finite Element Model
S	Matrix of sensitivities of frequencies to parameters
$\Delta\bar{\mathbf{p}}$	Vector of predicted changes in parameters to update the model
$\Delta\bar{\mathbf{f}}$	Vector of differences between test frequencies and model frequencies
$\mathbf{W}_f$	Weight matrix applied to frequency vector
$\mathbf{W}_p$	Weight matrix applied to reduce parameter changes
$COV\Delta\bar{\mathbf{p}}$	Covariance matrix for parameters
m	Number of frequencies
n	Number of parameters
PESTDY	Parameter Estimation for Structural Dynamics
SSP	Statistically Significant Parameters
σ_f^2	Variance of the $\Delta\bar{\mathbf{f}}$ vector calculated as $\Delta\bar{\mathbf{f}}^T\Delta\bar{\mathbf{f}}/(m-n)$
STD	Standard Deviation

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INTRODUCTION AND MOTIVATION

The IMAC has as one of its major focus technologies the reconciliation of Finite Element Models (FEMs) with results from modal test data. Typically, the strongest element of the reconciliation is the attempt to match (or at least reconcile) the frequencies of the FEM with the modal test frequencies. One approach is to utilize the design sensitivities of particular physical parameters of the FEM to predict how much the parameters should be changed to enable the FEM to more closely match the test frequencies[1]. This approach is popular because it allows the analyst to develop physical insight to the hardware, which may be of significant value for future design changes, particularly at the prototype stage. The problem can be cast into a linear formulation to be solved with the least squares approach

$$\mathbf{S}\Delta\bar{\mathbf{p}} = \Delta\bar{\mathbf{f}} \qquad (1)$$

where **S** is the matrix of sensitivities of the frequencies to the parameters, $\Delta\bar{\mathbf{p}}$ is the vector of changes required to the parameters and $\Delta\bar{\mathbf{f}}$ is the vector of differences in the FEM and test frequencies. To get a unique solution, $\Delta\bar{\mathbf{p}}$ must be shorter than $\Delta\bar{\mathbf{f}}$ so that the system of equations is overdetermined. (Also, S must be of full column rank). In a Bayesian formulation, the analyst can assign weights to the various frequencies and weights to the parameters.

$$[\mathbf{S}^T\mathbf{W}_f\mathbf{S}+\mathbf{W}_p]\Delta\bar{\mathbf{p}} = \mathbf{S}^T\mathbf{W}_f\Delta\bar{\mathbf{f}} \qquad (2)$$

Weights on the frequencies, $\mathbf{W}_f$, increase the effort of the solution to reduce more highly weighted frequency errors, and weights on the parameters, $\mathbf{W}_p$, make the more highly weighted parameters resistant to change. These matrices are usually square and diagonal. A major point of the philosophy of the following approach is that we desire the model to tell us as much as possible about itself, so we do not include parameter weights, $\mathbf{W}_p$, in the analysis for this work. (Parameter weights are usually derived by

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the analyst's judgment and can significantly bias the outcome of the analysis. The analyst can apply judgment about the parameter weights *after* the parameter selection analysis is complete). We do allow frequency weights, since the analyst may be much more interested in the model matching certain frequencies than others. The basic starting equation for this work is then

$$\mathbf{W}_f \mathbf{S} \Delta \bar{\mathbf{p}} = \mathbf{W}_f \Delta \bar{\mathbf{f}} \quad (3)$$

which is still of the standard form $\mathbf{A}\bar{\mathbf{x}} = \bar{\mathbf{b}}$. For a well-conditioned least squares solution of $\mathbf{A}\bar{\mathbf{x}} = \bar{\mathbf{b}}$, we like for vector **b** to be very long (corresponding to many frequencies being compared between the FEM and test) and for vector **x** to be comparatively short (i.e. a relatively small number of parameters). Generally there is not a lot of control over how many frequencies are being compared, so we would like to solve for as few parameter changes as possible. There are a myriad of parameters in most complex finite element models, so this raises the question: Which parameters are both important and in error? If a parameter is not important, but in error, it makes little difference in the final solution. If a parameter is important, but not in error, then there is no need to change it in the FEM, and therefore no need to include it in the parameter change vector. So how can we determine which parameters in the FEM are both important to the solution *and* in error? With some assumptions, there is a way to approach this using statistics.

THE STANDARD DEVIATION OF THE MEAN OF EACH PARAMETER

First we make an assumption that will be evaluated later. Assume that the FEM is the best possible fit of the model form selected, so that any errors in $\Delta\bar{\mathbf{f}}$ are because of random measurement errors or model form problems. Then a standard deviation (STD) can be calculated for the differences in frequencies. This STD can be related through equation (3) to give a STD of the mean of each parameter included in $\Delta\bar{\mathbf{p}}$. The math is shown here, but the important point is the relationship between the STD of the frequencies and the STD of the parameter means in equations (9) and (10). Premultiplying equation (3) by $\mathbf{S}^T$ yields

$$\mathbf{S}^T \mathbf{W}_f \mathbf{S} \Delta\bar{\mathbf{p}} = \mathbf{S}^T \mathbf{W}_f \Delta\bar{\mathbf{f}} . \quad (4)$$

Solving for $\Delta\bar{\mathbf{p}}$ gives

$$\Delta\bar{\mathbf{p}} = [\mathbf{S}^T \mathbf{W}_f \mathbf{S}]^{-1} \mathbf{S}^T \mathbf{W}_f \Delta\bar{\mathbf{f}} . \quad (5)$$

Now postmultiply by $\Delta\bar{\mathbf{p}}^T$.

$$\Delta\bar{\mathbf{p}}\Delta\bar{\mathbf{p}}^T = [\mathbf{S}^T \mathbf{W}_f \mathbf{S}]^{-1} \mathbf{S}^T \mathbf{W}_f \Delta\bar{\mathbf{f}}[[\mathbf{S}^T \mathbf{W}_f \mathbf{S}]^{-1} \mathbf{S}^T \mathbf{W}_f \Delta\bar{\mathbf{f}}]^T \quad (6)$$

$$\Delta\bar{\mathbf{p}}\Delta\bar{\mathbf{p}}^T = [\mathbf{S}^T \mathbf{W}_f \mathbf{S}]^{-1} \mathbf{S}^T \mathbf{W}_f \Delta\bar{\mathbf{f}}\Delta\bar{\mathbf{f}}^T \mathbf{W}_f^T \mathbf{S}[\mathbf{S}^T \mathbf{W}_f \mathbf{S}]^{-T} \quad (7)$$

Take the expected value of (7), and assume the expected value of $\mathbf{W}_f \Delta\bar{\mathbf{f}}\Delta\bar{\mathbf{f}}^T$ becomes the weighted STD of the frequency error vector times the identity matrix. The expected value of $\Delta\bar{\mathbf{p}}\Delta\bar{\mathbf{p}}^T$ is then the estimated covariance matrix for the parameters.

$$COV\Delta\bar{\mathbf{p}} = [\mathbf{S}^T \mathbf{W}_f \mathbf{S}]^{-1} \mathbf{S}^T \sigma_f^2 \mathbf{I} \mathbf{W}_f^T \mathbf{S}[\mathbf{S}^T \mathbf{W}_f \mathbf{S}]^{-T} \quad (8)$$

Because the bracketed matrices are symmetric (8) reduces to

$$COV\Delta\bar{\mathbf{p}} = \sigma_f^2 [\mathbf{S}^T \mathbf{W}_f \mathbf{S}]^{-1} . \quad (9)$$

The diagonals of the covariance matrix of $\Delta\bar{\mathbf{p}}$ are the variances of the parameters. Reference [2] provides another concise explanation of the derivation of the covariance matrix (without the weighting matrix). The STD of the <u>mean</u> of each parameter is just the square root of each variance divided by m where m is the number of rows in equation (2). So the estimated vector of STDs of the means of the parameters is

$$\bar{\sigma}_{mean}(\Delta \mathbf{p}) = sqrt(\mathbf{diag}(COV\Delta\bar{\mathbf{p}})/m) \quad (10)$$

In equation (10) the frequencies as a group are most sensitive to changes in the parameters with low STDs of the mean. From equation (9) it can be seen that in general the inverse of a large sensitivity number will give a low variance and resulting STD. This does not give any information about which parameters may be in error. So how do we use this to determine which parameters are both important and in error in our FEM?

DETERMINING THE IMPORTANT PARAMETERS USING THE STD OF THE PARAMETER MEAN AS A BASIS

Now we calculate the actual changes in the parameters from equation (5). Remember that we made an assumption that the FEM was the best fit of the model to the data in order to calculate the STD of the parameters above. If this was a good assumption, we would expect that the change in the parameters from equation (5) would not deviate "very far" from the initial parameter values. How much is "very far" can be determined statistically using the results from equation (10). A typical value chosen to represent "very far" by statisticians is 2 STD of the mean of the parameters. If the FEM really was a best fit, then there should be about a 95% probability that equation (5) would not yield any parameter changes that were more than 2 STD of the mean away from the initial value for each parameter. Therefore, if equation (5) predicts changes greater than 2 STD of the mean away from the parameter, the assumption that the FEM is a best fit is an *erroneous assumption* and such parameters should be changed. The relative importance of these parameters is calculated with a z-score, which is the value of each parameter change divided by that parameter's STD of the mean.

$$z = \Delta p / \sigma_{mean}(\Delta p) \quad (11)$$

So the z-score is just the number of STDs of the mean which equation (5) predicts the parameter should be adjusted. If the z-scores for all parameters are low, then the FEM is as good as it can reasonably be with the parameters chosen for the calculation in (5) and (10) above. If this is not acceptable to the analyst, then other parameters must be chosen which are more important, or the FEM model form must be changed to be more representative of reality. This is very important information.

At this point the analyst can make key decisions to answer the following questions. Should I continue with the model reconciliation utilizing some or all of these parameters? Should I go back and select other parameters because these do not appear to be important? Should I change the FEM form to make it more representative of reality? Answering these questions appropriately can save a lot of wasted effort on a poor model, or on FEM runs utilizing many unimportant or wrong parameters. The value of this type of analysis, sometimes called analysis of variance, is that with one number, the z-score, the analyst can see which parameters are both important and in error.

An understandable explanation of the basic concepts explained above, sometimes called "tests of significance", is given in reference [3]. A more thorough explanation of estimation of multiple parameters for engineers is provided by Benjamin and Cornell [4]. At previous IMACs, others have presented more complete work on analysis of variance [5] and Bayesian estimation techniques to determine weighting matrices [6].

SYSTEMATIC IMPLEMENTATION

At Sandia National Laboratories, a code with the acronym PESTDY (Parameter Estimation for STructural DYnamics) implements the Bayesian Estimation in a MATLAB-based set of routines. The systematic approach to determine the important parameters is implemented in a module called SSP (Statistically Significant Parameters) as follows. First equation (1) is normalized so that the $\Delta\bar{\mathbf{p}}$ changes will be fractional changes required to be applied to each initial parameter value. The $\Delta\bar{\mathbf{f}}$ values are differences between the model and test frequencies divided by the initial model frequencies. Then sensitivity matrices, **S**, are calculated for every parameter that is uncertain and possibly important. Frequency weights, $\mathbf{W}_f$, are set based on the analyst's interest in certain frequencies. If there are no specifically important frequencies, the identity matrix is used. Then the analysis described above is run for one parameter, that is, equations (5) and (10) are solved where the parameter vector has a length of one. The results are saved and the analysis is run for the next parameter. This proceeds until all parameters have been analyzed. The parameter with the highest z-score is then declared the most important parameter *and is used for the rest of the analysis.* The same process is repeated for a parameter vector length of two utilizing the most important parameter in combination with every other parameter. Two z-scores are produced for every set of two parameters. The lower of the two z-scores is retained from every set. The set with the largest low z-score determines the second most important parameter, and this parameter is used for the rest of the analysis. This process is repeated with sets of three parameters and so on until the largest low z-score is below a user specified value, typically two. When the low z-scores get below this value, the model is telling the analyst that little additional improvement to the model can be obtained with additional parameters, i.e. all the important parameters have been identified.

ADDITIONAL TOOLS HELPFUL FOR PARAMETER SELECTION

In addition to the approach for calculating z-scores, a plot of the predicted final frequency STD is generated vs the number of parameters used in the analysis based on the z-score selection technique described above. In some instances it has been found that an obvious knee in the curve shows that the number of parameters could be limited to less than those selected using a criterion of a low z-score of two. Next, a calculation is made to determine which parameters have highly correlated effects with those that are chosen to be important from the z-score analyses. The correlation calculation from Branham [2] is given from calculations on the **S** matrix with only the two columns of interest.

$$\mathbf{b} = [\mathbf{S}^T \mathbf{S}]^{-1} \tag{12}$$

$$\mathbf{c} = \begin{bmatrix} 1/\sqrt{\mathbf{b}(1,1)} & 0 \\ 0 & 1/\sqrt{\mathbf{b}(2,2)} \end{bmatrix} \tag{13}$$

$$\mathbf{cor} = \mathbf{c} * \mathbf{b} * \mathbf{c} \tag{14}$$

The correlation coefficient is the off diagonal term of the **cor** 2x2 matrix. All possible combinations of two parameters are analyzed. A list of other parameters with correlation coefficients above .8 absolute value is printed with each important parameter from the z-score analyses. Parameters with high correlation values could produce results similar to the important parameters with which they are paired. Another way to state it is that two parameters with a correlation coefficient near 1 have frequency sensitivity vectors that are close to parallel. This is important, since experience has shown that sometimes parameters correlated with those chosen as important from the z-score analyses are actually the major cause of the model's inaccuracies. The analyst then has the option of including these correlated parameters in the model reconciliation with (or in lieu of) the important parameters. A final tool that helps decide on the appropriate parameters is a plot of the sensitivities of all the important parameters. Sometimes this will lead to further culling. In the PESTDY code, at the end of the SSP module, the sensitivity matrix can be trimmed to the sensitivity vectors of the parameters that the analyst has decided are important. Then parameter weights may be applied and the Bayesian estimation completed.

REQUIREMENTS AND LIMITATIONS OF THE APPROACH

The basic requirement is to determine the sensitivity matrix of all possible important parameters. This can be performed fairly automatically in some codes such as MSC/NASTRAN. In other codes a finite difference approach is required where there is a baseline eigenvalue run, and then a run for a small deviation of each parameter. This approach requires *n*+1 eigenvalue solutions of the code where *n* is the number of parameters being considered. Of course, then the frequency differences between the test and initial FEM must be calculated.

There are several important limitations of the approach. The foremost is that a poor FEM may not yield much information, particularly if an important physical phenomenon is not modeled at all. If this is the case, no important parameters may be evidenced in the analysis, or worse, a parameter that is not truly in error, but helps to change a frequency with large error, may be identified as important. This is where engineering judgment and other methods to validate parameters, such as simple measurements, are of value. Another limitation is the fact that the analyses cannot distinguish between errors in highly correlated parameters. If these parameters are parallel stiffnesses, a more basic measurement of each of the parallel stiffnesses may be required. If these parameters are not stiffnesses in parallel, sometimes mode shape information is valuable in determining where the major error lies. Finally, these analyses tend to focus on a relatively small number of parameters. (This is a weakness as well as the major strength.) If there are truly many parameters with large errors, this approach focuses on the smallest number of parameters that can be adjusted to remedy the frequency differences. Therefore, one parameter may be correcting for more than its share of the error.

ANALYTICAL EXAMPLE

To demonstrate, let us consider a simple analytical example which is not even a structural dynamics problem, but will make it easy to illustrate the process. We arbitrarily declare that the response of some system is:

$$y=1+x/2+\sin x+\cos x . \qquad (15)$$

We measure the response accurately at 11 equally spaced points between x=0 and 7. Figure 1 shows the x-y plot of this "measured data".

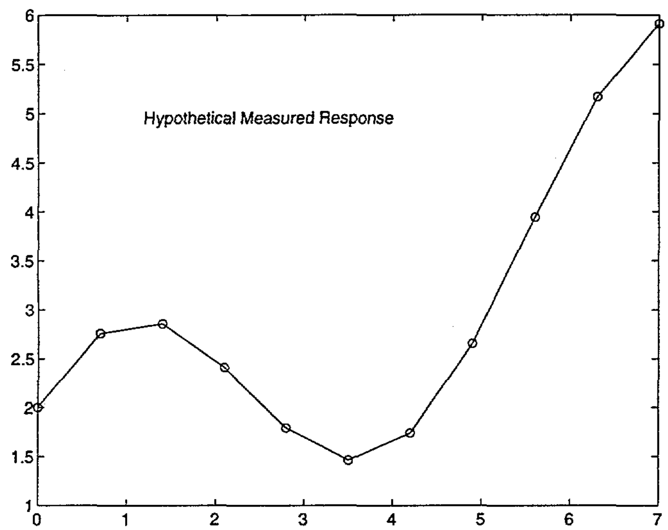

Figure 1 - Accurately "Measured" System Response

We have a model that we have generated to represent this system which is simply the polynomial series

$$y=p_1+p_2x+p_3x^2+p_4x^3+p_5x^4 \qquad (16)$$

where the least squares fit yields the parameters p_1=1.97, p_2=2.61, p_3=-1.94, p_4=.42 and p_5=-.027. This fit of the model response is plotted along with the "measured" system response in figure 2. This comparison simply shows that it is possible with this model form to achieve a reasonable, though not perfect, fit to the measured data.

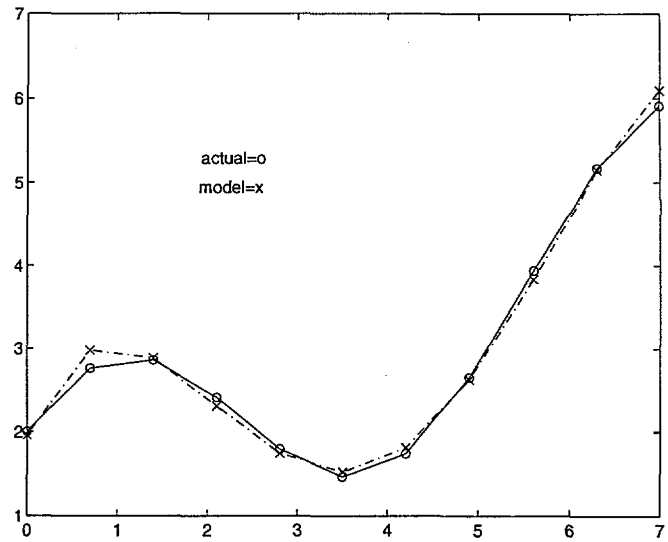

Figure 2 - Best Fit Model Response Compared with "Measured" Response

Now, let us consider the typical situation in which there is some uncertainty about the parameters. For sake of argument suppose that our initial estimate of parameter p_2 is 10% high and parameter p_4 is 10% low. Let us use the approach that has been described to determine which parameters should be most important to update in our system identification code, PESTDY. We assume that we have no advance knowledge of which parameters need to be updated. Figure 3 shows a plot of our initial model compared with the "measured" data. The comparison shows that the model is significantly in error.

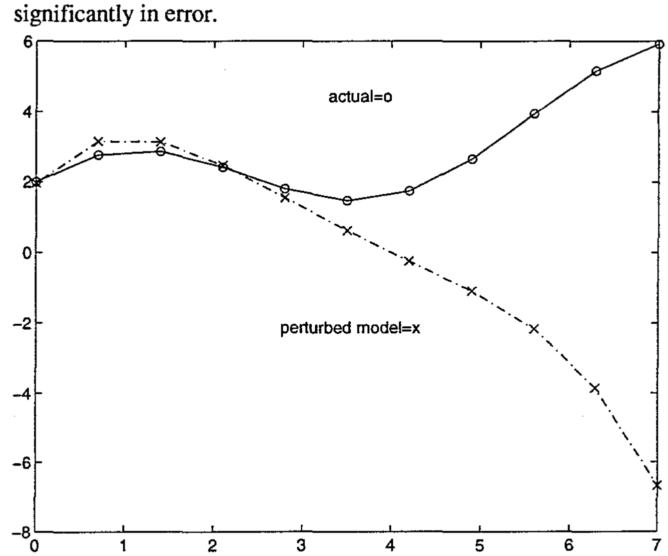

Figure 3 - Initial Model vs. "Measured" Data

The sensitivity of each y with respect to each parameter is generated. We run the PESTDY code to predict updated parameters. In the process the SSP module is run to determine which parameters are important to update. The results of the z-score analysis show that only one parameter survives with a z-score > 2, and that is p_5 with a z-score of 10.5. The SSP module has selected the wrong parameter! (Of course, we do not know this yet). However, that is not all the data provided by SSP. It also shows that p_5 is highly correlated with p_4 with a correlation coefficient of .993. This information shows that it could also be p_4 that is in error, or that it could be a combination of p_4 and p_5. That is all that the analysis can tell us. Let us say that we decided to ignore the correlation and proceed with the solution using only p_5. PESTDY is run and the change for p_5 is calculated as -23.1%. This change is put into the model and the new comparison is shown in figure 4. As a matter of fact, it turns out that updating p_5 by itself provides a better fit than updating p_4 by itself, which is the reason that the SSP module selected p_5 over p_4.

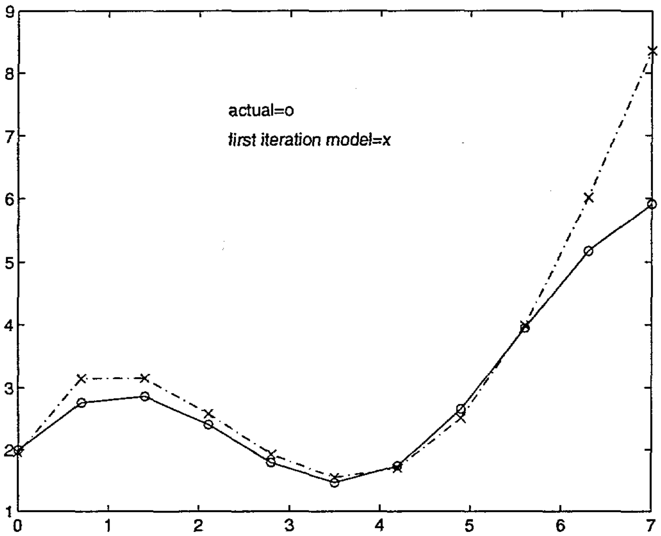

Figure 4 - Comparison of First Iteration Of Updated Model

Now we will proceed with the second iteration. Sensitivities are calculated again, and we execute the PESTDY code. This time the SSP module is run, and the new results show that three parameters survive the z-score > 2 requirement. These are p_5, p_4 and p_2. So now SSP has selected the two parameters that should be changed, p_4 and p_2, in addition to the parameter that we wrongly modified in the first iteration, p_5. In figure 5 the plot of the STD of the difference between the measured response and the model response is plotted vs the number of parameters included in the analysis based on the z-score approach.

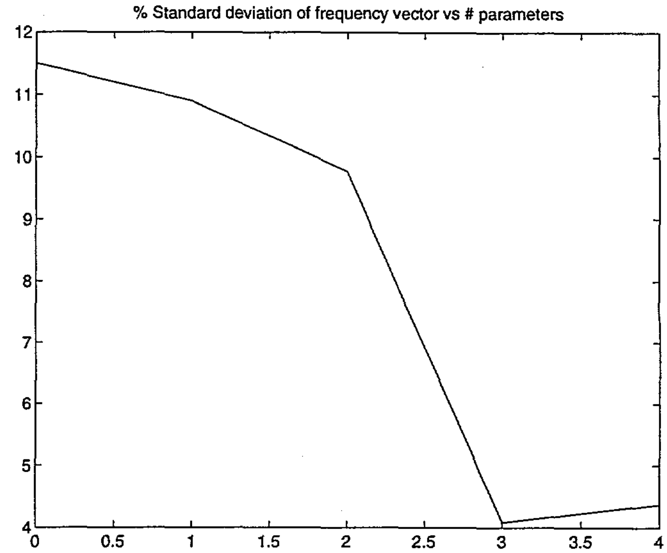

Figure 5 - STD of Predicted Frequencies with Addition of More Parameters Using Z-score Selection Criteria

It can be seen that this plot confirms the z-score analysis, since with the addition of a fourth parameter the STD actually starts to increase. (This can happen because the variance of the responses is the total squared error/(m-n) where m is the number of responses and n is the number of parameters being evaluated. The total squared error may continue to decline with the addition of parameters, but at some point the denominator declines faster).

Figure 6 shows the resulting model response after the parameters are updated the second time. After this update the parameters match the best fit parameters listed immediately after equation (16).

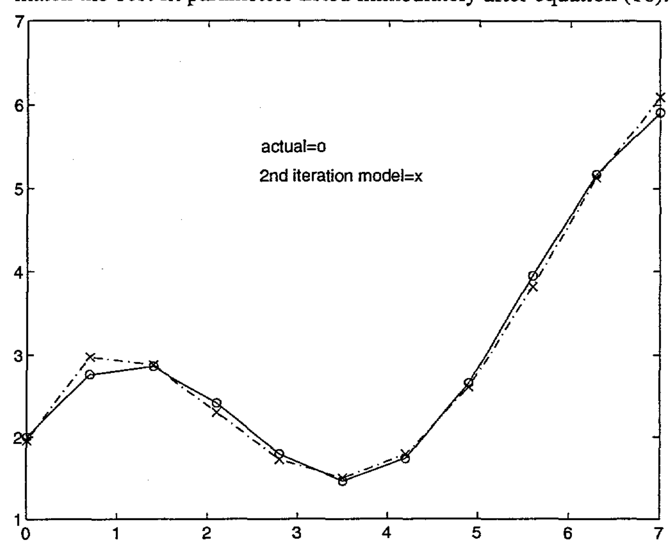

Figure 6 - Comparison of 2nd Iteration Of Updated Model

This example demonstrates two important points. The first point is that as the model becomes more accurate, the z-score approach to parameter selection works better. This is partially because the *linear* estimates of parameter STD are more accurate near the point of best fit of the model to the measured data. The second point is that the method can make wrong decisions and select highly correlated parameters as most important. That is why the calculation of high correlations between the selected parameters and other parameters is valuable information for the analyst.

Now let us pose one more hypothetical scenario using the same data for the "measured" response. Let us suppose that we developed only a cubic polynomial model this time, but we estimated the four parameters perfectly and got a least squares fit to the data. This comparison is shown in figure 7.

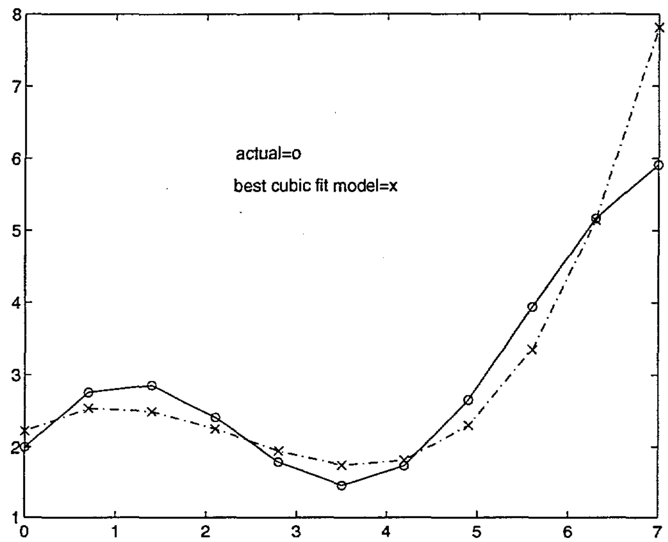

Figure 7 - Best Cubic Polynomial Fit to "Measured" Data

At this point we do not "know" that we have the best fit that this model form can provide, so we run PESTDY to begin the model updating process, since some of the modeled responses are as high as 30% off. The execution of the SSP module yields no parameters with a z-score > 2. This shows that none of the considered parameters are worth the effort to adjust, because the model form does not have the capability to fit the data any better. If the analyst is not satisfied with the model, such information tells the analyst that the model form must be changed or that the parameters evaluated are not the important ones in the model. In this case the former is true, since all the parameters have been considered.

In a real application, the analyst selects several uncertain parameters to evaluate from possibly tens or hundreds in a complex FEM. If the SSP module indicates that there are no important parameters, the analyst may need to go back and include some more that were missed the first time. At least the SSP module has stopped the model updating process before time is wasted on futile FEM updating runs. One might say that, for the examples shown here, it would be easier to simply solve for the least squares solution. This is true, but in real applications, there are usually more parameters to investigate than there are response frequencies. To get a unique solution, the number of parameters must be reduced to a number less than or equal to the number of frequencies, and this is part of the reason for the approach taken.

EXPERIENCE FROM ACTUAL APPLICATIONS

The author has experience utilizing the approach with four actual reconciliations of FEMs with modal test data. There were between 5 and 16 modes utilized for each application. Some general experience from these can be provided. In some cases there were points in the updating process where the PESTDY predictions indicated that the chosen parameters did not have the capability to bring the model to a point of satisfactory accuracy, so efforts were made to go back and find other parameters to evaluate. This led to finding parameters that significantly improved the model.

In another significant experience, the most important parameter selected by the z-score criterion was highly correlated with three other parameters. Through materials tests it was found that three of the four correlated parameters had large errors. Any method based on the design sensitivity approach prefers the more frequency sensitive parameters to less frequency sensitive correlated parameters. All parameters highly correlated with parameters surviving the z-score analysis should be considered in the model updating process. Where distinctions cannot be made, further testing to identify the correlated parameters should be performed.

CONCLUSIONS

Statistical tests of significance have been applied to FEM parameters used in the Bayesian estimation process for FEM reconciliation. The particular method has been designed to reduce a large number of possible parameters to a number that is less than the number of responses being matched to make the least squares solution as robust as possible. The goal is to determine the parameters that are *both* important *and* in error. This goal may not be met if: 1. the model form does not represent the physical phenomena; 2. the model response is far from the true system response; 3. some parameters are highly correlated to another important parameter. The parameter correlations are calculated to provide knowledge that the third hindrance to the goal may be an important issue. Other tools utilized to help in parameter selection are plots of STD of frequencies vs the number of parameters analyzed and plots of frequency sensitivity for statistically important parameters.

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