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COMPARISON OF DIFFERENT PROCEDURES FOR ITERATIVE FITTING OF BUBBLE CHAMBER EVENTS INVOLVING NEUTRAL PARTICLES

by

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DISCLAIMER

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COMPARISON OF DIFFERENT PROCEDURES FOR ITERATIVE FITTING OF BUBBLE CHAMBER EVENTS INVOLVING NEUTRAL PARTICLES*

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ABSTRACT

The goals of the new method tested in this report were twofold. The first goal was to reduce the computation time required to calculate the derivative matrices which occur in the iteration process associated with the kinematical analysis of bubble chamber data. The second goal was to reduce the number of "fake fits" which are obtained when using the present kinematical analysis computer program. Reducing the number of "fake fits" also reduces the time required to do an ionization check of the data. The effectiveness of the new method in accomplishing these two goals is limited. The computation time required to do the iteration procedure was reduced. However, while reducing the number of "fake fits", the new method also lost a portion of the real fits.

The new method has two versions. The two versions resulted from two different ways of doing the calculation of errors associated with variables computed in the iteration procedure. Version 2 did better than version 1 in attaining the desired goals. The undesirable feature of the old method of doing the kinematical analysis was that it obtained too many "fake fits." Version 1 reduced the number of "fake fits" by 67%. This reduction is helpful in easing the job of ion checking. However,

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due to the rigidity of error assignments, version 1 lost up to 32% of the real fits. This loss cannot be rationalized by the time savings associated with the computation in the iteration procedure and with the ionization check.

Version 2 reduced the number of "fake fits" by 47% and at the same time retained at least 85% of the real fits. Although the reduction of "fake fits" by version 2 is not as great as that for version 1 the retention of real fits is better. During the experiments used to test the new method, version 2 represented a compromise between the leniency of the old method and the rigidity of version 1. In some cases the losses sustained by version 2 may be counterbalanced by the time savings obtained. For example, if the number of events expected for a desired final state is large, one could balance the 15% loss of fits with the reduction in time required to run the program on a computer. However, even a maximum 15% loss is not acceptable if one has a small number of fits. In that case the old method is preferable and the extra computation and ion checking time is necessary.

INTRODUCTION

In a particle reaction at high energy it is possible to produce more than one final state. For example when K^- mesons interact with protons some of the possible final state configurations involving two outgoing charged particles are:

$$K^- p \longrightarrow K^- p \qquad \text{(elastic scattering)}$$

$$\longrightarrow K^- p\, \pi^o$$
$$\longrightarrow K^- \pi^+ N$$
$$\longrightarrow \pi^- p\, \bar{K}^o$$
$$\longrightarrow \pi^- K^+ \Xi^o$$
$$\longrightarrow \pi^- \pi^+ \Lambda^o \qquad \text{(one neutral particle)}$$
$$\longrightarrow \pi^- \pi^+ \Sigma^o$$
$$\longrightarrow K^- K^+ \Lambda^o$$
$$\longrightarrow K^- K^+ \Sigma^o$$

$$\longrightarrow K^- \pi^+ N\, \pi^o \qquad \text{(two or more neutral particles)}$$
$$\longrightarrow \text{etc.}$$

When analyzing data accumulated from the observation and measurement of particle interactions, it is necessary to determine the final state into which a particular event belongs. A high energy physicist uses several criteria to decide which configuration is most probable. For example, in all interactions the laws of momentum and energy conservation must be satisfied. These two laws give four equations or four constraints relating the variables associated with the incoming and outgoing particles. When applying these relations to bubble chamber data, one has to realize that only charged particles can be seen in a

bubble chamber. Therefore, if a neutral particle does exist in the final state it will not be seen and consequently cannot be measured. This problem may be circumvented by using three of the four constraint equations to solve for the three variables associated with the unseen particle. Since three of the constraints were used to solve for the missing variables only one constraint is left to be satisfied. By analogy one sees that it is impossible to solve the case where two neutral particles exist in the final state because there are not enough constraints to determine the missing variables. Hence if an interaction has only charged particles in the final state, it is referred to as four constraint (4C) event. $K^- p \rightarrow K^- p$ is an example of a 4C event. Likewise a final state configuration containing a neutral particle is referred to as a 1C event. There are eight such final states listed in the example above. Finally, any final state containing two or more neutral particles is said to be unconstrained. In general, four minus the number of unmeasured variables is the number of constraints.

The high speed computer aids a physicist in the analysis of the large amounts of bubble chamber data. The analysis "may be accomplished in roughly three stages. The first stage may consist of an analysis of the measurements in order to determine the most probable values of the various quantities -- such as coordinates, space angles, curvature or momentum, and energy or velocity -- to be assigned each particle both before and after the interaction .... In the second stage, the output of such a 'one-track' analysis or spatial reconstruction program may be used in conjunction with kinematical requirements to interpret the interaction under consideration. Such a kinematical analysis may be used in

the choice of one of several possible interpretations, ... Finally, in the third stage, the results from separate events may be combined and the usual statistical analysis performed upon a group of events" (Berge et al., 1). It is during the kinematical analysis or second stage that such criteria as the momentum and energy constraint equations are used to determine whether or not a particular configuration could be a fit. In its turn, each of the possible constrained final states is tried as a working hypothesis. Once a final state obtains a fit, it must be quantitatively ascertained how good the fit is. The mathematical quantity χ^2 is used to determine the "goodness" of a fit. Usually not all of the final states which get a fit are retained. The fits with χ^2's greater than a predetermined value are rejected. Generally the limit is set lenient enough to insure that every possible real fit is retained. Due to the errors associated with the measured data, it is possible for more than one final state to get a fit for a particular event. For the purposes of this report the fits other than the true one are called "fake fits." The large quantity of "fake fits" which accumulate in the present kinematical analysis requires a physicist to spend additional time checking the ionization of the tracks in each event. The accumulation of "fake fits" is greater for the 1C case than it is for the two constraint (2C), three constraint (3C) and 4C cases. Therefore, taking the K^-p interaction cited above, there will be more "fake fits" associated with the final states involving one neutral particle than there will be in the case of elastic scattering.

One would like to develop a system which would reduce the number of "fake fits" but, at the same time, retain the real fits. This report

describes a method which reduces the number of "fake fits" by 47% and still retains at least 85% of the desired fits for a particular 1C final state. This method eases the job of ion checking and reduces the computation time involved in the iteration process. The new version does not alter the calculation in the 4C case. However, in the 1C, 2C and 3C cases it calculates the quantities and errors associated with the missing variables before the kinematical analysis begins. The program logic then proceeds as if it were processing a 4C case. The result of this change appears in the χ^2 value for the fit. Consequently, conclusions about the effectiveness of the new version were made by comparing the χ^2 distributions of the old and new methods.

The kinematical analysis program used for the tests was a local version of GUTS (Kernan et al., 2). The modification of GUTS was tested on 1C and 4C data only. However, through analogy the new method could be used in the 2C and 3C cases. In testing two different error calculations were used, and they produced different results.

NOTATION

In this report the following notation conventions are used:

P The total number of outgoing particles plus one (if there is an incoming particle); for the program GUTS, $2 \leq P \leq 10$.

L The number of analytical constraints to be applied at the interaction vertex. Here $0 \leq L \leq 4$.

I The number of measured variables. $I = 3P + (L - 4)$, and for GUTS $2 \leq I \leq 30$.

ϕ_q The azimuthal angle of the qth particle at the vertex, measured from some arbitrary axis.

$\tan \lambda_q$ The "projected curvature" of the qth particle, defined by $k_q = [P_q \cos \lambda_q]^{-1}$.

P_q The momentum of the qth particle, Mev/c.

π_x The x components of the momentum summed over all measured tracks, defined by $\pi_x = \sum_q^{\text{meas.tracks}} (\pm)_q \cos \phi_q / k_q$.

π_y The y component of the momentum summed over all measured tracks, defined by $\pi_y = \sum_q^{\text{meas. tracks}} (\pm)_q \sin \phi_q / k_q$.

π_z The z component of the momentum summed over all measured tracks, defined by $\pi_z = \sum_q^{\text{meas.tracks}} (\pm)_q \tan \lambda_q / k_q$.

$(\pm)_q$ Equals +1 (-1) if the particle is outgoing (incoming).

x_i Any measured variable (ϕ_q, $\tan \lambda_q$, k_q).

G_{ij} The error matrix comprised of the errors, δx_i, associated with the measured variable, x_i, given by $\langle \delta x_i \ \delta x_j \rangle_{ave} = G^{-1}$.

m As a superscript indicates a measured quantity.

THEORETICAL DISCUSSION

The mathematical formulation used in the local version of GUTS cited in the introduction was obtained from an article by Berge, Solmitz and Taft (Berge et al., 1). This formulation involves the assigning of probable values to k_q, ϕ_q and $\tan \lambda_q$ for each track. The optimum set of variables x_i are obtained when the function

$$\chi^2 = \sum_{\substack{i=1 \\ j=1}}^{I} (x_i - x_i^m)\, G_{ij} (x_j - x_j^m) \qquad (1)$$

subject to the constraint equations

$$f_\lambda (x_i) = 0 \qquad \lambda = 1, 2, \ldots L, \qquad (2)$$

is minimized. L Lagrange multipliers α_λ are introduced and the problem is reduced to finding the stationary value of the function.

$$M = \sum_{\substack{i=1 \\ j=1}}^{I} (x_i - x_i^m)\, G_{ij} (x_j - x_j^m) + 2 \sum_{\lambda=1}^{L} \alpha_\lambda\, f_\lambda (\vec{x}) \qquad (3)$$

This is solved by a simple iteration which involves the equations

$$\frac{\partial M}{\partial x_i} = 2 \sum_{j=1}^{I} G_{ij} (x_j - x_j^m) + 2 \sum_{\lambda=1}^{L} F_{i\lambda} (\vec{x})\, \alpha_\lambda = 0 \qquad (4)$$

$$i = 1, 2 \ldots I,$$

$$\frac{\partial M}{\partial \alpha_\lambda} = 2\, f_\lambda (\vec{x}) = 0 \qquad \lambda = 1, 2, \ldots L, \qquad (5)$$

where

$$\frac{\partial f_\lambda (x)}{\partial x_i} = F_{i\lambda} (\vec{x}) \qquad (6)$$

For all cases (0C, 1C, 2C, 3C, 4C), $f_\lambda (\vec{x})$ is a function of the measured

variables only (ie $x_i = x_i^m$). However the derivative matrix $F_{i\lambda}(\vec{x})$ is a function of the measured and whatever unmeasured variables are assumed to exist. That is, in the 1C case when a neutral particle is assumed to be missing the quantities k, ϕ and tan λ are unknown. The derivative matrix has for the measured variables entries of the form

$$F_{i4} = \frac{\partial f_4}{\partial x_i} \qquad i = 1, 2, \ldots I-3, \qquad (7)$$

and for the unmeasured variables entries of the form

$$F_{i4} = \frac{\partial f_4}{\partial x_i} \left(\sum_{j=1}^{3} \frac{\partial x_j}{\partial y_j} \right) \qquad i = I-2, I-1, I, \qquad (8)$$

where $y_1 = \pi_x$, $y_2 = \pi_y$ and $y_3 = \pi_z$.

In order that these derivatives may be evaluated GUTS has to calculate the unknown variables (k, ϕ, and tan λ) at the start of each iteration. On the other hand, in the 4C case there are, by definition, no unmeasured quantities and the derivative matrix has entries of the form

$$F_{i\lambda} = \frac{\partial f_\lambda}{\partial x_i} \qquad i = 1, 2, \ldots I, \lambda = 1, 2, \ldots L. \qquad (9)$$

These are relatively simple, and in addition the program is not required to calculate any unknown variables at the start of each iteration. This results in less computation time when calculating the derivative matrix in the 4C case than for the 1C case.

Furthermore when attempting a 1C fit GUTS has more freedom to adjust the tracks in order to minimize χ^2 than it does in the 4C case. One might expect this because of the fewer constraints that need to be satis-

fied while minimizing χ^2 in the 1C case. This freedom leads to "fake fits" (fits obtained by GUTS which are later rejected by an ionization check of the events). Intuitively one feels that if the 1C case could be approached as a 4C case there would be fewer "fake fits" and the calculation procedure would be less time consuming. Such an approach is suggested by A. G. Wilson when he is describing a kinematical analysis program used at the Rutherford High Energy Laboratory (Wilson, 3).

For a 1C case the approach is the following. First, solve the three momentum constraint equations for the unmeasured variables plus their associated errors. Second, one considers these calculated quantities as if they were measured data. Finally, the iteration involved in minimizing χ^2 is carried out as if it were a 4C case. This reduces the computation time because the unknown variables do not have to be calculated at the start of each iteration. Hopefully, this approach also reduces the number of "fake fits."

In the case of a outgoing neutral particle the equations for the unmeasured variables become (Berge et al., 1)

$$\phi = \tan^{-1}(\pi_y/\pi_x) \tag{10a}$$

$$k = \{\pi_x^2 + \pi_y^2\}^{-1/2} \tag{10b}$$

$$\tan\lambda = -k\,\pi_z \tag{10c}$$

The errors $\delta\phi$, δk and $\delta(\tan\lambda)$ were calculated in two different ways. The difference in calculation constitutes the difference in the two versions which are compared with the original method later in the report. The first is from differential calculus.

$$\delta V(\vec{x}) = \sum_{i}^{\text{indep variables}} \frac{\partial V}{\partial x_i} \delta x_i \qquad (11)$$

As stated in Beers (4) the deviation δV is good to the first order of approximation. This is consistent with the assumption in GUTS "that the constraint functionsare linear over the region covered by the errors" (Berge et al., 1).

The second method is suggested by Beers (4)

$$P_V = \left[\sum_{i}^{\text{indep variables}} \left(\frac{\partial V}{\partial x_i}\right)^2 P_{x_i}^2 \right]^{1/2} \qquad (12)$$

where P_V represents the probable error in the quantity V. The equation as stated by Beers in the reference cited has a correlation coefficient which ranges in value from -1 to +1. When measurements are known a priori to be independent the coefficient is set to zero. Since GUTS makes the assumption that "individual particles are not correlated with one another," (Berge et al.,1) the coefficient has been set equal to zero for this experiment. Again note that the error expression in equation 12 is consistent with the assumption in GUTS concerning linear constraint functions.

For the purposes of comparison errors calculated using equation 11 will be called version 1 and those calculated using equation 12 will be called version 2. The error equations for version 1 are:

$$\delta\phi = \frac{\pi_x}{\pi_x^2 + \pi_y^2} \left\{ \sum_{q}^{\text{meas. tracks}} (\pm)_q \left(\frac{\cos\phi_q}{k_q} \delta\phi_q - \frac{\sin\phi_q}{k_q^2} \delta k_q \right) \right\}$$

$$+ \frac{\pi_y}{\pi_x^2 + \pi_y^2} \left\{ \sum_{q}^{\text{meas. tracks}} (\pm)_q \left(\frac{\sin\phi_q}{k_q} \delta\phi_q + \frac{\cos\phi_q}{k_q^2} \delta k_q \right) \right\} \qquad (13a)$$

$$\delta k = \left\{\pi_x^2 + \pi_y^2\right\}^{-3/2} \left\{\pi_x \sum_q^{\text{meas. tracks}} (\pm)_q \left(\frac{\sin\phi_q}{k_q}\delta\phi_q + \frac{\cos\phi_q}{k_q^2}\delta k_q\right) - \pi_y \sum_q^{\text{meas. tracks}} (\pm)_q \left(\frac{\cos\phi_q}{k_q}\delta\phi_q - \frac{\sin\phi_q}{k_q^2}\delta k_q\right)\right\} \quad (13b)$$

$$\delta(\tan\lambda) = -\pi_z\delta k + \left\{\pi_x^2 + \pi_y^2\right\}^{-1/2} \left\{\sum_q^{\text{meas. tracks}} (\pm)_q \left(\frac{\tan\lambda_q}{k_q^2}\delta k_q - \frac{\delta(\tan\lambda_q)}{k_q}\right)\right\} \quad (13c)$$

While the error equations for version 2 are:

$$\delta\phi = \left[\frac{\pi_y^2}{(\pi_x^2+\pi_y^2)^2} \sum_q^{\text{meas. tracks}} (\pm)_q \left\{\left(\frac{\sin\phi_q}{k_q}\delta\phi_q\right)^2 + \left(\frac{\cos\phi_q}{k_q^2}\delta k_q\right)^2\right\} + \frac{\pi_x^2}{(\pi_x^2+\pi_y^2)^2} \sum_q^{\text{meas. tracks}} (\pm)_q \left\{\left(\frac{\cos\phi_q}{k_q}\delta\phi_q\right)^2 + \left(\frac{\sin\phi_q}{k_q^2}\delta k_q\right)^2\right\}\right]^{1/2} \quad (14a)$$

$$\delta k = \left[\frac{\pi_x^2}{(\pi_x^2+\pi_y^2)^3} \sum_q^{\text{meas. tracks}} (\pm)_q \left\{\left(\frac{\sin\phi_q}{k_q}\delta\phi_q\right)^2 + \left(\frac{\cos\phi_q}{k_q^2}\delta k_q\right)^2\right\} + \frac{\pi_y^2}{(\pi_x^2+\pi_y^2)^3} \sum_q^{\text{meas. tracks}} (\pm)_q \left\{\left(\frac{\cos\phi_q}{k_q}\delta\phi_q\right)^2 + \left(\frac{\sin\phi_q}{k_q^2}\delta k_q\right)^2\right\}\right]^{1/2} \quad (14b)$$

$$\delta(\tan\lambda) = \left[\pi_z^2(\delta k)^2 + (\pi_x^2+\pi_y^2)^{-1}\left[\sum_q^{\text{meas. tracks}} (\pm)_q \left(\frac{\delta(\tan\lambda_q)}{k_q}\right)^2 + \sum_q^{\text{meas. tracks}} (\pm)_q \left(\frac{\tan\lambda_q}{k_q^2}\delta k_q\right)^2\right]\right]^{1/2} \quad (14c)$$

EXPERIMENTAL PROCEDURE

The new approach was tested on data obtained from K^-p interactions at 4.6 Bev/c. Of the possible final states listed in the introduction, the ones used in the testing were

$$K^-p \longrightarrow K^-p \qquad \text{(elastic scattering--4C)}$$

$$\left.\begin{array}{l} \longrightarrow K^-p\,\pi^o \\ \longrightarrow K^-\pi^+N \\ \longrightarrow \pi^-p\,\bar{K}^o \\ \longrightarrow \pi^-K^+\Xi^o \\ \longrightarrow \pi^-\pi^+\Lambda^o \end{array}\right\} \text{(one neutral particle--1C)}$$

The events were originally measured and ion checked in connection with an experiment by Kang et al. (5). The experimental procedure consisted of three steps which were as follows.

Step 1 600 events which originally obtained a 4C fit (corresponding to an elastic scatter) were used as the data in steps 1 and 2. The 600 fits were assumed to contain no "fake" 4C fits.[1] Under this assumption these 4C fits were used as a standard to which the results in steps 1 and 2 were compared.

In step 1 the 600 events were processed by GUTS as if it were 1C data. That is, a subroutine, EUREKA, inserted in GUTS first deleted the measured information associated with the proton track. Then EUREKA set

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[1] William J. Kernan and John Ullman, Iowa State University, Ames, Iowa. Ion checking. Private communication. 1967. Ion checking indicated that events not involving $K^-p \leq 2\%$. William J. Kernan and Lee S. Schroeder, Iowa State University, Ames, Iowa. Background estimates. Private communication. 1967. Background estimates indicated events involving $K^-p\pi^o \leq 2\%$.

up the necessary logic and conveyed the remaining measured data to GUTS. GUTS proceeded to do a normal 1C kinematical analysis of this data. The purpose of step 1 was to test how well a normal 1C calculation could reproduce known results.

Step 2 The new method was tested on the 4C data. A 1C fit was again simulated by deleting the measured information associated with the proton track. However, this time, EUREKA took the measured information of the remaining two tracks and using equations 10, 13 and 14 calculated the missing variables plus corresponding errors for the proton track. Then the measured and calculated information were conveyed to GUTS. GUTS proceeded to do a normal 4C kinematical analysis of the combined data. The purpose of step 2 was to test how well the new method could reproduce known results.

Step 3 The data used in step 3 was not the same 600 events used in steps 1 and 2. The new data consisted of 622 new events. When originally processed by GUTS, most of new events obtained fits for more than one of the possible final state configuration. In fact some events obtained a 4C fit in addition to the 1C fits. Consequently, there were 1335 1C fits associated with the 622 events.

In step 3 the new events were processed by GUTS using equations 10, 13 and 14 to supply the variables and errors for the assumed missing neutral. That is, using the information from all three of the measured tracks, EUREKA calculated the projected curvature, k, the azimuthal angle, ϕ and the tangent of the latitude, $\tan\lambda$, plus the associated errors for the assumed missing neutral particle. In addition, EUREKA set up the logic for a normal 4C fit. GUTS then processed the combined measured

and calculated data as a standard 4C case.

The results of an ionization check were used to determine which of the original 1335 fits were "fake." The results of the ionization check and a sample of the 622 events were used to determine which of the methods (old or new) did a better job of rejecting "fake fits" and retaining real fits.

Note that in steps 2 and 3 the error calculation was done in two different ways (see p. 8). Consequently, in the discussion of the results three rather than two sets of data will be compared.

DISCUSSION OF RESULTS

The importance of χ^2 is not its absolute value but in the relative values among fits for a particular set of data. Therefore it is not valid to compare the χ^2 distributions obtained from different sets of data. Nor is it meaningful to compare χ^2 distributions resulting from 1C and 4C minimizations performed on the same data. However, it is instructive to notice the appearance of 1C and 4C distributions relative to one another. The theoretical χ^2 distribution for various numbers of constraints is shown in Figure 1 (Melissinos, 6, p. 466).[1] In Figure 1 the peaks of the various distributions become broader as the number of constraints increases. Also, as the constraints increase the value of χ^2 at which the peak occurs increases. In the case of a χ^2 fit in this report, the reason the peak shifts to the right is that GUTS must simultaneously minimize χ^2 and each constraint equation. Therefore with an increasing number of constraints the program has a decreasing amount of freedom to adjust the tracks in order to minimize χ^2. In particular, the 1C distribution is sharply peaked at $\chi^2 = 0$ whereas the 4C distribution appears to be flatter and has a peak at $\chi^2 = 2.0$. One would like experimental results to be similar to the theoretical predictions.

The original χ^2 distribution of 600 events used in steps 1 and 2 is shown in Figure 2. The histogram agrees quite well with the theory. The distribution has a relatively broad peak occurring at a χ^2 between 1.5 and 2.0. Likewise when the same 600 events are processed in the 1C

[1] Mellssinos refers to ν as the number of degrees of freedom whereas this report refers to the same quantity as the number of constraints.

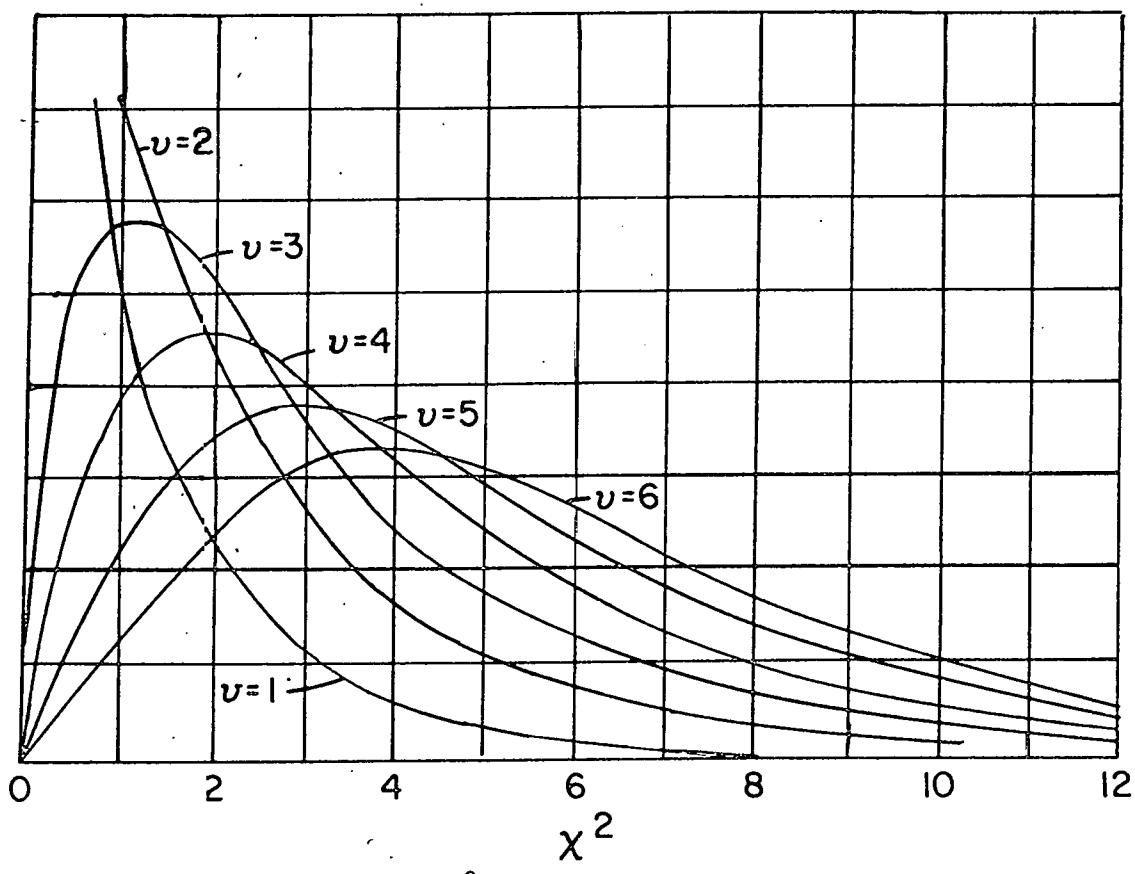

Figure 1. The theoretical χ^2 distribution for various constraints

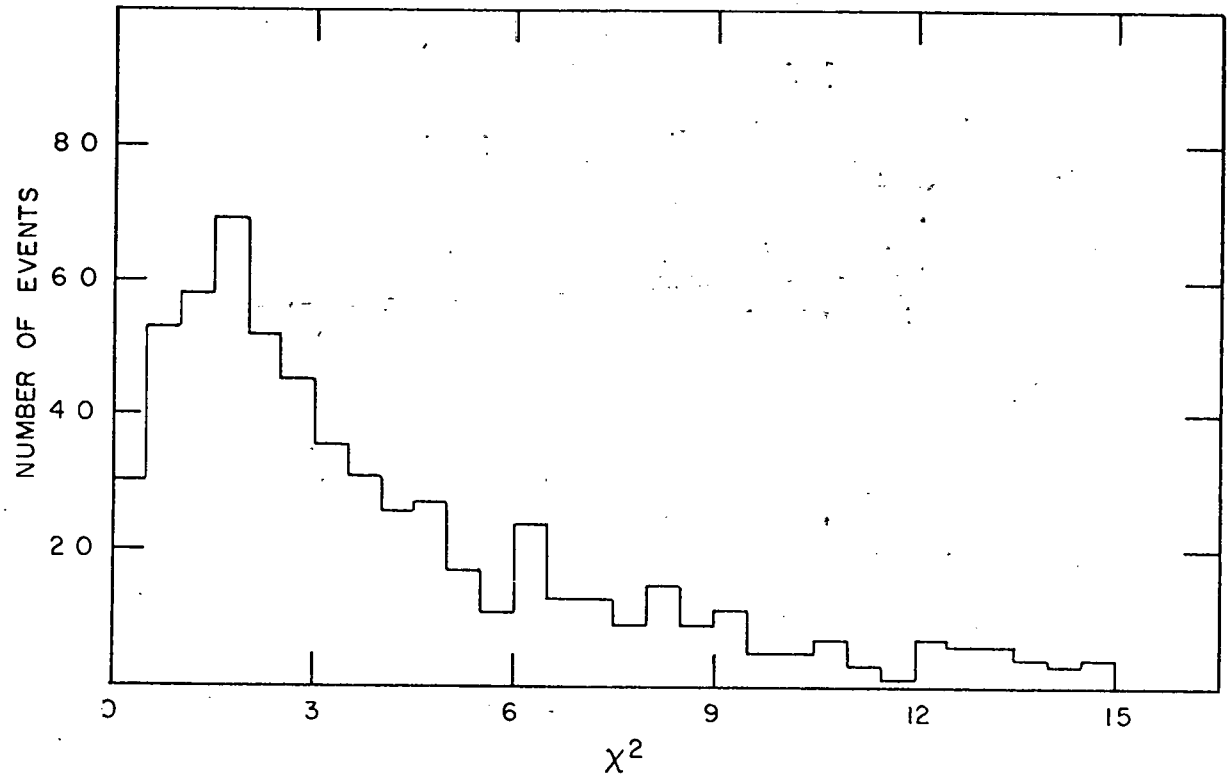

Figure 2. The χ^2 distribution of the original 4C events

mode -- as described in step 1, p. 11 -- the resulting χ^2 distribution agrees with the theoretical prediction. Figure 3 displays the 1C χ^2 distribution. In addition all events which obtained fits in the 4C mode also obtained fits in the 1C mode. This agreement gives one confidence that the data does not contain an abnormality such as a systematic error. Also, since none of the original 4C fits were lost in the 1C fitting procedure, one concludes that the 1C mode is effective in reproducing known results.

Step 2 represented the first test for the new method. As mentioned before on p. 8 errors corresponding to the calculated variables were computed in two differing ways. The χ^2 distribution resulting from the application of version 1 -- as described in step 2, p. 12 -- is shown in Figure 4. Since version 1 is a modified 1C calculation, it is valid to compare the distributions in Figures 3 and 4. Although the distribution in Figure 4 has the general features of a normal 1C distribution, it is not as sharply peaked as the distribution in Figure 3. Also, version 1 rejected 101 events which obtained a $\chi^2 \geq 15$. The χ^2 distribution resulting from the application of version 2 shows similar but a less pronounced effect when compared with the original χ^2 distribution. Version 2's χ^2 distribution shown in Figure 5 is not as sharply peaked as the original distribution but is sharper than version 1's distribution. In addition, version 2 rejected only 33 events. As noted earlier the χ^2 distribution flattens out with an increasing number of constraints. Versions 1 and 2 also tend to flatten the distribution. It appears the modified 1C case has the same effect as increasing the number of constraints. That is, the modification restricts the freedom GUTS has to

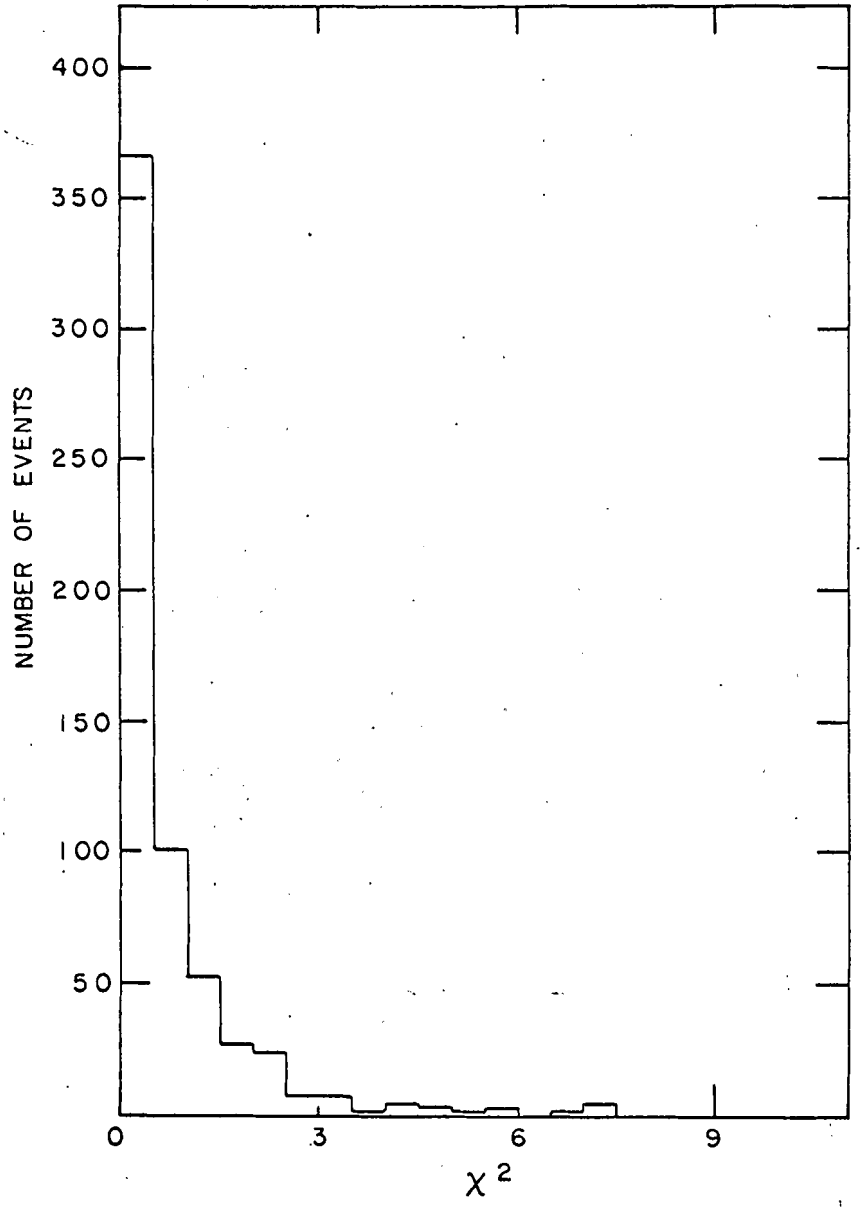

Figure 3. The χ^2 distribution of 4C events processed by normal 1C method

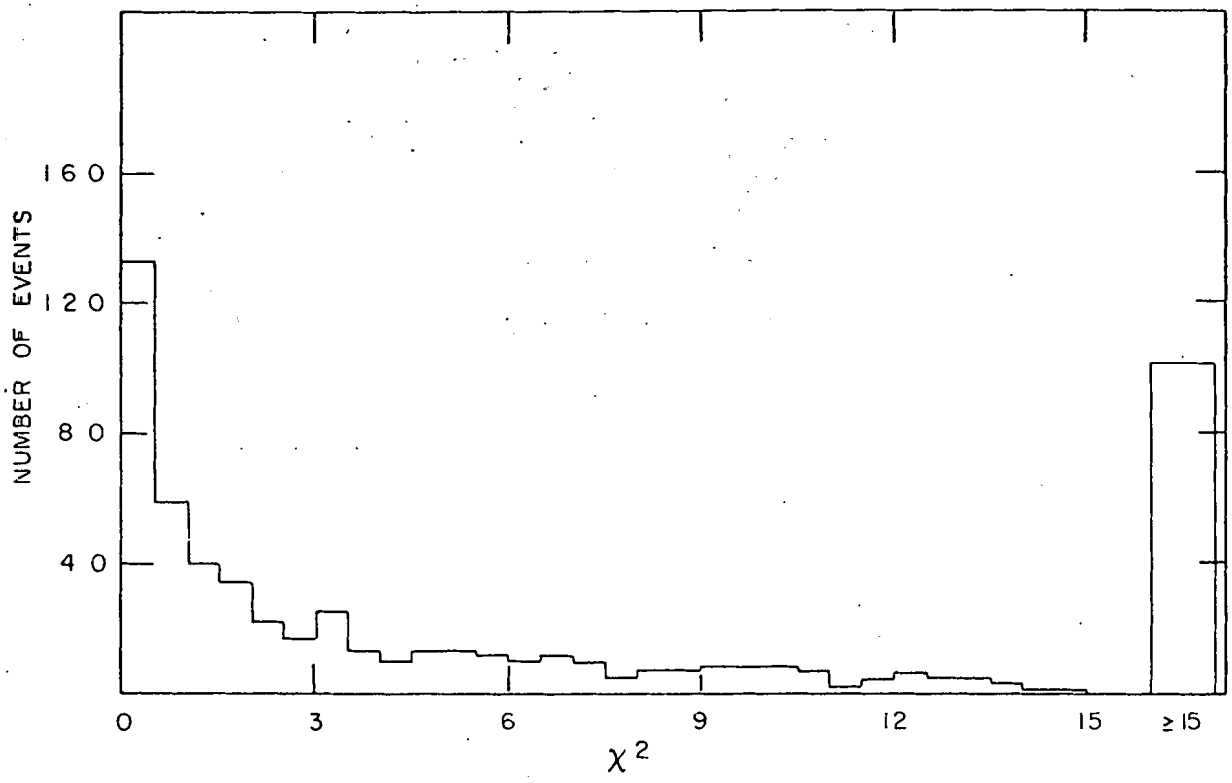

Figure 4. The χ^2 distribution of 4C events processed by version 1

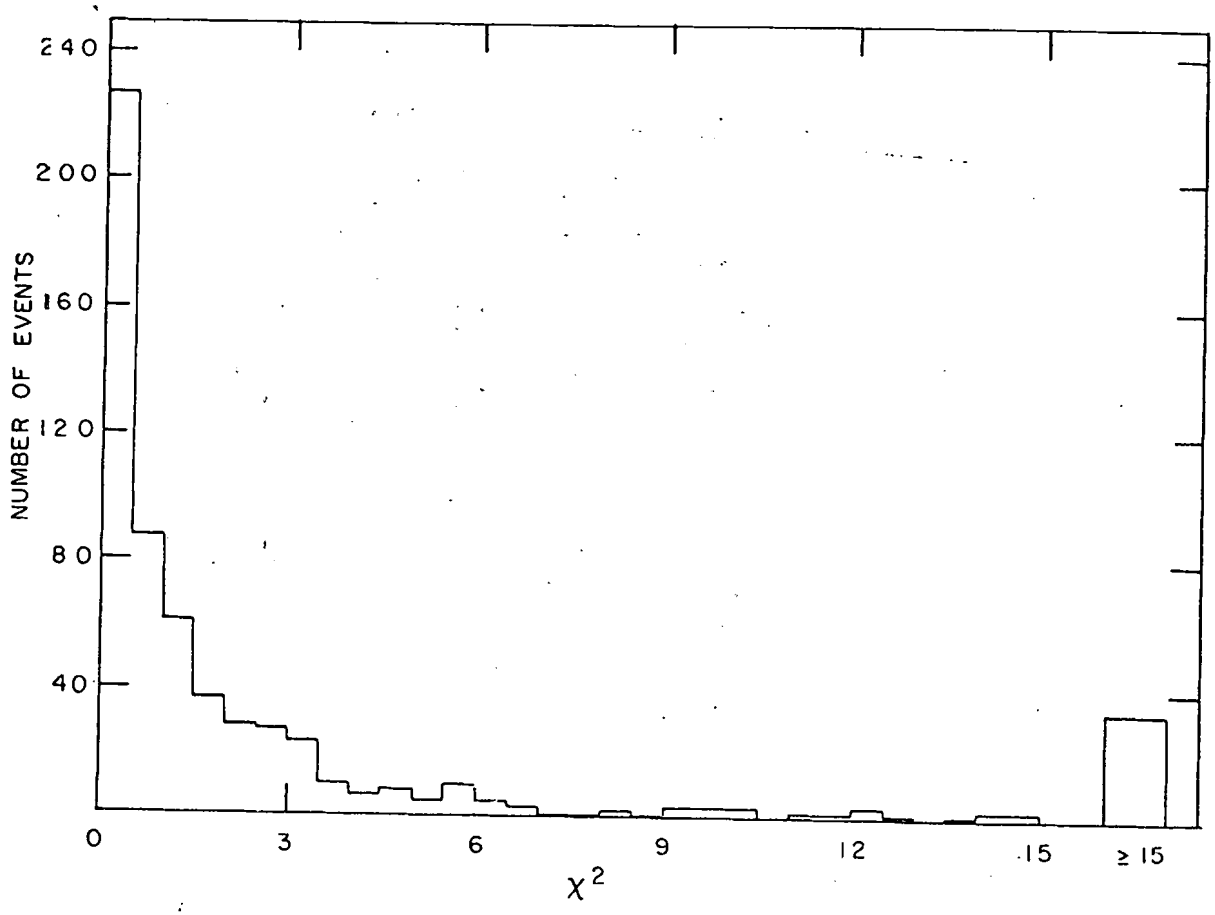

Figure 5. The χ^2 distribution of 4C events processed by version 2

adjust the tracks in order to minimize χ^2.

The 101 and 33 rejected fits represent a maximum loss because the original 600 events were assumed to have only 4C fits. This assumption may be in error as much as 4% (footnote p. 11). For 600 events this represents 24 events. Although these 24 events cannot account for 101 rejections by version 1, they could account for a majority of the 33 rejections by version 2. Assuming 12 of the rejected fits belonged originally to the contaminated sample leaves one with 89 and 21 out of 588 fits which were rejected by version 1 and 2.

The ability of the new method to reproduce known results was limited. The limitation was more severe for version 1 -- where ~ 15% of the fits were lost -- than for version 2 -- where ~ 3.5% of the fits were lost. However, the ability of the new method to flatten out the χ^2 distribution may be helpful in culling out the "fake fits" in a normal 1C calculation involving a neutral particle. It appears these "fake fits" may be culled out at the expense of losing 3.5% or 15% of the real fits. While 15% is too great a number to be lost, a 3.5% loss could be tolerated if the number of events one has to work with is large. Whether or not this will be the performance of versions 1 and 2 is the test of step 3.

Before continuing, a point concerning the χ^2 limit should be discussed. 89 fits under version 1 and 21 fits under version 2 were rejected because their χ^2 value was greater than 15. One might think a solution to the rejection problem is to raise the χ^2 limit set in the last step of the iteration. However, a study of Figure 6 indicates this will not alleviate the problem. Figure 6 is a graph of the integral of the theoretical χ^2 probability function versus χ^2 for various numbers of

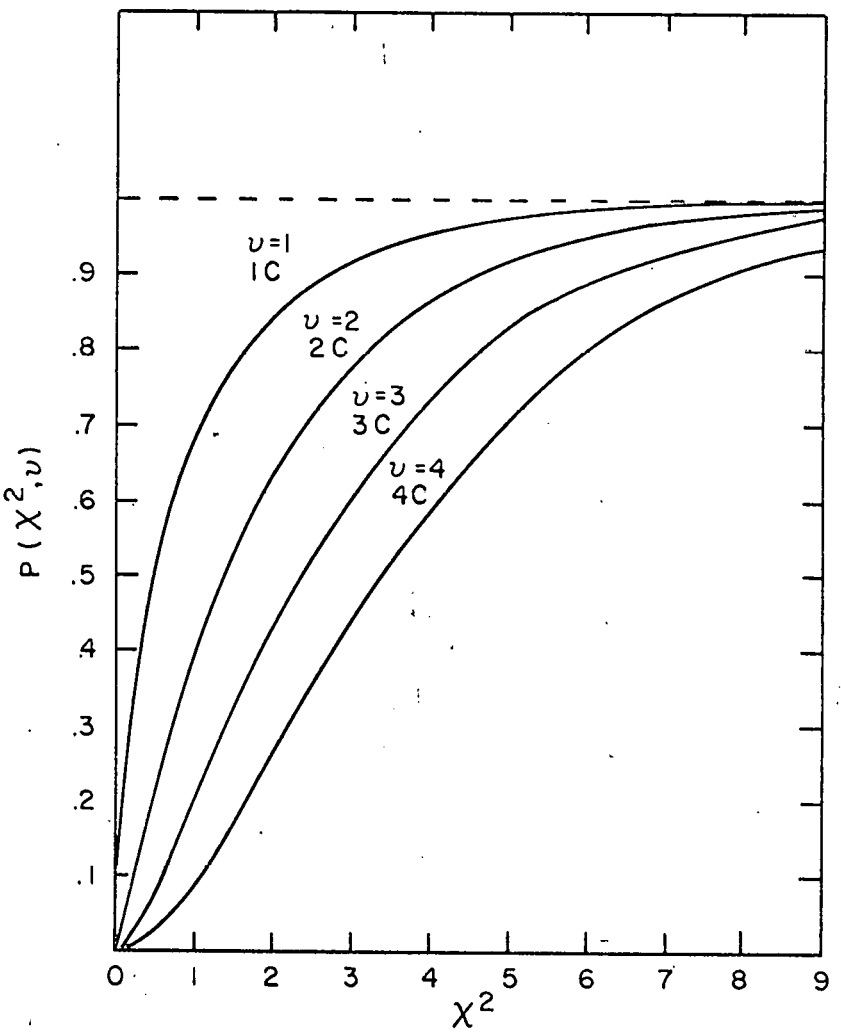

Figure 6. The integral of the theoretical χ^2 probability function

constraints. Given data which give χ^2 distributions such as the ones shown in Figure 1, the interpretation of Figure 6 is as follows. For a particular constraint and a particular χ^2 limit, $P(\chi^2, \nu) \times 100$ represents the percentage of possible fits one will have obtained after processing the given data. For example with a χ^2 limit set at 9 for the 1C case ~ 99.99% of the possible fits will be obtained. Likewise, with $\chi^2 = 9$ for the 4C case ~ 94% of the possible fits will be obtained. Therefore, since the χ^2 limit in steps 1 and 2 was set at 15, raising the limit will not help solve the problem.

In step 3 the χ^2 value was set at 9 since at that value ~ 99.99% of the possible fits will be obtained. Also in step 3 -- as explained on p. 12 -- the data is different than in steps 1 and 2. Figures 7, 8 and 9 show the χ^2 distributions which resulted from the application of original GUTS, version 1 and version 2 respectively. The original χ^2 distribution (Figure 7) agrees quite well with the theoretical prediction for a 1C case. The shape and peak are exactly as one expects. Also, as expected from the results of step 2, versions 1 and 2 (Figures 8 and 9) have a flattened distribution and have rejected a portion of the fits. The rejections are shown at the right of the corresponding histogram. Before becoming alarmed at the number of rejections, one must realize that the original fits used in step 3 contained many "fake fits." In it would be a boon if all the rejections were "fake." An ionization check of the data revealed that 807 of the original 1335 fits were "fake." The distribution of these 807 "fake fits" is shown in Figure 10. How well the two new versions rejected the "fake fits" can be determined from Figures 11 and 12. As can be seen from Figure 11 version 1 reduces the

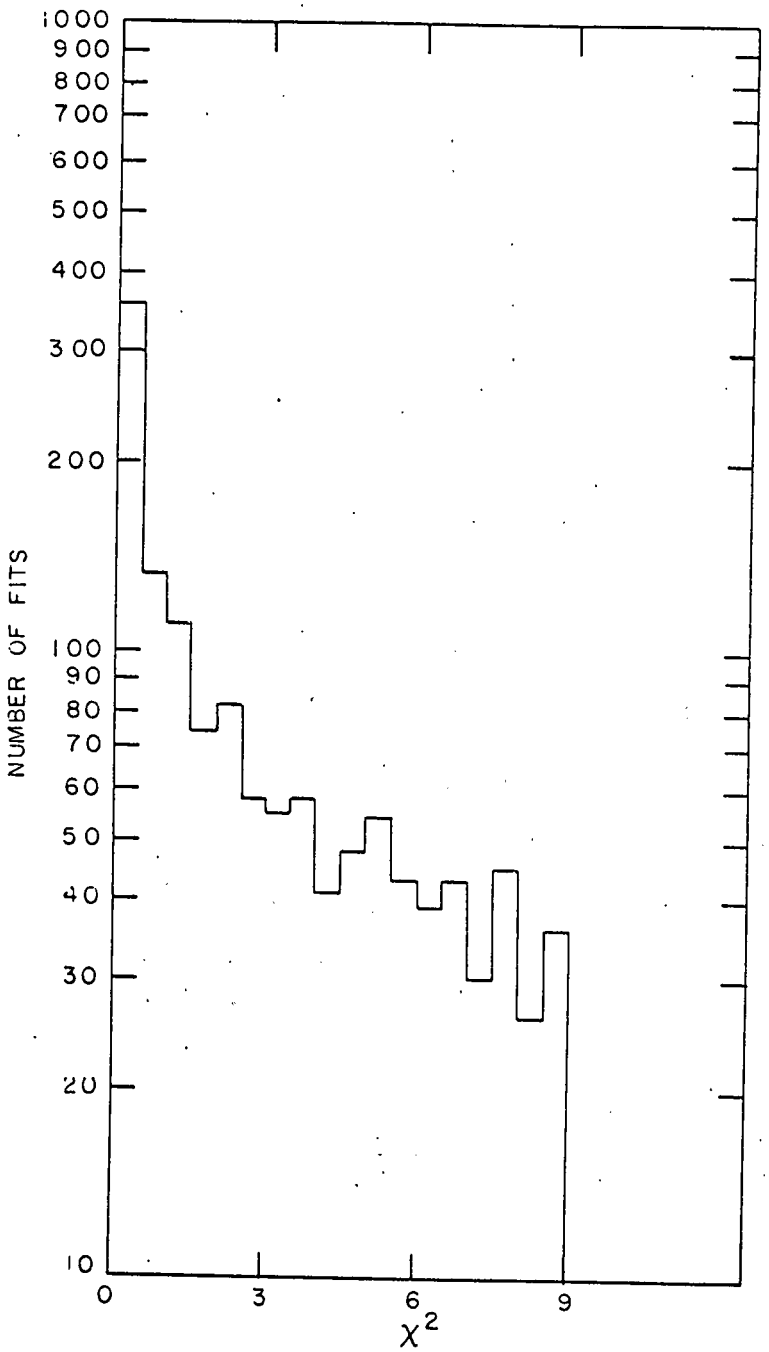

Figure 7. The χ^2 distribution of 1C fits originally obtained by GUTS

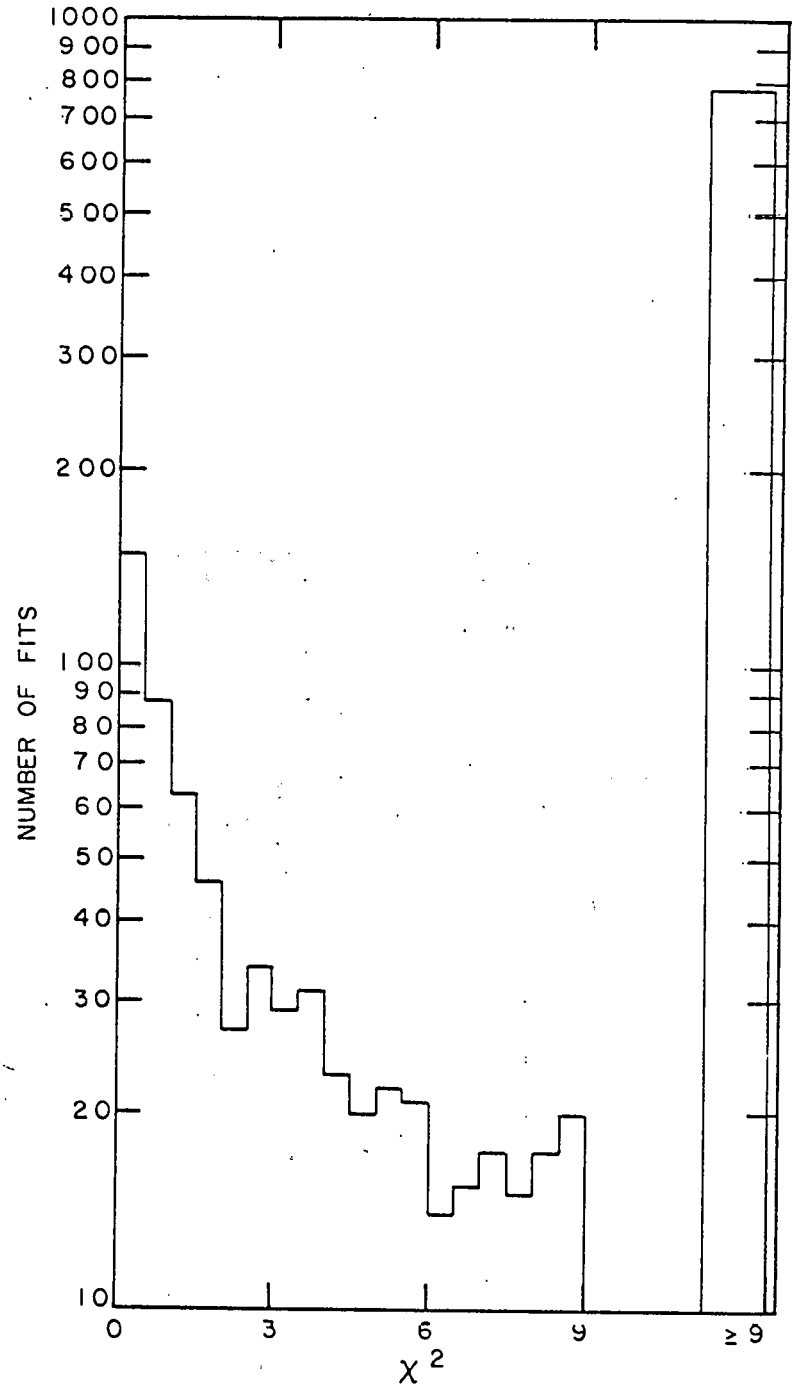

Figure 8. The χ^2 distribution of 1C fits after being processed by version 1

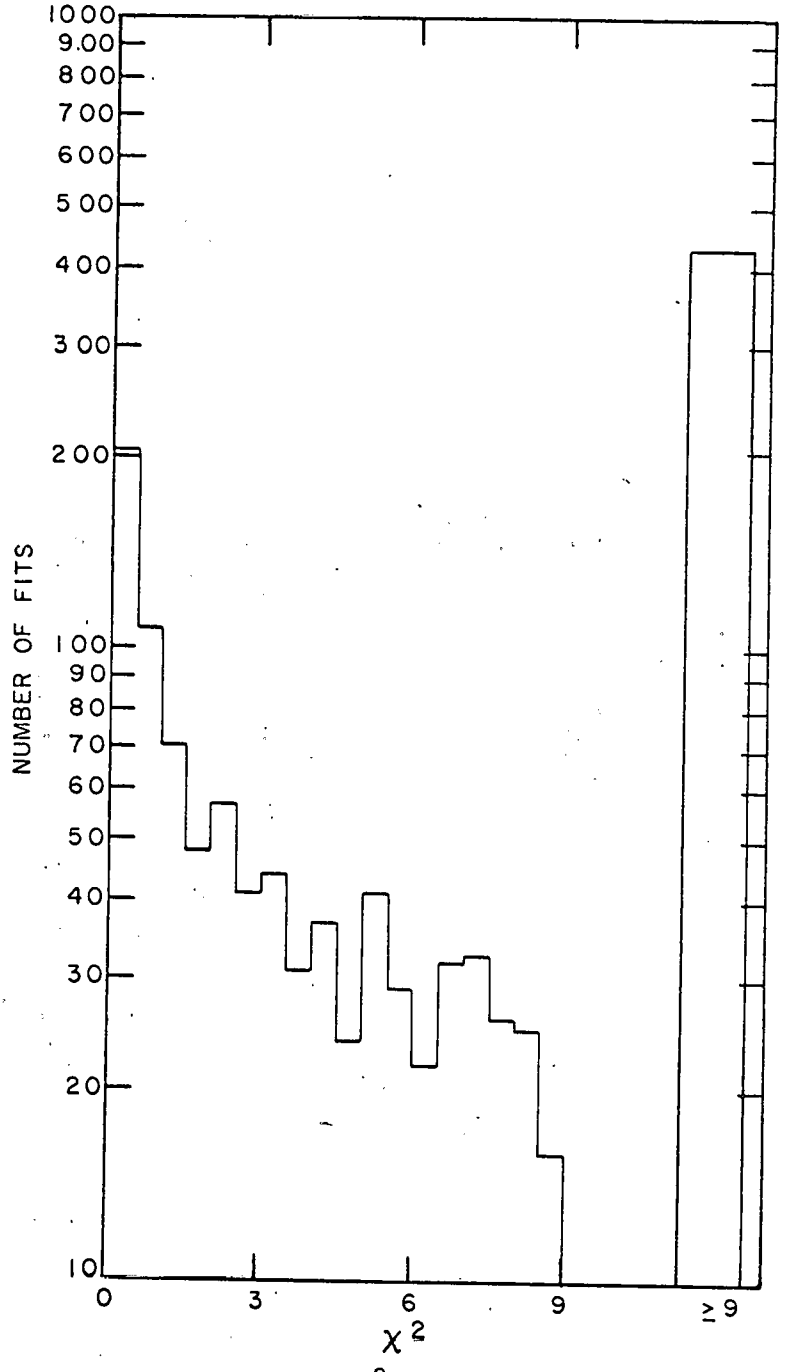

Figure 9. The χ^2 distribution of 1C fits after being processed by version 2

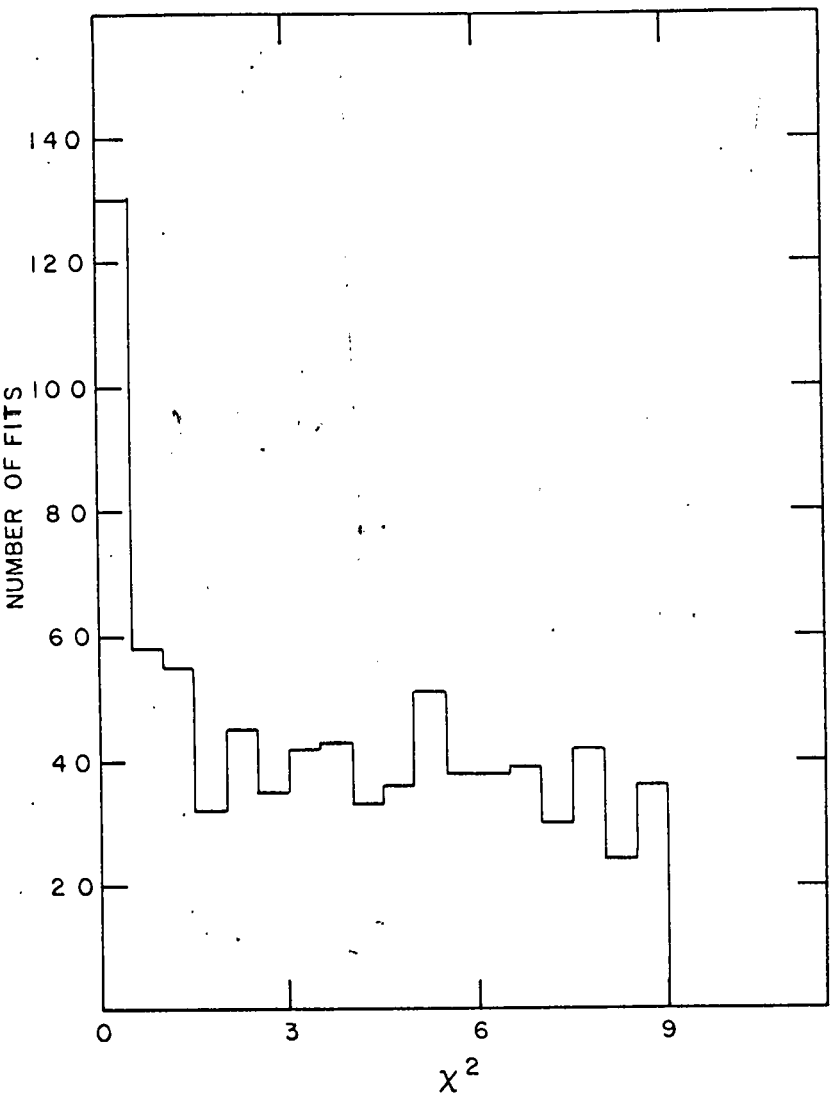

Figure 10. The χ^2 distribution for the 1C fits which were rejected by the ionization check

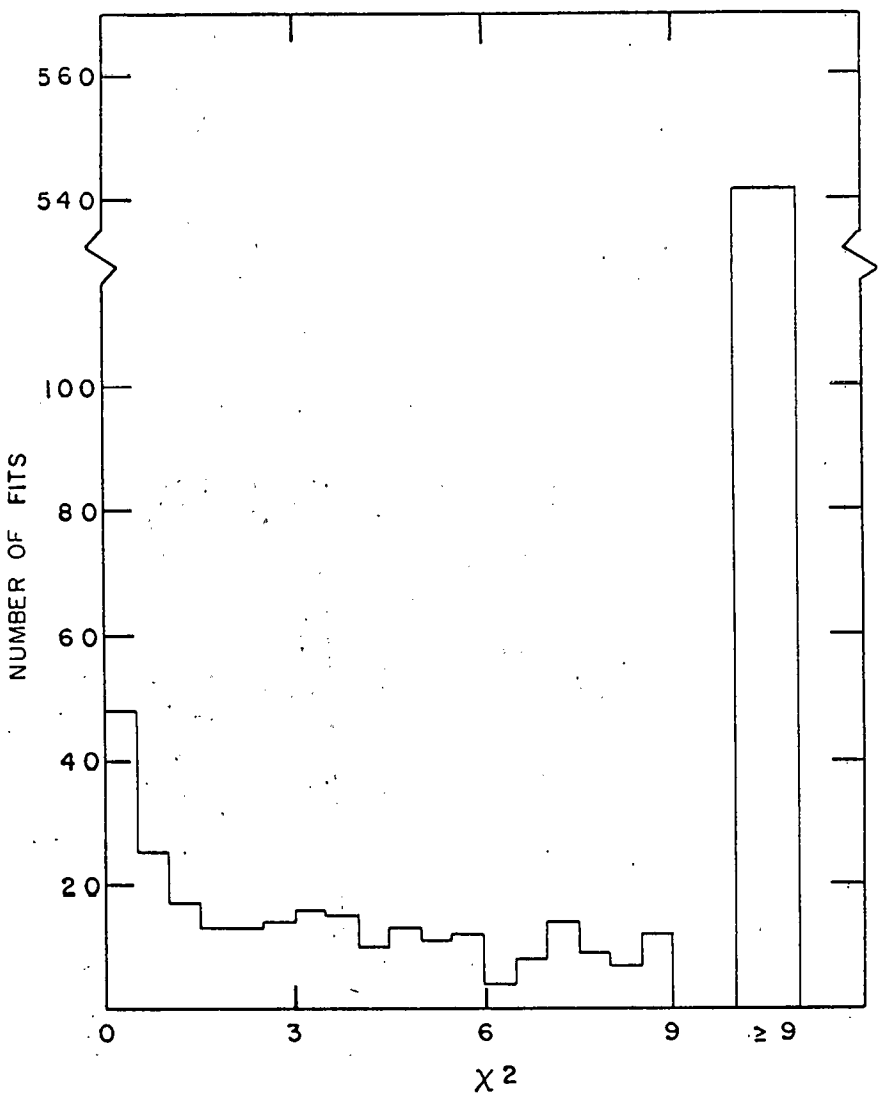

Figure 11. The χ^2 distribution of the IC fits rejected by the ionization check after they were processed by version 1

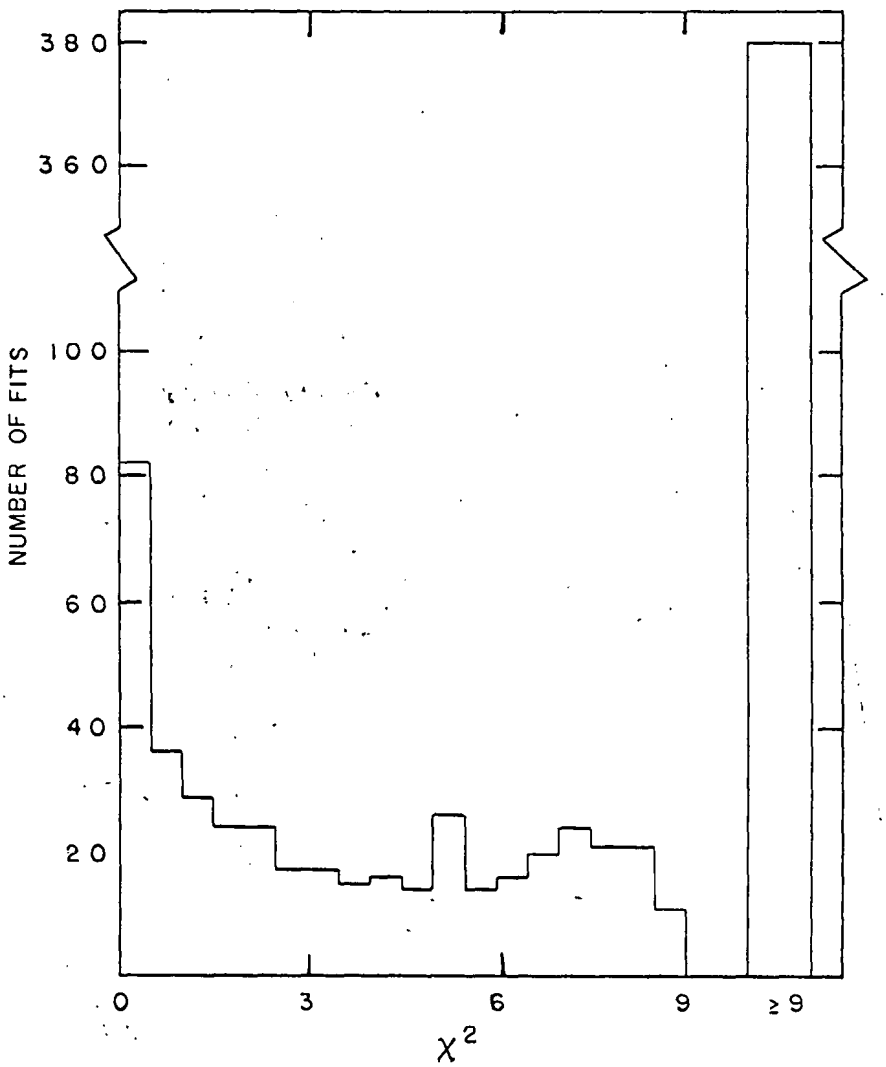

Figure 12. The χ^2 distribution of the IC fits rejected by the ionization after they were processed by version 2

number of "fake fits" from 807 to 265. Whereas from Figure 12 one sees version 2 reduces the "fake fits" to 427. These numbers represent 67% and 47% reductions in the number of "fake fits." The difference in the number of rejections comes from the magnitude of the errors involved. The errors in version 1 have smaller magnitudes than do the ones in version 2. Therefore the errors which version 1 conveys to GUTS do not allow as much freedom to adjust the tracks in order to minimize χ^2 as do the ones which are calculated by version 2. Hence the number of rejections for version 1 is greater.

An ideal situation would have existed if the new versions had rejected only the "fake fits." But, as expected from the results of step 2, an ionization check revealed this was not the situation. In fact, 187 of the fits originally obtained by GUTS and which were ion consistent were rejected by version 1. Similarly 66 of the fits were rejected by version 2. These numbers are quoted to point out that not only "fake fits" but real fits were rejected by by the new versions. They are not to be taken as the exact number of rejections. To understand this one must realize that an ionization check does not indicate one final state configuration as the only one probable for the event. Instead it is possible for an event to have more than one configuration and to be ion consistent. However, only one of the possible configurations was the real final state for the interaction. Hence, the ionization check cannot be used as an absolute standard by which to determine how many events were rejected by the new versions. To determine this one, one needs a data sample in which only one final state configuration is known to exist. Such a data existed in the 622 events processed for step 3. These 622 events were part of the data

Kang used for his paper (5). To determine $K^{*}(891)$ and $K^{*}(1420)$ productions he selected a sample of events which had the final configuration $K^{-}\pi^{+}N$. The contamination in the sample was small ($\sim$10%) (Kang, 5, p. 2). 184 events of his final sample were contained in the 622 used in step 3. Therefore, these 184 events were used to further test the effectiveness of versions 1 and 2. The results are shown graphically in Figures 13, 14 and 15. Figure 13 shows the original GUTS χ^2 distribution of $K^{-}\pi^{+}N$ fits. It has the theoretically predicted shape for a 1C event. The fact that the distribution does not contain a fit having a χ^2 greater than 6.0 is another indication of the "cleanness" of the sample. The performance of version 1 is shown in Figure 14. Version 1, as expected, spreads the distribution considerably. In addition, it rejected 58 or 32% of the original fits. The performance of version 2 was better. As seen in Figure 15 it retained the shape of the original distribution better and only rejected 28 or 15% of the original fits. The loss of 58 and 28 fits represent a maximum loss because up to 10% of the rejections may be attributed to contamination in the sample. That is, events which did not really have $K^{-}\pi^{+}N$ in the final state could be a portion of the events which were rejected by the new versions. Therefore, version 1 may be responsible for only 22% of the rejections. Similarly version 2 may be responsible for as little as 5% of the rejections. However, one cannot be sure what portion of the 10% were in the fits rejected by the new versions. Therefore the rejections can only be stated in terms of maximum losses.

In the case of version 1 a maximum loss of 32% is too great. This loss cannot be rationalized by the time savings associated with the

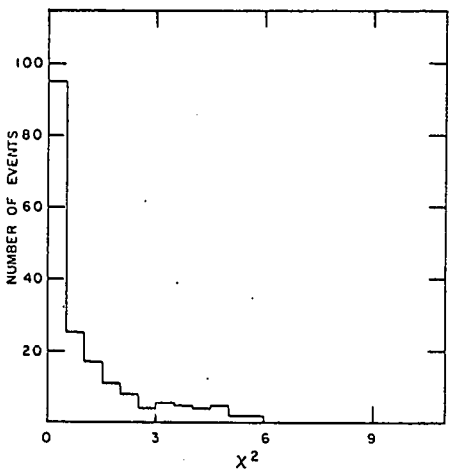

Figure 13. The original χ^2 distribution of the $K^-\pi^+N$ final state sample

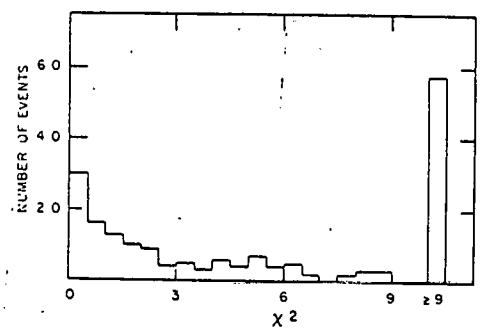

Figure 14. The χ^2 distribution resulting after the application of version 1 to the $K^-\pi^+N$ final state sample

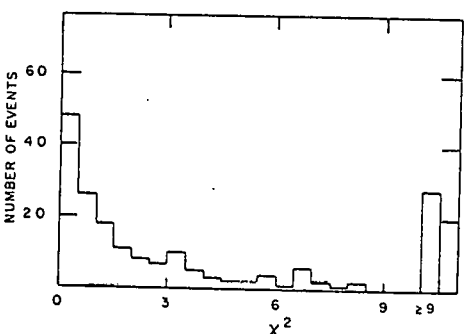

Figure 15. The χ^2 distribution resulting after the application of version 2 to the $K^-\pi^+N$ final state sample

computation in the iteration procedure and with the ionization check. However the maximum 15% loss by version 2 may be rationalized in some cases. If dealing with a large number of events the time savings may be great enough that one could accept the 15% loss. Even a maximum 15% loss is not acceptable if one is dealing with a small number of fits. In that case the old method is preferable and the extra computation and ion checking time are necessary.

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