

P H Y S I C S

1. E L E M E N T A R Y P A R T I C L E T H E O R Y

✓ Explicit Wave Functions for any Spin⁽¹⁾: Y. Frishman* and E. Gotsman

A general method is given for calculating the exact form of the helicity amplitudes of a wave-function for any spin. Explicit expressions of the wave-functions for spin $J = 1, 3/2, 2$ and $5/2$ are listed. Examples of helicity amplitudes for the production of $J = 3/2^+$ and $J = 5/2^+$ isobars in proton-proton scattering are computed.

Reference:

1. FRISHMAN, Y. and GOTSMAN, E., Phys. Rev. 140, B1151 (1965), and Israel AEC Report, IA-1040 (1965)

A Bootstrap Calculation for the Mass of the Intermediate Vector Boson⁽¹⁾:
E. Gotsman and D. Horn**

Assuming that the weak interactions are mediated by a neutral intermediate vector boson, W_0 , a bootstrap type calculation is carried out to determine the mass of the particle (in the neutrino anti-neutrino channel). The requirement of self-consistency between input and output parameters leads to the result that the mass of the W_0 is approximately one thousand times the proton mass.

Reference:

1. GOSTMAN, E. and HORN, D., Phys. Rev. Letters, 15, 714 (1965)

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Tests of Relativistic SU(6) in Proton - Antiproton Annihilation : H. Harari
and H. J. Lipkin *

The proton - antiproton annihilation at rest into two mesons is proposed as a critical test of the various relativistic generalisations of the SU(6) theory. It is shown that every relativistic extension of SU(6) in which mesons with finite momenta are still in the 35-dimensional representation of SU(6) leads to a ratio of 16 between the production rates of charged and neutral K mesons. This ratio is clearly in contradiction with experimental data, and symmetry breaking effects must be introduced in order to reconcile the theoretical predictions with experimental results.

Reference:

1. HARARI, H. and LIPKIN, H. J., Phys. Letters, 15, 286 (1965)

Γ_0 Selection Rules in SU(12) Proton - Antiproton Annihilation : H. Harari
H. J. Lipkin * and S. Meshkov **

The operators $\Gamma_{op'}$, $\Gamma_{on'}$, $\Gamma_{o\lambda'}$, are defined by the operation of the Dirac matrix γ_0 on the SU(3) triplet of Dirac quarkspinors p' , n' , and λ' respectively. It is shown that these operators represent quantities conserved in SU(12), forbidding various processes. In particular, particles at rest are eigenstates of these operators. The annihilation, at rest, of the pp system into some multimeson final states is forbidden. The process $\bar{p} + p \rightarrow K^0 + \bar{K}^0 + \pi^+ + \pi^-$ is forbidden, while the processes :

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$$\bar{p}+p \rightarrow K^+ + K^- + \pi^+ + \pi^-$$

$$\bar{p}+p \rightarrow K^0 + K^- + \pi^+ + \pi^0$$

$$\bar{p}+p \rightarrow \bar{K}^0 + K^- + \pi^- + \pi^0$$

are allowed, at rest. At finite energies the cross sections of processes which are forbidden at rest are reduced by factors of v/c . At low velocities this may be detected experimentally.

Reference:

1. HARARI, H., LIPKIN, H. J. and MESHKOV, S., Phys. Rev. Letters, 14, 845 (1965)

Meson and Baryon Masses in SU(6) Symmetry⁽¹⁾: H. Harari and H. J. Lipkin*

The question of deriving mass formulae for the baryon and meson supermultiplets in the SU(6) symmetry model is studied. It is shown that the experimental values of meson and baryon masses are in disagreement with the assumption that the mass splittings in both SU(6) supermultiplets are given by the expectation values of the operators having the same definite transformation properties under SU(6). It is further argued that simple tadpole-type symmetry breaking mechanisms tend to give a definite linear combination of SU(3) singlet and octet terms of the mass operator, which transforms like the 405 representation of SU(6). Hence such theories are not capable of explaining the mass spectrum of the low-lying mesons and baryons.

Reference:

1. HARARI, H. and LIPKIN, H. J., Phys. Rev. Letters, 14, 570 (1965)

* Weizmann Institute of Science, Rehovoth

Electromagnetic Interactions and the Subgroups of $SU(6)^{(1,2)}$: H. Harari

Various sets of relations concerning the electromagnetic properties of the constituents of the $SU(6)$ supermultiplets were recently derived by various authors. A systematic study of these relations is presented, using subgroups of $SU(6)$ which are generalizations of the U-spin subgroup of $SU(3)$. It is shown that two different assumptions on the transformation properties of the electromagnetic contributions to the particle masses are not in contradiction with present data.

References:

1. HARARI, H., Phys. Rev., 139, B1323 (1965)
2. HARARI, H., Israel AEC Report IA-1023 (1965)

Intrinsic Spin and $U(12)$ Theory⁽¹⁾ : H. Harari

A complete classification of all the possible spin states of a system of any number of quarks and anti-quarks within the framework of $U(12)$ is presented. A one-to-one correspondence is shown to exist between all finite irreducible representations of $U(12)$ and the pair of numbers (N, J) where N is the total number of basic spinors in a system with a given total spin J . A set of $Sp(4)$ selection rules which are valid in any $U(12)$ invariant theory is derived.

Reference:

1. HARARI, H., Phys. Rev. Letters, 14, 1100 (1965)

Experimental Tests of Broken $U(12)$ and Approximate $U(6) \times U(6)^{(1)}$:

H. Harari and H. J. Lipkin *

It is shown that predictions based on certain subgroups of $U(12)$ remain valid both in broken $U(12)$ (to all orders in kinetons and simple derivative coupling) and in the Dashen Gell-Mann non-chiral $U(6) \times U(6)$

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approximate symmetry. Examples are also given of new predictions which follow from the W -spin collinear subgroup.

Reference:

1. HARARI, H. and LIPKIN, H. J., Phys. Rev. (in press)

Algebraic Aspects of Regge Recurrences⁽¹⁾: E. Gotsman and Y. Ne'eman*

The algebraic structure of Regge recurrences is examined and its relation to symmetry schemes underlying various models is established. The relationships between Spectrum Generating Algebras (S.G.A.) and the Regge non-compact sub-algebras are discussed for the Coulomb potential, the three-dimensional oscillator, the signature which appears in exchange potentials, as well as for the $U(6,6)$ symmetry of hadron physics.

Reference:

1. GOTSMAN, E. and NE'EMAN, Y., Math. Phys. (in press)

Three-Particle Final States and Unitary Symmetry⁽¹⁾: I.M. Bar-Nir* and H. Harari

A large number of new predictions of $SU(3)$ are derived for processes with three outgoing particles. All the reactions treated involve scattering of charged particles on protons, with no more than one neutral particle in the final state. They can all be measured by the present experimental techniques. A detailed comparison between the available data and the predictions is presented. In a few cases, disagreement is found, but when we allow symmetry breaking contributions within the matrix elements which describe the processes the data are always consistent with the predictions. The general problem of comparing the predictions of a broken symmetry with scattering data is discussed and various possibilities for performing such comparisons are suggested.

Reference:

1. BAR-NIR, I. M. and HARARI, H., Phys. Rev. (in press)

* Physics Department, Tel Aviv University

2. REACTOR PHYSICS

One-Dimensional Multigroup Perturbation Codes : M. Lemanska and S. Yiftah

The PERTR^(1,2) code was adapted to solve the material replacement problem for ZPR-III assemblies and for the fast, small, critical assemblies Jezebel, Godiva and Topsy. The results obtained for the central and non-central reactivity contributions of various samples were compared with the experimental values. In addition, the reactivity coefficients were computed and the results compared with the experimental data^(3,4) and with the values computed by other investigators^(5,6). The dependence of the computed results on the interval division in the perturbing sample and on the change of the fluxes resulting from the presence of the sample were considered.

A comparison was made between the perturbation method and the direct method for treating the central reactivity contribution problem.

The PERDIF⁽¹⁾ code was adapted to compute the Doppler effect for large fast breeder power reactors. The results were compared with those given by the diffusion code.

References:

1. LEMANSKA, M., and YIFTAH, S., Israel AEC Semi-Annual Report Jan.-June 1964, IA-984, p. 34.
2. LEMANSKA, M. and YIFTAH, S., Israel AEC Semi-Annual Report July-Dec., IA-1021, p. 17.
3. LANG, J.K., LOEVENSTEIN, W.B., BRANYAN, L.E., BRUNSON, G.S., KIVU, F.S., OKRANT, D., RICE, R.E. and THALGOTT, F.W., Fast Neutron Power Reactor Studies with ZPR-III A/CONF.15/8/598, July 1958.
4. ENGLE, L.B., HANSEN, G.E. and PAXTON, H.C., Nucl. Sci. and Eng. 8, 543 (1960).
5. HANSEN, G.E. and MAIER, Cl., Nucl. Sci. and Eng. 8, 532 (1960).
6. DAVEY, W.G., A Comparison of Experimental and Calculated Prompt Neutron Lifetimes and Central Reactivity Coefficients in ZPR-III Assemblies and their Relationship to other Reactor Parameters. ANL-6682, June 1963.

Fundamentals of Fast Reactor Calculations : M. Segev and S. Yiftah

A complete group-diffusion theory was developed for the calculation of fast reactors. The main features are set out below.

- a) The P_1 approximation for the Boltzman equation is formally, and without any physical assumptions, reduced to group equations and group cross sections.
- b) The exact meaning of flux separation (into energy and spatial parts) for reactor calculations is elucidated.
- c) It is shown that all (not only the transport) effective group-diffusion cross sections are higher (typically by 3-5%) than the corresponding effective group-physical ones.
- d) Approximate formulae are derived for the intergroup transfer parameters in elastic scattering. These are simple to apply and for $A \geq 10$ yield values which are within 1% of the exact ones.
- e) A theorem is proved which shows "Placek oscillations" in the collision density for elastic slowing-down in any moderator (excluding H) with any absorption and scattering laws.
- f) The asymptotic slowing-down equation is expanded in "slowing-down" moments. This expansion describes various problems such as anisotropy of scattering and scattering in mixtures.
- g) Simple analytical forms are derived for the solution of the slowing-down equation for typical homogeneous fast reactor mixtures. These solutions take into account the complicated nature of slowing-down such as the narrow and dense resonances of heavy elements; the resonances of structural elements and the mixtures of several elements.
- h) The above analytical formulae for the flux were compared with a numerical solution of the slowing-down equation coded for the Philco-2000. The comparisons show a very good agreements between the results. A full report is being written.

Collision Density for Typical Fast Reactor Conditions : M. Segev and S. Yiftah

A search for analytical solutions of the slowing-down equation under fast reactor conditions was motivated by the following considerations.

- a) In calculating effective cross sections, a constant collision density is assumed. Disregarding absorption this assumption is valid for heavy isotope resonances which are mostly narrow resonances (N.R.). The flux decay towards lower energies, caused by absorption, is to a certain extent taken into account by averaging the effective cross sections over some gross behaviour of the flux between the group limits. However such a fixed behaviour cannot account for changes in the rate of decay due to changes in the relative amount of absorbers.
- b) Neither narrow resonance (N.R.) nor wide resonance (W.R.) techniques apply to resonances of the lighter elements such as Fe, Na, O, C. The Elmoe Code finds the flux in such cases by iterations, through hundreds of groups. It is an involved technique since any change in the relative amount of the light elements calls for a recalculation of the flux and hence of the "coarse" (i.e. usual) group cross sections. Also the lack of an explicit relation between the amount of light elements and the flux is a disadvantage.
- c) An analytical knowledge of the collision density in a fast reactor will enable one to calculate group cross sections for wider, and hence less, groups than usual, thus reducing the time for a criticality computation. This is particularly important for two-dimensional calculations.

A later report will describe in detail the steps leading to an approximate and simple analytical solution of the slowing-down equation under fast reactor conditions. This solution depends explicitly on the relative amounts of each isotope in the mixture, on the isotope masses and on the energy-dependent cross sections. It does not depend on anything else; in particular, no iterations are involved. It thus describes in an analytic way the flux decay caused by absorption and the flux depressions under light-element resonances.

It is found that, in the energy range of fast reactors and for typical mixtures, the collision density under a scattering resonance of a heavy isotope is quite constant. Under a symmetrical resonance of a light or structural element it is depressed in an asymmetrical way. Such asymmetries were found and estimated in an iterative way by Goldstein⁽¹⁾ for intermediate resonances (I.R.).

For the sake of comparison and checking, a numerical solution to the slowing-down equation was coded for the Philco-2000 and run for several typical mixtures. The analytic solution was found to give a good estimate of the flux and a very good estimate of effective cross sections.

Reference:

1. GOLDSTEIN, R., Nucl. Sci. and Eng. 19, 359 (1964)

Sodium Void and Doppler Effects in Large Fast Power Reactors⁽¹⁾: S. Yiftah, L. Gitter and Y. Ilamed

Calculations were made in order to study the sodium void and Doppler effects in a series of fast reactors which differ in core volume, isotopic content (type) of Pu fuel, and chemical composition of fuel (metal, oxide, carbide). The sodium void and Doppler effects were obtained from criticality factors computed for 486 spherical systems. Details of the computations and results are given in ref.1. Since the calculations were undertaken in connection with parametric studies, the computational results were analysed to find analytical expressions useful for these studies. The following conclusions were drawn.

For a given reactor the sodium void effect at normal temperature, as a function of sodium leakage, is described to a good approximation by a parabola with negative curvature. Three criticality factor calculations are enough to obtain the parameters of the parabola.

For a particular composition and type of fuel, the parameters of the parabola are functions of the core volume. Using the parameters obtained, the sodium void effect for any volume can be calculated by interpolation.

The sodium void effect for any initial sodium content can be calculated as a simple difference, if the sodium void effect vs. leakage is known for one initial sodium content. For any given leakage the effect is more negative for lower initial sodium content.

The concept of "stationary volume" can be defined as the volume for which any change in sodium density (leakage or addition of sodium) will have a negative effect on the reactivity of the reactor. The stationary volume for any initial sodium content can be read directly from the graph of X_m (the leakage for which the sodium void effect is maximal) vs. volume, as obtained for one initial sodium content. Stationary volumes increase with decrease of initial sodium content.

For any given assembly at any temperature the sodium void effect becomes more positive as the volume increases. The stationary volume is the maximal volume for which the sodium void effect is non-positive at any leakage at normal temperature. For any leakage the negative Doppler effect increases with volume. For a given volume the Doppler effect does not change much with sodium leakage.

With respect to composition of fuel it appears that the oxide fuels have the most negative sodium void effect and consequently the largest stationary volumes. For large volumes with a given sodium content the Doppler effect is also most negative for oxide assemblies.

With respect to fuel type, the presence of greater amounts of the higher isotopes causes a more negative sodium void effect and thus the corresponding stationary volume is larger. The presence of higher isotopes also causes a more negative Doppler effect for any particular sodium content.

Thus oxide assemblies allow for the largest reactors with combined non-positive reactivity effects, and the change in the isotopic composition of the fuel occurring during operation will not affect this safety characteristic of the reactor unfavourably.

Reference:

1. YIFTAH, S., GITTER, L. and ILAMED, Y., Israel AEC Report, IA-1057, (1965)

Effect of Changes in Cross Section Increments on the Total Doppler Effect in Fast Reactors : S. Yiftah, L. Gitter and Y. Ilamed

We recently calculated the Doppler effect in large fast reactors⁽¹⁾. The cross section increments which enter into these calculations are affected by many complex factors, and it is possible that some of these may have been inadequately taken into account.

In order to obtain a better understanding of the dependence of the change in criticality factor, $\Delta c/c$, on the increments of the cross sections, $\Delta\sigma$, criticality changes were computed for a number of systems, with several different reasonable sets of $\Delta\sigma$ for each system. The various sets of $\Delta\sigma$ were computed by Dr. G. Shaviv by simplified Doppler calculations, taking into account effects that had previously been neglected. Computations were also carried out for increments equal to half the given increments $\Delta\sigma$; in addition the Doppler effect for each energy group ("local" effects) were calculated.

The following conclusions seem to be warranted from inspection of the computed reactivities and flux integrals of the systems:

- (i) In general, in the range of variation of $\Delta\sigma$ considered, i.e. for "small" $\Delta\sigma$ ($\Delta\sigma \ll \sigma$), the Doppler coefficient is additive in $\Delta\sigma$:

$$\frac{\Delta c}{c} (\Delta_1 \sigma_f + \Delta_2 \sigma_f, \Delta_1 \sigma_a + \Delta_2 \sigma_a) = \frac{\Delta c}{c} (\Delta_1 \sigma_f, \Delta_1 \sigma_a) + \frac{\Delta c}{c} (\Delta_2 \sigma_f, \Delta_2 \sigma_a)$$

For increasing $\Delta\sigma$ the Doppler coefficient becomes subadditive:

$$\frac{\Delta c}{c} (\Delta_1 \sigma_f + \Delta_2 \sigma_f, \Delta_1 \sigma_a + \Delta_2 \sigma_a) < \frac{\Delta c}{c} (\Delta_1 \sigma_f, \Delta_1 \sigma_a) + \frac{\Delta c}{c} (\Delta_2 \sigma_f, \Delta_2 \sigma_a)$$

- (ii) The contribution to the Doppler effect from neutrons of the lowest energy groups is small in metal assemblies, and high in carbide and oxide assemblies, the contribution being higher in oxide than in carbide assemblies. This is apparently due to the difference in the flux distribution. In metal assemblies there is a relatively small flux in the lowest energy group where the increase in cross section is relatively high, while in carbide and oxide systems there is a marked shift toward lower energies.
- (iii) The contribution to the Doppler effect from the lowest energy group decreases with the sodium leakage. This again seems to be due to the shift of the spectrum of the flux towards higher energies with the leakage of sodium.

A detailed report with tables of the computed results is in preparation.

References:

1. YIFTAH, S., GITTER, L. and ILAMED, Y., Israel AEC Report IA-1057 (1965)

Results of Doppler Coefficient Calculations for Fast Reactors and Comparison of Different Methods⁽¹⁾ : Part I by R. Froelich** and K. Ott**, Part II by G. Shaviv and S. Yiftah

Doppler coefficients of dilute fast reactors were calculated by the successive k method and by perturbation theory. The results obtained by the two methods were compared and discussed in detail over the temperature range

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** Kernforschungszentrum Karlsruhe

of interest. The Doppler coefficient during excursions was treated by taking into account the space distribution of the additional energy release and the dynamic spectra for reasonable asymptotic periods.

Reference:

1. FROELICH, R., OTT, K., SHAVIV, G. and YIFTAH, S., Paper Presented at International Conference on Safety, Fuels and Core Design in Large Fast Power Reactors, Argonne, October 11-13, 1965

Effect of Pu Isotopic Composition on the Doppler Coefficient in Fast Reactors : G. Shaviv and S. Yiftah

The effect of the plutonium isotopic composition on the Doppler coefficient was examined in fast reactors having different chemical compositions of the fuel and different core volumes. It was found that for a given core volume and chemical composition the absolute value of the Doppler coefficient increases with increase in the amount of high plutonium isotopes (Pu^{240} , Pu^{241} and Pu^{242}).

Monte Carlo Investigation of the Heterogeneity of Fast Reactor Cores : G. Shaviv and S. Yiftah

In a reactor the effective cross section for a reaction over a given energy group is usually defined as that value which when multiplied by the group flux gives the reaction rate in the group under consideration. Thus for the energy group between E_g and E_{g+1}

$$\tilde{\mu}_x = \frac{\int_{E_g}^{E_{g+1}} \phi \mu_x dE}{\int_{E_g}^{E_{g+1}} \phi dE} \quad (1)$$

Eq. (1) can be written in the form

$$\tilde{\mu}_x = \frac{\int_{E_g}^{E_{g+1}} \frac{F \mu_x}{\mu_t} dE}{\int_{E_g}^{E_{g+1}} \frac{F}{\mu_t} dE} \quad (2)$$

Using eq. (2) effective cross sections were calculated by means of a Monte Carlo code adapted from the NYU program which calculates resonance capture in thermal reactors⁽¹⁾. This method is based on the fact that the collisions recorded in a Monte Carlo history in an element R of phase space (volume, energy, direction) give an estimate of the collision density integrated over R. Consequently, appropriate summations of (μ_x / μ_t) and $(1/\mu_t)$ at these collisions provide estimates of the reaction rates and the integrated flux over R, which are the terms required to evaluate equation (2).

The Monte Carlo results for a typical fast reactor lattice confirm the fact that core heterogeneity has only a slight influence on the effective group cross sections in fast reactors. In the calculations a number of effects which are normally ignored in analytical studies of this problem were included. The statistical inaccuracies inherent in the Monte Carlo method are small enough not to invalidate the general conclusions reached.

Reference:

1. RICHTMYER, R. D., VAN-NORTON, R. and WOLFE, A., The Monte Carlo Calculation of Resonance Capture in Reactor Lattices, Proc. 2nd UN Conf. Geneva 16, 180 (1958)

Monte Carlo Investigation of the Interaction between the Resonances of Fertile and Fissile Isotopes in Fast Reactor Cores : G. Shaviv and S. Yiftah

The problem of the interaction between the resonances of the fertile and fissile isotopes in fast reactor cores and its influence on the effective and group effective cross sections has been treated analytically by various

authors^(1,2). In all analytic methods the simplifying assumption is made that the total collision density is constant. This assumption is generally a good approximation for a single resonance which is narrow compared with the energy degradation occurring in the collisions of a neutron with the atoms of the medium. However fluctuations in the total collision density may arise due to interaction between resonance sequences belonging to different isotopes. The aim of this work was to see whether these fluctuations have any effect on the interaction between the resonances of the fertile and fissile isotopes in fast reactors.

The effective cross sections were calculated by the Monte Carlo method, in which it is not necessary to assume that the total collision density is constant over the resonance. Table 6 summarizes the results obtained for the effective cross section of Pu at two energies and two temperatures. It can be seen that the effect of the resonances of U²³⁸ on those of Pu²³⁹ is to reduce the effective absorption and fission cross sections by a few per cent. The results are in good general agreement with those obtained by Codd and Collins⁽¹⁾ who used different conditions and energies. This shows that the assumption of constant collision density is adequate for the situation discussed here.

TABLE 6
Effect of the interaction between the U²³⁸ and Pu²³⁹
resonances on the effective cross section of Pu²³⁹

E (keV)	T (°K)	Effective cross section for fission (Fermi)		Effective cross section for absorption (Fermi)	
		without U ²³⁸ resonances	with U ²³⁸ resonances	without U ²³⁸ resonances	with U ²³⁸ resonances
4-5	300	2.179±0.0034	2.043±0.0029	1.216±0.0017	1.167±0.0015
	1500	2.241±0.0028	2.087±0.0031	1.247±0.0014	1.197±0.0015
9-10	300	2.291±0.0034	2.036±0.0037	0.7792±0.0017	0.7658±0.0016
	1500	2.327±0.0030	2.054±0.0032	0.7868±0.0015	0.7711±0.0016

The large errors inherent in the Monte Carlo method prevent any definite conclusions with regard to the temperature variation of the effect studied. An investigation of the temperature effect must await the compilation of a Monte Carlo code aimed specifically at the study of temperature variations.

References:

1. CODD, J. and COLLINS, P.J., Some Calculations Concerning the Influence of Resonance Overlapping on the Doppler Effect in a Dilute Fast Reactor, Proc. of the Conf. on Breeding, Economics and Safety in Large Fast Power Reactors, Oct. 7-10, 1963, ANL-6792.
2. HWANG, R.N., An Improved Method of Doppler Effect Calculation for Fissile Materials in the Intermediate Energy Region, *ibid.*

3. NUCLEAR PHYSICS

During the past year three new experiments have been installed in Beam Tube 1 of the IRR-1 reactor. Two of them - capture gamma ray spectroscopy, and Mössbauer studies with capture gamma rays - use the newly completed external neutron beam (see p. 312). The third experiment utilises a radioactive gas loop, the gas being irradiated at the core face, for fluorescence gamma scattering experiments. Mössbauer studies during the year have been centered on the use of I^{129} to investigate the structure of molecular crystals, and on the study of phase transitions in ferro-electric materials. The Mössbauer source, 70 min Te^{129} , is prepared in the reactor. The range of the low temperature Mössbauer experiments has recently been extended by acquiring a Cryo-Tip open cycle Joule-Thompson cryostat, using commercial hydrogen.

Resonant Scattering: G. Ben-David, B. Arad, Y. Schlesinger

(I) Resonant Scattering from Highly Excited Nuclear Levels

The fifty resonance levels so far discovered were re-examined, and the effective cross sections evaluated more precisely, making use of improved stability of the electronic equipment, and more accurate methods of calculation. The results have been submitted for publication⁽¹⁾. It may be possible to achieve a large increase in the number of measurable resonances by increasing the flux of gammas and using detectors of higher resolution. A brief summary of the results was presented at the Antwerp conference on nuclear structure⁽²⁾.

(II) Measurement of the 7.64 MeV Level in Tl by Self-Absorption Technique

The strong resonance observed for the 7.64 MeV level in Tl permitted measurement of the level width using the self-absorption method. For a target thickness of 3.81 g/cm^2 , and resonant and non-resonant absorber thicknesses of 14.85 g/cm^2 each, the self-absorption coefficient was re-measured and a value of 0.17 ± 0.2 obtained. Making use of the known temperature dependence of the resonance, and the measured effective cross section, the level width was estimated assuming the Doppler approximation for the cross section. A value of $\Gamma/\alpha g \cong 0.6 \text{ eV}$ was obtained, where α is the natural abundance of the resonant isotope and g the ratio of the spins of the excited and ground states. Accurate computation of the cross section requires the use of a computer programme, which has just been completed (see next paragraph).

(III) Evaluation of the Doppler Integral $\psi(x,t)$ using Computer Methods

Accurate analysis and interpretation of experimental resonant scattering data requires calculation of the Doppler integral in a very broad region of the parameters (x,t) . The existing compilation of this function⁽³⁾ does not cover the whole region required. In addition, the Doppler and Lorentzian approximations used in the calculations can cause large errors if not applied strictly in the appropriate regions of x and t . It was found possible to calculate the integral using a particularly simple numerical procedure, to an accuracy as high as $1 \text{ in } 10^8$, which is far better than the accuracy of any previous compilation in the required range of parameters. The sub-routine PSI (x,t) is now being used for general analysis of the experimental data, and will be described in an IA-report. In Table 7 the values obtained for the ψ -function are compared with those given in the published tabulations.

TABLE 7

Comparison of values of the ψ -function from different tabulations

β	Ref.	X				
		0.0	0.1	1.0	10.0	100.0
10^{-3}	C	1.000000	.990099	.500000	.009901	.000010
	D	.9999987	.99009853	.50000012	.009900990	.000009999
1.0	A	.75783054	.75510317	.5401622	.01004972	
	B	.75957	.75684	.54082	.010049	
	C	.757872	.755143	.540145	.010050	.000100
	D	.75787296	.75514314	.54014446	.01004971	.00010000499
10^2	A	.017527		.017525	.017354	.006524
	C	.017526	.017526	.017525	.017354	.006535
	D	.0175263108	.01752629	.0175245	.0173548	.00653508
10^4	D	.00017722551	.0001772255	.0001772255	.00017722533	.00017720779

A - Rose, Miranker, Leak, Rosenthal and Henriekson. WAPD.SR.506
Vol. I and II (1954).

B - Seth and Tabony. TID.21304 (1964).

C - Dandeu, Gautier, Guillerniu, Olivie and Roche. CEA.R2824 (1965).

D - Present work.

(IV) Coincidence Measurement of the 7.53 MeV Level in Tellurium

The complex scattered spectrum found in the scattering of Ni capture gamma rays from a Te scatterer has been examined by a coincidence technique, in an attempt to identify the inelastic scattering components. Both a two-dimensional pulse height analyser and a one-dimensional multichannel analyser gated by the sum-coincidence pulse were used. The preliminary results are in agreement with the assumption that inelastic scattering gives rise to the 6.7 MeV and 5.8 MeV components. A further

study of this resonance, using lithium drifted germanium detectors, will be performed to establish finally the inelastic energy levels and isotopes responsible for the resonance.

References:

1. BEN-DAVID, G., ARAD, B., BALDERMAN, J. and SCHLESINGER, Y., Israel AEC Report IA-1050 (1965) and Phys. Review, April 1966, (in press).
2. BEN-DAVID, G., ARAD, B., BALDERMAN, J. and SCHLESINGER, Y., Intern. Conf. on the Study of Nuclear Structure with Neutrons, Antwerp, Belgium, July 1965.
3. DANDEU, Y., OLIVIE, G. and ROCHE, F., Table de L'integrale $\psi(\beta, x)$, Rapport No. 43 bis, Section Aut. de Calcul Electr.

Measurement of the Mean Life of the First and Second Excited States in Na²³:

R. Moreh and J. Balderman

An experimental set-up is now ready for the measurement of the lifetime of the first (0.44 MeV) and second (2.07 MeV) excited states of Na²³, using the resonance fluorescence technique. The levels of Na²³ are populated by Ne²³, which is obtained from the Ne²²(n, γ) reaction by irradiating spectroscopically pure neon in the IRR-1 reactor.

The thermal neutron flux around the activation chamber is $\sim 10^{13}$ n/cm² sec. To obtain an equilibrium Ne²³ source, the gas is circulated through a loop by means of an ordinary rotary pump, the oil contamination being removed using a liquid air trap containing glass wool. The cylindrical copper source container (diameter 40 mm, length 60 mm, wall thickness 1.5 mm) is placed inside a large perspex chamber surrounded by a shielding installation to reduce the high background of γ -rays. The sodium scatterer was prepared by melting the metal under paraffin oil, then cooling it and pressing the solid sodium into the appropriate shape under oil and covering it with 0.3 mm PVC foil.

The positions of several energy levels in O^{18} and O^{19} were established, and the angular distribution of proton groups leading to several levels in O^{17} , O^{18} and O^{19} were measured.

The experimental results were analysed using the Bulter-Born theory and an estimate was obtained of the amplitudes of the simple shell model configuration and deformed states in several levels of O^{18} . The results compare fairly well with recent theoretical estimates by Federman and Talmi⁽⁵⁾.

(III) Gamma Transitions in O^{18} *

The levels at 5.25 and 5.37 MeV in O^{18} have been investigated using the $O^{17}(d,p\gamma)O^{18}$ reaction. The 5.25 MeV level was found to decay $41 \pm 6\%$ to the 1.98 MeV level and $59 \pm 6\%$ to the ground state. The 5.37 MeV level was found to decay $89 \pm 3\%$ to the 1.98 MeV level and $11 \pm 3\%$ to the 3.92 MeV level, with an upper limit of 3% and 2% to the 3.55 MeV and ground state respectively. The results are in agreement with the assignment of 2^+ to the 5.25 MeV level and 3^+ to the 5.37 MeV level.

References:

1. MOREH, R., Nucl. Phys. 70, 293 (1965).
2. MOREH, R. and DANIELS, T., Nucl. Phys., 74, 403 (1965).
3. JAFFE, A.A., et. al, Proc. Phys. Soc. 76, 914 (1960)
4. MIDDLETON, R. and PULLEN, D.J., Nucl. Phys. 51, 50 (1964)
5. FEDERMAN, P. and TALMI, I., Phys. Letters 15, 165 (1965)

Mössbauer Studies of Various Ferrite Structures: T. Rothem and D. Kadem

A Mössbauer spectrometer of good resolution and linearity has been built. Its saw-tooth drive is provided by the Intertechnique 400 channel analyser.

The resolution is 0.2 mm/sec, as measured with a thin absorber of natural Fe.

*

The experimental work was carried out at the University of Manchester.

The comparison scatterer was made by filling a ring-shaped aluminium container with the appropriate weight of MgO, matched for both Compton scattering and pair production to give the same effect as the sodium scatterer.

Energy Levels of Oxygen Isotopes: R. Moreh

(I) Study of the (t,p) Reaction on O^{16} , O^{17} and O^{18} using 5.55 MeV Tritons ^{*(1,2)}

The (t,p) reaction on O^{16} , O^{17} and O^{18} was studied at a triton bombarding energy of 5.55 MeV. Special attention was given to the reaction $O^{17}(t,p)O^{19}$. The position of several energy levels in O^{19} was established. Good agreement was obtained in some cases between the measured proton angular distributions and the predictions of the plane wave double stripping theory. In other cases the contribution of other reaction modes seems to be appreciable.

The spin and parity of the ground and 3.153 MeV levels in O^{19} obtained by the $O^{17}(t,p)$ reaction were determined unambiguously to be $5/2^+$, arising from the $d_{5/2}^3$ and $d_{5/2}^2 s_{1/2}^2$ configurations respectively. The 0.093 and 2.775 MeV levels were identified with the $3/2^+$ and $9/2^+$ levels respectively of the $d_{5/2}^3$ configuration.

The results for the $O^{16}(t,p)O^{18}$ reaction agreed with those of Jaffe et al. ⁽³⁾, who used tritons of 5.50 MeV. In the $O^{18}(t,p)O^{20}$ reaction the spin and parity assignments for the ground and first three excited levels of O^{20} were in essential agreement with those of Middleton et al. ⁽⁴⁾ who used tritons of 10 MeV.

(II) The Level Structure of O^{18} and O^{19} by Stripping Reactions ^{*}

The (d,p) reactions on O^{16} , O^{17} and O^{18} were investigated at a deuteron energy of 5.56 MeV, with a view to studying the O^{18} energy levels.

The Q values of the ground state transitions in the reactions $O^{17}(d,p)O^{18}$ and $O^{18}(d,p)O^{19}$ were determined to be 5.820 ± 0.010 and 1.734 ± 0.010 MeV respectively.

* The experimental work was carried out at the University of Manchester.

Mössbauer Effect in Molecular Iodine Crystals⁽¹⁾ : M. Pasternak,
A. Simopoulos*, Y. Hazony

The Mössbauer effect of the 27.7 keV transition in I^{129} has been studied in molecular iodine crystals. From the quadrupole splitting of the transitions (see Fig.8) the ratio $Q_{26.8}/Q_{gnd}$, the quadrupole coupling $e^2 q Q_{gnd}$, and the isomer shift were determined. The sign of the quadrupole coupling shows that the sign of the iodine field gradient is positive. From the isomer shift of I_2 and from those of alkali iodides, a linear dependence of the isomer shift on the number of p-holes was derived. The intensities of the peaks give a strong indication of an angular dependence of the recoilless fraction $f(\theta)$. The θ_D of the I_2 crystal, as well as that of the KI crystal, was derived from the temperature dependence of the recoilless fraction.

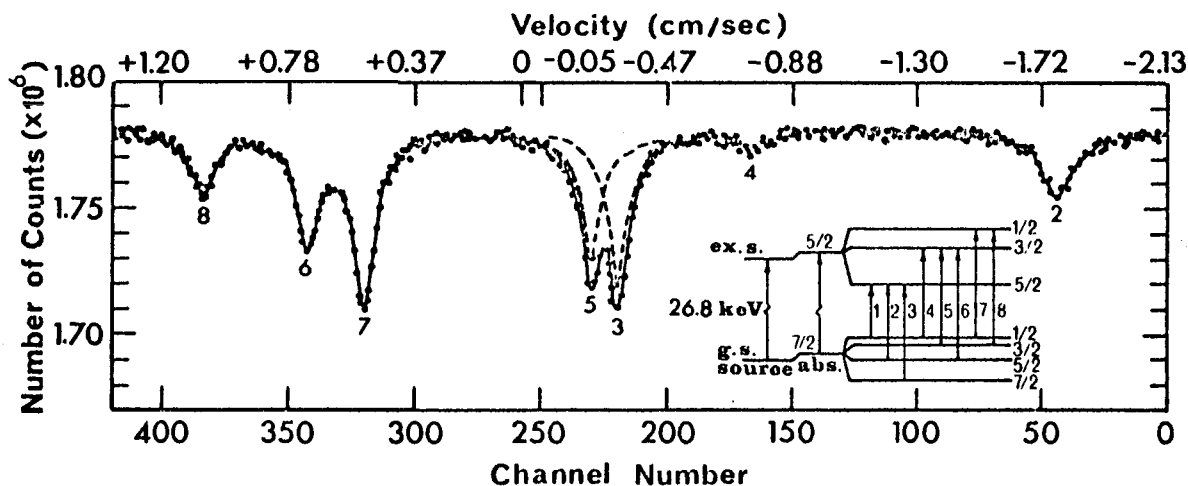


Fig.8

Quadrupole splitting of I_2 (molecular iodine crystal),
measured with a Zn^{66} - Te^{129} source. Both source and
absorber were at liquid nitrogen temperature

Reference :

1. PASTERNAK, M., SIMOPOULOS, A., HAZONY, Y., Phys. Rev., 140 6A 1892, (1965)

* IAEA fellow, on leave from the Nuclear Research Center "Democritos",
Athens, Greece

Measurements have been performed on ferrite absorbers of ZnFe_2O_4 , CuFe_2O_4 and CdFe_2O_4 . The results indicate that the internal magnetic field of a Fe nucleus varies according to the site at which it is located in the ferrite crystal lattice. Zinc ferrite, for instance, which is a normal spinel where Fe atoms are all situated in octahedral sites, gave no magnetic splitting either at room temperature or at liquid nitrogen temperature. It did show, however, a significant quadrupole splitting (0.32 mm/sec).

Measurements are in progress to determine the behaviour of these compounds at lower temperatures.

Absolute Recoilless Fraction of ZnTe: S. Bukshpan and M. Pasternak

The absolute recoilless fraction, f , of ZnTe was measured at 104°K using a cubic absorber - CuI^{129} . The experiment was performed by measuring the Mössbauer efficiency at increasing absorber thicknesses, in order to find the "saturation" absorption depth.

The absorption depth $\epsilon(0)$ is related to f_s according to the following formula⁽¹⁾

$$\epsilon(0) = f_s \left(1 - \exp - \frac{T_a}{2} \cdot J \left(i \frac{T_a}{2} \right) \right)$$

where $T_a = f_a n_a t_a \sigma_0$. Due to the behaviour of the Bessel function $J(i \frac{T_a}{2})$, $\epsilon(0)$ was saturated at $\sim f_s \times 0.85$. The following values were obtained:

$$f(\text{ZnTe}) = 0.21 \pm 0.01$$

$$f(\text{CuI}) = 0.10 \pm 0.01$$

By plotting the absorber line width as a function of the absorber thickness, the half-life of the 27.7 keV level was derived. It was found to be $\frac{T_{1/2}}{2} = 15$ nsec, which is shorter by 10% compared with the data obtained by the conventional electronic technique⁽²⁾.

References:

1. MARGULIES, S. and EHRMAN, J.R., Nuclear Instr. 12, 131 (1961).
2. DEWAARD, H., private communication

Mössbauer Effect in I_2Cl_6 : M. Pasternak

The I_2Cl_6 molecule has a planar structure. The field gradient acting on the iodine nucleus is produced by the p electron lone pairs directed perpendicular to the I-Cl bond.

An I_2Cl_6 absorber was prepared by direct combination of chlorine and I^{129}_2 at $-75^\circ C$. The quadrupole coupling at $90^\circ K$ was determined from the positions of the quadrupole lines. It was found to be $e q Q = + 2144$ Mc/sec, which agrees to within 1% with the data obtained by the N.Q.R. technique. The plus sign indicates that the field gradient is produced by the lone electron pairs. The isomer shift related to the ZnTe source was found to be ± 0.35 cm/sec, indicating that the I-Cl bonds are chiefly produced by p electrons. The quadrupole peak intensities give no indication of anisotropy in the recoilless fraction.

Mössbauer Effect in Ferroelectric Compounds of Iodine : S. Bukshpan

A study has been initiated of the Mössbauer effect in ferroelectric iodine compounds, specifically NH_4I , $(NH_4)_3IO_6$, $Ag_2H_3IO_6$ and BiSI. Preliminary measurements made with NH_4I show a very low effect near the temperature of phase transition. Attempts will be made to get a higher effect by using a source with higher efficiency; for instance, a solid solution of V and 5% Te. Experiments to prepare such a source are in progress. Of the other compounds, BiSI has been prepared by a chemical process. It has a low transition temperature ($-160^\circ C$) and can be used with the standard ZnTe source.

Mössbauer Effect of Co^{57} Impurities Diffused in LiOH Crystals: A. Simopoulos *

The excitation of approximately localized modes in light crystals doped with heavy impurities has been theoretically predicted and experimentally observed by the infrared technique⁽¹⁾. On the basis of theoretical calculations⁽²⁾ for the use of the Mössbauer technique to study these modes, we have started a project in this direction. Experiments with the conventional Mössbauer technique (zero phonon transitions) are being conducted using LiOH crystals as a host matrix and Co^{57} as impurity.

Results with single crystals show anisotropy of the binding of the impurity atoms in accordance with the anisotropy of the crystal. The overall spectrum suggests the existence of two impurity sites in the crystal, one corresponding to an Fe^{3+} ion (the decay product of Co^{57}) and the other probably to an Fe^+ ion. Work is in progress to relate the atomic motion of the host atoms to that of the impurity and to establish, with more confidence, the existence of the Fe^+ site. The temperature dependence of the recoil-free fraction will give information about the existence of approximately localized modes.

Three other related projects are in the planning stage :

- a) Further experiments with $\text{Mg}(\text{OH})_2$ and $\text{Ca}(\text{OH})_2$ as host matrices.
- b) Study of vanadium hydrides doped with Co^{57} . These hydrides have been studied by neutron inelastic scattering and the results show new low-frequency modes not present in the spectrum of the vanadium metal. Such modes could also be detected by the Mössbauer technique.
- c) Study of KI crystals doped with LiI. Electrocalorimetric measurements⁽³⁾ have shown tunneling modes of the Li impurity inside the crystal. These modes amount to 1°K . Such a mode could be detected easily with a "one-phonon transition" experiment by using the Mössbauer technique.

References:

1. SIEVERS, A.J., Phys. Rev., Letters 13, 310 (1964)
2. BROUT, R. and VISSCHER, W.M., Phys. Rev. Letters 9, 54 (1962)
3. LOMBARDO, G. and POHL, R.O., Phys. Rev. Letters 15, 291 (1965)

IAEA fellow, on leave from the Nuclear Research Center "Democritos", Athens, Greece

of nucleons in the smaller sphere, A , and the ratio between the heavy and light spheres, α , which gives the degree of asymmetry. Mathematical saddle points were calculated in the two dimensional space of A and d for various values of α . The energies of these points are plotted as a function of α in Fig.9. The curves go through a minimum for α different from 1, indicating that the nuclear shape at the saddle point is asymmetric.

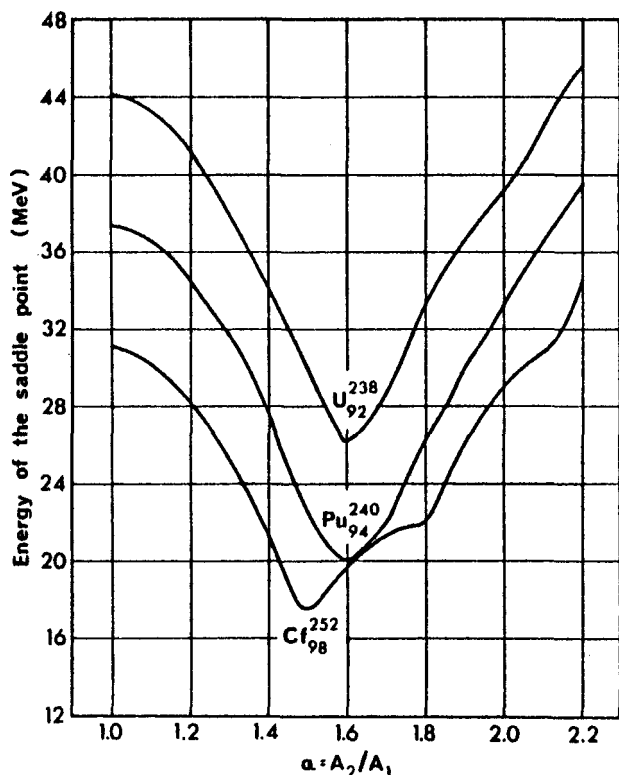


Fig. 9

Energy of saddle point as a function of the ratio α between the heavy and light spheres

Reference:

1. SWIATECKI, W., Proc. of the Second International Conference on Nuclidic Masses (1965) p.58, IAEA Vienna.

Alpha Particle Associated Fission in U^{235} : Y. Gozez, G. Ben-David, R. Moreh and A. Katase *

The preliminary experimental results, obtained with the apparatus described previously⁽¹⁾, showed excessive pile-up due to the high gamma fluxes present in the reactor thermal column. This was sufficient to distort

* Permanent address : Kyashu University, Japan.

Mean Square Displacement of Iodine in ICN: M. Pasternak and T. Sonnino

The ICN crystal is an interesting compound for measuring the atomic mean square displacement $\langle x^2 \rangle$. The crystal consists of covalently bonded ICN, forming linear chains bound by weak Van der Waals forces⁽¹⁾. The aim of the present experiment is to measure $\langle x^2 \rangle$ by depositing oriented crystals and measuring the absolute Mössbauer fraction f ($\theta = 90^\circ$).

Assuming $\langle r^2 \rangle = \langle x^2 \rangle_{\perp} \cos^2 \theta + \langle x^2 \rangle_{\parallel} \sin^2 \theta$, we have derived, using a computer, the relation between $\langle x^2 \rangle_{\perp} - \langle x^2 \rangle_{\parallel}$ and the ratio $\frac{R(\Delta m = 0)}{R(\Delta m = 1)}$, where $R(\Delta m)$ is the normalized quadrupole intensity for $\Delta m = 0$ and $\Delta m = \pm 1$ transitions. The values of f ($\theta = 90^\circ$) and f for randomly oriented crystals will be measured, and from these and the $R(\Delta m)$ ratios (Goldanskii effect⁽²⁾) the anisotropy of the recoilless fraction will be derived.

References:

1. WYCKOFF, R.W.G., Crystal Structures, Vol 1 Interscience, N.Y. 1948
2. GOLDANSKII, V.I., MAKAROV, E.F. and KHRAPOV, V.V., J. Exper. and Theor. Phys. 17, 508 (1963).

Saddle Point Calculations for Fission, Taking into Account Shell Effects:

E. Nardi, E. Cheifetz*, Y. Boneh*

Saddle point calculations which take into account shell effects have been carried out for fission in heavy nuclei. The nuclear shape at and around the saddle is described by two spherical clusters connected by a cylindrical neck. Shell effects are introduced by attributing to each spherical cluster the shell correction term for an equivalent nucleus, taken from the liquid drop mass formula of Swiatecki⁽¹⁾. The parameters describing the problem are the distance between the centers of the spheres, d , the number

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Weizmann Institute of Science, Rehovot.

the alpha spectrum considerably. The electronic set-up has now been modified by using biased amplifiers to cut out the low energy pulses. Improvements in the three-dimensional analyser have been effected, and data collection is continuing.

Reference:

1. GOZEZ, Y., BEN-DAVID, G., and MOREH, R., Israel AEC Semi-Annual Report July-Dec. 1964, IA-1021, p.25.

Neutron Emission in Triple Fission: E. Nardi and Z. Fraenkel*

Measurements are being continued in the experiment to determine the number of prompt neutrons emitted in the ternary fission of Cf^{252} . The pulse height data are recorded on an N.D.A. 4-dimensional analyser. Preliminary values of the total number of ternary fission neutrons have been obtained. A Monte-Carlo calculation was performed to compute the efficiency of the plastic scintillator and the time distribution for neutrons of different energies and incident angles. Specifically, a calculation was made for the geometry of the experiment (25 cm source-to-crystal distance). The results indicate the crystal to be relatively insensitive to differences in neutron energies.

Reference:

1. NARDI, E. and FRAENKEL, Z., Israel AEC Semi-Annual Report, July-Dec. 1964, IA-1021, p.27.

Evaporation Calculations: L. Meilen and Z. Fraenkel*

A computer programme for computing the evaporation of particles from a nucleus on the basis of the compound nucleus and statistical models has been improved and completed.

* Weizmann Institute of Science, Rehovot

The programme can be used for two different types of processes:

- a) Evaporation from nuclei bombarded with particles of medium energy
(<50 MeV)
- b) Evaporation from excited nuclei after a prompt cascade.

The programme can compute the competition between fission and neutron emission in heavy elements, and the evaporation of neutrons from the fission fragments.

The output table gives the spectrum of the residual nuclei and the spectra of the emitted particles.

Resonance Fluorescence of γ -Rays Using Gaseous and Liquid Sources :

T. Rothem and B. Locker *

An experimental arrangement has been completed for the study of resonance fluorescence of γ -rays using gaseous and liquid sources. The resonant nuclei are embedded in ferrites. Sources include Co^{60} in the form of CoCl_2 and Mn^{56} in the form of MnCl_2 . Resonance scattering measurements in $\text{NiO.Fe}_2\text{O}_3$ and $\text{CoO.Fe}_2\text{O}_3$ have been started.

Decay Schemes of $\text{Sn}^{121\text{m}}$ and $\text{Te}^{121\text{m}}$: B. Locker* and Y. Hazony

Coincidence measurements have been performed for a) the ~ 400 keV β -rays of a $\text{Sn}^{121\text{m}}$ source and the low energy gamma spectrum; b) the 1.1 MeV gamma rays of $\text{Te}^{121\text{m}}$ and the low energy gamma spectrum. Both measurements showed a gamma transition of ~ 9 keV. This transition is in cascade with a ~ 28 keV gamma transition to the ground state of Sb^{121} . The results have been prepared for publication.

The various methods used for the preparation and purification of the $\text{Sn}^{121\text{m}}$ source will be published as an IA report(IA-1081) .

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Bar-Ilan University

Study of (n,α) Reactions using Thermal Neutrons: G..Hazan, R. Moreh,
A. Katase* and G. Gozez.

Experiments have been continued on the (n,α) reaction induced by thermal neutrons in medium weight nuclei⁽¹⁾. A small reaction chamber was used, with 10 cm thick lead shielding walls to reduce background due to gamma rays. The whole arrangement, including preamplifier, was inserted into the thermal column by means of a special loading device. Modifications were introduced in the electronic circuit to reduce the effect of gamma pile-up which limited the sensitivity of the experiment for low cross section reactions. After fast differentiations, a biased amplifier is used to cut off the low energy signals, the stretched output being fed to a P.H.A. The apparatus can now be used to measure (n,α) cross sections as low as a few tens of μ -barns.

A preliminary 30 hour run with Mo^{95} has not shown any significant peak near the energy of the expected Q value for the ground state transition. This experiment is continuing. Further studies are planned for Xe, Hg, and Sm.

Reference

1. CHEIFETZ, E., GILAT, J., YAVIN, A.I., ASHERI, D., COHEN, S.G. and OPATOWSKY, A., Phys. Letters 1, 289 (1962).

* Permanent address : Kyashu University, Japan.

4. S O L I D S T A T E P H Y S I C S

During the past year an NMR Laboratory has been set up. It is equipped with a 15" electromagnet capable of producing a field of 23,000 gauss in a 2" air gap. A pulsed nuclear resonance spectrometer operating between 2 and 30 Mc has been built. The RF pulse output power is 5 kW over most of the range. This spectrometer is suitable for experiments in solids. Various other instruments for the NMR experiments in solids have also been built.

Nuclear Magnetic Resonance Study of Hydrogen Alloying in the Early Transition Metals (Group V B) ⁽¹⁾ : D. Zamir

The Knight shifts and spin lattice relaxation times of vanadium and niobium in the VH_x and NbH_x systems respectively have been measured. From the observed temperature dependence of the relaxation time the contribution of the conduction electrons to the nuclear relaxation was found. A comparison of the Knight shifts and relaxation times of V and Nb in the hydrides with those for the corresponding alloys, VCr_x and $NbMo_x$, shows a great similarity between the two types of systems. Some of the implications of this similarity have been examined. Using a Korringa-type relation for Nb in the NbH_x system, the type of conduction electrons giving rise to the Knight shift and relaxation rate was investigated.

Reference:

1. ZAMIR, D., Phys. Rev. 140, A 271 (1965)

Infrared Spectra of Small Ionic Crystals * : R. Englman and R. Ruppin

The long wavelength part of the dispersion curves for a thin disc-shaped rocksalt type crystal is calculated by means of an approximate model,

* This research was sponsored by the Air Force Materials Laboratory, Research and Technology Division, AFSC, through the European Office of Aerospace Research, United States Air Force, Contract AF 61 (052) - 753.

which employs cyclic boundary conditions. It is found that for wavelengths of the same order as, or smaller than, the thickness of the crystal, significant shifts in the frequencies of the optical branches occur.

Absorption frequencies and oscillator strengths are calculated as a function of the thickness of the specimen and of the angle of incidence of the infrared radiation. It is found that, in addition to the usual transverse optical frequency, the crystal absorbs at other frequencies which are either less than the transverse optical frequencies, or between the transverse and the longitudinal frequencies or greater than the longitudinal frequencies.

Recently we have developed an approach which can be applied to finite solids of very general shapes in order to obtain those vibrational modes and frequencies which are affected by the presence of the boundary.

The Operator for Spin-Forbidden Transitions^{*} : R. Englman

With the Dirac equation as a basis we have shown that the matrix element of the dipole moment, taken between four-component spinors, gives the fundamental transition amplitude for spin-forbidden transitions. It was found that very frequently it is sufficient to take the same matrix element, but only between the large components, those which are the solutions of the Pauli-Hamiltonian. Finally, it was shown that because it excludes hole-excitations, the Pauli-Hamiltonian cannot possibly give a rigorous expression for the transition matrix.

* This research was sponsored by the Air Force Cambridge Research Laboratories through the European Office of Aerospace Research, United States Air Force, Contract AF 61 (052) - 806.

Studies in the Vibronic Theory of Ferroelectrics^{*} : R. Englman and
C. Yatom

It was shown in ref. 1 that the vibronic theory of ferroelectricity reduces to a simple solvable form (the Wigner-Jaynes formulation) in the mean interaction approximation. We have now derived the parameters appearing in the Devonshire phenomenological expression for the free energy. The parameters depend on the temperature explicitly and are functions of several constants of the microscopic theory.

The results of Bell and Rupprecht⁽²⁾ for the dielectric constant $\epsilon = \epsilon_L + C(T - T_0)^{-1}$ in five perovskites show that ϵ_L is an order of magnitude smaller than C/T_0 . We investigated the consequences for ϵ_L of an electronic band scheme rather than a degenerate level scheme, and found that ϵ_L may be affected considerably provided that the Fermi-level is near to the crossing point of the energy bands or to the point (in k space) of closest approach of these bands. For the calculations to be significant for a real crystal, the distance between the Fermi-level and the crossing point or the point of closest approach must be of the order of the transition temperature.

References:

1. ENGLMAN, R., Israel AEC Report IA-994 (Chapter 4)
2. RUPPRECHT, G. and BELL, R. D., Rev. 135, A748 (1964)

* This research was sponsored by the Air Force Materials Laboratory, Research and Technology Division, AFSC, through the European Office of Aerospace Research, United States Air Force, Contract AF 61 (052)-753.

Covalency Effects in Trigonal Distortion* : R. Englman

The strength of the axial field, as derived from the splittings of the optical bands of paramagnetic metal ions in crystals of trigonal symmetry, fluctuates greatly from band to band. We have made a qualitative analysis of the effect on the assumption that, in addition to the (mostly) positive trigonal field splitting parameter acting on the metal ions, there is also a larger and negative trigonal field splitting parameter acting on the molecular orbitals formed from the ligand orbitals. Covalency mixing results in an effective splitting parameter that differs for each configuration (in the t^3 and e^2 subshell). Of equivalent configurations, the one possessing greater interelectronic interaction energy is likely to have smaller covalency coupling and consequently a more positive trigonal field splitting parameter.

It appears that covalency may result in the stabilization of the Jahn-Teller instability or even in a distortion in the opposite sense to that which obtains in the absence of covalency.

The Jahn-Teller Effect for Threefold Degeneracies : M. Caner and R. Englman

We have treated the case of a triply degenerate electronic state interacting with a set of triply degenerate vibrational modes.

The vibronic states (being sums of products of electronic states with vibrational states) have been classified according to the irreducible representations of the group (O or T_d) of the Hamiltonian. The eigenvalues, eigenvectors and certain transition probabilities have been computed.

* This research was sponsored by the Air Force Cambridge Research Laboratories through the European Office of Aerospace Research, United States Air Force, Contract AF 61 (052) - 806.

Intensity Calculations for Absorption Bands in Tetrahedral Symmetry :

Z. Yager

A general method has been developed for calculating the intensities of $d^n \rightarrow d^n$ transitions for paramagnetic ions in tetrahedral symmetries. Electric dipole transitions of this type would be forbidden in a crystal with inversion symmetry. In tetrahedral symmetry two mechanisms may be responsible for the transition: a) the odd static electric field, b) the odd vibrational motion of the neighbours. Either of these mechanisms causes an inmixture of charge transfer states in the even-parity metal states.

With this physical background we have classified both the electronic states and the vibrational motion by means of group theory, thereby also reducing the number of overlap integrals entering the transition probabilities. These overlap integrals are calculated for Slater-type wave functions of the metal and the ligand. They, as well as other related quantities, have been tabulated. Numerical calculations have been made for Ni^{2+} in ZnO .

Calculation of Jacobi Polynomials and Angular Momentum Transformation
Matrix Elements⁽¹⁾ : M. Gazith

Angular momentum transformation matrix elements were calculated directly from Jacobi polynomials by using an appropriate form of the explicit expression for these polynomials. The use of this form of the

This research was sponsored by the Air Force Cambridge Research Laboratories through the European Office of Aerospace Research, United States Air Force, Contract AF61(052) - 806

Jacobi polynomials obviates the need for lengthy calculations by recurrence formulae. Matrix elements for $j = 3$ and 5 were calculated and tabulated. The results obtained were used for numerical calculations by Z. Yager (see previous item).

Reference :

1. GAZITH, M., Israel AEC Report IA-1067 (1966)

A Theoretical Model for the Motion of Krypton in a Clathrate⁽¹⁾ : B. Barnett and Y. Hazony

A theoretical model of the potential inside a hydroquinone clathrate is suggested on the basis of data obtained from Mössbauer effect experiments and specific heat measurements. The experimental evidence suggests that the guest particle lies in a narrower potential well at low temperatures than at high ones. The three dimensional interpretation of the model has been given as well as thermodynamic considerations regarding the relationship between the experimental specific heats and the theoretical C_V . A comparison between the experimental and theoretical specific heats is made for the hydroquinone-krypton clathrates.

Reference :

1. BARNETT, B. and HAZONY, Y., J. Chem. Phys. 43, 3462 (1965) and Israel AEC Report IA-1019 (1965)

$\text{NH}_4\text{H}_2\text{PO}_4$ Phase Transition Studies by Infrared Absorption: I. Pelah, E. Wiener (Avnear) and S. Levin

The phase transition of antiferroelectric $\text{NH}_4\text{H}_2\text{PO}_4$ (ADP) was studied by measuring the infrared absorption spectrum in the wavelength region 250 cm^{-1}

to 4000 cm^{-1} . The changes in the spectrum which occurred on crossing the antiferroelectric Curie temperature (148°K), were compared with those found earlier⁽¹⁾ for the ferroelectric transition point (123°K) of KDP (KH_2PO_4). Above their Curie temperature the spectra of ADP and KDP are very similar, except of course for the addition of the NH_4 characteristic lines in the case of ADP. Below the phase transition the spectra differ considerably: the PO_4 lines are split in a different manner, especially the broad peak at 250cm^{-1} to 450 cm^{-1} which is split into 6 lines in ADP instead of 4 as in KDP. Changes are also in the NH_4^+ lines on cooling below the Curie temperature. The positions of the hydrogens in the short O-H...O bonds are not known as yet⁽²⁾ for the low temperature ADP phase. It is most important for an understanding of the antiferroelectric phenomena that these positions should be established. Taking into account the static and dynamic crystal effects on the symmetry, our results indicate⁽⁴⁾ that below the Curie temperatures the symmetry of the PO_4 group is C_1 in $\text{NH}_4\text{H}_2\text{PO}_4$ and C_2 in KH_2PO_4 .

This may support Nagamiya's⁽³⁾ theory on the different arrangement of the hydrogens in the PO_4 tetrahedra. Similar results were found when comparing the spectra of KH_2AsO_4 and $\text{NH}_4\text{H}_2\text{AsO}_4$ above and below their Curie temperatures. The effect of phase transition on the infrared spectra of the deuterated compounds of $\text{NH}_4\text{H}_2\text{PO}_4$ were studied for different percentages of deuteration in order to determine the role played by the hydrogens and to aid in the assignment of the infrared lines.

The results support our previous idea⁽¹⁾ that in the upper phase there is a low frequency hydrogen mode which changes drastically on phase transition. This mode is explained as resulting from a splitting of the ground level of the proton in a slightly asymmetric double minimum potential well, where tunneling of the proton takes place.

References:

1. IMRY, Y., PELAH, I. and WEINER, E., J. Chem. Phys. 43, 2332 (1965)
2. TENZER, L., FRAZER, B.C. and PEPINSKY, R., Acta Cryst. 11, 505 (1958)
- KEELING, R.O. and PEPINSKY, R., Z. Kristallogr. 106, 236 (1955)

3. NAGAMIYA, T., Prog. Theor. Phys. 7, 275 (1954)
4. WIENER (Avnear), E., PELAH, I. and IMRY, Y., ADP Phase Transition Studied by Infrared Absorption, 8th European Congress on Molecular Spectroscopy. Copenhagen, Denmark, August 14-20, 1965.

A New Interpretation of the Quasielastic Scattering of Slow Neutrons in Water⁽¹⁾: I. Pelah and Y. Imry

A refinement was made in our previous simple model⁽²⁾ for liquid water which depicted it as a "gas" of aggregates of a few hundred molecules each. In the limit where the aggregates do not interact, the model gave the same neutron scattering as from a "gas" of molecular weight about 300 times that of H_2O . The "Doppler shift" in scattering from this "gas" gives the right order of magnitude for the quasielastic broadening and its dependence upon momentum transfer⁽²⁾.

In the present work the interactions between the aggregates are taken into account. The system is described as a "quasisolid" composed of the aggregates as units, with a characteristic temperature which is presumably low (because of the large aggregates). Neutron scattering from this system for small energy transfers will be the sum of many phonon contributions. Machine calculations of the differential cross section for scattering of slow neutrons from this model were made using Sjölander's⁽³⁾ phonon expansion and assuming a simple frequency distribution for the "phonons". It was found that for high values of the Debye Waller exponent, $2W$, the cross section tends to the gas approximation. For $2W \approx 3$, which is obtained for $\theta \approx 3^\circ K$, the scattering cross section consists of a series of peaks whose total envelope is given by the gas approximation. This is found to agree with the experimental results of Bajoric et al.⁽⁴⁾ and to give qualitatively the right temperature behaviour - contrary to former diffusion models. The physical implications of the model are discussed.

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Paramagnetic Resonance Spectrum of Co^{1+} in Single Crystals of Calcium Oxide⁽¹⁾:

W. Low* and J.T. Suss

The investigation of the paramagnetic resonance spectrum of Co^{1+} in CaO was completed. Co^{1+} was obtained by gamma irradiation of crystals containing Co^{2+} . The resonance results are:

$$A = (31.6 \pm 0.3) \times 10^{-4} \text{ cm}^{-1}; \quad g = 2.2743 \pm 0.0008 \text{ at } 20^\circ\text{K} \text{ and} \\ g = 2.2756 \pm 0.0008 \text{ at } 12^\circ\text{K}.$$

A comparison was made of the internal magnetic field due to effective core polarization for Co^{1+} in MgO , Co^{1+} in CaO and Ni^{2+} in MgO . The core polarization for similar ions was found to be nearly independent of the crystal host.

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E.S.R. Studies of Radiation Effects in Simple Oxides⁽¹⁾: W. Low* and J.T. Suss

Gamma irradiation of single crystals of MgO , CaO and SrO , causes a number of effects which can be measured by means of electron spin resonance.

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In addition to valence changes of iron group impurities in $\text{MgO}^{(2)}$ and $\text{CaO}^{(3),(4),(5)}$ we have also observed H_1 centers (holes trapped near a positive ion vacancy) in CaO and SrO . These centers are not stable at room temperature. They have axial symmetry, with $g_{\parallel} = 2.0020$, $g_{\perp} = 2.0717$ in CaO , and $g_{\parallel} = 1.9999$, $g_{\perp} = 2.0757$ in SrO . Similar hole centers in $\text{MgO}^{(6)}$ were stable at room temperature.

A number of additional centers have been found in crystals of CaO and SrO . One of the most interesting is a center found at 20°K which can be expressed by means of a spin Hamiltonian :

$$\mathcal{H} = g_{\parallel} \beta H_z S_z + g_{\perp} \beta (H_x S_x + H_y S_y) + A S_z I_z + B S_y I_y + D \left[S_z^2 - \frac{1}{3} S(S+1) \right]$$

with $g_{\parallel} = 2.0010$, $g_{\perp} = 2.1232$, $A = 1.9 \times 10^{-4} \text{ cm}^{-1}$, $B = 3.6 \times 10^{-4} \text{ cm}^{-1}$, $D \sim 0.5 \times 10^{-4} \text{ cm}^{-1}$. This center is partially stable at 77°K and could arise from a hole trapped near a Na impurity.

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Electroluminescence and Dark-Current in Semiconducting Diamonds* : A. Halperin**,
V. Bar and J. Levinson.

The electroluminescence and conductivity of some specimens of semi-conducting diamonds (type IIb) were investigated.

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** Hebrew University, Jerusalem.

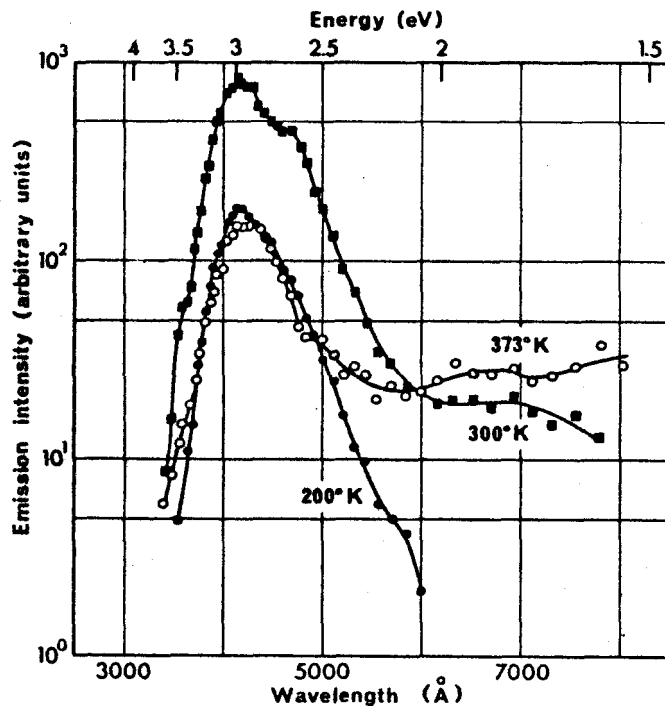


Fig. 10

Dependence of the electroluminescence emission spectrum on temperature, with 300 V applied across specimen C_5 .

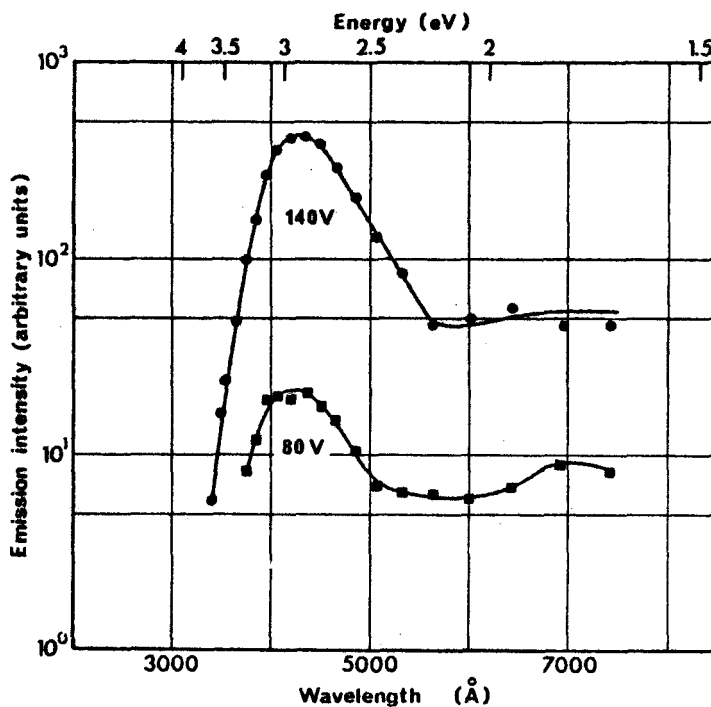


Fig. 11

Dependence of the electroluminescence emission spectrum on the applied voltage at room temperature in specimen C_2 .

All the crystals showed p-type direction of rectification. The dependence of the current and the emitted light on the voltage and temperature was measured, as well as the spectrum of the emitted light. In most of the experiments short square voltage pulses were applied to the crystals. In some experiments, especially at low voltages and low temperatures, both DC and AC voltages were applied.

The electroluminescence was found to rise exponentially on warming the crystal, with an activation energy of 0.3 - 0.4 eV, which is close to the activation energy obtained from dark-current measurements⁽¹⁾.

The plot of $\log I$ against $\frac{1}{\sqrt{V}}$ (where I is the intensity of the emitted light and V the applied voltage) gives a constant slope up to about 100 volts across the crystals (about 300 volts per cm). At higher voltages the curve becomes steeper. It is interesting that electroluminescence has been observed with only 1.8 volts across the crystal, which is less than one third of the forbidden gap.

The emission spectrum consists of two broad bands, one in the blue-violet region⁽²⁾ with a maximum around 4400 Å, and the other in the orange-red region (5500-8000 Å) with several maxima. Figure 10 shows the temperature dependence of the emission spectrum of one specimen (C_5 in our notation). When the temperature rises the ratio between the intensity of the blue band and that of the red band decreases. Figure 11 shows the dependence of the emission spectrum on the voltage for specimen C_2 . The "red" band is more pronounced at low voltages.

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Thermoluminescence in Semiconducting Diamonds*: A. Halperin** and R. Chen

The thermoluminescence glow curves of semiconducting diamonds have been measured in earlier work⁽¹⁾. It was shown that after excitation with UV light (2250 Å) at liquid air temperature the glow obtained during the warm-up of the crystal consists of a weak peak below 150°K, a very strong composite peak near 250°K and weaker glow peaks at higher temperatures. The emission was found to be blue-green. The most efficient excitation wavelength was a sharply defined peak near 2250 Å.

In the present work the thermoluminescence was examined in greater detail, and the following new information obtained:

- a) The blue glow peaks are accompanied by red peaks at about 175°K and 280°K.
- b) The blue peaks could be excited not only by wavelengths near the absorption edge (2250 Å) but also by longer wavelengths up to about 4000 Å.
- c) The excitation spectrum for the red peak (at 280°K) extended to wavelengths up to 6000 Å and more.
- d) The dependence of the intensity of the main thermoluminescence peak on the total dose of the exciting light was also examined in detail. For excitation with light near the edge (2200 to about 2600 Å) the intensity of the 250°K peak was strictly proportional to the excitation dose. For excitation with wavelengths of 2700 to nearly 4000 Å the dependence was superlinear, being proportional to the second power of the dose at 3000 Å and to the third power at 3600 - 3700 Å. For wavelengths above 4000 Å only the red peak (at 280°K) could be excited, and its intensity was strictly proportional to the dose. Fig.12 shows some representative curves of the maximum intensity of the 250°K peak as a function of the dose for some

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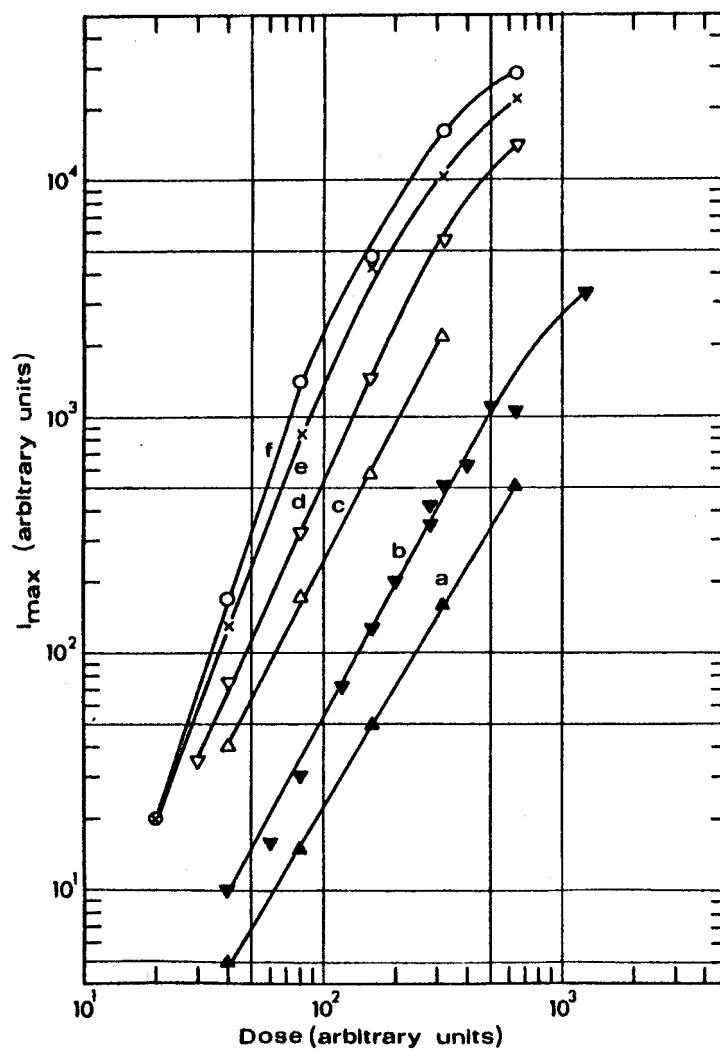


Fig. 12

Intensity of the main peak as a function of the irradiation dose Type IIb diamond (specimen C_3)

- a) 2800 Å, initial slope (n) = 1.7
- b) 3000 Å, n = 1.9
- c) 3200 Å, n = 2.1
- d) 3400 Å, n = 2.3
- e) 3500 Å, n = 2.7
- f) 3600 Å, n = 3.0

wavelengths of excitation. On the log-log scale used the slope gives the power of the dose to which the intensity is proportional (at low doses).

e) The blue peak at about 150°K was found to depend linearly on the dose at low doses and sublinearly at higher ones. At a certain dose the intensity of the peak reaches a maximum and when higher doses are used, the intensity of the peak drops. This seems to be the result of competition between excitation and bleaching effects. Bleaching effects are also observed when illuminating the crystal with visible or infrared light after excitation in UV.

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5. P L A S M A P H Y S I C S

Radiation by Growing Plasma Oscillations : S. Cuperman

The electromagnetic radiation emitted from bounded plasmas by growing (in time) plasma oscillations was studied. Such growing oscillations occur when for some reason a plasma instability is set up. Specifically, the growing plasma oscillations arising from the nonconvective electrostatic instability in an infinitely long current carrying cylindrical plasma of finite radius were considered.

Using the linearized equations of motion, the fields and the dispersion relation corresponding to free boundaries were obtained. Under certain circumstances the solution of the dispersion equation leads to complex values for the frequency, the wave number being kept constant. The phase velocity of the propagating field is near the constant axial velocity of the electrons, thus being smaller than the velocity of light.

An approximation for the electromagnetic power density, valid for the case in which the rate of growth γ is much smaller than the real part of the frequency, was obtained and its asymptotic expression considered. The particular value of the e.m. power density for the stationary regime (i.e. vanishing rate of growth) was found. This quantity is vanishing if the phase velocity of the propagating fields is smaller than the velocity of light. However, with $\gamma \neq 0$, the electromagnetic power density builds up from zero to nonvanishing values.

Theory of the Buildup of a Beam-Generated Plasma: S. Cuperman

Recent experiments on beam - plasma interactions in which the plasma is produced by the interaction of a monoenergetic electron beam

with neutral hydrogen gas have shown an unusual behaviour of the degree of ionization of the neutral gas as a function of the neutral gas density. The electron density of the plasma initially increases as a linear function of the density of the neutral background gas; then an abrupt jump occurs, corresponding to a density rise of 300 - 400%, or even much larger if a mirror magnetic confinement is present. Following this discontinuity, the plasma electron density again rises linearly with the neutral background. During the unusual rise, intense radiofrequency activity is observed. Moreover, the emerging electron beam spreads considerably and diffuses in the direction of lower energies. Under certain circumstances beam-electrons with energies several times larger than the initial one are observed.

An attempt is being made to derive the theory of this enhanced ionization by electron beam - plasma interaction. The theory involves the non-linear interaction between the strongly time dependent microscopic electric fields (which appear when a beam - plasma electrostatic instability is set up), and the beam particles and cold plasma particles.

Review of Experimental and Theoretical Studies on Plasma Formation by Laser Beams : J. Joskowicz^{*}

Several experimental and theoretical studies have been carried out over the past three years on ionisation and gas breakdown phenomena associated with laser beams. Experiments proved to be perfectly concordant, but the theories were contradictory at first. However, a crystallisation seems to be on the way: gas-breakdown is essentially governed by inverse bremsstrahlung, whereas the first ionisations are due to multiphoton absorption. The present work summarizes the various theories on plasma formation by laser beams.

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M A T H E M A T I C S

On the Structure of T \underline{A} - Algebras : Y. Ilamed

The idea of studying the structure of T \underline{A} - algebras (Taussky \underline{A} algebras) was suggested by Olga Taussky⁽¹⁾ who proposed studying the structure of an algebra whose composition law is given by the generalised Jordan product $MN + N^* M^*$, where M and N are complex n by n matrices and $*$ denotes the complex conjugate and transpose operation.

Corresponding to an associative algebra \underline{A} we define a T \underline{A} algebra by the composition law ab

$$ab = \frac{1}{2} (a.b + b^*.a^*) \quad (1)$$

where $a.b$ denotes multiplication in \underline{A} and $*$ denotes a one-to-one linear mapping of the algebra \underline{A} onto itself such that

$$(a.b)^* = b^*.a^*, \quad (a^*)^* = a \quad (2)$$

Denoting by $[a,b]$ the commutator of a,b , namely $[a,b] = ab - ba$, and by (a,b,c) the associator of a,b,c , namely $(a,b,c) = (ab)c - a(bc)$, the following identities are valid in T \underline{A} - algebras

$$(ab)^* = b^* a^* \quad (3)$$

$$(ab)^* = ab \quad (4)$$

$$[a,b] = [b^*, a^*] \quad (5)$$

$$(a,b,c) + (c^*, b^*, a^*) = 0 \quad (6)$$

$$[ab,cd] = 0 \quad (7)$$

$$((ab)(ab), cd, ab) = 0 \quad (8)$$

$$(a,b,c) = (b,a,c^*) + (a^*, c,b) \quad (9)$$

$$(a,b^*,c) + (b,c^*,a) + (c,a^*,b) = 0 \quad (10)$$

$$(a,bc,d) + (d,bc,a) + (bc)[ad^*] = 0 \quad (11)$$

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Hamilton-Cayley Theorem for Matrices with Non-Commutative Elements⁽¹⁾ :

Y. Ilamed

In this paper an example is given to show how the Hamilton-Cayley theorem for a matrix with non-commutative elements can be used to obtain identities in the enveloping algebra of a given Lie algebra.

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Fourth Order Three Point Difference Scheme for Solving Linear Second Order Differential Equations : A. Gersten

The differential equation $y''(x) + A(x)y'(x) + B(x)y(x) = C(x)$ with its boundary conditions is approximated by finite differences with an accuracy of $O((\Delta x)^4)$. The formula connects entities for three values of the argument and can be applied in some cases of regular singularities.