UNIVERSITY OF CALIFORNIA — BERKELEY

RADIATION LABORATORY

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

Contract No. W-7405-eng-48

FOCUSING IN LINEAR ACCELERATORS

Edwin M. McMillan

August 24, 1950

Berkeley, California

INSTALLATION:	No. of Copies
Argonne National Laboratory	8
Armed Forces Special Weapons Project	1
Atomic Energy Commission - Washington	2
Battelle Memorial Institute	1
Brush Beryllium Company	1
Brookhaven National Laboratory	4
Bureau of Medicine and Surgery	1
Bureau of Ships	1
Carbide and Carbon Chemicals Division (K-25 Plant)	4
Carbide and Carbon Chemicals Division (Y-12 Plant)	4
Chicago Operations Office	1
Columbia University (J. R. Dunning)	1
Columbia University (G. Failla)	1
Dow Chemical Company	1
H. K. Ferguson Company	1
General Electric, Richland	3
Harshaw Chemical Corporation	1
Idaho Operations Office	1
Iowa State College	2
Kansas City Operations Branch	1
Kellex Corporation	2
Knolls Atomic Power Laboratory	4
Los Alamos Scientific Laboratory	3
Mallinckrodt Chemical Works	1
Massachusetts Institute of Technology (A. Gaudin)	1
Massachusetts Institute of Technology (A. R. Kaufmann)	1
Mound Laboratory	3
National Advisory Committee for Aeronautics	1
National Bureau of Standards	3
Naval Radiological Defense Laboratory	2
New Brunswick Laboratory	1
New York Operations Office	3
North American Aviation, Inc.	1
Oak Ridge National Laboratory	8
Patent Branch (Washington)	1
Rand Corporation	1
Sandia Corporation	1
Santa Fe Operations Office	2
Sylvania Electric Products, Inc.	1
Technical Information Division (Oak Ridge)	15
USAF, Air Surgeon (Lt. Col. R. H. Blount)	1
USAF, Director of Armament (Captain C. I. Browne)	1
USAF, Director of Research and Development (Col. R. J. Mason, Fred W. Bruner)	2
USAF, Eglin Air Force Base (Major A. C. Field)	1
USAF, Kirtland Air Force Base (Col. Marcus F. Cooper)	1
USAF, Maxwell Air Force Base (Col. F. N. Moyers)	1
USAF, NEPA Office	2
USAF, Office of Atomic Energy (Col.H.C. Donelly, A.A. Fickel)	2
USAF, Offutt Air Force Base (Col. H. R. Sullivan, Jr.)	1
USAF, Wright-Patterson Air Force Base (Rodney Nudenberg)	1

INSTALLATION	No. of Copies
U. S. Army, Atomic Energy Branch (Lt. Col. A. W. Betts)	1
U. S. Army, Army Field Forces (Captain James Kerr)	1
U. S. Army, Commanding General, Chemical Corps Technical Command (Col. John A. MacLaughlin thru Mrs. Georgia S. Benjamin)	1
U. S. Army, Chief of Ordnance (Lt. Col. A. R. Del Campo)	1
U. S. Army, Commanding Officer, Watertown Arsenal (Col. Carroll H. Deitrick)	1
U. S. Army, Director of Operations Research (Dr. Ellis Johnson)	1
U. S. Army, Office of Engineers (Allen O'Leary)	1
U. S. Army, Office of the Chief Signal Officer (Curtis T. Clayton thru Maj. George C. Hunt)	1
U. S. Army, Office of the Surgeon General (Col. W. S. Stone)	1
U. S. Geological Survey (T. B. Nolan)	2
USAF, Director of Plans and Operations (Col. R.L. Applegate)	1
U. S. Public Health Service	1
University of California at Los Angeles	1
University of California Radiation Laboratory	5
University of Rochester	2
University of Washington	1
Western Reserve University	2
Westinghouse Electric Company	4
Naval Medical Research Institute	1
University of Rochester	1
California Institute of Technology (R. F. Bacher)	1

TOTAL   139

Information Division
Radiation Laboratory
University of California
Berkeley, California

FOCUSING IN LINEAR ACCELERATORS

Edwin M. McMillan

Radiation Laboratory, Department of Physics
University of California
Berkeley, California

August 24, 1950

It is well known that phase stability and first-order focusing are incompatible in a simple linear accelerator without foils or grids in the path of the particles.[1] However, various attempts have been made to design more complicated field shapes and time variations that will circumvent this limitation. A general proof that the limitation exists in all cases was made by the author in 1945 but was not published at that time. This proof is given below.

A particle of charge e moves parallel to the z-axis with velocity v and is acted on by fields $\underset{\sim}{E}$ and $\underset{\sim}{H}$, which are periodic in time and nearly periodic along the z-axis. The time period is T and the corresponding repeat length L is equal to vT. Changes in velocity and direction during one repeat length will be neglected; this is what we mean by the term "first order." The time corresponding to the position z is then given by $t_o + z/v$, where t_o is an arbitrary starting time.

The focusing effect depends on the x and y force components F_x and F_y, whose mean values are given by:

$$\left.\begin{array}{l} \overline{F}_x = (e/vT) \displaystyle\int_o^L \left[E_x - (v/c)H_y \right] dz, \\[4mm] \overline{F}_y = (e/vT) \displaystyle\int_o^L \left[E_y + (v/c)H_x \right] dz, \end{array}\right\} \tag{1}$$

the field components being evaluated at the position and corresponding time

---

(1) See for example J. C. Slater, Rev. Mod. Phys. 20, 473 (1948)

of the moving particle. If the line of motion is chosen so that both these forces vanish, the restoring force constants toward this line are:

$$k_x = - d\overline{F}_x/dx, \\ k_y = - d\overline{F}_y/dy \Bigg\} \tag{2}$$

From (1) and (2) we get:

$$k_x + k_y = (e/vT)\int_o^L \Big[- (\partial E_x/\partial x + \partial E_y/\partial y) + (v/c)(\partial H_y/\partial x - \partial H_x/\partial y) \Big] dz. \tag{3}$$

With the aid of the Maxwell equations div $\underset{\sim}{E} = 0$, curl $\underset{\sim}{H} = (1/c)\partial\underset{\sim}{E}/\partial t$, this becomes:

$$k_x + k_y = (e/vT)\int_o^L \Big[\partial E_z/\partial z + (v/c^2)\partial E_z/\partial t \Big] dz \tag{4}$$

Finally, since $\partial/\partial z = d/dz - (1/v)\partial/\partial t$, we can write:

$$k_x + k_y = (e/vT)\Big[E_z \Big]_o^L - (e/v^2 T)(1 - v^2/c^2)\int_o^L (\partial E_z/\partial t)\,dz. \tag{5}$$

Next consider the energy gain ΔW during one period, given by:

$$\Delta W = e\int_o^L E_z\,dz. \tag{6}$$

This depends on the starting time t_o, and its rate of change with t_o is:

$$d(\Delta W)/dt_o = e\int_o^L (\partial E_z/\partial t)\,dz. \tag{7}$$

Combining (5) and (7), we obtain the relation:

$$k_x + k_y = (e/vT)\Big[E_z \Big]_o^L - (1/v^2 T)(1 - v^2/c^2)d(\Delta W)/dt_o \tag{8}$$

Now, in order to have phase stability, $d(\triangle W)/dt_o$ must be positive, since an increasing t_o means that the particle is too slow and therefore has an energy deficiency. In order to have focusing, both k_x and k_y must be positive; this is clearly incompatible with the above requirement unless some help is obtained from the first term on the right of (8). This term depends on the difference in E_z experienced by the particle on leaving and entering the repeat length, and is obviously zero in the absence of foils or grids. With a foil placed so that the field strength on one side is zero, it is determined by the field strength on the other side at the instant the particle enters the foil. (The repeat length must be taken as ending at the foil, since the field equations used are not valid inside a conductor.) In the case of a grid, the effect is essentially the same, even though there is no field discontinuity through a grid opening; the focusing force can then be considered as arising from the charge lying between the equilibrium path and the displaced path.

This work was sponsored by the Atomic Energy Commission.