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The Equivalent, Fundamental–Mode Source

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Abstract—*In 1960, Hansen[1] analyzed the problem of assembling fissionable material in the presence of a weak neutron source. Using point kinetics, he derived the weak source condition and analyzed the consequences of delayed initiation during ramp reactivity additions. Although not clearly stated in Hansen's work, the neutron source strength that appears in the weak source condition actually corresponds to the equivalent, fundamental–mode source.*

In this work, we describe the concept of an equivalent, fundamental–mode source and we derive a deterministic expression for a factor, g, that converts any arbitrary source distribution to an equivalent, fundamental–mode source. We also demonstrate a simplified method for calculating g* in subcritical systems. And finally, we present a new experimental method that can be employed to measure the equivalent, fundamental–mode source strength in a multiplying assembly. We demonstrate the method on the zero-power, XIX-1 assembly at the Fast Critical Assembly (FCA) Facility, Japan Atomic Energy Research Institute (JAERI).*

I. INTRODUCTION

In any multiplying system, prompt fission chains must be initiated by a neutron source. This source can be in the form of an external neutron source (e.g., a start-up source), an intrinsic source caused by spontaneous fission or (α,n) reactions in the fuel, or *delayed neutrons* produced by precursors generated during previous prompt fission chains. When the reactivity of the system is below prompt critical, each prompt fission chain that is initiated is destined to eventually die out. In fact, even when the system is superprompt critical, most prompt fission chains are still destined to die out; only occasionally will a prompt fission chain actually diverge. This is clearly illustrated by an experiment performed by Wimett et al.[2] in 1960. Using the Godiva–II burst assembly, a series of 94 superprompt–critical bursts were performed in which the average *time–to–initiation* was measured to be approximately 3 seconds, with a maximum of approximately 13 seconds. Although the intrinsic neutron source of Godiva–II was only 70 n/s, because of the preburst operating history, the effective source strength (which includes the delayed neutron contribution) actually corresponded to approximately 1000 n/s at the time each burst was fired. On an average, 3000 source neutrons appeared in the system prior to the first persistent chain that caused

the burst to occur. Since the probability of any neutron source to cause an initial fission is approximately 40%, it follows that, on an average, 1,200 fission chains were initiated prior to the first persistent chain. In the case of the burst that was delayed by 13 seconds, 13,000 source neutrons appeared in the system initiating 5,200 prompt fission chains which ultimately died-out before the first persistent prompt fission chain caused the burst.

In the Godiva experiment, the reactivity of the system was increased to 0.05$ above prompt critical and held at that reactivity until the burst was eventually initiated by a source neutron. It is easy to imagine a scenario in which reactivity is being ramped into a system that has a large excess reactivity. If the neutron source of the system is too weak, then the potential exists to reach a point well above prompt critical before the first persistent chain can be initiated. This, in turn, could lead to a serious criticality accident.

In 1960, Hansen[1] analyzed the problem of assembling fissionable material in the presence of a weak source. In that work, he defined the weak source condition as

$$\frac{2Q\tau^{+}}{\bar{\nu}_{p}\Gamma_{2}} \ll 1 \quad , \tag{1}$$

where Q is the neutron source strength, τ^{+} is the adjoint-weighted, neutron–removal lifetime, $\bar{\nu}_{p}$ is the average number of prompt neutrons released per fission, and Γ_{2} is the neutron dispersion factor (also known as the Diven factor).[3,4] Although it was not clearly stated by Hansen,[1] the neutron source strength, Q, that appears in Eq. (1) actually corresponds to the *equivalent, fundamental–mode source*, which is the adjoint-weighted source strength normalized to the average importance of a fission neutron.

In related work, Spriggs[5] showed that neutron noise experiments, such as Rossi-α and Feynman's variance-to-mean experiments,[6,7] could be performed in subcritical assemblies if the following condition was satisfied.

$$\left(\frac{2Q\tau^{+}}{g\bar{\nu}_{p}\Gamma_{2}}\right)\left(1-\frac{1}{\rho_{\$}}\right) < 100 \quad , \tag{2}$$

where g is a spatial factor derived from transport theory[8–10] and $\rho_{\$}$ is the reactivity of the system in the units of \$ (note, $\rho_{\$}<0$ when the system is subcritical; hence, the minus sign in front of the $1/\rho_{\$}$ term.). The ability to perform neutron noise experiments is a direct indicator that non-Poissonian statistical fluctuations in the neutron population are still discernible; the neutron population has not yet reached a level in which prompt fission chains overlap to such an extent that an *apparent* persistent neutron population results. This point doesn't occurs until the mean time between

equivalent, fundamental–mode source neutrons (including delayed neutrons) is much shorter than the average duration of a prompt fission chain.

Because of its potential importance in the field of criticality safety and reactor start–up operations, it is essential to understand the equivalent, fundamental–mode source. In this work, we describe the concept of an equivalent fundamental–mode source, and we derive a deterministic expression for a factor, g^*, that converts any arbitrary source distribution to an equivalent fundamental–mode source. We also demonstrate a simplified method for calculating g^* in subcritical systems using deterministic and Monte Carlo methods. And finally, we present a new experimental method that can be used to measure the equivalent, fundamental–mode source strength in a multiplying assembly. We demonstrate the method on the zero-power, XIX-1 assembly at the Fast Critical Assembly (FCA) Facility, Japan Atomic Energy Research Institute (JAERI).

II. THEORY

In a subcritical assembly in equilibrium with a fixed external/intrinsic neutron source, the neutron loss rate must be equal to the source rate plus the integral fission neutron production rate. That is,

$$\frac{N}{\tau} = S + \int \chi_f \bar{\nu}_t \Sigma'_f \Phi' d\Omega' dE' \; d\Omega dV dE \quad , \tag{3}$$

where

$\Phi = \Phi(\boldsymbol{r}, \Omega, E)$ = angular flux,
$\Sigma'_f = \Sigma'_f(\boldsymbol{r}; \Omega', E' \rightarrow \Omega, E)$ = macroscopic fission cross section,
$\bar{\nu}_t = \bar{\nu}_t(\boldsymbol{r}, E')$ = the average of the total number of neutrons released per fission,
χ_f = fission spectrum (normalized to 1.0),
N = the total, unweighted, neutron population, and
τ = the unweighted, neutron–removal lifetime.

The neutron loss rate, N/τ, in the above expression is the sum of the leakage rate and the absorption rate. In integral form, this can be expressed as

$$\frac{N}{\tau} = \int \Omega \cdot \nabla \Phi \; d\Omega dV dE + \int \Sigma_a \Phi \; d\Omega dV dE \quad , \tag{4}$$

where the total, unweighted, neutron population, N, can be defined in terms of the angular flux, Φ,

and the neutron velocity, v, as

$$N = \int \frac{\Phi}{v} \, d\Omega dV dE \quad . \tag{5}$$

From Eqs. (4) and (5), it follows that the unweighted, neutron–removal lifetime, τ, is defined as

$$\tau \equiv \frac{\int \frac{\Phi}{v} \, d\Omega dV dE}{\int \Omega \cdot \nabla \Phi \, d\Omega dV dE + \int \Sigma_a \Phi \, d\Omega dV dE} \quad . \tag{6}$$

[A more detailed explanation of the definition of a neutron-removal lifetime can be found in Spriggs et al.[11]]

The multiplication, M, of a subcritical system is defined as the neutron loss rate divided by the external/intrinsic neutron source rate, S.

$$M = \frac{N}{\tau S} \quad . \tag{7}$$

By dividing Eq. (3) by S, we note that the average number of fission neutrons produced per source neutron is equal to $M - 1$. Furthermore, if the source S is distributed identical in all respects to the fission source distribution (i.e., angle, energy, and space), then the multiplication, M, will be related to the effective multiplication factor of the system, k_{eff}, as

$$M = \frac{1}{1 - k_{eff}} \quad , \tag{8}$$

The quantity $1/(1 - k_{eff})$ is defined as the fundamental–mode multiplication, M_o, since it represents the multiplication that would occur if the source S were distributed as the fundamental mode. However, in nature, external/intrinsic sources usually occur as uniformly distributed sources, such as intrinsic sources produced by spontaneous fissioning of one or more of the isotopes contained in the fuel, or as point sources that have been placed in or near the assembly, such as an external start-up source. In general, a uniformly distributed intrinsic source or an external

point source placed in or near an assembly will produce a system multiplication that can differ significantly from the fundamental–mode multiplication, M_o.

Because it is customary in reactor physics to express most quantities in terms of the effective multiplication factor, k_{eff}, we modify Eq. (8) by including a factor, g^*, that allows us to express the actual multiplication produced by an arbitrary source distribution in terms of the fundamental–mode multiplication. That is,

$$M = g^* M_o = \frac{g^*}{1 - k_{eff}} \quad . \tag{9}$$

We can derive a deterministic expression for g^* from the steady-state transport equation. When written in terms of the angular flux, Φ, the transport equation is.

$$\Omega \cdot \nabla \Phi + \Sigma_t \Phi = \int \Sigma'_s \Phi' d\Omega' dE' + \int \chi_f \bar{\nu}_t \Sigma'_f \Phi' d\Omega' dE' + \chi_s s \quad , \tag{10}$$

where

$\Sigma_t = \Sigma_t(\boldsymbol{r}, E)$ = total macroscopic cross section,
$\Sigma'_s = \Sigma'_s(\boldsymbol{r}; \Omega', E' \rightarrow \Omega, E)$ = macroscopic scattering cross section,
χ_s = external/intrinsic source spectrum,
$s = s(\boldsymbol{r}, \Omega)$ = source distribution per unit volume per unit angle, and
$\chi_s s = \chi_s(E)\, s(\boldsymbol{r}, \Omega)$ = source distribution per unit volume per unit angle per unit energy.

[As is customary in k-eigenvalue problems, any $(n,2n)$, $(n,3n)$, etc. reactions occurring in the system are accounted for by altering the scattering and absorption cross sections such that

$$\Sigma_s = \Sigma_{so} + 2\Sigma_{2n} + \ldots \quad , \tag{11}$$

and

$$\Sigma_a = \Sigma_{ao} - 2\Sigma_{2n} - \ldots \quad , \tag{12}$$

thus, preserving the total macroscopic cross section, Σ_t.]

Equation (10) is merely a statement of neutron conservation as applied to an infinitesimal

element of direction, energy, and space. If it is integrated over all phase space, it becomes a statement of neutron conservation for the integral system.

$$\int \Omega \cdot \nabla \Phi \; d\Omega dV dE + \int \Sigma_t \Phi \; d\Omega dV dE - \int \Sigma'_s \Phi' d\Omega' dE' \; d\Omega dV dE =$$

$$\int \chi_f \nu \Sigma'_f \Phi' d\Omega' dE' \; d\Omega dV dE + \int \chi_s s \; d\Omega dV dE \quad . \tag{13}$$

From the definition of the total cross section, the difference between the total interaction rate and the total scattering rate is equal to the total absorption rate.

$$\int \Sigma_t \Phi \; d\Omega dV dE - \int \Sigma'_s \Phi' d\Omega' dE' \; d\Omega dV dE = \int \Sigma_a \Phi \; d\Omega dV dE \quad , \tag{14}$$

where we again stress that the absorption cross section, Σ_a, includes all reactions with the exception of the $(n,2n)$, $(n,3n)$, etc. reactions multiplied by their respective multiplicities. Hence, we can rewrite Eq. (13) as

$$0 = P - L + S \quad , \tag{15}$$

where P represents the unweighted, neutron production rate due to fission,

$$P = \int \chi_f \nu \Sigma'_f \Phi' d\Omega' dE' \; d\Omega dV dE \quad , \tag{16}$$

L represents the unweighted, neutron loss rate due to leakage and absorption (i.e., N/τ),

$$L = \int \Omega \cdot \nabla \Phi \; d\Omega dV dE + \int \Sigma_a \Phi \; d\Omega dV dE \quad , \tag{17}$$

and S is the unweighted, external/intrinsic source rate,

$$S = \int \chi_s s \; d\Omega dV dE \quad . \tag{18}$$

Although Eq. (13) represents a neutron balance for the system, it does not take into account the importance in the multiplication process of a source or fission neutron born in a certain angle, at a certain energy, or at a certain location in space. Therefore, implicit in Eq. (13) is the assumption that all neutrons have equal importance even though neutrons born in the center of an assembly have a much greater probability of causing multiplication in the system than neutrons born at the outer edge of the assembly. To include the effect of neutron importance in the transport equation, we multiply Eq. (13) by the adjoint angular flux, $\Psi(r,\Omega,E)$, which is a direct measure of a neutron's importance in the multiplication process. The adjoint angular flux satisfies the equation

$$-\Omega \cdot \nabla \Psi + \Sigma_t \Psi = \int \Sigma'_s \Psi' d\Omega' dE' + \int \chi_f \nu \Sigma'_f \Psi' d\Omega' dE' \quad . \tag{19}$$

Integrating over all phase space leads to an adjoint-weighted transport equation of the form

$$0 = P^+ - L^+ + S^+ \quad , \tag{20}$$

where P^+ is the adjoint-weighted, neutron production rate due to fission,

$$P^+ = \int \Psi \chi_f \nu \Sigma'_f \Phi' d\Omega' dE' \; d\Omega dV dE \quad , \tag{21}$$

L^+ is the adjoint-weighted, neutron loss rate,

$$L^+ = \int \Psi \Omega \cdot \nabla \Phi \; d\Omega dV dE + \int \Psi \Sigma_a \Phi \; d\Omega dV dE \quad , \tag{22}$$

and S^+ is the adjoint-weighted, neutron source rate,

$$S^+ = \int \Psi \chi_s s \; d\Omega dV dE \quad . \tag{23}$$

By dividing Eq. (20) by P^+, we obtain

$$0 = 1 - \frac{L^+}{P^+} + \frac{S^+}{P^+} \quad . \tag{24}$$

If both the forward and adjoint fluxes in Eqs. (21), (22), and (23) are obtained from a k-eigenvalue solution, then the ratio P^+/L^+ is identically equal to k_{eff}. Therefore, Eq. (24) becomes

$$0 = k_{eff} - 1 + \frac{k_{eff} S^+}{P^+} \quad . \tag{25}$$

Next, we multiply and divide the last term on the right-hand side of Eq. (25) by the unweighted quantities S, P, and N. This leads to

$$0 = k_{eff} - 1 + \left[\frac{PS^+}{SP^+}\right]\left[\frac{k_{eff} N}{P}\right]\left[\frac{S}{N}\right] \quad . \tag{26}$$

As also pointed out by Spriggs et. al,[11] the unweighted, removal lifetime, τ, can also be written in terms of the fission production rate by noting that k_{eff} times the neutron loss rate must be equal to the neutron production rate.

$$k_{eff} \frac{N}{\tau} = \int \chi_f \nu \Sigma'_f \Phi' d\Omega' dE' \; d\Omega dV dE = P \quad . \tag{27}$$

Hence, the unweighted, removal lifetime, written in terms of the unweighted, neutron population and the unweighted, fission production rate, is identically equal to

$$\tau = \frac{k_{eff} N}{P} \quad . \tag{28}$$

Using the above definition of τ, Eq. (26) reduces to

$$N = \frac{\tau g^* S}{1 - k_{eff}} \quad , \tag{29}$$

where g^* is defined as

$$g^* = \frac{PS^+}{SP^+} \quad . \tag{30}$$

Written in terms of the angular fluxes, g^* corresponds to

$$g^* = \frac{\int \Psi \chi_s s \; d\Omega dV dE}{\int \chi_s s \; d\Omega dV dE} \times \frac{\int \chi_f \nu \Sigma'_f \Phi' d\Omega' dE' \; d\Omega dV dE}{\int \Psi \chi_f \nu \Sigma'_f \Phi' d\Omega' dE' \; d\Omega dV dE} \quad . \tag{31}$$

The physical meaning of g^* is easily seen by noting that the ratio of S^+/S is equal to the average importance of a source neutron, and the ratio of P^+/P is the average importance of a fission neutron. Consequently, g^* is the ratio of the average importance of a source neutron to the average importance of a fission neutron.

$$g^* = \frac{\overline{\Psi_s}}{\overline{\Psi_f}} \quad , \tag{32}$$

which is synonymous with the ratio of the actual multiplication, M, to the fundamental–mode

multiplication, M_o. Obviously, when the source is distributed as the fundamental mode, g^* is identically equal to 1.0.

Although Eq. (29) is written in terms of the unweighted quantities N, τ, and S, we could just as easily have multiplied and divided Eq. (25) by the adjoint-weighted, neutron population, N^+, and the unweighted, source, S, to obtain

$$\frac{N^+}{\tau^+} = \frac{\overline{\Psi}_s S}{1 - k_{eff}} \quad , \tag{33}$$

where τ^+ is the adjoint-weighted, removal lifetime (see Spriggs et al.) defined by

$$\tau^+ = \frac{k_{eff} N^+}{P^+} \quad . \tag{34}$$

The quantity N^+/τ^+ is equal to the adjoint-weighted, neutron loss rate and, by comparison with Eq. (29), is equal to the average importance of a fission neutron times the unweighted loss rate.

$$\frac{N^+}{\tau^+} = \overline{\Psi}_f \frac{N}{\tau} \quad . \tag{35}$$

It follows, therefore, that the equivalent, fundamental–mode source, $g^* S$, is the adjoint–weighted source, $S^+ = \overline{\Psi}_s S$, divided by the average importance of a fission neutron. When the adjoint fluxes are normalized such that $\overline{\Psi}_f = 1$, than the adjoint-weighted, source strength is identically equal to the equivalent, fundamental–mode source.

III. CALCULATION OF g^*

In principle, the evaluation of Eq. (31) is rather straightforward when using a deterministic code to estimate the forward and adjoint angular fluxes from a *k*-eigenvalue solution. One can easily develop a post–processor code that will perform the integrals in Eq. (31).

For subcritical systems, a simpler method exists. In most deterministic and Monte Carlo codes, the user has the option to perform a *k*-eigenvalue solution or a *fixed-source* calculation. Using the fixed-source option, one can obtain the actual multiplication of the system, M, corresponding to any arbitrary source distribution. When combined with a *k*–eigenvalue solution, g^*

can be determined directly from

$$g^* = M(1 - k_{eff}) \quad . \tag{36}$$

Furthermore, when multiple sources are in the system, the equivalent, fundamental–mode source is the sum of the equivalent, fundamental–mode sources of each of the constituent sources. That is

$$Q = g^* S = g_1^* S_1 + g_2^* S_2 + \ldots \quad . \tag{37}$$

A separate solution for g_i^* can be obtain for each source constituent. The effective g^* of a multiple–source distribution is simply a weighted average of the individual values of g_i^* for each of the constituent sources.

$$g^* = \frac{\sum_i g_i^* S_i}{\sum_i S_i} \quad . \tag{38}$$

IV. NUMERICAL EXAMPLE

Consider a bare, spherical system comprised of 51.2 Kg of ^{235}U and 2.75 Kg of ^{238}U containing a 100 n/s ^{252}Cf start–up source in the center of the assembly. Using the deterministic transport code ONEDANT[12] and the standard 16–group Hansen–Roach cross section set,[13] the effective multiplication factor, k_{eff}, of this system was calculated to be 0.9914 when the outer radius of the sphere was 8.85 cm. It is well known that both ^{235}U and ^{238}U undergo spontaneous fission; ^{235}U produces 0.01 n/s per Kg and ^{238}U produces 13.6 n/s per Kg (see Table I). For this system, the ^{235}U produces a total of 0.5 n/s, which are uniformly distributed over the volume of the assembly, and the ^{238}U produces a total of 37.4 n/s, which are also uniformly distributed over the volume of the assembly. The spontaneous fission spectra for ^{235}U, ^{238}U, and ^{252} Cf are listed in Table II.

The equivalent, fundamental–mode source for this fictitious system was determined by

running three fixed-source problems: 1) a ^{235}U spontaneous fission source distributed uniformly over the volume of the assembly, 2) a ^{238}U spontaneous fission source distributed uniformly over the volume of the assembly, and 3) a ^{252}Cf point source located at the center of the assembly, and a k-eigenvalue solution to determine k_{eff}. Because the ^{235}U and ^{238}U spontaneous fission spectra are similar, the multiplication calculated by ONEDANT was found to be 98.76 for both sources. Hence, g^* for both the ^{235}U and ^{238}U sources corresponds to

$$g^* = 98.76\,(1 - 0.9914) = 0.85 \quad .$$

The multiplication produced by the ^{252}Cf point source was calculated to be 204.7. So, g^* for the ^{252}Cf point source located in the center of the assembly corresponds to

$$g^* = 204.7\,(1 - 0.9914) = 1.76 \quad .$$

When combined, the equivalent, fundamental–mode source of the system turned out to be

$$Q = 0.85\,(0.5 + 37.4) + 1.76\,(100) = 208.2 \text{ n/s},$$

with an effective g^* of

$$g^* = \frac{208.2}{0.5 + 37.4 + 100} = 1.51 \quad .$$

From a physical standpoint we can interpret these results as follows. A uniformly distributed $^{235}U + ^{238}U$ spontaneous fission source producing 37.9 n/s in this spherical assembly will produce the same total neutron multiplication as an equivalent, fundamental–mode source of strength 32.2 n/s (i.e., 0.85 x 37.9). A ^{252}Cf point source emitting 100 n/s in the center of the assembly will produce the same total neutron multiplication as an equivalent, fundamental–mode source of strength 176 n/s (i.e., 1.76 x 100). The combined sources ($^{235}U + ^{238}U + ^{252}Cf$) of 137.9 n/s will produce the same total neutron multiplication as an equivalent, fundamental–mode source of strength 208.2 n/s.

V. k_{eff}–DEPENDENCE OF g^*

The factor g^* is dependent on k_{eff}. However, depending on how the source is distributed, this dependence may be relatively weak. For example, using the numerical example from the previous section, we varied the radius of the uranium sphere from 8.85 cm to 0.85 cm in 1–cm decrements; k_{eff} varied from 0.9914 to 0.0933, respectively. As can be seen in Fig. 1, g^* for the uniformly distributed spontaneous fission source varies from 0.85 to 0.99 as k_{eff} decreases. In comparison, g^* for a ^{252}Cf point source located at the center of the assembly decreases from 1.76 to 1.03 over the same range in k_{eff}.

As another example, we calculated g^* for a reflected system in which the uranium core used in the previous example was surrounded by a spherical graphite reflector 12 cm in radius. At a core radius of 7.0 cm, $k_{eff} = 0.9842$. When the core radius was reduced to 1.0 cm (while maintaining the outer radius of the graphite at 12 cm), k_{eff} decreased to 0.1136. For the case of a uniformly distributed source, g^* increased from 0.949 to 0.995 as k_{eff} decreased. For the case of a point source located at the center of the assembly, g^* decreased from 1.49 to 1.04 (see Fig. 1).

VI .MEASUREMENT OF THE EQUIVALENT, FUNDAMENTAL–MODE SOURCE

The equivalent, fundamental–mode source can be easily measured in a multiplying system. This is accomplished by placing a neutron detector in or somewhere near the core and observing the change in the count rate produced by placing a *calibrated* neutron source at some known location within the system. The count rate of the detector is proportional to the unweighted, neutron loss rate and the detector efficiency, ε, defined as

$$\varepsilon = \frac{C}{(\frac{N}{\tau})} \quad , \tag{39}$$

where C is the detector count rate.

Based on the above definition of the detector efficiency, we can rewrite Eq. (29) as

$$C = \frac{\varepsilon g^* S}{1 - k_{eff}} \quad . \tag{40}$$

Assume for the moment that we wish to measure the equivalent, fundamental–mode

source strength of an intrinsic source present in a critical assembly containing a large amount of one or more isotopes that undergo spontaneous fission. If the system reactivity is adjusted to be just slightly subcritical, then the detector count rate produced by the intrinsic source distribution will correspond to

$$C_i = \frac{\varepsilon g_i^* S_i}{1 - k_{eff}} \quad , \tag{41}$$

where C_i is the detector count rate at that subcritical configuration and S_i is the intrinsic source strength.

If we then place a calibrated point source at some known location within that system and we reestablish the same subcritical configuration, the detector count rate produced by the intrinsic source plus the point source will now correspond to

$$C_{pi} = \frac{\varepsilon (g_i^* S_i + g_p^* S_p)}{1 - k_{eff}} \quad , \tag{42}$$

where C_{pi} is the new count rate, S_p is the strength of the point source, and g_p^* corresponds to the g^*–value at the location of the point source.

Taking the ratio of Eq. (42) to Eq. (41) and solving for $g_i^* S_i$ leads to

$$g_i^* S_i = \frac{g_p^* S_p}{\left(\frac{C_{pi}}{C_i} - 1\right)} \quad . \tag{43}$$

Note that we have assumed that the efficiency of the detector does not change upon insertion of the point source. In far subcritical systems, this assumption may not be valid. However, if the system is just slightly subcritical, then the multiplication of the system will be high enough to excite the fundamental mode. When this occurs, the flux distribution will be determined by the fission source distribution and will be relatively insensitive to the actual source distribution. Therefore, the detector efficiency will remain essentially constant for both source distributions used during the measurement.

Furthermore, in the above derivation, we have optimistically assumed that the observed count rates, C_i and C_{pi}, are produced only by neutrons. However, most neutron detectors are

somewhat sensitive to gamma rays—which are always present in a multiplying system. When using fission chambers, this sensitivity can be effectively eliminated by adjusting the lower-limit discriminator to be high enough to detect only those pulses that are produced by the fission fragments generated within the chamber when a neutron is detected. However, when using ^{3}He or BF_3 detectors, it is much more difficult to discriminate out all gamma rays without simultaneously reducing the ability to detect neutrons as well. Consequently, one must be able to determine the gamma ray background count rate, $C\gamma$, in order to obtain the correct values for C_i and C_{pi} to be used in Eq. (43). This can be accomplished if these measurements are repeated at several different subcritical configurations in the vicinity of delayed critical. Eq. (43) can be modified as follows.

When written in terms of reactivity [i.e., $\rho_\$ = (k_{eff} - 1)/\beta k_{eff}$], Eq. (40) becomes

$$C = \varepsilon g^* S - \frac{\varepsilon g^* S}{\beta \rho_\$} \quad . \tag{44}$$

where β is the effective delayed neutron fraction. When $\rho_\$ \rightarrow -\infty$ (i.e., $k_{eff} = 0.0$), the count rate must approach $\varepsilon g^* S$, which is the count rate that would be produced by an unmultiplied source in that particular source/detector geometry. If we define $\varepsilon g^* S$ as C_o, we can write Eq. (44) as

$$C = C_o - \frac{m}{\rho_\$} \quad . \tag{45}$$

where m is $\varepsilon g^* S/\beta$. Obviously, $C_o = \beta m$ and since β is no larger than 1%, we expect C_o to be negligible relative to $-m/\rho_\$$ when $\rho_\$$ is in the vicinity of delayed critical. Therefore, to a first approximation, we can state that

$$C_i - C_{io} = -\frac{m_i}{\rho_\$} \approx C_i \quad , \tag{46}$$

and

$$C_{pi} - C_{pio} = -\frac{m_{pi}}{\rho_\$} \approx C_{pi} \quad . \tag{47}$$

From these two equations, we can see that the ratio C_{pi}/C_i is approximately equal to the ratio of m_{pi}/m_i. Therefore, Eq. (43) can also be written as

$$g_i^* S_i = \frac{g_p^* S_p}{\left(\frac{m_{pi}}{m_i} - 1\right)} \quad . \tag{48}$$

The slopes m_{pi} and m_i can be readily determined by plotting the detector count rate as a function of the inverse reactivity of the system and then performing a least-squares fit of these data to determine the y-intercept, C_o, and the slope, m, for both source configurations. If the detector system happens to be sensitive to gamma rays, then C_o will be noticeably greater than βm. When this occurs, we can infer the gamma ray background to be

$$C_\gamma = C_o - \beta m \quad . \tag{49}$$

However, determining the gamma ray background is for information purposes only. When evaluating Eq. (48), we presume (albeit, somewhat optimistically) that the gamma ray background doesn't change much with the reactivity of the system over a small range of reactivity in the vicinity of delayed critical. We expect the slopes m_{pi} and m_i to be relatively independent of the background count rate, whatever it may be.

We now illustrate this technique on a real system.

VII. INTRINSIC SOURCE MEASURMENT AND CALCULATION FOR THE XIX-1 CORE

Measurement

The experimental procedure described in the previous section was performed on the zero-power XIX-1 assembly located at the Fast Critical Assembly (FCA) facility operated by the Japan Atomic Energy Research Institute (JAERI). The XIX-1 assembly is a multi-region system comprised of an inner core fueled with highly-enriched ^{235}U metal. The core is surrounded by an inner blanket (referred to as the 'soft blanket') containing a significant amount of depleted uranium-oxide and sodium, and an outer blanket (referred to as the 'depleted blanket') containing only depleted uranium metal. Cross-sectional views of the core are shown in Figs. 2, 3, and 4.

For this measurement, four ^3He detectors were inserted into the soft blanket region of the assembly (see Fig. 2). Count rates from these detectors were obtained at three different subcritical

configurations when the system was driven by just the intrinsic source. The reactivity ranged from –0.072$ to –0.892$. A calibrated ^{252}Cf source was then inserted into the center of the assembly and the count rates from the ^{3}He detectors were obtained at eight different subcritical configurations over the same reactivity range. A least-squares fit of count rate vs. inverse reactivity was performed on these two sets of data.

With just the intrinsic source driving the system, it was found that $m_i = -55.8 \pm 1.3$ \$/s and $(C_{io} + C_{i\gamma}) = 22 \pm 10$ cps (see Fig. 5). The y-intercept is obviously too large; from the measured slope and the measured value of the effective delayed neutron fraction, β=0.00729, the y-intercept should have been approximately 0.4 cps if the detectors were only counting neutrons; consequently, from Eq. (49), we can infer that the gamma ray background, $C_{i\gamma}$, for these ^{3}He detectors during the intrinsic source measurements was approximately 21 cps.

A similar process was repeated for the count rates obtained with the ^{252}Cf source in the center of the assembly. The least-squares fit yielded $m_{pi} = -500.6 \pm 3$ \$/s and $(C_{pio} + C_{pi\gamma}) = 39 \pm 25$ cps. This y-intercept is also too high; C_{io} should have been approximately 3.7 cps; so, the gamma ray background present during the measurements with the Cf source in the center of the assembly increased to approximately 35 cps.

The ratio of the slopes is

$$\frac{C_{pi}}{C_i} \approx \frac{m_{pi}}{m_i} = \frac{500.6}{55.8} = 8.97 \pm 2.4\% \quad .$$

Using the procedure described earlier in this report for calculating g^* in a subcritical assembly, g_p^* for the XIX-1 assembly was calculated to be 1.68 in the vicinity near delayed critical. And, as determined from a previous source calibration performed at JAERI, the strength of the ^{252}Cf source used to perform the experiment was measured to be 97,600 ± 1% n/s on the day of the measurement. Substituting these values into Eq. (43) yields an equivalent, fundamental–mode *intrinsic* source strength of

$$g_i^* S_i = \frac{1.68 \times 97,600}{8.97 - 1} = 20,600 \pm 3\% \text{ n/s} \quad . \tag{50}$$

Calculation

Using TWODANT,[12] the equivalent, fundamental–mode intrinsic source strength was calculated using the data listed in Tables I and II. The intrinsic source was separated into its various constituents so that the neutron spectrum from each spontaneous fission source and the (α,n)

source could be properly accounted for in each region of the assembly. The results of this analysis is shown in Table III.

Although there is a 30% difference between the measured and calculated equivalent, fundamental–mode source strength, this difference is not particularly significant for this system. A small increase of 2% in g^* in the soft blanket (i.e., 0.0896 to 0.1096) and a 2% increase in g^* in the depleted blanket (i.e., 0.0277 to 0.0477) would increase the calculated effective intrinsic source strength to 20,800 n/s—which would compare very favorably with the measured value. A 2% increase in these two values of g^* is very plausible considering the fact that the 16–group Hansen–Roach cross section set used to perform these calculations is not well suited for this particular system. This is best seen by comparing the measured adjoint–weighted neutron lifetime to the calculated value. When calculated using the 16–group Hansen–Roach cross section set, the adjoint-weighted neutron lifetime was determined to be 550 ns, which is 21% lower than the measured value of 700 ns. Because the neutron lifetime varies as the inverse of the absorption cross section, it follows that the 16–group Hansen–Roach cross section set overpredicts the absorption rate in this particular system. Hence, in accordance to the calculations, source neutrons born in the soft and depleted blankets will have a much harder time penetrating those regions then they would in reality. This shorter mean–free path, in turn, yields a calculated g^* that will be lower than the actual value.

Finally, we note that the measured value of the effective g^* corresponds to

$$g^* = \frac{20,600}{343,160} = 0.060 \quad ,$$

which shows that the equivalent, fundamental–mode source is only 6% of the total neutron source present in this system.

The XIX-1 assembly is a prime example of the importance of understanding the equivalent, fundamental–mode source. If we were to evaluate the weak–source condition using the total source strength present in the system, we would obtain

$$\frac{2S\tau}{\bar{\nu}\Gamma_2} = \frac{2 \times 343,160 \times 700x10^{-9}}{2.6 \times 0.80} = 0.23 \quad ,$$

which marginally satisfies the condition of << 1. However, when we use the equivalent, fundamental–mode source strength of 20,600 n/s, then the weak–source condition becomes 0.0138, which clearly satisfies the condition of << 1.

VIII. DISCUSSION

In addition to the applications previously mentioned in this manuscript, the factor g^* also has an impact on one of the most basic experimental techniques used in the field of nuclear engineering—the classical *approach–to–critical* experiment using a $1/M$ plot. In this experiment, a source in place in an assembly and a reference count is taken with a nearby detector. Then a small amount of fuel is added to the assembly and a new count is obtained. The ratio of the reference count to the new count (i.e., the inverse of the multiplication, M) is plotted as a function of the amount of fuel in the system and then extrapolated to the point where $1/M = 0.0$ to obtain an estimate of the critical mass. In principle, if this process is repeated for a series of small mass additions, the $1/M$ plot will be a linear function of the amount of mass in the system assuming each mass addition produces the same Δk_{eff}. In general, this is not a very good assumption; if the system is loaded with fuel from the center out, then the initial mass additions will produce a much greater change in k_{eff} than will the later mass additions at the outer surface of the assembly.

For the sake of argument, let's assume that we are able to load a new system in such a way as to produce equal changes in k_{eff}. Since $1/M = (1 - k_{eff})/g^*$, the $1/M$ plot will be linear with k_{eff} only if g^* is constant with k_{eff}. This, of course, will usually not be the case. Using the numerical example presented in Section V, when a point source was positioned in the center of that assembly, g^* increased as the radius of the assembly was increased. As shown in Fig. 6, the $1/M$ plot for this example is noticeable non-linear with k_{eff}. However, in this particular instance, the curve is considered to be conservative from the standpoint that an extrapolation to $1/M = 0.0$ using any two successive points underestimates the critical mass. Nevertheless, it is conceivable to perform an approach–to–critical experiment in which g^* decreases as k_{eff} increases, thereby, producing a non-conservative estimate of the critical mass. Hence, when preparing to perform an approach–to–critical experiment, it may be helpful to understand how g^* is going to change as k_{eff} increases.

IX. CONCLUSIONS

The factor g^* is the parameter that converts any arbitrary source distribution to an equivalent, fundamental–mode source. The equivalent, fundamental–mode source, in turn, is the effective source strength that is multiplied by the factor $1/(1 - k_{eff})$ to yield the correct neutron production rate in the system. k_{eff} in this expression corresponds to the k-eigenvalue of the system and is, by definition, independent of the source distribution.

The equivalent, fundamental–mode source has many applications in reactor kinetics, particularly point kinetics, as well as being an important parameter in criticality safety and neutron–noise experiments. In subcritical systems, the equivalent, fundamental–mode source is easily calculated using deterministic or Monte Carlo methods and can be readily measured in real critical assemblies using the technique described in this work.

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Table I: Spontaneous Fission Data[14–18]

Isotope	Spont. Fission Decay Const. (s^{-1})	$\bar{\nu}$ [a]	S ($\frac{n}{s\text{-Kg}}$)	Watt[b] Spectrum Coef. A	Watt Spectrum Coef. B
^{232}Th	2.81E-29	2.14 ± 0.20	1.56E-04	0.5934	8.0298
^{233}U	8.28E-26	1.76 ± ?	0.38	0.8548	4.0321
^{234}U	1.47E-24	1.81 ± ?	6.83	0.7712	4.9245
^{235}U	2.19E-27	1.86 ± ?	0.01	0.7747	4.8523
^{236}U	8.82E-25	1.90 ± 0.05	4.27	0.7352	5.3575
^{237}U	2.41E-26	1.87 ± ?	0.12	0.6931	5.9940
^{238}U	2.68E-24	2.00 ± 0.02	13.6	0.6483	6.8106
^{239}U	3.96E-25	2.04 ± ?	2.03	0.7356	5.2608
^{236}Pu	1.05E-17	2.12 ± 0.14	5.70E+07	0.9883	3.1039
^{237}Pu	1.07E-21	1.88 ± ?	5.09E+03	0.9546	3.3077
^{238}Pu	4.63E-19	2.21 ± 0.06	2.59E+06	0.8478	4.1693
^{239}Pu	2.73E-24	2.16 ± ?	14.9	0.8853	3.8027
^{240}Pu	1.92E-19	2.151 ± 0.005	1.04E+06	0.7949	4.6893
^{241}Pu	3.67E-25	2.25 ± ?	2.07	0.8425	4.1515
^{242}Pu	3.26E-19	2.141 ± 0.006	1.74E+06	0.8192	4.3667
^{243}Pu	1.10E-23	2.43 ± ?	66.2	0.7354	5.3872
^{244}Pu	3.29E-19	2.29 ± 0.19	1.86E+06	0.6947	6.0037
^{252}Cf	2.57E-10	3.768 ± 0.012	2.31E+15	1.0250	2.9260

a. Those values of ν that have a ? as an uncertainty were interpolated from measured data (cf. Ref.).
b. Watt Fission Spectrum:

$$f(E) = C\exp(-\frac{E}{A})\sinh\sqrt{BE} \quad .$$

Table II: Spontaneous Fission Spectrum

Group[a]	^{235}U	^{238}U	$(\alpha,n)^b$	^{252}Cf
1	0.186	0.140	0.011	0.275
2	0.364	0.362	0.307	0.353
3	0.174	0.190	0.036	0.149
4	0.179	0.200	0.237	0.146
5	0.083	0.093	0.342	0.067
6	0.013	0.014	0.064	0.010
7	0.001	0.001	0.003	0.000
8–16	0.0	0.0	0.0	0.0

a. Corresponding to Hansen-Roach group structure.
b. This spectrum was calculated specifically for the soft blanket region of the XIX-1 core using the SOURCE code.[14]

Table III: Equivalent Fundamental–Mode Intrinsic Source

Region	Isotope	Mass (Kg)	S (n/s)	g^*	g^*S (n/s)
Core	^{235}U	153.0	2	0.994	2
	^{238}U	11.7	159	0.991	158
Soft Blanket	^{235}U	11.7	<0.1	0.0916	0
	^{238}U	5,841.2	79,400	0.0896	7,114
	(α,n)[a]	—	772	0.0796	61
Depl. Blan.	^{235}U	38.8	<0.4	0.0285	0
	^{238}U	18,713.7	255,000	0.0277	7,064
			$\sum S =$ 343,160		$g_i^* S_i =$ 14,399

a. The uranium in the soft blanket is in the form of uranium-oxide plates. The alpha particles from the uranium interact via (α,n) reactions with other elements in the soft blanket (primarily the ^{18}O and ^{23}Na) to produce an additional source of intrinsic neutrons.

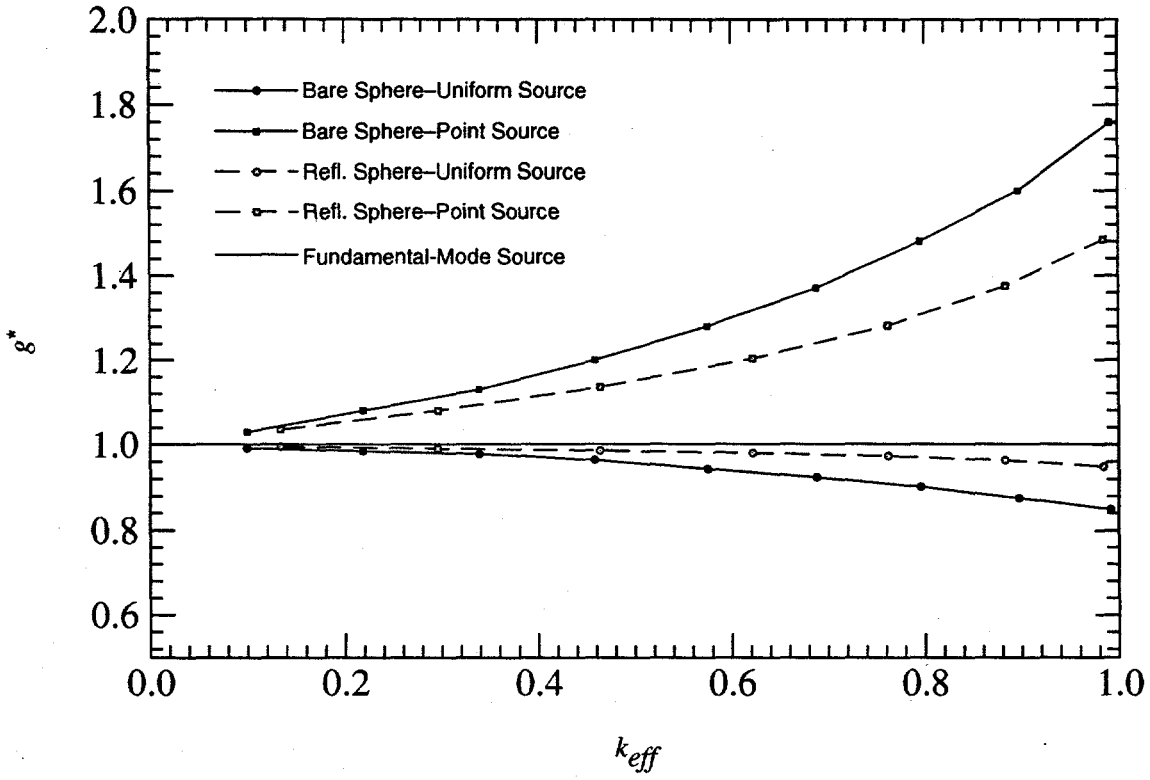

Fig. 1. k_{eff}–dependence of g^* for four different cases: 1) a bare sphere with a uniformly distributed spontaneous fission source, 2) a bare sphere with a point source at the center of the assembly, 3) a reflected sphere with a uniformly distributed spontaneous fission source, and 4) a reflected sphere with a point source at the center of the assembly.

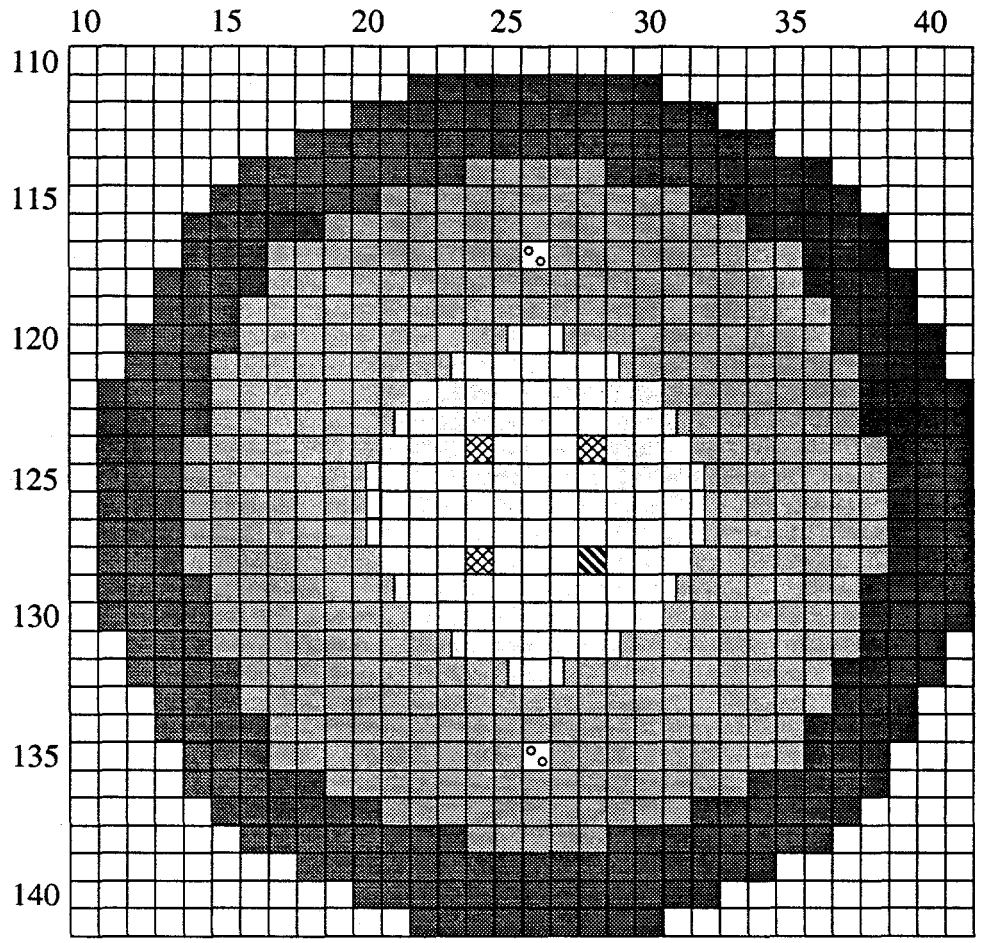

Fig. 2. Cross-sectional view of FCA XIX-1 assembly. Two pairs of He detectors are located in positions 26-118 and 26-135.

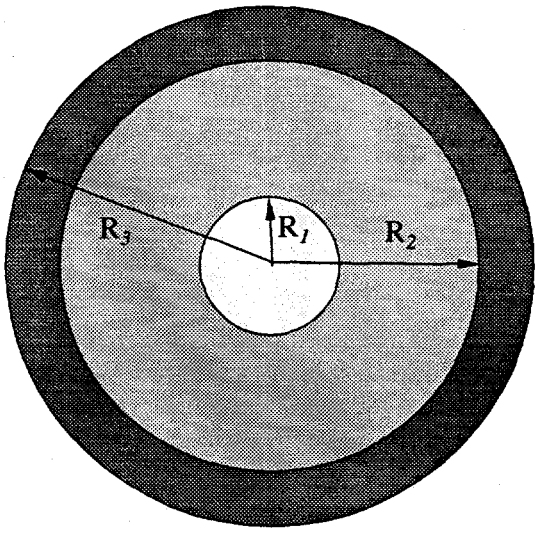

Fig. 3. Effective radii of the three regions of the XIX-1 Assembly. $R_1 = 32.958$ cm, $R_2 = 68.30$ cm, $R_3 = 86.36$ cm.

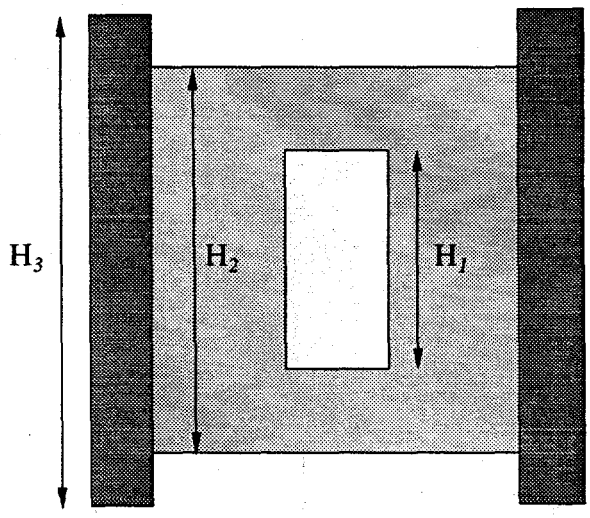

Fig. 4. Effective heights of the three regions of the XIX-1 Assembly. $H_1 = 50.80$ cm, $H_2 = 121.92$ cm, $H_3 = 132.08$ cm.

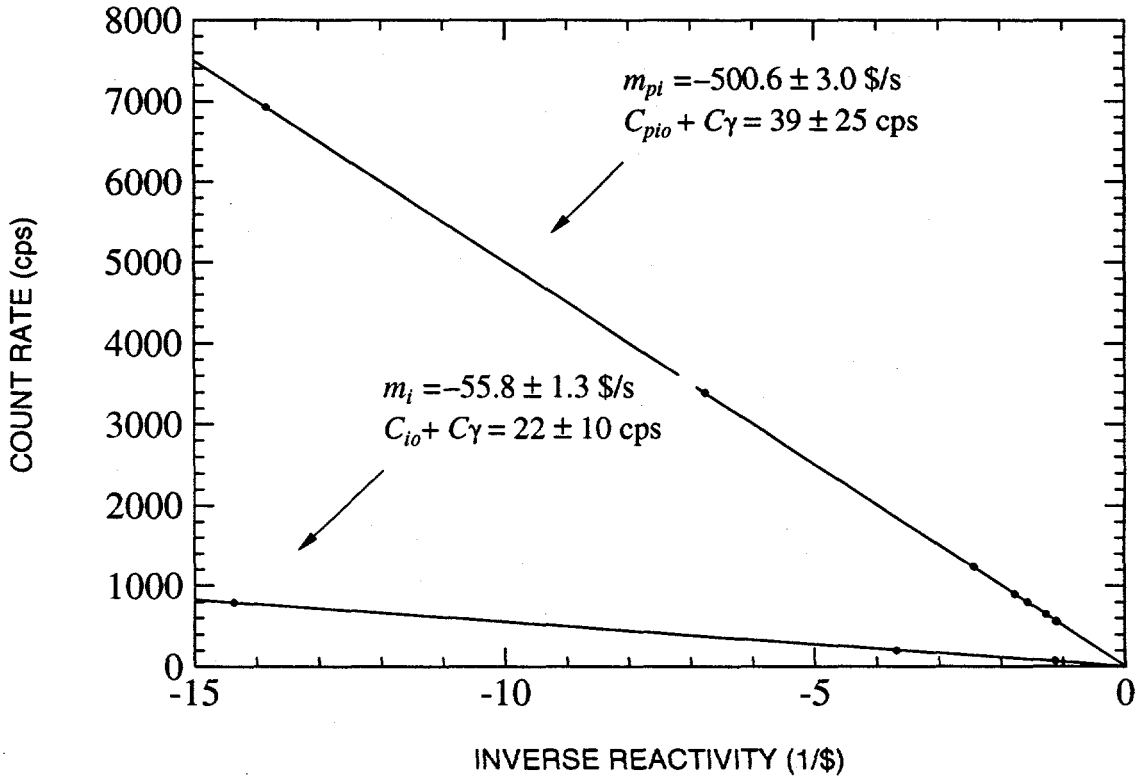

Fig. 5. Plot of the count rate of the four detectors summed together vs. reactivity with only the intrinsic source present.

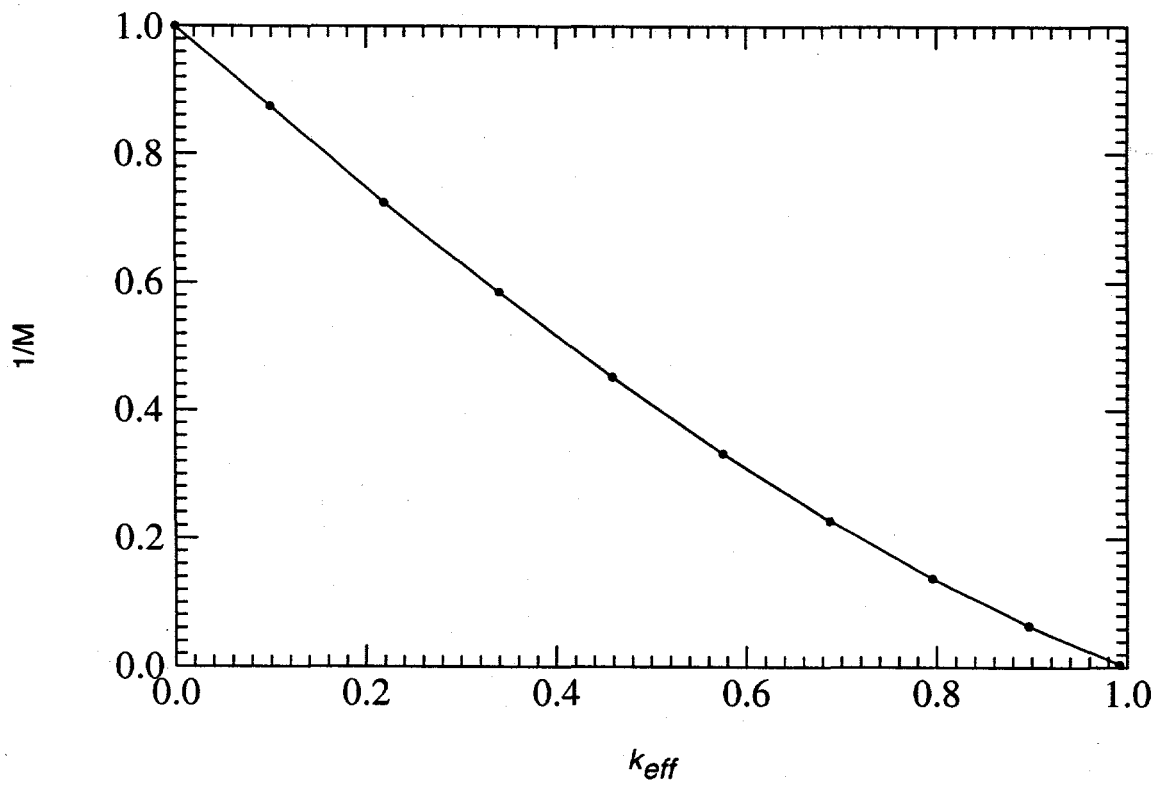

Fig. 6. $1/M$ plot vs. k_{eff} for a spherical, uranium assembly with a ^{252}Cf point source positioned in the center of the assembly.