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BRIEF REPORT ON THE PRESENT STATUS OF MESON THEORY OF NUCLEAR FORCES

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Brief Report on the Present Status of Meson Theory of Nuclear Forces*

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I. Introduction

The meson theory of nuclear forces has had a long albeit not a conspicuously successful history. Starting with Yukawa's first attempt in 1935 to explain the strength, short range, and saturation character of nuclear forces on the basis of charged and neutral field quanta of intermediate mass, the vicissitudes of the subject have been numerous and exasperating. A major obstacle to progress for a long time was the wrong identification of the elusive cosmic ray μ meson as the meson responsible for nuclear forces. This blind alley was followed for a dozen years and lack of knowledge of the specific properties of the μ meson left room for a plethora of meson theories of nuclear forces-- based on different assignments of spin and parity to the meson and corresponding to different types of coupling to the nucleon (direct vs. gradient coupling, single vs. meson pair coupling, single coupling vs. a mixture of couplings, etc.). When finally the π meson was discovered in 1947

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at Bristol and its properties determined within the next few years by means of the high energy machines in the U.S., it became clear that if nuclear forces in reality possess a field-theoretic origin, the pion field must be the field chiefly responsible. The observation of the three charge states of the pion (all of them associated with zero spin and odd parity) and the large nucleonic production and absorption cross sections have given strong support to the qualitative features of Yukawa's original hypothesis.

The working out of a quantitatively correct meson theory of nuclear forces still remained, however. Fortunately, independent of the developments in meson physics, great progress had been registered in the allied domain of quantum electrodynamics. A series of important papers (Tomanaga, Schwinger, Feynman, Dyson) led to the formulation of the renormalization method for treating problems of quantum electrodynamics. Despite the occurrence of infinities in the higher order approximations of quantum electrodynamics, it is possible to renormalize the charge and mass of the electron so that finite and experimentally verified answers to well-defined problems can be derived. The same mathematical techniques have been applied to meson fields and it has been shown (Matthews, Salam) that only the scalar field with scalar coupling and the pseudoscalar field with pseudoscalar coupling can be renormalized for both charged and neutral quanta (with one small qualification which can easily be handled--see below). Since the pion has turned out to be a pseudoscalar particle, existing in three charge states, it follows that a renormalizable meson field theory of nuclear forces exists in the sense of quantum electrodynamics provided one assumes pseudoscalar coupling of the pion to the nucleon.** As we shall see, the whole problem of constructing a mathematically correct meson theory of nuclear forces would be greatly simplified if the pion were a scalar particle--although, of course, nature would look entirely different.

The crucial problem of the meson theory of nuclear forces can, therefore, be formulated as a series of three subquestions: (1) Is the pseudoscalar pion coupled to the nucleon via pseudoscalar coupling?* (2) If the answer to (1) is affirmative, what are the predictions of the renormalizable

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* We shall assume throughout our discussion that the coupling is symmetrical as far as the isotopic spin dependence is concerned; this immediately implies that the nuclear forces are charge-independent (Kemmer).

**A good deal of work has, however, been done on nuclear forces yielded by the non-renormalizable pseudoscalar meson theory with pseudovector coupling by the Japanese school (see Taketani et al.)

PS(PS) theory concerning nuclear forces? (3) Do the quantitative predictions of this theory agree with experiment? These questions are interrelated; one can only decide whether pseudoscalar coupling is correct by performing a full-fledged calculation of the two-nucleon interaction with all divergences properly removed and by confronting the resulting interaction with existing experimental data.

While in principle the procedure would appear to be straightforward, the fulfillment of the above program has turned out to be an enormously difficult task. Some of the difficulties can be spelled out by comparing the PS(PS) theory with the S(S) theory which is the only other type of renormalizable symmetrical meson theory of nucleon forces. The PS(PS) theory involves the odd Dirac γ_5 operator which is large (of order unity) for positive $\rightleftarrows$ negative energy transitions but small (of order v * where v is the nucleon velocity) for positive $\rightarrow$ positive or negative $\rightarrow$ negative transitions. This is in contrast to the S(S) theory which involves the operator 1 and is obviously of order unity for positive $\rightarrow$ positive and negative $\rightarrow$ negative energy transitions and small for the positive $\rightleftarrows$ negative transitions. Furthermore, Wick's old argument that the range of nuclear forces is $\frac{1}{\mu}$ (μ is the meson mass) implies that the effective range in $2n^{th}$ order, in the sense of perturba-

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* We shall throughout set $\hbar = c = 1$

tion theory, is $\frac{1}{M\mu}$. Since the lowest (second) order nuclear force involves the emission of one meson by one nucleon in a positive energy state and its reabsorption by the other nucleon in a positive energy state, the second order force is--apart from the factors involving the coupling constant--smaller than the fourth order force (which permits positive $\rightleftarrows$ negative transitions) in the PS(PS) theory but larger in the S(S) theory. The consequence is that the observed strength of the two-nucleon interaction requires a large PS(PS) coupling constant ($G^2/4\pi \approx 10-15$) where G is the pseudoscalar pion-nucleon coupling constant--cf. below) in contrast to the much smaller S(S) coupling constant ($g^2/4\pi \approx 0.1-0.2$, where g is the scalar meson-nucleon coupling constant). The quantity $g^2/4\pi$ is still much larger than the electromagnetic coupling constant since nuclear forces are after all much stronger than electromagnetic forces. However, the fact that $g^2/4\pi < 1$ would at least justify treating the second order force as a first approximation in the S(S) theory whereas a similar starting point for the PS(PS) theory is manifestly absurd. (Of course, apart from the fact that the pion is a pseudoscalar particle, it can be demonstrated that the S(S) theory is completely incapable of explaining nuclear forces--sub-questions (2) and (3) above can be answered for the S(S) theory!). The minimum starting point for the PS(PS) theory is, therefore, a calculation of the second and fourth order force within the framework of some weak coupling approximation (perturbation method, Tamm-Dancoff or any other method equivalent to an expansion in powers of the coupling constant). Actually, as we shall see below, the large size of the PS(PS) coupling constant renders even such a starting point most dubious.

II. Non-adiabatic theory. Definition of various approximations.

Let us now examine the progress which has been made in constructing a renormalizable PS(PS) meson theory of nuclear forces with a method which is somewhat of an improvement over the usual perturbation approximation, namely the non-adiabatic method, either in its non-covariant (Tamm-Dancoff) or covariant (Bethe-Salpeter) forces. In the non-adiabatic treatment of the two-body problem, the state vector of the system is defined by means of a set of probability amplitudes $a_\nu^{(m,n)}$ of the free states where m mesons and n nucleon pairs are present; the two initially interacting nucleons are treated separately, and ν is a variable which specifies the momenta, spins, isotopic spins, etc., of the particular free state which is considered. The set $[a_\nu^{(m,n)}]$ satisfies a system of simultaneous integral equations and if one eliminates all the amplitudes except $a_\nu^{(0,0)}$ by means of successive substitutions, one obtains the general eqration which can be written in the center-of-mass system as follows:

$$(E - 2E_p)\, a^{(0,0)}(\underline{p}) = \frac{1}{(2\pi)^3} \int K(\underline{p}, \underline{p}'; E)\, a^{(0,0)}(\underline{p}')\, d\underline{p}' \qquad (1)$$

in which E is the total energy of the system and $E_p = \sqrt{p^2 + M}$. The kernel $K(\underline{p}, \underline{p}'; E)$ is expressed as a power series of G^2 :

$$K(\underline{p}, \underline{p}'; E) = \sum_{n=1}^{\infty} K_{2n}(\underline{p}, \underline{p}'; E) \qquad (2)$$

Eq. (2) can easily be rewritten in coordinate space:

$$\left(\frac{\Delta}{M\rho} + \epsilon\right) \phi^{(0,0)}(\underline{r}) = \int U(\underline{r}, \underline{r}'; E)\, \phi^{(0,0)}(\underline{r}')\, d\underline{r}' \qquad (3)$$

where $\phi^{(0,0)}(\underline{r}) = \frac{1}{2\pi} \sqrt{\frac{2M\rho}{\pi}} \int \frac{1}{\sqrt{2E_p + E}}\, a^{(0,0)}(\underline{p})\, e^{i\underline{p}\cdot\underline{r}}\, d\underline{p}$ (3a)

$\rho = 1 + \epsilon/4M$, $\epsilon = E - 2M$; and:

$$U(\underline{r}, \underline{r}'; E) = \sum_{n=1}^{\infty} U_{2n}(\underline{r}, \underline{r}'; E) = \sum_{n=1}^{\infty} \frac{1}{(2\pi)^3} \int \frac{\sqrt{(2E_p+E)(2E_{p'}+E)}}{4M\rho} K_{2n}(\underline{p}, \underline{p}'; E)\, e^{i(\underline{p}\cdot\underline{r} - \underline{p}'\cdot\underline{r}')}\, d\underline{p}\, d\underline{p}'$$

In general, $U(\underline{r}, \underline{r}'; E)$ is a non-local interaction operator and only reduces to a static potential in certain limiting cases. One such limiting case is when $|\underline{r} + \underline{r}'| \gg \frac{1}{M}$

which yields:

$$V(\underline{r}, \underline{r}'; E) \longrightarrow \delta(\underline{r} - \underline{r}')\, V_{2n}\left(\frac{\underline{r} + \underline{r}'}{2}\right)$$

Other limiting cases will be enumerated below. In order to define the static approximation and also to distinguish the other types of approximation (perturbation vs. non-perturbation, adiabatic vs. non-adiabatic), we write down the explicit form of $K_2(\underline{p}, \underline{p}'; E)$. K_2 receives its contribution from essentially one diagram (cf. Fig. 1) since the other contribution is merely obtained by relabeling the two nucleons. One finds

$$K_2(\underline{p}, \underline{p}'; E) = -G^2\, \underline{\tau}_1 \cdot \underline{\tau}_2\, \frac{(M+E_p)(M+E_{p'})}{4E_pE_{p'}}\, \frac{\sigma_1 \cdot \underline{S}\ \ \sigma_2 \cdot \underline{S}}{\omega(p, p')[\omega(p, p') + E_p + E_{p'} - E]} \quad (4)$$

where $\omega(p, p') = \sqrt{|\underline{p} - \underline{p}'|^2 + \mu^2}$, $\underline{S} = \dfrac{\underline{p}'}{M + E_{p'}} - \dfrac{\underline{p}}{M + E_p}$

The various types of approximation are now defined as follows:

Adiabatic approximation: set $E_p = E_{p'}$ in Eq. (4).

Perturbation approximation: set $E = 2E_p$ or $2E_{p'}$ in Eq. (4).

Static approximation: set $E_p = E_{p'} = M$ in Eq. (4).

It is evident that the static is a special case of the adiabatic approximation which allows for the motion of the nucleon but neglects its recoil after meson emission or absorption. Moreover, the Tamm-Dancoff method leaves room for an improvement over the perturbation method since one need not set $E = 2E_p$ or $2E_{p'}$ (Van Hove). Finally, it should be noted that non-perturbation corrections to K_2 can be transformed by a mathematical device into non-adiabatic perturbations corrections of the same order plus static corrections of higher order. (Lévy).

III. Discussion of the second-order force.

The usual calculation of the second-order PS(PS) two-nucleon interaction assumes $E_p = E_{p'} = M$, $E = 2M$ (perturbation-static approximation) so that K_2 reduces to:

$$K_2 = - \frac{G^2 \underline{\tau_1} \cdot \underline{\tau_2} \, \underline{\sigma_1} \cdot \underline{k} \, \underline{\sigma_2} \cdot \underline{k}}{4M^2 \omega^2} \tag{5}$$

where $\underline{k} = \underline{p}' - \underline{p}$ The Fourier transform of K_2 is:

$$V_2(\underline{r}) = \frac{1}{3} \frac{G^2}{4\pi} \left(\frac{\mu}{2M}\right)^2 \underline{\tau_1} \cdot \underline{\tau_2} \left\{ \underline{\sigma_1} \cdot \underline{\sigma_2} + S_{12} \left[1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right] \frac{e^{-\mu r}}{r} \right\} - \frac{G^2}{3} \underline{\tau_1} \cdot \underline{\tau_2} \, \underline{\sigma_1} \cdot \underline{\sigma_2} \, \delta(\underline{r}) \tag{6}$$

Eq. (6) is a well-known result, derived many years ago, and contains a spin-dependent central force plus a contact interaction term in addition to the strongly singular tensor force. The perturbation-static approximation is only justified for weak binding or low kinetic energies $\ll \mu$ (in scattering problems)--and at distances $\gg \frac{1}{M}$. Even for weak binding or low kinetic energy, the static approximation will break down at distances $\lesssim \frac{1}{M}$. Indeed, the attractive $\frac{1}{r^3}$ singularity of Eq. (6) would not permit the existence of stationary states if it were employed down to the origin. It can be shown easily that the term proportional to $\delta(\underline{r})$ in Eq. (6) becomes smeared out by non-static effects in the neighborhood of the origin and takes the form:

$$U_c^{(2)}(\underline{r},\underline{r}') = -\frac{G^2}{3}(\underline{\tau}_1\cdot\underline{\tau}_2)(\underline{\sigma}_1\cdot\underline{\sigma}_2)\frac{M^2}{rr'}K_1(Mr)K_1(Mr') \qquad (7)$$

$U_c^{(2)}$ operates only on S-states and behaves like a repulsive $\frac{1}{r^2}$ potential at small distances. A detailed study of the complete expansion of V_2 $(\underline{r},\underline{r}')$ shows, however, that the second order interaction behaves, near the origin, more like a $\frac{1}{r}\log r$ repulsive potential.

If the binding between two nucleons were strong or the kinetic energy were large compared to μ but small compared to M, the adiabatic approximation could serve as a reasonable starting point but neither the static nor the perturbation approximations would be justified. The adiabatic approximation enables us to write K_2 as a function of $|\underline{r}|$ and $|\underline{r}'-\underline{r}|$ but the nuclear interaction V_2 still remains non-local in character over most of the interesting range. Finally, for relativistic nucleon energies, no approximations in the expression for K_2 are permissible and the interaction is non-local over its entire range. It should be remarked that the Møller scattering formula follows from Eq. (4) in the perturbation approximation; that is, setting $E = 2E_p = 2E_{p'}$ in Eq. (4) yields the Møller formula.

To sum up: the rigorous second-order nuclear interaction, valid at all energies and distances, is a non-local interaction which acts as the kernel of the integro-differential equation (3); in principle, this equation can be solved by numerical means once the boundary conditions are specified. The second-order nuclear interaction becomes a local static potential at sufficiently large distances ($\gg \frac{1}{M}$) or at sufficiently low kinetic energies ($\ll \mu$). At very small distances ($\lesssim \frac{1}{M}$), the second-order interaction becomes a quasi-static repulsive $\frac{1}{r}\log r$ potential operating exclusively on the S-states of the two-nucleon system. At intermediate distances and at moderate or high energies, corrections to the static approximation must certainly be taken into account in order to obtain quantitatively correct answers.

IV. Discussion of the fourth-order force.

If one had reason to suppose that K_2 contains the bulk of the nuclear interaction,* it would be sensible to

* It would still be necessary to take into account the radiative corrections to the second-order interaction; we shall let this point pass (see below).

spend a considerable effort to obtain solutions of Eq. (3) (by electronic computers if necessary). However, we have already pointed out that it is an essential feature of pseudoscalar coupling that the matrix elements which simply create or annihilate a meson are, in the non-relativistic limit, much smaller than those which, in addition, create or annihilate a pair of nucleons. This implies that if $G^2/4\pi > 1$ (which is indeed the case), the fourth order nuclear interaction should be larger than the second order, even in the static approximation. It is, therefore, essential to work out K_4 and to examine its contribution to the nuclear interaction in the various approximations. This is easier said than done. First of all, there are 36 (non-covariant) finite diagrams to consider in evaluating K_4 (in contrast to 2 for K_2), namely 4 which are iterations of the second order diagrams, 8 which are not second-order iterations but still contain no nucleon pairs, 12 which contain one intermediate nucleon pair and 12 which contain two intermediate nucleon pairs. Secondly, radiate corrections now play an essential role and supply corrections of order $G^2/4\pi$ to the fourth order interaction even in the static approximation, in contrast to the second order interaction where radiative corrections are of order $\left(G^2/4\pi\right)\left(\mu/2M\right)^2$ in the static approximation. Finally, the fourth order interaction is the first to be affected by the special type of divergence associated with meson-meson scattering which is not removed by the mass and charge renormalization procedures characteristic of quantum electrodynamics; this has the consequence that a new arbitrary constant is introduced into the fourth order interaction. We proceed to examine each of these points in somewhat greater detail.

With sufficient patience, one can write down the 36 contributions to K_4. One soon finds that in the static approximation, the properties of the γ_5 operator are such that the largest contributions arise from those diagrams which contain the maximum number of transitions of the nucleons from positive to negative energies together with the minimum number of nucleon pairs in intermediate states (this statement actually holds in each order). In fourth order, these diagrams (cf. Fig. 2) contain two virtual nucleon pairs in intermediate states with one intermediate state containing no nucleon pairs at all. The exact contribution of the two diagrams in Fig. 2 can be written down; however, we content ourselves with recording the perturbation-static approximation, namely:

$$K_4(=) - \frac{G^4}{2(2\pi)^6\,\omega_k\,\omega_{k'}\,(\omega_k+2M)(\omega_k+\omega_{k'})(\omega_{k'}+2M)} \tag{8}$$

which is independent of spin and isotopic spin. At this point, the usual procedure is to neglect ω compared to 2M in the denominator and to compute the static potential in what we shall call the "incomplete static" approximation; one finds

$$V_4^{(a)} = -\frac{6}{\pi}\left(\frac{G^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^2\frac{1}{\mu r^2}K_1(2\mu r) \tag{9}$$

which is the potential one would derive in the static approximation from the pion-pair term $\frac{G^2}{4\pi}\frac{\mu}{2M}\psi^*\phi^2\psi$ (ψ and $\underline{\phi}$ are the nucleon and pion field amplitudes respectively) in the canonically transformed PS(PS) Hamiltonian. This fact (which has been pointed out, among others, by Lepore) is demonstrated by the following alternative expression of $V_4^{(a)}$.

$$V_4^{(a)} = \frac{3iG^2}{8M^2}\int_{-\infty}^{\infty}\Delta_F^2(\underline{x},t)\,dt \tag{10}$$

where

$$\Delta_F(x) = \frac{-2i}{(2\pi)^4}\int\frac{e^{ikx}\,d^4k}{k^2+\mu^2} \tag{11}$$

is the Meson propagation function defined by Feynman. Eq. (9) shows that $V_4^{(a)}$ is of order $\left(G^2/4\pi\right)^2\left(\mu/2M\right)^2$ with a range $1/2\mu$, whereas V_2 is of order $\left(G^2/4\pi\right)\left(\mu/2M\right)^2$

with a range $1/\mu$; hence, at distances $\lesssim 1/2\mu$, $V_4^{(a)}$ is $G^2/4\pi$ times as large as V_2. One should further remark that the "incomplete static" potential has a $1/r^3$ singularity at small distances in contrast to the $1/r$ singularity of the static potential defined by the Fourier transform of Eq. (8). If one tries to improve on the "incomplete static" potential by expanding $(\omega+2M)^{-1}$ in powers of $\omega/2M$, the first-order correction becomes:

$$\delta V_4^{(a)} = \frac{12}{\pi^2}\left(\frac{G^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^3\frac{1}{\mu r^2}\left[K_1(\mu r)\right]^2 \qquad (12)$$

which possesses a $1/r^4$ singularity at small distances. A method of correction which augments rather than diminishes the nature of the singularity at small distances must be handled with extreme caution. However, if one restricts oneself to distances $\gtrsim 1/\mu$, it is true that $\delta V_4^{(a)} \approx \left(\frac{\mu}{2M}\right) V_4^{(a)}$.

In the above sense, the one-pair contributions to the static fourth order interaction are of order $\mu/2M$ compared to $V_4^{(a)}$ while the no-pair contributions are of order $(\mu/2M)^2$.

If one wishes to know the fourth order interaction at smaller distances $\left(<\frac{1}{2\mu}\right)$, the stronger singularities associated with some of the one-pair terms cause them to become comparable to, or at least important corrections to $V_4^{(a)}$. Moreover, at distances $\leq 1/2\mu$, the one-pair terms $\left[\text{of order } \left(\frac{G^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^3\right]$ are comparable to the second order static potential $\left[\text{of order } \frac{G^2}{4\pi}\left(\mu/2M\right)^2\right]$ since $\frac{G^2}{4\pi}\,\mu/2M \sim 1$. Consequently, a self-consistent derivation of the second plus fourth order nuclear interaction at distances $\gg \frac{1}{M}$ requires full knowledge of the non-static corrections to the fourth order two-pair contribution (to first order in $\mu/2M$) plus the static contributions of the fourth order one-pair terms and the second order terms in addition to the static contribution of the fourth-order two-pair terms. At small distances $(\lesssim 1/2M)$, the smeared out contact interaction present in the no-pair terms of the fourth order interaction behaves like a $1/r^3$ repulsive quasi-potential acting only in S-states so that the sum of the second and fourth order interactions seems to remain repulsive at small distances for S states.

We have stated that the radiative corrections affect the fourth order interaction in an essential way. Of course, there are also radiative corrections to the second order interaction. However, the second order radiative corrections are easily calculated by making use of Dyson's prescriptions for replacing the meson propagation function Δ_F by Δ_F', the nucleon propagation function S_F by S_F' and the interaction operator γ_5 by Γ_5 in the covariant Salpeter-Bethe equation which must now replace the non-covariant Tamm-Dancoff equation for the purposes of carrying out the renormalization. It happens that in the static limit, the second order potential can be written with radiative corrections in the form:

$$V_2' = V_2\left[1 + \alpha \, G^2/4\pi \, (\mu/2M)^2 + \cdots\right] \tag{13}$$

where α is a function of G, M, and μ and is of order unity. The situation is different in the case of the fourth order interactions. Not only must one make the normal radiative corrections characteristic of the second order but one must in addition take account of finite radiative diagrams (an illustration of such a diagram is given in Fig. 3) and the meson-meson scattering types of diagrams (cf. Fig. 4). The normal (renormalizably infinite) radiative corrections plus the finite radiative corrections to the fourth order two-pair interaction turn out, in the static approximation, to be proportional to $V_4^{(a)}$; however, the corrected potential must now be written in the form:

$$V_4^{(a)\prime} = V_4^{(a)}\left[1 + \beta \, G^2/4\pi + \cdots\right] \tag{14}$$

where β is a well-determined function of G, M and μ and is again of order unity. The meson-meson scattering types of diagrams will only yield a finite result if one introduces a term of the form $\lambda \phi_i^{*2} \phi_i^2$ (λ is a completely arbitrary constant) into the pion-nucleon interaction Hamiltonian. The additional correction supplied by these diagrams can then be written, in the static limit in the form (Bonneway):

$$V_4^{(a)''} = V_4^{(a)} \left[\lambda c + \gamma \left(G^2/4\pi \right)^2 + \cdots \right] \tag{15}$$

where c and γ are well-determined functions of λ in addition to $G, M,$ and μ . Radiative corrections to the fourth order one-pair terms must also be taken into account but a proper calculation of these corrections, even in the static approximation, has not as yet been carried out. It should be noted that a complete expression for the fourth order static potential to order $\left(G^2/4\pi\right)^2 \left(\mu/2M\right)^3$ would require a knowledge of the first-order (in $\mu/2M$) non-static radiative corrections to $V_4^{(a)}$ (as well as the first-order non-static corrections to $V_4^{(a)}$ itself.)

V. Remarks on higher order terms.

The above discussion underlines the incomplete state of present calculations of the second and fourth order nuclear interactions. Needless to say, the calculations of the sixth and higher order interactions are in a much more rudimentary state. The important point to settle is whether the higher orders are comparable to the second and fourth orders, due account being taken of the smaller range associated with each successively higher order. Klein has attempted to answer this question by summing certain classes of terms to all orders. In particular, he considers in a specified order the "incomplete static" approximation in that order, plus the terms in lower orders to which the ($\omega/2M$) corrections and non-static adiabatic corrections contribute in the same order. In this manner, the $K^{(a)}_{4n}$'s containing only nucleon pairs, are summed, leading to the "incomplete static" potential:

$$V^{(a)} = -\frac{3\mu}{2\pi}\int_0^\infty \frac{k\,dk}{(1+k^2)^{1/2}} \tan^{-1}\left\{\frac{4\delta^2 \sin(2kx)/x^2}{1-4\delta^2\cos(2kx)/x^2}\right\} \qquad (16)$$

where $\delta = G^2/4\pi \;\; \mu/2M$, $x = \mu r$

The $K^{(b)}_{4n}$'s containing ($2n-1$) nucleon pairs, are summed together with the appropriate corrections to the $K^{(a)}_{4n}$'s ; the resulting series is:

$$V^{(b)} = \left[1-4\delta^2 e^{-2x}/x^2\right]^{-1} 6\mu\delta^2(\mu/2M)(1+x^{-1})^2 e^{-2x}/x^2 \qquad (17)$$

Finally, the K'_{4n+2}'s are summed in static approximation with the result:

$$V_{4n+2} = \left[1-4\delta^2 e^{-2x}/x^2\right]^{-1} 4\mu/3\,(\underline{\tau}_1\cdot\underline{\tau}_2)\,\delta^3(\mu/2M) \qquad (18)$$

$$\times\ (1+x^{-1})^2 e^{-3x}/x^2\,\left[\underline{\sigma}_1\cdot\underline{\sigma}_2 + S_{12}\right]$$

It is seen that the condition for convergence of the above three sums is $\chi_0 e^{2\chi_0} \gtrsim 2\delta$ which yields $\chi_0 \geq 0.56$ for $G^2/4\pi = 10$ and $\chi_0 \geq 0.87$ for $G^2/4\pi = 15$, to mention only two reasonable limits for the coupling constant. Since the larger value of the coupling constant is probably closer to the truth (at least on the basis of the pion-nucleon scattering theory and data), Klein's result implies that the higher order static interactions are comparable to the second and fourth order static interactions at distances comparable to $1/\mu$. This throws considerable doubt on the sum of the second and fourth order contributions as a suitable static potential (valid at distances $\gg \frac{1}{M}$). While Klein's convergence criterion is probably too rigid because of the artificially strong singularities introduced by his expansion of the denominators of the $K_{2n}{}'R$ in powers of $\omega/2M$ (cf. above), it must be admitted that his work indicates that the predictions of the PS(PS) theory are, in a strict sense, unknown even in the static approximation (apart from the incidental fact that the radiative corrections to the higher orders have not been calculated).

VI. Comparisons with low energy experimental results.

Nevertheless, despite the above qualifications which must always be borne in mind, a great deal of labor has gone into comparing the predictions of the second order static nuclear potential plus certain crude versions of the fourth order static potential with the experimental data at low and moderate energies. It seems worthwhile to briefly mention this work. First, Lévy used V_2 plus a V_4^{L} given by:

$$V_4^{L} = V_4^{(a)} + \delta V_4^{(a)} \qquad (19)$$

The V_4^{L} term represents the sum of $V_4^{(a)}$ plus what he had erroneously calculated to be the contribution of the two-pair and one-pair diagrams to order $(G^2/4\pi)^2(\mu/2M)^3$. Since the second and fourth order interactions become repulsive at sufficiently small distances ($\lesssim \frac{1}{M}$), Lévy assumes that Eq. (17) is valid down to $r = r_c$ (radius of the repulsive core) which he treats as an adjustable parameter. Fitting the deuteron binding energy and the singlet scattering length at low energies, he determines the values of $G^2/4\pi$ and r_c, namely $G^2/4\pi = 10$, $r_c = 0.38/\mu$. He then proceeds to calculate the singlet and triplet effective ranges, the percentage of D wave and the quadrupole moment of the deuteron. The first three quantities agree well with experiment whereas the quadripole moment turns out to be 20% too small. The disagreement with the deuteron quadripole moment is a serious matter--as Blatt and Kalos have recently shown--unless one argues that meson exchange effects can influence the static quadripole moment (Villars, Deser). Finally, Lévy calculates the n-p scattering cross section at 40 Mev with the above values of $G^2/4\pi$ and r_c and finds that while the absolute value of the cross section is in reasonable accord with experiment, the differential cross section is not sufficiently anisotropic. The n-p calculations were carried out by assuming the same repulsive core for all states, performing an exact calculation of the S phase shifts and using the Born approximations for the P and D phase shifts but neglecting the coupling of the $^3P_2$ to the $^3F_2$ state.

Further calculations were performed by Martin and Verlet of the p-p scattering cross sections at 18 and 32 Mev, making precisely the same approximations and assumptions as Lévy made for the 40 Mev n-p scattering. While excellent agreement was found with the experimental data, the calculations were criticized by Wick on the ground that the Born approximation is not justified for the P and D phase shifts (particularly the $^3P_0$ phase shift). New calculations of the p-p scattering were, therefore, undertaken by Martin and Verlet who, in place of the Lévy potential, took a static potential of the form:

$$V = V_2 + \lambda V_4^{(a)} \tag{20}$$

where λ was regarded as an arbitrary parameter (in an attempt to simulate the radiative corrections); $G^2/4\pi$ was varied accordingly but it was still assumed that $r_c = 0.38/\mu$. Exact calculations of the p-p scattering cross sections at 18 and 32 Mev with Eq. (20) showed that the 18 Mev data still exhibit reasonable agreement with the theoretical predictions and are rather insensitive to the value of λ. On the other hand, the previous agreement with the 32 Mev data disappears and the situation is not improved by taking account of the non-perturbation correction (i.e. $E \neq 2M$) and of the coupling of the $^3P_2$ to the $^3F_2$ state. Moreover, the magnitude of the p-p cross section at 32 Mev is only attained by choosing $\lambda > 1$, which is contrary to the damping effects of the two-pair static potential expected on the basis of Wentzel's calculations. Indeed, Blatt and Kalos have demonstrated, by means of detailed calculations on an electronic computer, that a static potential of the form (20) with $G^2/4\pi$, λ and r_c allowed to vary arbitrarily, will not yield a fit of all the low energy data. They fix the three adjustable parameters by fitting the binding energy and quadripole moment of the deuteron and the singlet scattering length and then find that the predicted effective ranges are outside the experimental errors by 20%). As we have already remarked, exact fitting of the deuteron quadripole moment is a strong constraint and the effective ranges can easily be duplicated if some leeway is permitted in the fitting of the quadripole moment.*

We thus see that a completely satisfactry fit of the low energy nuclear data is not achieved with any of the static potentials partially derived on the basis of the PS(PS) theory. We underline the word partially since none of the potentials used for the numerical calculations is actually the total potential predicted by the PS(PS) theory (including radiative corrections, proper treatment of the repulsive core, etc.) in a well-defined approximation. Nevertheless, it is interesting that the qualitative features of even the partial $V_2 + \lambda V_4^{(a)}$ PS(PS) potential are in accord with the low energy data: the long range tensor force in V_2 is needed for the quadripole moment, the short-range Wigner force in $V_4^{(a)}$ helps to keep the admixture of D wave in the deuteron small while allowing the quadripole moment to stay large and the repulsive core is then useful to preserve the correct effective range. A short-range singular Wigner force is also needed to give to the singlet central force the same degree of singularity as the

*A calculation using the potential of Eq. (20) has also been done by Jastrow, who reported good agreement with all the low energy data--rather insensitive to the value of r_0 -- provided that slightly different core radii are assumed for singlet and triplet states.

triplet central force, and thus help in fitting the singlet scattering length with the same coupling constant as the one required for the deuteron binding energy. It would seem that the total PS(PS) potential should retain these features and, to that (limited) extent, the outlook is not entirely negative. However, it cannot be asserted at this time that the PS(PS) theory is capable of explaining the low energy data. Moreover, we have seen that trouble sets in when we consider moderate energies such as 32 Mev for p-p scattering and 40 Mev for n-p scattering. Again the disagreement found cannot be used as an argument <u>against</u> the PS(PS) theory since the total static potential was not used in the calculations and only weak attempts were made to estimate the non-static corrections.

VII. Situation at high energy.

When we come to high energy nucleon-nucleon scattering, one can fairly say that not even a partial PS(PS) theory has been confronted with the experimental data. The non-static corrections to the second and fourth order interactions must certainly become important at energies of 300-400 Mev but no serious attempt has been made to work them out. Moreover, at these high energies, the distinction between the two-pair, one-pair and no-pair terms becomes increasingly less important since the effective energies of the meson quanta exchanged between the nucleons increase (i.e., the parameter is no longer $\mu/2M$, but something like $\omega/2M$ where ω is the meson energy); one can put it another way by saying that the distances from which the major contributions to the scattering phase shifts ~~arisse~~ arise become smaller $(< 1/\mu)$ as the energy increases. By the same token, the higher order interactions become more and more comparable to the second and fourth order interactions so that the expansion parameter instead of being $\left(\frac{G^2}{4\pi}\right)\left(\frac{\mu}{2M}\right)$ is more like $G^2/4\pi$. Despite these substantial qualifications and in view of our ignorance of anything more than a partial static potential in the second and fourth orders, it would still be of interest to examine the consequences of a static potential like Eq. (20) for the high energy nucleon-nucleon scattering. While no quantitative calculations have been performed, the following qualitative predictions of Eq. (20) (assuming a repulsive core only in S-states, which was not the case for the p-p and n-p calculations at moderate energies--cf. above) should be noted: (1) the isotropy of the unpolarized p-p cross section could be explained by the combination of the tensor force in the triplet states and the repulsive core in the S-state (Noyes and Cammitz). (2) the strong polarization effect produced in p-p scattering* could be explained

* A full discussion of the polarization experiments at high energies will be found in the Proc. of the Fourth Annual Rochester Conference on High Energy Nuclear Physics, 1954.

by the strongly singular tensor force with a gradual cutoff (not the repulsive-core type cutoff for the higher states--cf. Goldfarb and Feldman). Parenthetically, we note that the PS(PS) theory predicts a spin-orbit force in fourth order, namely:

$$V_{s.o.}^{4} = -6\left(G^2/4\pi\right)^2\left(\mu/2M\right)^4\left(\mu^3 r\right)^{-1}\frac{\partial}{\partial r}\left[K_1(2\mu r)/r^2\right]\underline{L}\cdot\underline{S} \tag{21}$$

with $\underline{L} = \underline{r} \times \underline{p}$; $\underline{S} = \frac{1}{2}(\underline{\sigma_1} + \underline{\sigma_2})$

While the coefficient in front of $V_{s.o.}^{(4)}$ is small, the strong singularity leads to an appreciable spin-orbit force at small distances. Since the smaller distances are important for the high energy scattering, Eq. (21) may lead to a significant polarization effect in addition to the tensor force. (3) the PS(PS) potential (20) will probably not fit the unpolarized n-p scattering if the experimental cross-section is really quite symmetric (albeit strongly anistropic) about 90° (in the c.m. system). The experimental situation still remains to be clarified. (4) the fact that the polarization effect in n-p scattering is not greater than in p-p scattering--despite the presence of twice as many triplet states for the n-p system as compared to the p-p system--could be explained if the tensor force is chiefly responsible for the polarization since the $\underline{\tau_1}\cdot\underline{\tau_2}$ factor in front of the tensor force leads to opposite signs for the isotopic singlet states (which only operates for the n-p system) and isotopic triplet states (which operate for both the n-p and p-p systems). These qualitative expectations may not be borne out by quantitative calculations but on the face of it, there seems to be a great deal of potential good in the PS(PS) theory. In conclusion, we must confess that our original question of whether the pion field can provide the basis for a correct meson theory of nuclear forces has been left unanswered.

One of the authors (R.E.M.) was a Guggenheim Fellow when this report was written.

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FIGURES

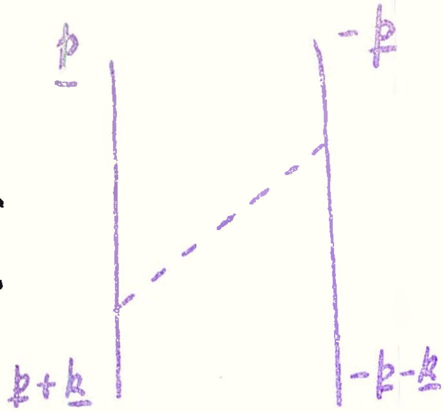

FIG 1: Second-order diagram.

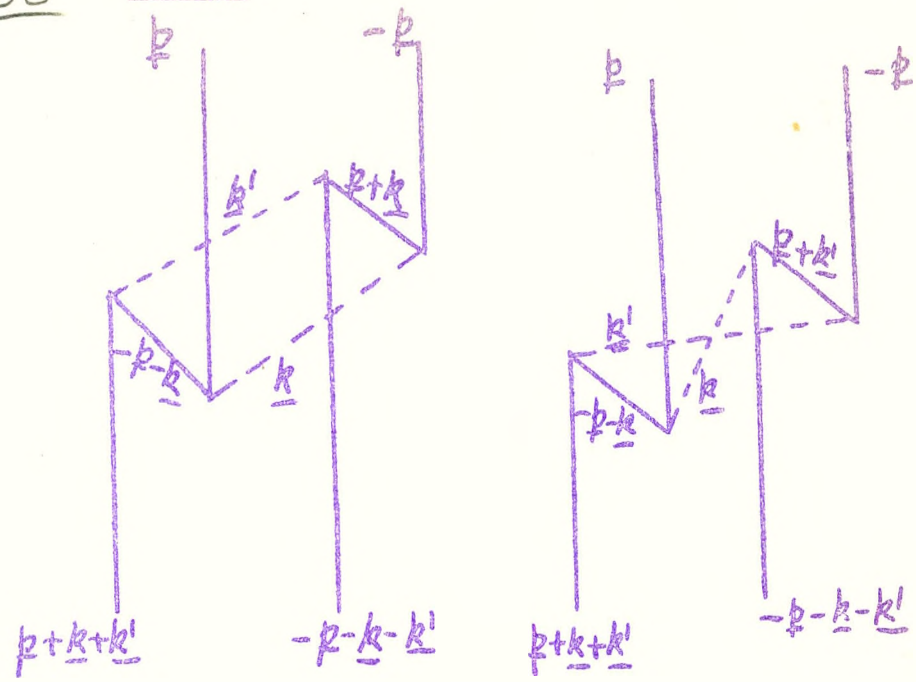

FIG 2: Leading fourth-order diagrams in static approximation.

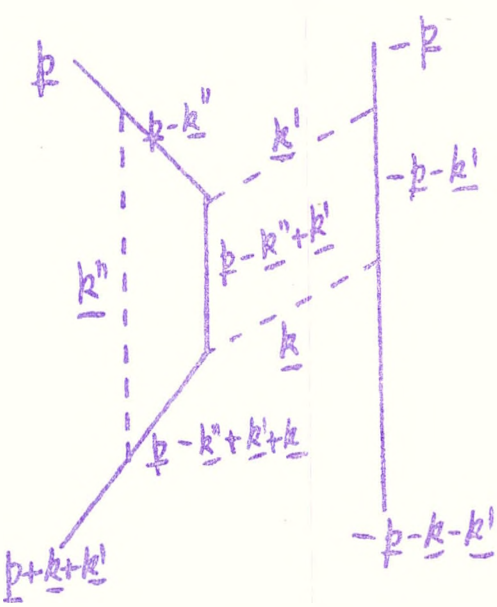

FIG 3: Finite radiative correction to fourth-order interaction.

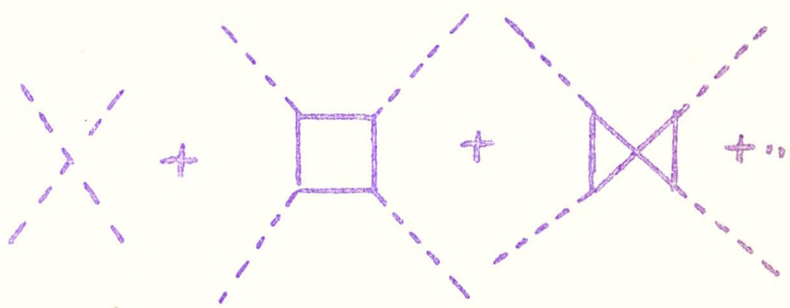

FIG 4: Meson-meson scattering type diagram