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DISPERSIVE TEMPORAL COMPRESSION OF LASER PULSES AS AN
ALTERNATIVE SOLUTION TO THE SELF FOCUSING PROBLEM
IN LASER AMPLIFIER CHAINS*

Robert A. Fisher and W. K. Bischel

November 5, 1973

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ABSTRACT

In response to the discovery of a nonlinear index of refraction (n_2) in Nd:glass laser media, experimenters have traditionally expanded the beam diameter (and reduced the intensity) to alleviate the self focusing tendency. We propose, instead, to reduce the peak intensity by injecting a lower intensity and longer-duration pulse into the amplifier chain. We predict that the glass nonlinearity will impress upon the pulse a chirp suitable for efficient temporal compression. This may result in more efficient laser operation. Related schemes have been used in radar systems. As an example, we have calculated the evolution of a 1 nsec (full 1/e duration) temporally Gaussian pulse in a 2 m long Nd:glass laser chain. For a chain averaged intensity of 2 GW/cm^2 , we calculate that the pulse could be subsequently compressed (by a series of Gires-Tournois interferometers) to 125 psec with good stability against input pulse amplitude noise.

*This work was performed under the auspices of the U. S. Atomic Energy Commission.

I. INTRODUCTION

For efficient and reliable operation of a high energy short pulse Nd:glass laser system, there are many reasons for reducing the light intensity in the laser material. We wish to present here a pulse compression scheme consisting of the use of longer duration and less intense pulses in the laser amplifier chain, followed by the use of a dispersive structure to temporally compress the pulses after they exit the laser chain. This dispersive structure must have a frequency dependent group delay such that lower frequencies have a greater transit time. This scheme is pictorially depicted in Fig. 1. In order to prepare the pulses for compression, we take advantage of the nonlinear index of refraction of the host glass.

Duguay, Hansen, and Shapiro¹ have presented evidence which has brought much attention to the nonlinear index of refraction of the host glass in Nd:glass lasers. Here the glass index of refraction is written

$$n = n_0 + \frac{1}{2} n_2 \mathcal{E}^2$$

where n_2 approximately equals 1.7×10^{-13} esu and \mathcal{E} is the electric field envelope function. This nonlinear index of refraction is responsible for self focusing² (in which a bright beam may constrict and intensify, and may subsequently damage optical components), and self phase modulation³ (in which the spectral content of a pulse broadens as the pulse propagates). As pointed out by Bliss,⁴ large scale self focusing tendencies can be reduced somewhat by certain design considerations. Marburger and co-workers⁵ have addressed the problem of small scale self focusing in laser systems due to statistical fluctuations of the index of refraction. The self focusing tendency is readily alleviated by increasing the beam cross sectional area to reduce the peak intensity.

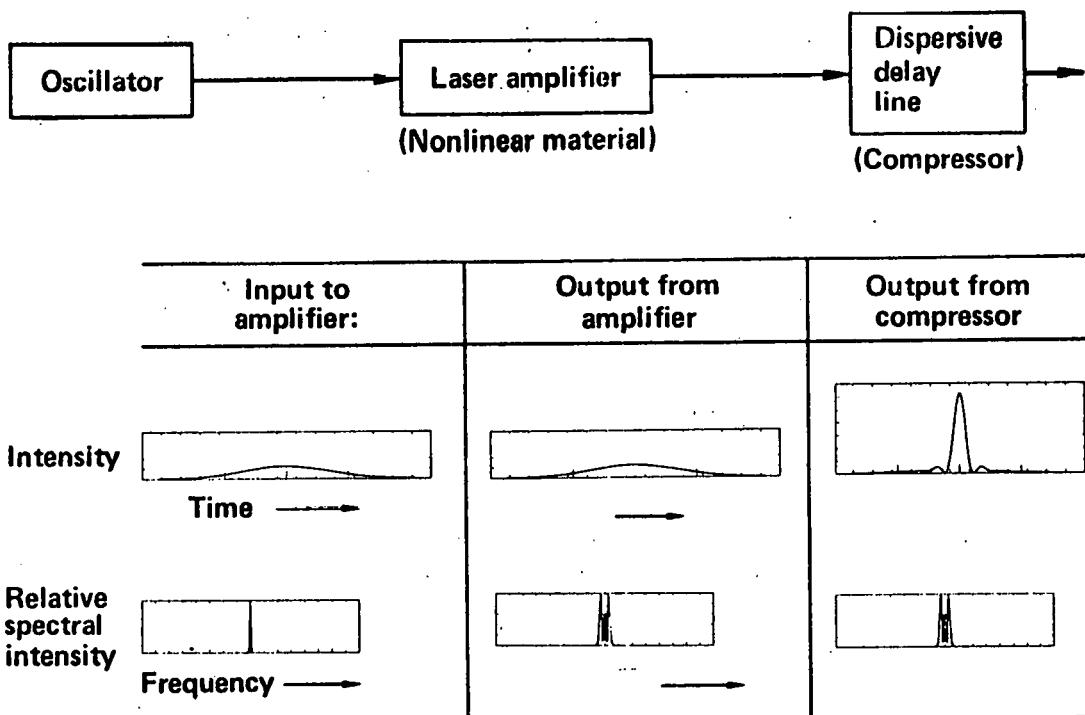


Fig. 1. A pictorial diagram of the pulse compression scheme. The pulse spectrum is broadened due to self phase modulation in the laser amplifier. The pulse is then compressed by a dispersive delay line which takes advantage of the self phase modulation "chirp."

The above considerations of the effects due to the glass nonlinearity have a profound (and costly) effect on high power, ultra short pulse laser chain design, and there is strong interest in developing laser glasses with a lower nonlinear index (n_2). We wish to point out that there may be a wide variety of circumstances in which we find that the concern over n_2 related engineering problems may to a large extent be unnecessary. We base this contention on our observation that although self focusing is detrimental to the faithful operation of a laser amplifier chain, self phase modulation can be useful if the pulse emitted from the laser chain acquires a "chirp" suitable for subsequent dispersive compression.

II. PULSE COMPRESSION SCHEME

The scheme of "chirp radar"⁶ was first made public in 1960. This development resulted in far more efficient operation of a high resolution radar set because the peak power limited transmitters could generate longer duration pulses with more energy. With the appropriate chirp impressed on the outgoing pulse, the reflection (signal) could be temporally compressed by passage through a suitably dispersive circuit, thus optimizing the effective power handling capability of the transmitter.

Analogous compression of chirped optical pulses was independently proposed by Gires and Tournois⁷ and by Giordmaine, et al.⁸ It has been suggested⁹ that unchirped laser pulses may be temporally compressed by first passing them through a nondispersive optical Kerr material, and subsequently compressing them by passage through an anomalously dispersive structure. Some

experimental verification of this scheme was presented by Laubereau,¹⁰ but the two photon fluorescence technique used to detect reshaping has been shown to be quite unsuitable in this regard.¹¹ Duguay and Hansen¹² have successfully performed a related compression scheme by rf modulation of cw mode-locked laser pulses and subsequent dispersive compression.

We are pointing out here that a Nd:glass laser chain may serve as the optical Kerr material or "chirper" in the compression scheme of Ref. 9. We address our attention to Nd:glass laser chains suitable for preliminary laser fusion experiments. Fusion experiments with laser heated pellets will require 10^4 J laser pulses with durations of approximately 100 psec.¹³ Since it has been estimated¹⁴ that the cost of capacitors, flashlamps, and glass for such a 10^4 J Nd:glass laser system depends linearly upon the reciprocal of the laser pulse duration, the savings can be considerable. We also expect that this pulse compression technique could improve by an order of magnitude the temporal resolution of the laser moon-ranging effort.¹⁵

In the absence of dispersion, the preparation of self-phase modulated plane wave laser pulses for subsequent dispersive compression is fairly well understood.⁹ Let us first consider the example where self-phase modulation of an initially temporally Gaussian pulse in nondispersive undoped glass causes a chirp to be impressed on the pulse. R , the ratio of pulse durations before and after optimal delay line compression is estimated by comparison of spectral widths of input and output pulses. From Appendix A, we find:

$$R = \frac{T_{\text{original}}}{T_{\text{compressed}}} = 0.86 K_0 \ell \delta n_{\text{max}} + 1. \quad (1)$$

Here $K_0 = 2\pi/\lambda$ (λ is measured in vacuum), l is the propagation length, and δn_{\max} is the maximum nonlinear index change. Note that R is not a function of pulse duration.

Prior to compression, the electric field is written as

$$E(t) = \frac{1}{2} \mathcal{E}(t) \{ \exp [-i(\omega_0 t - k_0 z - \delta\phi)] + \text{c.c.} \}, \quad (2)$$

where ω_0 is the laser frequency, $k_0 = \omega_0 n_0 / c$, and $\delta\phi$ is the phase perturbation. The envelope function $\mathcal{E}(t)$ is a real quantity. The compressed complex electric field of the pulse is discussed in Appendix B, and is given by⁹

$$E_c(t) = \frac{e^{-i\omega_0 t}}{2\pi} \iint_{-\infty}^{\infty} dt' d\Omega \mathcal{E}(t') \exp[i[\Omega(t' - t) + \delta\phi + Q_2\Omega^2 + Q_3\Omega^3 + \dots]] \quad (3)$$

where $\Omega \equiv \omega - \omega_0$ and Q_2 , Q_3 , etc. are coefficients determined by the details of the compressor. If a pulse has a positive "linear chirp," a negative group dispersion ($Q_2 < 0$) can compensate for the chirp in order to compress the pulse. In most cases the $Q_3\Omega^3$ term can be neglected.

III. A 2 GW ONE NSEC EXAMPLE

Since staging will tend to keep the peak intensity relatively constant, we consider a pulse with a chain averaged peak intensity on-axis of 2 GW/cm^2 . This corresponds to peak index change given by $\delta n_{\max} = 9.2 \times 10^{-7}$. Inserting this value into Eq. (1), we find $R = 10.3$ if the pulse passes through 2 m of glass. If the original pulse had a duration of 1 nsec, the optimally compressed pulse would be approximately 97 psec in duration.

The above estimate is somewhat imprecise. It assumes that the entire bandwidth of the self phase modulated pulse will participate in pulse compression. Since the chirp due to self phase modulation is not a linear ramp, there will be some reduction from this estimate. The estimate also does not reflect a spatial average over transverse modes, and there will also be some deviation because the glass and the amplifying ions are somewhat dispersive which alters the collection of nonlinear phase change as the pulse propagates. Although the latter effect is quite small, its influence will be included.

In order to more accurately estimate the dispersive compressibility of plane wave 1 nsec (full 1/e intensity duration) Gaussian laser pulses, we have performed computer simulations of propagation in Owens-Illinois¹⁶ ED-2 glass amplifier chains using a previously described¹⁷ algorithm. The plane wave calculation includes: the nonlinearity of the glass¹ ($n_2 = 1.7 \times 10^{-13}$ esu), the dispersion of the glass¹⁸ [$\lambda^3 (d^2 n / d\lambda^2) = 0.01845 \mu\text{m}$], and the linear properties of the amplifying ions. The gain at line center was chosen $g(0) = 0.09 \text{ cm}^{-1}$, and T_2 for the homogeneously broadened line was chosen as $T_2 = 0.042 \text{ psec}$. (Since the value of n_2 is only known to within 20%, the results presented here may undergo a small refinement as the value becomes better known.)

Staging was modeled by increasing a distributed loss until the peak pulse intensity remained relatively independent of position along the chain. For the 1 nsec pulse, this stability condition was achieved at $\alpha_{\text{loss}} = 0.089 \text{ cm}^{-1}$. Since the nonlinear phase important to dispersion-free self phase modulation is equal to $\frac{1}{2} K_0 n_2 \int_0^{\lambda} E^2(z) dz$ (which is proportional to the chain averaged intensity), this staging modeling does not significantly detract from the accuracy of

the calculation because it keeps the chain averaged intensity correct. Similarly, $g(0)$ is chosen as a chain averaged quantity, and this provides the correct influence of dispersion due to the resonance. Dispersive compressibility was calculated using Eq. (3) with $Q_3 = 0$ and $Q_4 = 0$ after 2 m of laser glass propagation, and is shown in Fig. 2. 1.06 μm input pulses of 1 and 5 GW/cm^2 were compared to the 2 GW/cm^2 case. The "relative compression settings" in each figure are proportional to the values of Q_2 , and our convention is described in Section VI. The 5 GW/cm^2 pulse is only shown for comparison because it will saturate the transition. Saturation was not considered in this calculation. If saturation were included, the resultant reshaping in time will substantially modify the chirp on the pulse. As can be seen, all three pulses develop sufficient chirp for efficient compression, yet we will focus our attention on the 2 GW/cm^2 case. If the slowly rising leading edges were unwanted, they might be somewhat eliminated by passage through a suitable saturable absorber.¹⁹ The optimally compressed pulse has a full 1/e duration of 125 psec. Thus we can improve the crude compression ratio estimate in Eq. (1) by replacing the multiplier (0.86) by the empirical value 0.65. Although the duration of the peak pulse in this example is reduced by a factor of 8, its peak intensity is only increased by a factor of 6.2. This implies that the efficiency of compression is $\sim 76\%$. We expect the case of Nd:YAG oscillator pulses (1.064 μm) in a glass chain to give quite similar results because the dispersion due to the resonance is at maximum only 10% of $(d^2n/d\lambda^2)$ due to the glass.

IV. STABILITY AGAINST AMPLITUDE NOISE

A. Periodic Amplitude Modulation

Since the temporal shape of the input pulses may not be as smooth as in

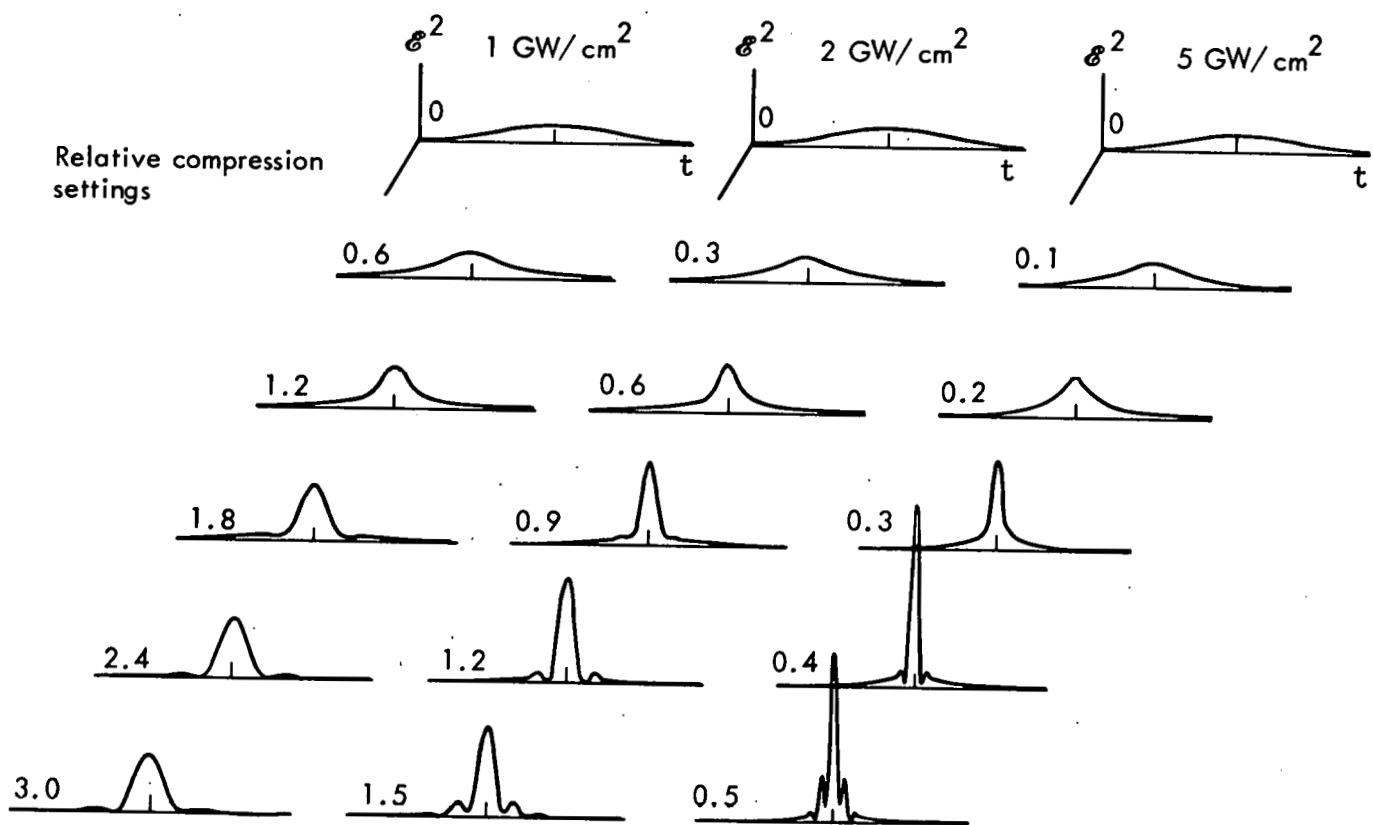


Fig. 2. Calculated dispersive compressibility of pulses emanating from a 2m Nd:glass laser chain. Cases are shown for input peak intensities of 1, 2, and 5 GW/cm^2 . Spatial effects due to the radial intensity profile are not considered. Relative compression settings in all figures are proportional to Q_2 (in Eq. (3)), with a setting of unity corresponding to $Q_2 = 6.4 \times 10^{-21} \text{ sec}^2$. In each case, the initial full 1/e intensity duration is 1 nsec.

the calculated cases presented here, we have repeated the above calculations with 10% (peak-to-peak intensity) sinusoidal ripple impressed upon the input pulse. Results are shown in Fig. 3. It can be seen that these considerations do not substantially reduce our interest in the 2 GW/cm^2 case.

1. The 2 GW/cm^2 case. In the case of compressing 2 GW/cm^2 chain averaged peak intensity pulses, we observe that for dispersive delay settings far less than the optimally compressing setting, the impressed ripple becomes accentuated in the compressed pulse. Each ripple, however, corresponds to a richer spectral content, and is quite smoothly dispersed (over-compressed) by the optimally compressing dispersive delay. At optimum compression, the 10% ripple case is indistinguishable from the smooth case (Fig. 2) in the time span we present. It is somewhat unfortunate, however, that during self phase modulation the spectrum of the pulse develops sharp sidebands (at the modulation frequency) well separated from the smooth pulse self-phase modulated spectrum (Fig. 4). These sidebands apparently have a spectral intensity dependent growth rate, and are $\sim 1/20$ th the height of the peak spectrum by the time the pulse exits the laser chain. At the optimally compressing dispersive delay (the setting 1.2 in Figs. 2 and 3), these "renegade" spectral components are delayed and advanced by approximately 1.72 nsec with respect to the center of the compressed pulse. These temporal features (not within the field of view on Fig. 3) are approximately the height and shape of the secondary lobes which appear on the 2 GW/cm^2 pulse after it has been optimally compressed.

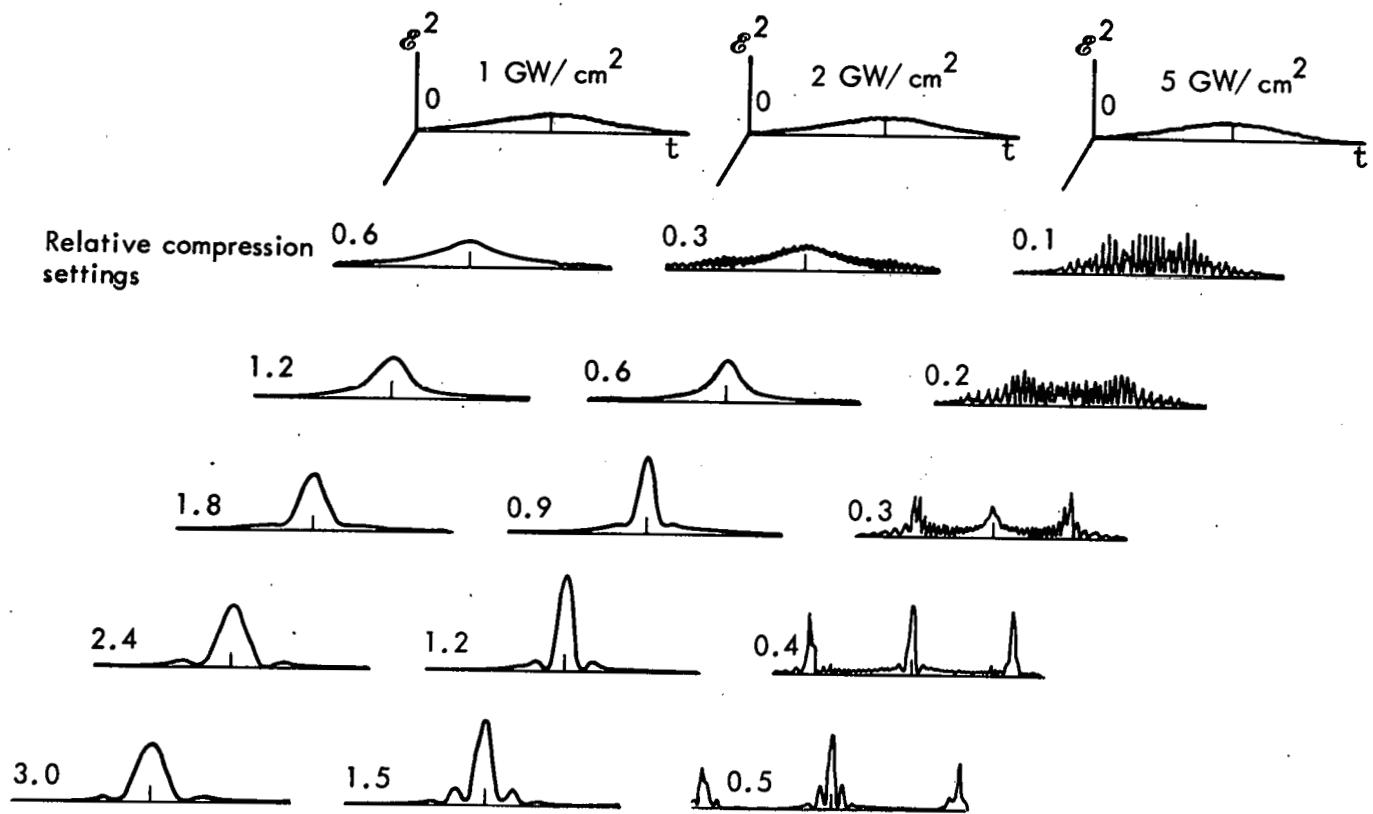


Fig. 3. A repeat of the calculation of Fig. 1 with the inclusion of 10% peak-to-peak amplitude ripple on the pulse which enters the laser chain. Note that compression of ripples occurs for far smaller compressor settings, so that the unwanted features are again temporarily spread out at the optimally compressing dispersive delay setting, except in the 5 GW/cm^2 case.

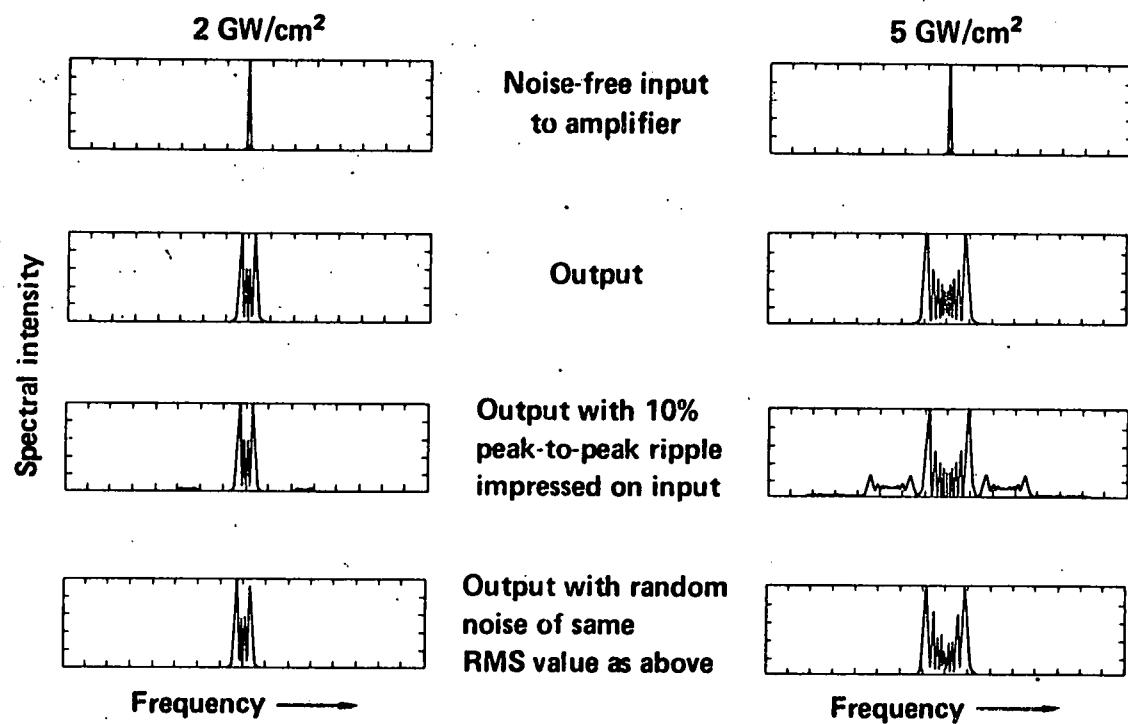


Fig. 4. Comparison of calculated spectra for pulses emanating from laser amplifier chain. Cases of noise free input, 10% peak-to-peak ripple, and random noise of the same RMS deviation as in the 10% ripple are presented for chain averaged peak intensities of 2 GW/cm^2 and 5 GW/cm^2 .

2. Comparison to other average intensities. In the 5 GW/cm^2 case, the precursors are prominent features, and are within the field of view because less dispersive delay is used in compressing the pulses. The precursor in the 2 GW/cm^2 case will have to be blocked by a suitable gate or saturable absorber. Thus we conclude that periodic amplitude modulation is somewhat deleterious to this high power glass laserpulse compression scheme because it can (if there is sufficient depth of modulation on the input pulse) cause undesirable precursors to come out of the compressor. With the exception of producing these precursors, these periodic amplitude changes do not affect the compressed pulse shapes.

B. Random Amplitude Modulation

We have repeated the above calculations with the inclusion of a random amplitude modulation. The modulation was generated by a multiplication of the original Gaussian pulse shape function at time N by the factor $[1 + \alpha A_N]$. Here α is an adjustable constant and $A_N = \sum_1^N R_i$, where $\{R_i\}$ is a set of random numbers distributed between the ranges + 1. and - 1. The pulse was filtered by a 0.4 \AA bandwidth Lorentzian filter, and the value of α was then adjusted until the calculated RMS deviation from the original Gaussian was equal to the calculated RMS deviation for the 10% peak to peak periodic amplitude ripple previously discussed. The spectral filtering did not significantly reduce the RMS deviation. We have calculated the passage of such randomly modulated pulses through the 2 meter laser chain, and have studied their compressibility. We find, as before, that for far less than optimally compressing dispersive delay settings, the noise is accentuated, but at optimally compressing dispersive delay, the noise is not discernable. There is also no perceptible precursor, and the spectrum as seen in Fig. 4 does not exhibit sidebands as had been observed in the periodic modulation case.

As we had pointed out before, the growth rate of a sideband is spectral intensity dependent, and the spectrally broader random noise did not have suitable peak spectral amplitude to get significantly amplified during self-phase modulation in the laser chain.

From the comparison of the above results for periodic and random amplitude modulation, we conclude that although random amplitude modulation can be well tolerated, efforts must be made to reduce periodic amplitude modulation in order to provide a pulse suitable for precursor-free dispersive compression.

V. COMPRESSOR CONFIGURATIONS

Since the pulses we wish to compress have a positive chirp, the group dispersion of the compressor must be anomalous so that lower frequencies are delayed more. By this we mean that Q_2 in Eq. (3) must be negative. For propagation through an optically clear dispersive material, this requires $(d^2n/d\lambda^2) < 0$. (In all figures, a relative compression setting of 1.0 corresponds to $Q_2 = -6.4 \times 10^{-21} \text{ sec}^2$.) The compressor we will focus our attention on is a Gires-Tournois interferometer.⁷ Other potential candidates found to be less desirable include a grating pair, a gas or liquid which absorbs at a wavelength slightly greater than 1.06 μm , and possibly an electron gas. We will discuss only the Gires-Tournois interferometer and the grating pair here.

A. The Gires-Tournois Interferometer

The Gires-Tournois interferometer is a modified Fabry-Perot interferometer with unequal reflectivities. One surface is 100% reflecting, one surface is partially reflecting, and the device is obviously used in reflection. All light incident on the device is reflected, so there are no fringes. Two

different Fourier components, however, can suffer very different phase shifts. The first reflection upon entry to the device may be either reinforced or cancelled by contributions from the first few "zig zags" between plates. Thus some colors are reflected "right away," while others are reflected only after several internal zig zags. This gives a frequency dependent group delay. The theory of this device has been presented elsewhere,⁷ and an English translation of this reference is included in this note as Appendix C. We calculate that Q_2 for this device is given by

$$Q_2 = \frac{1}{2} \frac{\partial^2 \phi}{\partial \omega^2} \Big|_{\omega=\omega_0} = - \frac{t_0^2 (1 - r^2) 2r \sin \delta}{2(1 + r^2 - 2r \cos \delta)^2} \quad (4)$$

where ϕ is the phase shift introduced by the compressor, r is the square root of the reflectivity for the partially transmitting coating, $t_0 = 2sn_0 \cos \theta/c$ (s is the spacing between reflecting surfaces, n_0 is the index of refraction, θ is the angle of incidence, and c is the speed of light), and $\delta = (\omega_0 - \omega_R)t_0$ where ω_0 is the center frequency of the laser, and ω_R is the nearest resonant frequency of the cavity. Note that Q_2 can be either positive or negative (thus causing expansion or compression of the pulse) depending on whether $\omega_R > \omega_0$ or $\omega_R < \omega_0$. Since the configuration of the interferometer strongly depends on the pulse bandwidth BW (because compressing linear chirp efficiently requires that $Q_2 \gg Q_3 (BW/2) + Q_4 (BW/2)^2 + \dots$ where $Q_3 = 1/6[(\partial^3 \phi / \partial \omega^3) \Big|_{\omega=\omega_0}$ and $Q_4 = 1/24[(\partial^4 \phi / \partial \omega^4) \Big|_{\omega=\omega_0}$), we will focus our attention to the 2 GW/cm^2 case as seen in Fig. 2. We calculate that the optimal relative compression setting for this case is 1.2 (corresponding to $Q_2 = -7.6 \times 10^{-21} \text{ sec}^2$), and gives

a pulse compression ratio of about 8. Neglecting the influence of the Q_3 and Q_4 terms, this compression ratio would require sequential reflections from eight Gires-Tournois interferometers (front mirror reflectance at 49%, rear mirror reflectance at 100%, and t_0 equal to 15 psec). Although $Q_3 = 0$ at $\delta = 0.207$, we choose to operate at $\delta = 0.3$. The number of Gires-Tournois interferometers may undergo some change because optimization considerations concerning the Q_3 and Q_4 terms have not yet been fully analyzed. These terms, of course, would have a large effect on the precursors described in Section IV. We have performed compression calculations including the influence of the $Q_3\Omega^3$ and $Q_4\Omega^4$ terms for the above-mentioned device. We find that (because of the detrimental influence of the $Q_4\Omega^4$ term) optimal compression requires sequential reflection from nine interferometers. The slight reduction in compression ratio R and in peak intensity can be seen in Fig. 5. The detrimental influence of the higher order terms can be reduced without limit by suitable choice in the operating range (δ) of the interferometer.

It may be possible to tailor the pulse shapes inputted into the laser amplifier chain in order that the output chirp is well matched to the dispersive delay curve of far fewer interferometers. This could provide substantial savings. The configuration now envisioned is a set of four parallel Gires-Tournois interferometers facing a set of five interferometers separated from each other by a perpendicular distance h . The angle of incidence (θ) should be as small as possible, and is determined by the diameter of the beam d and the separation distance h such that $\tan\theta = d/2h$. This device could be tuned by changing the angle of incidence slightly, or by varying the pressure if the interferometers were air gapped. With air gapping, tuning (which must be done) can be interferometrically performed.

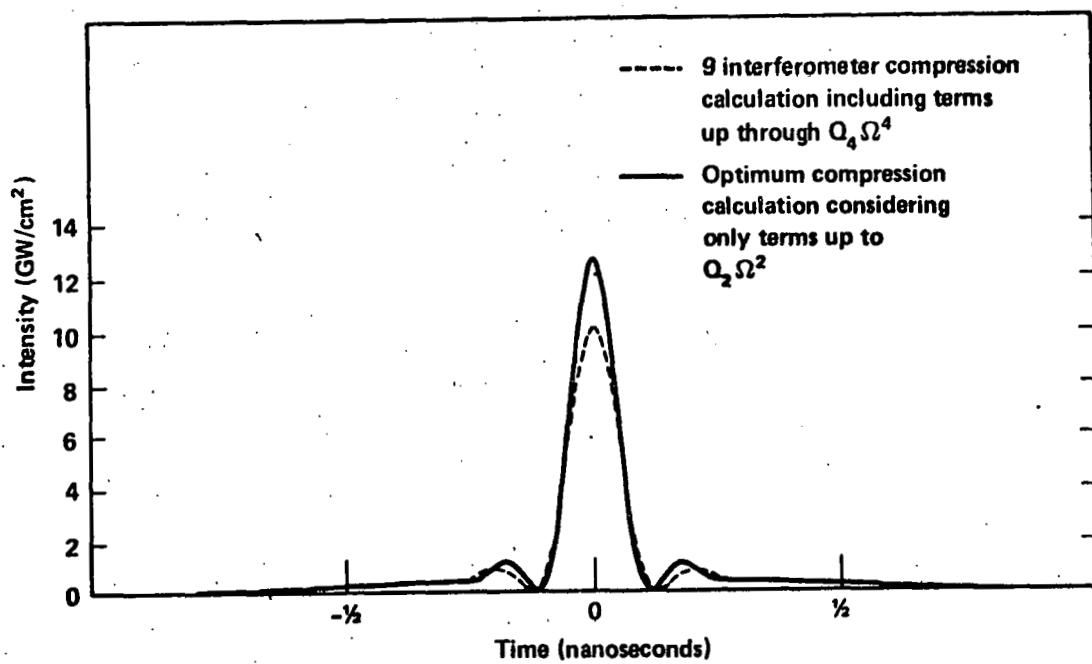


Fig. 5. Result for optimally compressed 2 GW/cm² pulse as shown in Fig. 2, compared to an actual calculated case considering terms up through $Q_4\Omega^4$. This case corresponds to reflections from nine interferometers under the conditions $\delta = 0.3$, $t_0 = 15$ psec, and $R = 0.49$.

It is essential to confirm that the response of the Gires-Tournois interferometer will not be obscured by the uncorrectable beam divergence introduced by the laser glass nonlinearity. It has been estimated²⁰ that for a chain averaged peak power of 10 GW/cm^2 , this divergence will be on the order of $\gamma = 30 \mu\text{rad}$. We estimate that for an angle of incidence of 6.6° (corresponding to $h = 2 \text{ m}$, $d = 46 \text{ cm}$), the change in δ of Eq. (4) is approximately $\Delta\delta \approx \omega_0(\gamma\theta + \gamma^2/2) t_0$. For the 2 GW/cm^2 case ($\gamma \approx 6 \mu\text{rad}$), the usable range of δ corresponds to $0.06 \leq \delta \leq 0.54$. This $\Delta\delta$ represents about 4% possible error in the exact delay seen by each Fourier component. Although this deviation is tolerable, this error can be minimized by reducing d , by increasing h , or by using the apodized chirper described in Sec. VII. We therefore conclude that this problem is not serious as long as the uncorrectable beam divergence does not become excessive.

There are many advantages of the Gires-Tournois interferometer when compared with the grating pair discussed below. Its main advantage is that its transmission efficiency is close to 100%. Secondly, it is less subject to damage in comparison to the grating pair. As long as the beam energy density is less than the damage threshold of bulk glass and good coatings it would be possible to build a more compact compressing device by focusing the beam down to a smaller size. An improvement in the coating technology may also allow a further size reduction of the device.

B. The Grating Pair Compressor

Another possible compressor configuration is a pair of parallel gratings. The theory of the dispersive delay due to the grating pair has been explained

elsewhere.²¹ We have calculated that a relative compressor setting of 1.0 (as used in all of our figures) corresponds to a 1200 λ/mm grating pair with an angle of incidence of 17.2°, angle of diffraction of 89°, and a slant height separation of 4 m. A grating pair has several drawbacks when envisioned as an actual compression device for laser fusion. First, the best grating blaze efficiency one can obtain is approximately 90%. Since such high angles of diffraction are involved here, there will be a shadowing effect because operation is so far from the Littrow condition. This could reduce the efficiency of a grating by as much as 2 orders of magnitude. Even if this problem were overcome, the combined efficiency of the grating pair would probably still be too low to warrant further consideration. Secondly, one must consider the possibility that gratings will damage at much lower intensities than good coatings. Although the damage threshold for gratings does not seem to be too well documented, one can surmise from the work of Bloembergen²² that the preponderance of sharp corners could severely reduce the ability of a grating to handle high light fluxes. This consideration would probably require unreasonably large surface gratings. One might solve this problem by piecing together several similar gratings, but the alignment problems and possible wavefront distortion would probably make this suggestion extremely hard to carry out.

VI. SPATIAL AVERAGING OVER TRANSVERSE STRUCTURE

Because the peak intensity is not constant across the spatial extent of the beam, we have averaged the above calculations over a Gaussian beam profile. For every plane wave pulse of peak intensity I_{max} , we write the compressed intensity as $C(I_{\text{max}}, t)$. The average over the spatial structure $\langle I \rangle$ is written

$$\langle I \rangle \propto \int_0^{\infty} C \left[I_0 e^{-(r/r_0)^2}, t \right] 2\pi r dr. \quad (5)$$

We have performed this averaging over a Gaussian beam profile with peak intensity on axis of 2 GW/cm^2 . The $Q_3 \Omega^3$ and $Q_4 \Omega^4$ terms were not included. The results are shown in Fig. 6, where it can be seen that significant compression still takes place. The optimally compressing dispersive delay is nearly 50% greater than in the 2 GW/cm^2 plane wave case, and this is because a greater dispersive delay is needed to compress the less intense portions of the pulse. The weaker spatial portions are responsible for the shoulder which appears to each side of the central peak, and some reduction of this shoulder could be effected by subsequent passage through a saturable absorber. Note that in the spatially averaged result, the efficiency is reduced. Although the pulse is compressed in time by a factor of 4.7, its peak is increased by 2.75. This corresponds to a 59% efficiency (compared to the 76% efficiency we found for the temporally Gaussian plane wave case). In practice, a far more flat topped (spatial) pulse is desired²⁰ (in preference to a Gaussian) in order that energy be extracted from a higher percentage of the amplifier cross sectional area. The reduction of peak power (through this compression scheme) will allow flat topped pulses to travel farther without self focusing, and we therefore expect that compressed shapes in practice will be more like the 76% efficient case in Fig. 2 than like the 59% efficient case in Fig. 6.

VII. ALTERNATIVE SCHEMES

Several variations of this scheme should be considered. First it must be pointed out that the Gaussian (in time) pulse shape is not optimum for generating a linear chirp. The leading and trailing edges of the pulse, for

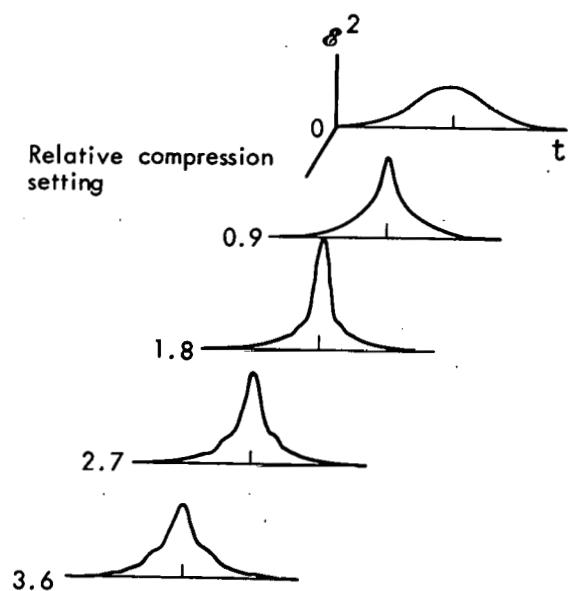


Fig. 6. Compression study of an average over a Gaussian transverse mode with peak chain averaged intensity on axis of 2 GW/cm^2 . Note that ample compression can be achieved. As in Figs. 2 and 3, Q_3 and Q_4 terms were not included.

instance, have a chirp with the wrong sign for compression. In the dispersionless plane wave case, the truly linear chirp is generated by a truncated parabola (in time). Clearly, if input pulses could be made closer to this shape, the efficiency of a pulse compression scheme would be substantially increased.

It is also possible that a more uniform chirp may be wanted across the transverse mode shape. If this is necessary, it may be possible to utilize a thin cell of CS_2 with windows made from plano-convex lenses. With the convex faces adjacent, the center of the beam will experience less path length of CS_2 than will the weaker edges of the pulse. With tailoring of the geometry, the phase front distortion can be reduced far below that introduced by a parallel sided CS_2 cell. This "apodized chirper" will not be as severe a source of unwanted self focusing as would be expected at first glance. One must be careful that the inclusion of CS_2 does not deplete the beam via 90° stimulated Raman scattering. At 2 GW/cm^2 the Raman gain is $\approx 15 \text{ cm}^{-1}$, so a CS_2 prechirper must be used early in the chain where the beam diameter is small, or might possibly be used in a "mosaic" structure at the end so that there is no long path with high Raman gain.

VIII. PROPOSAL

Since the experimental demonstration of these predictions need not await the construction of a 1 KJ chain, we have evaluated compression possibilities for the existing Lawrence Livermore Laboratory long path laser prior to the disks. In this system, the pulse duration is approximately 1 nsec, the peak on axis intensity averaged along the rod chain is 1 GW/cm^2 , and the length of glass

transit is approximately 5 m. Results of that calculation are shown in Fig. 7. No spatial average was performed here, so the signal depicted in Fig. 7 would be attained by trying to compress the portion of the beam which would pass through a centered aperture whose diameter were far less than any characteristic transverse dimension. Note that compression of the pulse should be readily demonstrable if the shape can be maintained down the chain. It is unfortunate that at present, the pulses inputted into the chain have large amplitude noise on them. In repeating the calculation of Fig. 7 with 50% impressed ripple, the resulting pulse cannot be dispersively compressed. An input pulse is required which is temporally smoother than the one presently injected into the amplifier chain. Although this example has been presented for the long path laser, any chain may be used for a verification of this scheme. The optimum compression ratio can be estimated from Eq. (1). Unpumped glass rods may be placed in the output for greater effective propagation lengths.

IX. CONCLUSIONS

We have presented an alternative means of operating high power short pulse laser amplifiers when they are to run near the limitations imposed by self focusing considerations. Our point is that self focusing can be avoided by increasing the time duration of the input pulse (and by correspondingly reducing the peak intensity). The chirp which the pulse acquires through self phase modulation can be suitable for efficient dispersive temporal compression. A computer calculated simulation was presented in which a 1 nsec (full 1/e intensity duration) pulse with peak power of 2 GW/cm^2 could

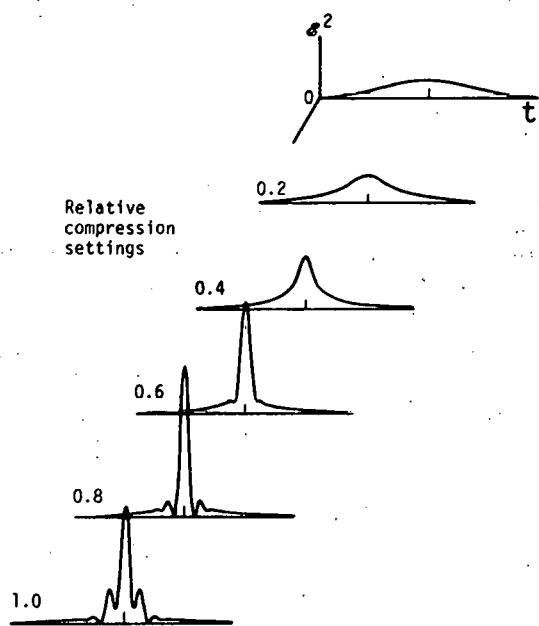


Fig. 7. The calculated compressibility of pulses emanating from the rod chain of the LLL Long Path Laser. Input conditions are: 1 nsec duration, 1 GW/cm^2 chain averaged peak intensity, and 5 m of propagation. These results correspond to trying to compress the spatially central portion of the beam.

be suitable for such a scheme after passage through a typical Nd:glass laser amplifier chain. The pulse can be compressed down to 125 psec in a suitably dispersive interferometer. Small fluctuations in amplitude (on the input pulse) were shown not to be of major concern in the scheme presented here. A calculated example was presented for an already existing laser chain so that this theoretical prediction might be verified. It is hoped that this note will help to initiate experimental verification of this pulse compression prediction.

Appendix A

Estimates of Spectral Growth in Dispersion Free Self Phase Modulation

In the absence of dispersion, an instantaneous Kerr effect perturbs the phase of the light pulse by the factor

$$\delta\phi(t_R) = \frac{K_0 \ln_2 \xi^2(t_R)}{2}, \quad (A-1)$$

where K_0 is the free space wave vector, ℓ is the distance of propagation, \ln_2 is the nonlinear index coefficient, and $t_R = t - n_0 z/c$. For a temporally Gaussian pulse we write the envelope ξ as

$$\xi^2(t_R) = \xi_0^2 e^{-(t_R/T_p)^2} \quad (A-2)$$

where T_p is one-half of the time between 1/e intensity points. The instantaneous frequency shift ($\Omega = -\partial\delta\phi/\partial t$) is given by

$$\Omega = -\frac{K_0 \ln_2 \xi_0^2}{T_p} \left[\frac{t_R}{T_p} e^{-(t_R/T_p)^2} \right] \quad (A-3)$$

The pulse full 1/e spectral bandwidth, BW, is given approximately by

$$\begin{aligned} BW &= \Omega_{\max} - \Omega_{\min} + BW_{\text{natural}} \\ &= \frac{K_0 \ln_2 \xi_0^2}{T_p} \left[\max(\delta e^{-\delta^2}) - \min(\delta e^{-\delta^2}) \right] + BW_{\text{natural}} \end{aligned} \quad (A-4)$$

where BW_{natural} is given by $2/T_p$.

The extreme values of $\delta e^{-\delta^2}$ occur when $\pm\delta^2 = 1/2$. We substitute the values $\delta \pm 1/\sqrt{2}$ to get

$$\begin{aligned} \text{BW} &= \frac{K_0 \ln 2 \xi_0^2}{T_p} \left[\frac{e^{-1/2}}{\sqrt{2}} + \frac{e^{-1/2}}{\sqrt{2}} \right] + \frac{2}{T_p} \\ &= \frac{K_0 \ln 2 \xi_0^2}{T_p} \left(\frac{2}{\sqrt{2e}} \right) + \frac{2}{T_p} \approx 0.86 \frac{K_0 \ln 2 \xi_0^2}{T_p} + \frac{2}{T_p} \end{aligned} \quad (\text{A-5})$$

Note that in the absence of nonlinearity ($n_2 = 0$), $\text{BW} = 2/T_p$.

This spectrally broadened pulse is still the same duration as it was when it entered the nonlinear material. The ratio of bandwidths after and before self phase modulation is found by dividing Eq. (A-5) by $2/T_p$ to get

$$R = \frac{\frac{0.86 K_0 \ln 2 \xi_0^2}{T_p} + \frac{2}{T_p}}{\frac{2}{T_p}} = 0.86 K_0 \xi_0^2 \frac{n_2 \xi_0^2}{2} + 1 \quad (\text{A-6})$$

Since $\delta n_{\max} = n_2 (\xi_0^2 / 2)$, Eq. (A-6) can be written

$$R = 0.86 K_0 \xi_0^2 \delta n_{\max} + 1 \quad (\text{A-7})$$

which is Eq.(1). The value of R represents the maximum possible pulse duration ratio if the compressing dispersive delay unit could be perfectly matched to the entire frequency shift curve.

Appendix B

Compression of Chirped Pulses with Dispersive Delay Lines

One can describe, in crude terms, a dispersive delay line as a passive element which delays different frequencies different amounts. This definition can be misleading because Fourier components are integrals over time. More accurately then, one might say that the speed of a pulse through a dispersive delay line will depend upon its center frequency.

The correct approach to the problem is to describe the dispersive delay line as a device which adds different phase shifts to different Fourier components. Since this is a linear device, the input is Fourier analyzed, the response of the device is found for each Fourier component, and the resultant pulse is reconstructed. Only a linear nonattenuating dispersive delay device is considered here, and thus the photographic spectrum $|E(\omega)|^2$ cannot be altered by such a device. The delay line transfer function can then be written as $\exp(iQ(\omega))$ where $Q(\omega)$ is real. For $\Omega = \omega - \omega_0$ we write for E_c , the complex compressed field

$$E_c = \frac{e^{-i\omega_0 t}}{2\pi} \iint_{-\infty}^{\infty} dt' d\Omega \mathcal{E}(t') \exp i[\delta\phi(t') + \Omega(t' - t) + Q(\Omega)] \quad (B-1)$$

Expanding $Q(\Omega)$ in a power series, i.e., $Q(\Omega) = Q_0 + Q_1\Omega + Q_2\Omega^2$, it is seen from Eq. (B-1) that Q_0 (the overall phase) can be neglected. The $Q_1\Omega$ term can be absorbed into the $\exp[i\Omega(t' - t)]$ by redefining t , and thus Q_1 corresponds to an equally uninteresting group delay. The $Q_2\Omega^2$ term is then the first important term in the series, and $Q(\Omega)$ can be effectively replaced

in Eq. (B-1) by $Q_2 \Omega^2$. One can then integrate over Ω to yield

$$E_C = \frac{e^{-i[(\pi/4) + \omega_0 t]}}{2\sqrt{\pi}Q_2} e^{-i(t^2/4Q_2)} \int_{-\infty}^{\infty} dt' \mathcal{E}(t') \exp \left[i \left(\delta\phi(t') - \frac{t'^2}{4Q_2} + \frac{t't}{2Q_2} \right) \right] \quad (B-2)$$

A linearly chirped pulse, for example, has a quadratic time dependence of $\delta\phi$ (i.e., $\delta\phi = -\beta t'^2$). Thus Q_2 of the dispersive delay line can be adjusted so that the first two terms cancel in the exponent of Eq. (B-2), which implies that $\delta\phi = -\beta t'^2 = (t'^2/4Q_2)$; or, equivalently, that $\beta^{-1} = -4Q_2$. If one further assumes that $\mathcal{E}(t')$ is a Gaussian with T_p being the half 1/e intensity width, then Eq. (B-2) can be evaluated to note that the intensity $I(t)$ is given by

$$I(t) \propto \exp \left[-t^2/(2\beta T_p)^2 \right] \quad (B-3)$$

which is a Gaussian pulse of duration $T_C \approx 4/\beta T_p$. Since βT_p is of the order of the pulse bandwidth, the pulse has been compressed to near its uncertainty limit. Thus, an increase in the linear frequency sweep (and, hence, in the bandwidth) increases the possible compression efficiency. If the frequency sweep is not linear, then it may not be possible to compress the pulse as well.

The converse of pulse compression is also of some interest. A very short pulse will come out longer and chirped (the time-reversed problem). This can be seen by letting $\mathcal{E}(t') \exp(i\delta\phi) = \delta(t - t_0)$. Inserting this into Eq. (B-2)

and performing the trivial integration, the only term quadratic in t is the exponent which precedes the integral. If a positively frequency-swept pulse is compressed to a short pulse, then a short pulse would be expanded and given a negative frequency sweep.

APPENDIX C

INTERFEROMETER USEABLE FOR COMPRESSION OF
FREQUENCY-MODULATED PULSES OF LIGHT

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Pages 6112-6115

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At the price of a certain experimental complexity, it is possible to emit frequency-modulated laser pulses. If the law of modulation is suitable, such pulses are compressible and one can expect to obtain luminous powers considerably higher than those produced by triggered lasers.

A nonmonochromatic wave is strongly disturbed in passing through a dispersive structure. If the characteristics of this structure are perfectly suited to the frequency modulation of the incident wave, the latter is presented at output in the form of a brief pulse whose peak power has been greatly increased. This technique, called the pulse-compression technique, is now commonly used in radars (since the ratio of the pulse duration before and after compression can reach 1,000). For simplification and also for a number of related reasons (possibility of translation of frequency produced by Doppler effect, for example), a linear frequency-modulation law is chosen, with the signal taking the form of a gate of duration T frequency-modulated linearly with a total deviation ΔF . It is theoretically possible to reduce the duration of this gate in the ratio $T\Delta F$.

Many dispersive structures, whether or not formed of a group of phase-shifting cells, have been suggested and used [1]. The

* Presented by Maurice Ponte; session of 15 June 1964.

interferometer described here is well suited to the domain of the optical frequencies, but could easily be extended to any other type of wave.

Let us consider a dielectric strip with plane and parallel surfaces of thickness e of index n immersed in a medium of index n' (Figure 1). One surface of this strip is made perfectly reflective. A beam of light of unit amplitude falling on the other surface is reflected by it and refracts at angle θ . This refracted wave undergoes, within the strip, a series of partial reflections of decreasing amplitudes. The waves coming from these reflections fuse to give a wave of amplitude

$$\psi e^{i\omega t} = \frac{-r + e^{-i\omega t_0}}{1 - re^{-i\omega t_0}} e^{i\omega t},$$

$e^{i\omega t}$ being the amplitude of the incident wave, to a constant equal to $(2en \cos \theta)/C$ (C , speed of light), and r the coefficient of reflection for the amplitude of the plane diopter.

The module of ψ is obviously unity, and its phase shift Φ in relation to the incident wave is given by

$$\operatorname{tg} \Phi = \frac{(1 - r^2) \sin \omega t_0}{2r - (1 + r^2) \cos \omega t_0}.$$

The group delay time of the wave ψ in relation to the incident wave defined by $t_n = -d\Phi/d\omega$ is written

$$t_n = \frac{t_0(1 - r^2)}{(1 - r^2) - 2r \cos \omega t_0} = t_0 \frac{1 + r}{1 - r} \frac{1}{1 + \frac{4r}{(1 - r)^2} \sin^2 \frac{\omega t_0}{2}}.$$

This delay time is maximum and equal to $t_0(1 + r)/(1 - r)$ for each of the resonance frequencies of the cavity formed by the strip (Figure 2). It varies nearly linearly with ωt_0 in a wide area around a point of inflection; it is this area which must be used as well as possible, since we have chosen a linear frequency law.

The pulse has a duration T and a frequency deviation ΔF centered around frequency F . The interferometer will be determined by its thickness e and reflecting power r . It is then necessary to evaluate what difference δt_n in relation to the tangent at the point of inflection it is possible to tolerate without notably decreasing the height of the compressed pulse. It can be shown [2] that this condition is fulfilled if

$$\frac{\delta t_n}{\Delta t_n} \leq \frac{1}{T \Delta F}.$$

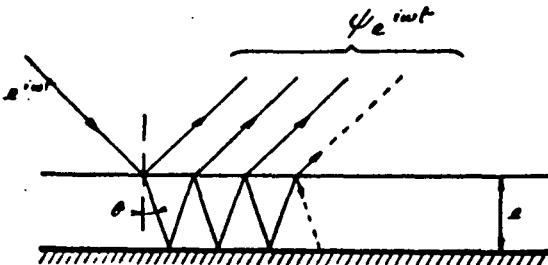


Figure 1 -- The incident wave of amplitude $\psi e^{i\omega t}$ undergoes, on the parallel-face strip, a series of partial reflections. All these partial waves together constitute a wave of amplitude $\psi e^{i\omega t}$.

Δt_n being the variation of the delay time between the extreme frequencies of the band ΔF . Depending on the characteristics of the pulse, it will be possible to use a more or less large portion of the curve t_R , and a glance at a table suffices for choosing the reflecting power r which ensures the largest product $\Delta t_n \Delta F$. At the end of

$$N = \frac{T}{\Delta t_n}$$

passages of light through the interferometer, the pulse has taken the width $(\Delta F)^{-1}$ below which it is impossible to compress it. These multiple reflections can be obtained by parallel structures of the type indicated in Figure 3.

An interferometer of the type described above can be suitable only for the low values of r . For larger values, it is necessary to use nonabsorbing multidielectric layers as partial reflector. The apparatus is calculated in the same manner, with care taken to introduce the correct phase shifts for reflection and transmission.

It remains to be pointed out that there are several methods for obtaining brief frequency-modulated pulses of light. Certain triggered lasers whose optical cavity contains electrically anisotropic molecules furnish frequency-modulated pulses themselves. If one desires -- and this is indispensable -- to stay in control of the modulation law, it suffices to place inside or outside the cavity a substance whose index of refraction is modifiable at will by means of an external force (electrical, magnetic, or mechanical, for example) [3].

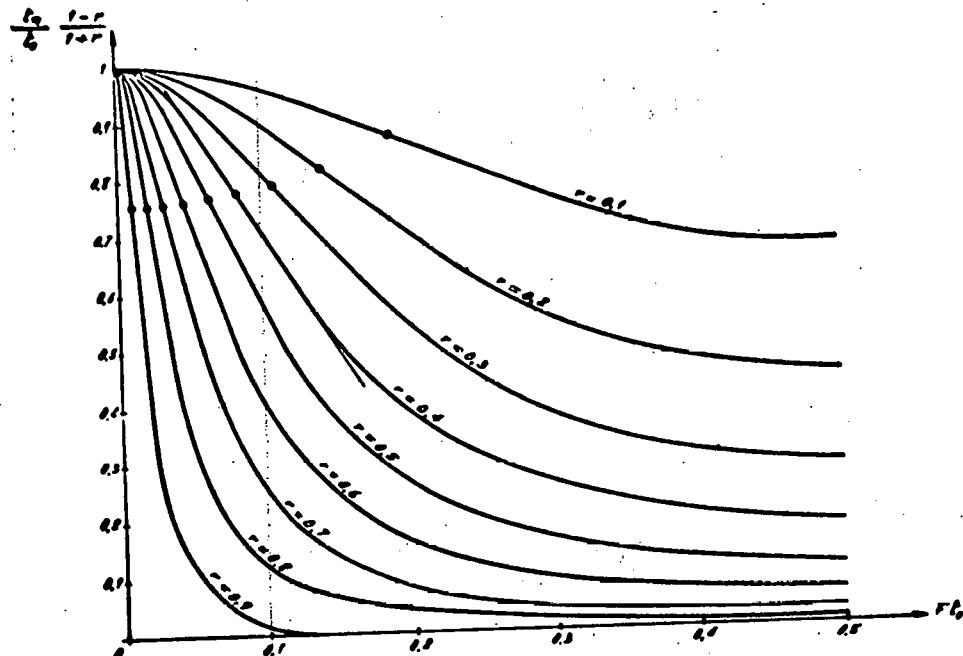


Figure 2 -- Group delay time standardized in function of the product Ft_0 . This delay time is linear in a certain area around the point of inflection.

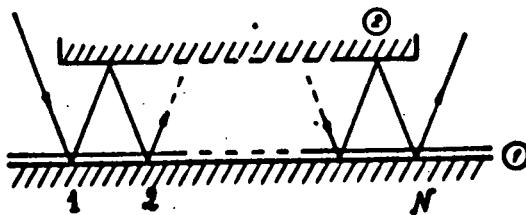


Figure 3 -- The interferometer (1) is surmounted by a reflector (2) which is parallel to it. The length of the whole arrangement fixes the number of reflexions N .

Laser pulse compression will perhaps provide a solution to the problem of very high powers, powers which it is not easy to obtain in the usual laser materials because of their manifold imperfections.

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