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An SU(3)-Symmetric Dual Model for Meson-Baryon Scattering
Based on Crossing Invariance* †

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I would like to talk today on "An SU(3)-Symmetric Dual Model for
Meson-Baryon Scattering Based on Crossing Invariance" as a first step
to construct a kind of dual model which explains the meson-baryon scat-
terings in all angles, for all energies.

Firstly, in order to find the FESR bootstraps solution for meson-
baryon scattering, let me start by constructing an SU(3)-symmetric dual
model for meson-baryon scattering.

As you already know, dual models so far constructed for meson-
baryon scattering have enjoyed a partial success as an approximation
to nature in the kaon-nucleon case (which has (s,t) or (u,t) dual term),

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but they have been less successful in the pion-nucleon case (which has the (s,u) dual term as well as (s,t) and (u,t) terms). The reason why it was less successful in the pion-nucleon case is that the inavailability of the crossing invariant duality solution of baryon exchange degeneracy (EXD) has prevented one from the construction of simple (s,u)⁴ terms satisfying the correct crossing invariance.

In order to construct a self-consistent dual model for meson-baryon scattering, a crossing invariant duality solution for baryon EXD scheme has to be found because the s and u channels are formally the same in meson-baryon scattering. A good solution at this level has recently been found by Eguchi and Fukugita within the framework of SU(3). As you have already heard from Eguchi, his solution has the following two simple EXD baryon spectra in the case of seven baryon trajectories:

$$(\underline{8+10})_\alpha \xleftarrow{\text{EXD}} (\underline{1+8})_\gamma \text{ for } \tau P = +\text{baryons (denoted as Sequence I)}$$

$$\underline{8}_\beta \xleftarrow{\text{EXD}} (\underline{8+10})_\delta \text{ for } \tau P = -\text{baryons (denoted as Sequence II)}$$

Here the two EXD sets are related to each other by s,u crossing, and the relative strength of sequence I and II baryon trajectories are fixed by crossing invariance. The octet F/D ratios of baryon couplings into meson-baryon are given by $(F/D)_\alpha = 2/3$, $(F/D)_\gamma = \infty$, $(F/D)_\beta = -1/3$, $(F/D)_\delta = -1/3$ and also the octet Reggeon F/D ratios in the t channel are given by $-7/3$ for the A' , $1/3$ for the A amplitude respectively, which are in reasonable agreement with experiments.

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Fukugita looked for general solutions in the more systematic way, and found families of crossing-invariant duality solutions within eight

trajectories, each being an SU(3) multiplet. In one family of his solutions all representation of $(\underline{3}^* \otimes \underline{3}) + (\underline{3}^* \otimes \underline{3})$ in the quark model appear and in the other all representation of $(\underline{3} \otimes \underline{3} \otimes \underline{3}) + (\underline{3} \otimes \underline{3} \otimes \underline{3})$ appear. The solution of type $(\underline{3}^* \otimes \underline{3}) + (\underline{3}^* \otimes \underline{3})$ is invariant under any crossing, hence is considered to correspond to the meson family, while the other solution correspond to the baryon family. His solution for baryon spectra implies the pattern of the EXD such as

$(\underline{8+10})_\alpha \xleftarrow{\text{EXD}} (\underline{1+8})_\gamma$, for $\tau p = +\text{baryons}$ $(\underline{1+8})_\beta \xleftarrow{\text{EXD}} (\underline{8+10})_\delta$, for $\tau p = -\text{baryons}$, and incorporated with the quark model where the SU(3) multiplets belong to $(\underline{56+70}, \text{even } L)$ and $(\underline{20+70}, \text{odd } L)$. In his duality scheme within eight trajectories, the pattern of the EXD is specified by the F/D value of either octet as a free parameter, and all of the Reggeon couplings in the t-channel have the nonet couplings and any isosinglet exchange conserves s-channel helicity independent of the choice of the free parameter. In particular when the F/D value of $\underline{8}_\alpha$ are set to be equal to $2/3$, $\underline{1}_\beta$ decouples from the solution (this corresponds to the absence of $\underline{20}$ in the quark model classification), which is the case in Ref. 5. (Hereafter, we use the crossing invariant duality solution of Ref. 5 in the actual calculation, for simplicity, but the generalization to the case of Ref. 6 is straightforward).

Now let me begin with constructing a simple SU(3)-symmetric dual model for meson-baryon scattering with the help of the crossing invariant duality solution. In order to construct the dual model, let us require the following:

- (i) crossing invariance for baryon spectrum,
- (ii) Regge asymptotic behaviors,
- (iii) correct pole structures (poles at $\bar{\alpha}_1 = 0, 1, 2, \dots$, etc)

$\bar{\alpha}_{II} = 1, 2, 3, \dots$ for the B amplitude, poles at $\bar{\alpha}_I = 1, 2, 3, \dots$, at $\bar{\alpha}_{II} = 1, 2, 3, \dots$ for the A amplitude, and at $\alpha_t = 1, 2, 3, \dots$ for mesons),

(iv) correct signatures for hadrons, in the case of linearly rising trajectories.

Let us now consider the meson-baryon invariant scattering amplitudes, A and B, constructed as sums of beta-function terms.

As we have already learned, (i) requires for the (s,u) dual term that the natural parity trajectories in the s(u) channel cross into the unnatural ones in the u(s) channel and vice versa, so the (s,u) dual term should be constructed in the form of $\frac{\Gamma(n-\bar{\alpha}_I(s))\Gamma(n-\bar{\alpha}_{II}(u))}{\Gamma(n-\bar{\alpha}_{II}(s))\Gamma(n-\bar{\alpha}_I(u))}$ or $\frac{\Gamma(n-\bar{\alpha}_{II}(s))\Gamma(n-\bar{\alpha}_I(u))}{\Gamma(n-\bar{\alpha}_I(s))\Gamma(n-\bar{\alpha}_{II}(u))}$ in contrast to the conventional approach. So, the term such as $\frac{\Gamma(s-\bar{\alpha}_{II}(s)-\bar{\alpha}_I(u))}{\Gamma(s-\bar{\alpha}_I(s)-\bar{\alpha}_{II}(u))}$ or $\frac{\Gamma(n-\bar{\alpha}_I(s)\Gamma(n-\bar{\alpha}_{II}(u))}{\Gamma(s-\bar{\alpha}_{II}(s)-\bar{\alpha}_I(u))}$ should be prohibited. Furthermore, the requirement (i) implies the following:

$$A'(s,u) \sim f_1 - f_2 \sim \underline{II}_s - \underline{I}_s = \underline{II}_u - \underline{I}_u \quad (1)$$

$$B(s,u) \sim f_1 + f_2 \sim \underline{II}_s + \underline{I}_s = -\underline{II}_u - \underline{I}_u \quad (2)$$

Thus, correct crossing is ensured, and

$$A(s,t) \sim \underline{II}_s - \underline{I}_s \quad (3)$$

$$A'(s,t) \sim \underline{II}_s + \underline{I}_s \quad (4)$$

Here the SU(3) structures of the residues of baryon trajectories in the s(u)-channel with sequence I and II are symbolically denoted as \underline{I}_s (\underline{I}_u) and \underline{II}_s (\underline{II}_u) for the (s,u) term, and I_s and II_s for the (s,t) term respectively. They are given explicitly in Eqs. (1)~(8) of Ref. 5 in the case of seven baryon trajectories. The A' amplitude is the usual one

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of Singh, and is related to A and B in the backward and forward direction as follows:

$$A'(s, u) \xrightarrow{s \rightarrow \infty, u \text{ fixed}} A(s, u) + MB(s, u) , \quad (5)$$

$$A'(s, t) \xrightarrow{s \rightarrow \infty, t \text{ fixed}} A(s, t) + \frac{S}{2M} B(s, t) . \quad (6)$$

Moreover, from the requirement (iv), the amplitude A(s, t) (or B(s, t)), must be restricted in such a way as to give the correct signatures to baryons when combined with A(s, u) (B(s, u)) terms. Because of these many restrictions, the construction of the dual model is not so trivial, but the outcome becomes very simple.

Using Eqs. (1), (2) and (3) together with the before-mentioned requirements (i), (ii), (iii), we are immediately led to the following expressions for A'(s, u), MB(s, u) and A(s, t):

$$\begin{aligned} A'(s, u) = & \lambda_A [\underline{II}_s (c - \alpha' u) B(1 - \bar{\alpha}_{II}(s), -\bar{\alpha}_I(u)) - \underline{I}_s (c - \alpha' s) B(-\bar{\alpha}_I(s), 1 - \bar{\alpha}_{II}(u))] \\ & + \lambda_A [\underline{II}_s (c_A - \alpha' s - \alpha' u) B(2 - \bar{\alpha}_{II}(s), 1 - \bar{\alpha}_I(u)) - \underline{I}_s (c_A - \alpha' s - \alpha' u) B(1 - \bar{\alpha}_I(s), 2 - \bar{\alpha}_{II}(u))] \end{aligned} \quad (7)$$

$$MB(s, u) = \lambda_B [\underline{II}_s (c_B - \alpha' u) B(1 - \bar{\alpha}_{II}(s), -\bar{\alpha}_I(u)) + \underline{I}_s (c_B - \alpha' s) B(-\bar{\alpha}_I(s), 1 - \bar{\alpha}_{II}(u))] , \quad (8)$$

$$A(s, t) = \lambda_A [\underline{II}_s (c_A - \alpha' s - \alpha' t) B(1 - \bar{\alpha}_{II}(s), 1 - \bar{\alpha}_t) - \underline{I}_s (c_A - \alpha' s - \alpha' t) B(1 - \bar{\alpha}_I(s), 1 - \bar{\alpha}_t)]. \quad (9)$$

In order to have the correct signatures for baryons, we obtain from Eq. (9) with (ii) and (iii),

$$A(s, u) = \lambda_A [\underline{\underline{I}}_s (c_A - \alpha' s - \alpha' u) B(1 - \bar{\alpha}_{II}(s), 1 - \bar{\alpha}_{II}(u)) - \underline{\underline{I}}_s (c_A - \alpha' s - \alpha' u) B(1 - \bar{\alpha}_I(s), 1 - \bar{\alpha}_{II}(u))]. \quad (10)$$

Note that Eqs. (8) and (10) are manifestly crossing invariant in the usual s, u crossing. Since Eqs. (7), (8) and (10) must satisfy Eq. (5), we must have the following constraint:

$$\lambda_A = \lambda_B, \quad c = c_B, \quad \lambda_A = 2\lambda_B. \quad (11)$$

Finally, the $B(s, t)$ amplitude is determined from Eq. (8) to give the correct baryon signatures together with the requirements (ii) and (iii), as

$$MB(s, t) = \lambda_B \underline{\underline{I}}_s [(c_B - \alpha' t) B(1 - \bar{\alpha}_{II}(s), 1 - \alpha_t) + c_{B2} B(2 - \bar{\alpha}_{II}(s), 1 - \alpha_t)] \\ + \lambda_B \underline{\underline{I}}_s [(c_B - \alpha' s) B(-\bar{\alpha}_I(s), 2 - \alpha_t) + c_{B1} B(1 - \bar{\alpha}_I(s), 1 - \alpha_t)]. \quad (12)$$

Since Eq. (4) must be satisfied, Eqs. (6), (9) and (12) give

$$c_{B1} = c_B + c_{B2} - 8\alpha' M^2 + \alpha_t(0) - 1. \quad (13)$$

It is to be noted here that (u, t) terms follow from (s, t) by s, u crossing, which must be even in A and odd in B.

Therefore, we have obtained the dual model for the invariant amplitudes $A(s, t)$, $A(s, u)$, $A(u, t)$, $B(s, t)$, $B(s, u)$ and $B(u, t)$ in terms of four parameters; λ_B , c_B , c_A , c_{B2} . Here we take the viewpoint that only the information of leading trajectories is reliable, and confine ourselves to the leading trajectories. Among them, c_A is not relevant to this leading-leading bootstrap. If the F/D ratios of Reggeon exchanges for the B amplitude are specified, the parameters are reduced to only two;

λ_B and c_B , which are fixed by normalizing the amplitude at the nucleon and the (3,3) resonance pole as

$$g^2 = \frac{25(c_B - \alpha' M^2)}{6\alpha' M} \lambda_B , \quad (14)$$

$$\Gamma_{\Delta_B} (J=\frac{3+}{2}) = -4\lambda_B \frac{M+\Delta}{M} \frac{E_\Delta + M}{6\pi M_\Delta^2} q_\Delta^3 . \quad (14')$$

So, we have determined the SU(3)-symmetric dual amplitudes for meson-baryon scattering. The important points to notice here are,

i) Our amplitude is very simple and is determined almost uniquely because of very stringent restrictions for invariant amplitudes implied by the crossing invariance.

ii) The helicity structure in the s-channel is correctly taken into account. Namely, the helicity amplitudes are related to f_1 and f_2 as $\bar{f}_{\pm\pm} = f_1 \pm f_2$ so the s-channel helicity non-flip and flip amplitudes are given respectively by $II+I$ and $II-I$. These amplitudes behave as,

$$\begin{aligned} \bar{f}_{++} &\sim \frac{1}{4\pi} \frac{M}{(s)^{\frac{1}{2}}} A' \quad \text{for } s \rightarrow \infty, \text{ with } t \text{ fixed} \\ &\sim \frac{1}{8\pi} \frac{(s)^{\frac{1}{2}}}{M} M B \quad \text{for } s \rightarrow \infty, \text{ with } u \text{ fixed} \end{aligned}$$

$$\begin{aligned} \bar{f}_{+-} &\sim \frac{1}{8\pi} A \quad \text{for } s \rightarrow \infty, \text{ with } t \text{ fixed} \\ &\sim \frac{1}{8\pi} A' \quad \text{for } s \rightarrow \infty, \text{ with } u \text{ fixed} , \end{aligned}$$

as it should from s-channel point of view.

iii) Our aim is to construct a dual model which satisfies several fundamental principles with the minimum number of free parameters and not to construct a phenomenological model to fit experiments precisely. So,

let us briefly test if our amplitudes give agreements with experiments as the first approximation.

In order to test our model with experiments, we shall confine ourselves to the pion-nucleon case which has rich experimental information but is considered to be the most difficult from the theoretical point of view. The octet F/D ratios both in the s,u channels and the t channel are in fair agreement with experiments, hence the predictions in other meson-baryon scattering should be as good as the pion-nucleon case. Let us begin with the πN Regge behaviours in the forward direction.

Predictions for forward πN scattering

The amplitudes A and B can be expanded in terms of the eigen-amplitudes A_λ^t and B_λ^t in the t-channel as

$$A_{b\bar{p} \rightarrow b'\bar{p}'}(s,t,u) = \sum_{\lambda} c(p,p';\lambda;\bar{b},b') A_{\lambda}^t(s,t,u) \quad (15)$$

and similar for the B. Here b and b' are octet baryons p and p' are octet pseudoscalar mesons, λ labels the SU(3) representation and $c(p,p';\lambda;\bar{b},b')$ is the product of SU(3) Clebsch-Gordon coefficients. As an example, we consider the $\pi^- p$ charge-exchange process. If the amplitude is given in the form of $2I_5 + 3I_8$, the t-channel F/D ratios for vector and tensor mesons are given by $F/D = -(1 + \frac{5}{2}r)/3(1 - \frac{r}{2})$. Therefore, $F/D = 1/3$ for $A(s,t)$, $F/D = -7/3$ for $A'(s,t)$ are obtained independently of t, in accord with Ref. 5. If we denote the F/D ratio at $t=0$ for $B(s,t)$ by $(F/D)_B$, Eq. (12) with Eq. (13) immediately leads us to

$$c_B + c_{B2} = 8\alpha'M^2 \quad \frac{5/3 - (F/D)_B}{7/3 + (F/D)_B} \quad (16)$$

We then obtain from Eqs. (6), (9), (11), (12), (13) and (16),

$$\frac{\text{Im}A_{p'}(v, t=0)}{v \text{Im}B_p(v, t=0)} \xrightarrow[v \rightarrow \infty]{} \frac{1}{2} \frac{(F/D)_B - 1/3}{(F/D)_B + 1} . \quad (17)$$

If we set: $(F/D)_B = 2/3$ implied by SU(6), Eq. (17) reduces to $1/10$ which is in remarkable agreement with the experimental value $1/11.6$. Therefore we fix $(F/D)_B$ to the SU(6) value $2/3$ hereafter. Then, the F/D ratio for the B amplitude is predicted to be $(64M^2 + 21t)/3(32M^2 - 3t)$ which is a slowly varying function of t . Equation (16) reduces to $c_B + c_{B2} = 8\alpha'M^2/3$.

We also predict the following for the p' parameters;

$\text{Im}A_{p'}(v, t=0)/v \text{Im}B_{p'}(v, t=0) = 1$, $\text{Im}B_{p'}(v, 0) = (3/5) \text{Im}B_p(v, 0)$ and $\text{Im}A_{p'}(v, 0) = 6 \text{Im}A_p(v, 0)$, which are again in fair agreement with experiments (see Table I).

If one gives $\frac{g}{4\pi} = 14.4$ and $\Gamma_{\Delta_0^3}(\frac{3}{2}) = 119$ MeV, Eqs. (14) and (14') fix the constants λ_B and c_B to be

$$\lambda_B = -15.95(\text{GeV})^{-1} , \quad c_B = -1.895\alpha'M^2 . \quad (18)$$

Therefore, our amplitudes predict the absolute magnitude of the p and p' parameters as is shown in Table I. Note that the predictions agree with the experimental values except for the overall scale factor 3.4, which is also the case for the conventional KN dual models. Thus, the daughter contributions must be suppressed in reality to reduce the overall magnitude of the amplitudes. We now proceed to the elastic widths of baryons.

Prediction for elastic widths of baryons

The width of a resonance of spin J and mass M_J on the leading baryon trajectory is given by our dual model as

$$\Gamma_{J\pm}(bp \rightarrow b'p') = \frac{\bar{E} + M}{8\pi M_J^2} \frac{q_J}{\alpha'} \frac{(s)^{\frac{1}{2}}}{\Gamma(J+1)} (\alpha' q_J^2)^{J-\frac{1}{2}} c(b, p; \mu; b' bp') b_{\mu, \pm}(M_J), \quad (19)$$

with

$$b_{II,+}((s)^{\frac{1}{2}}) = 2\lambda_B II_s \bar{\alpha}_{II}(s) - \lambda_B \frac{(s)^{\frac{1}{2}+M}}{M} II_s \bar{\alpha}_{II}(s) = -b_{II,-}(-(s)^{\frac{1}{2}})$$

$$b_{I,+}((s)^{\frac{1}{2}}) = -2\lambda_B I_s \bar{\alpha}_I(s) + \lambda_B \frac{(s)^{\frac{1}{2}+M}}{M} I_s(c_B - \alpha' s) = -b_{I,-}(-(s)^{\frac{1}{2}}).$$

Here the notation $J\pm$ denotes a state with a total angular momentum J and an orbital angular momentum $L = J\pm 1/2$. Our predictions for elastic widths of baryons are straightforward and listed for N^* resonances in Table II.

As is easily seen, even in the worse case ($\Gamma_{N\gamma}(\frac{3}{2}^+)$ and $\Gamma_{N\gamma}(\frac{7}{2}^+)$), the predicted values are small by only factor 2 compared with experimental values, and $\Gamma_{\Delta_0}(\frac{7}{2}^+)$ is 1.5 times as large. The absence of low mass $N_8(\frac{3}{2}^+)$ would not break the duality to a large extent as the elastic width is very small. The others are in reasonable agreement with experiments. As is usual, parity doublets appear but with small positive residues except for parity doublet of $N_{\alpha}(\frac{5}{2}^+)$.

Backward meson baryon scattering

The u -channel parity-conserving helicity amplitude $\Gamma_{bp \rightarrow b'p'}^{\pm}((u)^{\frac{1}{2}}, s)$ is given in our model by

$$F_{bp \rightarrow b'p'}^+((u)^{\frac{1}{2}}, s) = \sum_{\mu} c(b, p'; \mu; b', p) b_{\mu, \pm}((u)^{\frac{1}{2}}) R_{\mu}((u)^{\frac{1}{2}}, s) \quad (20)$$

with

$$R_{\mu}((u)^{\frac{1}{2}}, s) = - \frac{\pi}{\Gamma(\bar{\alpha}_{\mu}(u) + 1)} \frac{1 + \tau_e}{\sin \pi \bar{\alpha}_{\mu}(u)} (\alpha' s)^{\bar{\alpha}_{\mu}(u)}$$

Restricting ourselves to the $K^+ p$ backward scattering where the u-channel baryon EXD is so nice, we will test our prediction with those experiments. The $\Lambda_{\alpha} - \Lambda_{\gamma}$ EXD baryon trajectory turns out to be the dominant contribution to this process as $F_{pK^+ \rightarrow pK^+}^+((u)^{\frac{1}{2}}=0, s) \sim 34.9 s^{\bar{\alpha}_{\Lambda}(0)}$ which should be compared with $F_{pK^+ \rightarrow pK^+}^+((u)^{\frac{1}{2}}=0, s) \sim \pm 41 s^{\bar{\alpha}_{\Lambda}(0)}$ ($\bar{\alpha}_{\Lambda}(0) = -1.22$). This will encourage further investigation on more complicated cases like πN backward scattering. Especially, the presence of $\bar{\alpha}_{\Delta}(0)$ would explain the experimental absence of the dip at $u \sim -1.9 \text{ GeV}^2$ corresponding to $\alpha_{\Delta_6} = -3/2$.

We conclude with a few general remarks:

(i) We discussed mainly the pion-nucleon case to predict the Regge behaviours in the forward direction and elastic widths of baryons. However, the crossing invariant duality solution in Refs. 5 and 6 has a reasonable property of having realistic values for the octet F/D ratios of baryon couplings into meson-baryon and Reggeon couplings into baryon-antibaryon, hence the predictions in other meson-baryon scattering such as the kaon-nucleon case is as good as the pion-nucleon case.

(ii) The parity doublet problem still remains as an open question. At the expense of adding satellites a finite number of them can be eliminated but it must always be done by keeping Eqs. (1)-(4) in order to ensure the correct crossing invariance.

(iii) The common belief is that the leading-leading duality holds

good for A but not for B. On the contrary, we have explicitly constructed both the A and B amplitudes which manifestly satisfy the leading-leading duality with the help of the crossing invariant duality solution. Furthermore, our amplitude is much simpler compared with the previous approach and is determined almost uniquely because of very stringent restrictions for invariant amplitudes implied by the crossing invariance.^{2,3}

(iv) Last year, Eguchi and myself proposed the s-channel resonance model based on duality to explain exotic peaks as well as differential cross sections in all angles for $\pi\pi$ scattering. Similar procedure can be applied also for meson-baryon scattering.

We think that our simple approach will be very useful for the investigation of all the meson-baryon scattering and baryon-antibaryon annihilation into two mesons.

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Table I Predictions and Regge fits to πN scattering at $t=0$. The ρ Regge fits are taken from Ref. 10 ($\alpha_{\rho}(0) = 0.55$) and the p' from Ref. 11 ($\alpha_{p'}(0) = 0.51$). For the case of comparison, we factored out 3.4 in the predicted values.

Amplitude	Prediction		Experimental value	
$\text{Im}A_{\rho}'(v,0)$	$5.72(2Mv)^{0.55}$	(3.4)	$4.9(2Mv)^{0.55}$	
$v\text{Im}B_{\rho}'(v,0)$	57.2	"	57.2	"
$\text{Im}A_{\rho}(v,0)$	-51.5	"	-52.3	"
<hr/>				
$\text{Im}A_{p'}(v,0)$	$32.1(2Mv)^{0.51}$	(3.4)	$27.2(2Mv)^{0.51}$	
$v\text{Im}B_{p'}(v,0)$	32.1	"	27.2	"

Table II Predicted elastic widths for N^* baryons from Eq. (19) in the text. Experimental values are taken from the Particle Data Group, 1973. Experimental value of $\Gamma_{N_6}(\frac{7}{2}^+)$ are taken from C. Lovelace, in Proceedings of the Sixteenth International Conference on High Energy Physics, Chicago-Batavia, 1972 Vol. 3, p. 73.

N^* elastic width	Prediction (MeV)	Experimental value (MeV)
$\Gamma_{N_\alpha}(\frac{5}{2}^+)$	124	63 ~ 108
$\Gamma_{N_\gamma}(\frac{3}{2}^+)$	27.9	52.5 ~ 75
$\Gamma_{N_\gamma}(\frac{7}{2}^-)$	34.7	67.5 ~ 81.2
$\Gamma_{\Delta_\alpha}(\frac{5}{2}^+)$	61.9	34 ~ 59.5
$\Gamma_{\Delta_\delta}(\frac{3}{2}^+)$	119	119
$\Gamma_{\Delta_\delta}(\frac{7}{2}^+)$	189	76.5 ~ 121
$\Gamma_{N_\beta}(\frac{5}{2}^-)$	40.1	46 ~ 70
$\Gamma_{N_6}(\frac{3}{2}^+)$	7.4	-----
$\Gamma_{N_6}(\frac{7}{2}^+)$	11.8	30(CERN) 10(Saclay)

REFERENCES

1. K. Igi, University of Tokyo preprint UT-204, June 1973.
2. E.L. Berger and G.C. Fox, Phys. Rev. 188, 2120 (1969).
3. T. Inami, Prog. Theor. Phys. 45, 1903 (1971). A number of references to earlier works on meson-baryon Veneziano models are also found here.
4. The only possibility has been to weaken the hypothesis of leading-leading bootstrap to make it consistent with crossing invariance.
5. T. Eguchi, University of Tokyo preprint, UT-195, May 1973, Nucl. Phys. B (to be published) and also contribution to this symposium. References to earlier works on duality and symmetry are found here and in Ref. 6.
6. M. Fukugita, University of Tokyo preprint, UT-200, May 1973.
7. V. Singh, Phys. Rev. 129, 1889 (1963).
8. Therefore, c_A does not contribute to the baryon widths on the parent trajectory as well as the Regge behaviours both at fixed t and u in the leading order. It is, however, determined by the Adler's consistency condition.
9. P. McNamee and F. Chilton, Rev. Mod. Phys. 36, 1005 (1964).
10. V. Barger and R.J.N. Phillips, Phys. Rev. 187, 2210 (1969).
11. C. Michael, Springer Tracts in Modern Physics, Vol. 55, ed. G. Höhler, Springer Verlag 1970.
12. K. Igi and T. Eguchi, Phys. Letters 42B, 449 (1972); T. Eguchi and K. Igi "New Scheme of Dual Models and Exotic Peaks" UT-185, Phys. Rev. (to be published).