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HEALTH PHYSICS AND SAFETY DIVISION

INFORMAL REPORT

TEXT, FIGURES AND EXAMPLES FOR THE REFRESHER COURSE
"PRACTICAL STATISTICS FOR THE HEALTH PHYSICIST"

Joseph A. Ash

September 11, 1973

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UPTON, NEW YORK 11973

MASTER

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Presented at the 18th Annual Meeting of the Health Physics Society
June 17-21, 1973, Miami Beach, Florida

PRACTICAL STA TICS FOR THE HEALTH PHYSICIST*

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A B S T R A C T

The refresher course will provide a practical approach to the use of statistical concepts in interpreting data related to problems of applied health physics.

After a brief review of the theoretical probability distributions, emphasis will be placed on counting statistics, in particular the concept of the standard deviation. In this category the following topics will be covered: (1) calculation of the standard deviation, (2) propagation of errors (when two or more quantities are combined arithmetically or via a formula), (3) confidence limits, and (4) the null hypothesis.

The following topics will also be discussed: (1) Chi-Square in relation to checking the reliability of a counting instrument, (2) correction factors for deadtime losses, and (3) for the case of a decaying source.

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* Work carried out at Brookhaven National Laboratory under contract with the U. S. Atomic Energy Commission.

INTRODUCTION

Any measurement repeated under supposedly identical conditions will yield a variety of results. There are numerous reasons for these deviations; a few are:

1) reading errors,
2) alteration of supposedly similar conditions; for example, instrument drift, etc.
3) the random nature of certain processes, as the nature of radioactivity.

THE NORMAL DISTRIBUTION

Many random processes may be approximated by the normal distribution, that is, the curve known as the bell-shaped curve. Although also known as the Gaussian curve, the curve was first developed by de Moivre in 1733 and later by Gauss in the 1790's.

If one took many random samples from a population and plotted a frequency distribution, one would get a "normal distribution" curve approximating the shape of one of the curves shown in Fig. 1. The curve may be defined by the equation shown above the graph.

The normal distribution has two independent parameters, the mean μ and the standard deviation σ. The graphs represent normal distribution of a mean of 5 and standard deviations of 1, 2 and 3, respectively. The mean or average is represented on the graph by the abscissa (x value) with the largest ordinate. It is the most probable value. The standard deviation is a measure of how widely the data varies from the mean. Most of the practical statistical analysis is based on the normal distribution.

Nuclear events follow the Poisson probability distribution whose equation is shown in Fig. 2. The Poisson distribution has one parameter, the mean, similarly defined as the mean for the normal distribution. For practical purposes for means greater than 20, the Poisson distribution of mean μ can be approximated by a normal distribution of mean μ and standard deviation $\sqrt{\mu}$. This convergence is seen in Fig. 3. Thus all the properties of the normal distribution may be applied to radioactivity, providing the number of events is greater than 20.

The parameters, the mean and standard deviation are calculated as shown in Fig. 4. The decision to use N or N-1 depends on whether one is utilizing the entire population or a sample of the population for the calculation as shown in Fig. 5. Although not shown in Fig. 4, the statistical literature recognizes the difference between utilizing an entire population or just a sample of the population. μ and σ are usually reserved for the parameters calculated from the entire population, while $\bar{x}$ and s are reserved for the parameters calculated from sample populations.

SAMPLING

In counting samples and reporting data, one is interested in the true mean count rate. In order to obtain the true mean count rate, one would theoretically have to repeat the count an infinite number of times and use the average. This is impossible; thus one is concerned in estimating the mean count rate. Along with this estimated mean, one is interested in how well this estimated value approximates the true mean. This introduces the topic of sampling with parameters known as the sample mean and standard deviations of the mean. Although the concept has broad application, we will restrict ourselves to considering counting samples and count rates.

STANDARD DEVIATIONS OF COUNTING DATA

The standard deviation of a count rate is given by

$$\sigma = \sqrt{\frac{r}{t}}$$

where r = the count rate

t = the actual time counted

Note, as t becomes larger (a longer count is taken), the deviation of the count rate is decreased (see Figs. 6 and 7).

Usually one is interested in estimating the difference between two count rates, viz. a sample and a background. This brings up the topic of propagation of deviations, that is, when one determines an estimate of a mean via some mathematical function that involves parameters and their respective standard deviations. One must then calculate the associated standard deviation according to various prescribed rules.

The rules for subtraction and addition are given in Fig. 8, and an example shown in Fig. 9. The rules for multiplication and division are shown in Figs. 10 and 11; that for a constant in Fig. 12. Fig. 13 shows the generalized rule for the propagation of errors when an estimated mean is calculated via any smooth function.

Now that we know how to obtain the deviation of our net sample count, what is its significance? This can be seen in Figs. 14 and 15. Additional applications of practical problems are illustrated in Figs. 16, 17 and 18.

NULL HYPOTHESIS

Often one is interested in knowing if the difference between a sample and background is due to actual activity in the sample, or due to statistical fluctuations of the background.

The null hypothesis test, as other statistical tests, does not give absolute answers. It essentially reports the probability that the sample count rate is a result of statistical fluctuations of the background rate. If this probability is very small, then we assume the sample count is not a result of statistical fluctuations but is a result of activity present in the sample.

In using the null hypothesis, a confidence limit is chosen. Usually, and for the examples presented here, the 95% confidence limit is chosen. The difference of the sample and background rates is assumed to be 0. Then the probability of obtaining the actual difference is calculated. If this probability is less than or equal to 5%, we reject the hypothesis and assume the sample contains activity above background.

Since the 95% confidence limit corresponds to approximately two standard deviations, one need not compute the probability. If the net result differs by more than two standard deviations, one rejects the null hypothesis.

CHI-SQUARE

The Chi-square statistic determines the probability that the deviations observed in repetition of the observations follow that of the assumed distribution. It is useful in checking out counting equipment. Successive counts are taken with the instrument. The χ^2 statistic is calculated from the data as shown in Fig. 21; then one looks up the percentile values from a Chi-square table for the appropriate degree of freedom. The degree of freedom for the Poisson distribution is n-1 where n is the number of repetitions. It is advisable to make at least 10 repetitions. The table (Fig. 22) gives the area under the entire curve to the point in question. The area represents the probability of obtaining a Chi-square value of 0 to the value in question. Therefore the ideal percentile value is 0.5. Note a too small value for the χ^2 is just as bad as a too large number. In practice, at the 95% confidence limit, if the percentile value found on the table is between 0.025 and .975, the instrument is acceptable. A value less than 0.025 indicates the deviations in the data are greater than one would expect from statistics, probably due to some instability of the instrument. A value greater than .975 indicates the deviations of the sample data are less than expected from statistics, probably due to some oscillations or the counting of noise.

DEAD TIME

The topic of dead time and that of the decaying source does not pertain to statistics. However, being closely related to counting data and important to the health physicist, it has been included in this refresher course.

If two particles enter the counter in rapid succession in a Geiger counter, the avalanche of ions from the first particle paralyzes the counter. The positive ions are massive and slow moving. These form a sheath around the anode, thereby greatly decreasing the electric field intensity around the anode, making it impossible to initiate an avalanche by another ionizing particle. As the positive ion sheath moves toward the cathode, the field intensity increases until a point is reached when another avalanche could be started (see Fig. 23). The time required to attain this electric field intensity is called the dead time. However, after the end of the dead time, when another avalanche can be started, the output pulse from this avalanche is still relatively small and goes undetected as the electric field intensity is still not great enough to produce a Geiger pulse. When the output pulse is large enough to be passed by the discriminator and be counted, the counter is said to have recovered. This total time is the resolving time.

In other words, the resolving time is the minimum time that must elapse before a second particle may be detected. Typical resolving times of Geiger counters lie from 100-200 μsec. A proportional counter is much faster than a Geiger counter, as the avalanche is limited to a short segment of the anode. Their resolving times are much smaller than that for a Geiger counter, typically, in the order of microseconds. The resolving time can be gotten from an oscilloscope or by the two source method (see Fig. 24).

THE DECAYING SOURCE

The decaying source correction is necessary when the counting time is large compared to the half-life, about 5% or greater. The correction factor as well as its development is shown in Fig. 25.

DEFINITION OF SYMBOLS

μ	= mean
σ	= standard deviation
r	= count rate
t	= time
σ_r	= relative standard deviation
r_s, σ_s, t_s	= respective count rate, standard deviation, and time counted, for the sample (sample plus background)
r_{bkg}, σ_{bkg}, t_{bkg}	= respective count rate, standard deviation, and time counted for the background count
r_n, σ_n	= respective count rate and standard deviation of the count rate for the net sample; that is, total sample minus background
$\overline{X}$	= average
X_i	= sample data
N	= number of sample points
N_c	= total number of counts

NORMAL DISTRIBUTION

With a mean of μ
and a standard deviation of σ

$$P(n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(n-\mu)^2}{2\sigma^2}}$$

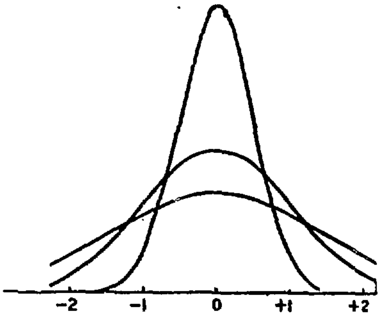


Figure 1*

*Reference: W.J. Dixon and F.J. Massey, Introduction to Statistics.
(Used with permission of McGraw-Hill Book Company)

POISSON DISTRIBUTION

With a mean of μ,

$$P(n) = \frac{(\mu)^n \times e^{-\mu}}{n!}$$

Figure 2

CONVERGENCE OF THE POISSON TO THE NORMAL DISTRIBUTION

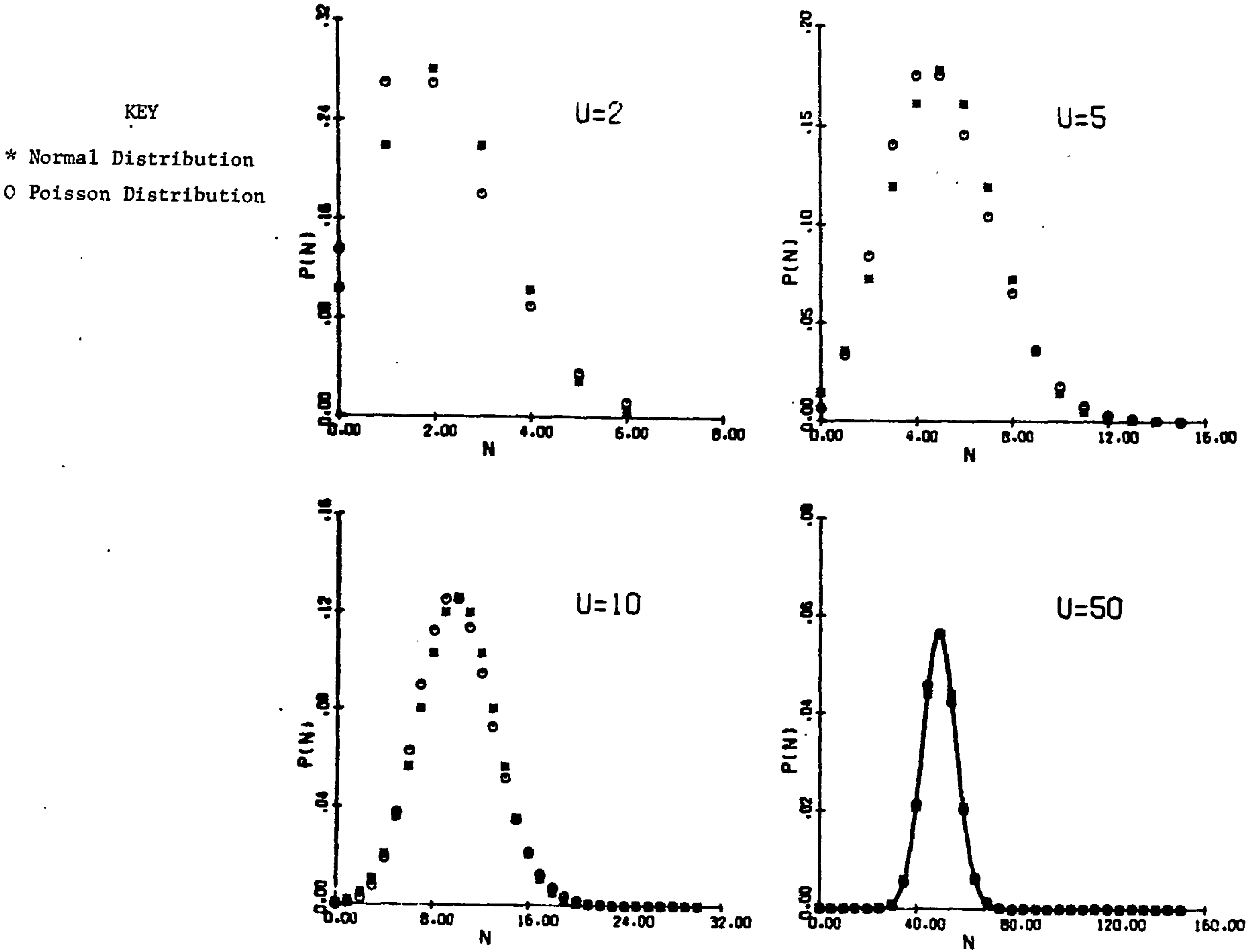


Figure 3

STANDARD DEVIATION FORMULAS

By Definition:

$$\sigma = \sqrt{\frac{\sum_i (X_i - \bar{X})^2}{N-1}}$$

By Computational Method:

$$\sigma = \sqrt{\frac{N\ (\sum_i X_i^2) - (\Sigma X_i)^2}{N\ (N-1)}}$$

Figure 4

SAMPLE CALCULATION OF THE STANDARD DEVIATION

Run	X	X^2
1	5,364	28,772,496
2	5,319	28,291,761
3	5,329	28,398,241
4	5,211	27,154,521
5	5,340	28,515,600
6	5,295	28,037,025
7	5,231	27,363,361
8	5,193	26,967,249
9	5,303	28,121,809
10	5,318	28,281,124
Σ	52,903	279,903,187

$\bar{X}$ = 5,290.30

ΣX = 52,903

ΣX^2 = 279,903,187

$(\Sigma X)^2$ = 2,798,727,409

$$\sigma = \sqrt{\frac{10 \times 27{,}903{,}187 - 2{,}798{,}727{,}409}{10 \times 9}}$$

$$\sigma = 58.16$$

$\sigma_{poisson}$ = 72.70 (for a single count of 5,290)

Figure 5

DEVIATION OF A COUNT RATE

Let r = count rate

t = total time the sample was counted

$$\sigma = \sqrt{\frac{r}{t}}$$

Figure 6

<u>Example</u>:

A 10 min. count resulted in 1,000 counts.

$$r = \frac{1000}{10} = 100 \text{ cnts/min}$$

$$\sigma = \sqrt{\frac{100}{10}} = 3.2 \text{ cnts/min}$$

Suppose a 1 min. count yielded 100 counts, then

$$r = 100 \text{ cnts/min}$$

$$\sigma = \sqrt{100} = 10 \text{ cnts/min}$$

Figure 7

PROPAGATION OF DEVIATIONS

Addition or Subtraction

If $y = a + b$

or $y = a - b$

$$\sigma_y = \sqrt{\sigma_a^2 + \sigma_b^2}$$

Figure 8

BACKGROUND SUBTRACTION

In general

$$\sigma_n = \sqrt{\sigma_s^2 + \sigma_{bkg}^2} = \sqrt{\frac{r_s}{t_s} + \frac{r_{bkg}}{t_{bkg}}}$$

Example:

A 5 min sample count yielded 510 counts, while a 1 hr bkg yielded 2,400 counts. Calculate the net sample count rate and standard deviation.

$$r_n = \frac{510 \text{ cnts}}{5 \text{ min}} - \frac{2400 \text{ cnts}}{60 \text{ min}} = 102-40 = 62 \frac{\text{cnts}}{\text{min}}$$

$$\sigma_n = \sqrt{\frac{102}{5} + \frac{40}{60}} = 4.6$$

62 ± 4.6 cnts/min

Usually 2 standard deviations are desired

62 ± 9.2

Figure 9*

*Reference: H. Cember, Introduction to Health Physics. (Used with permission of Pergamon Press, Inc.)

PROPAGATION OF DEVIATIONS

Multiplication and Division

For multiplication or division, it is convenient to use the relative deviations. Given $a \pm \sigma_a$ and $b \pm \sigma_b$

$$\sigma_{ra} = \frac{\sigma_a}{a}$$

$$\sigma_{rb} = \frac{\sigma_b}{b}$$

Figure 10

Multiplication and Division

If $y = a \times b$

or $y = \frac{a}{b}$

then $\sigma_{ry} = \sqrt{\sigma_{ra}^2 + \sigma_{rb}^2}$

$$\sigma_y = \sigma_{ry} \times y$$

Figure 11

PROPAGATION OF DEVIATIONS

Multiplying by a constant - multiply the deviation by the same constant.

Division by a constant - divide the deviation by the same constant.

This is useful when changing units or applying correction factors.

Figure 12

General Form:

If y is calculated from $a \pm \sigma_a$, $b \pm \sigma_b$, $c \pm \sigma_c$ ... via some function $y = f(a, b, c \ldots)$, then

$$\sigma_y = \sqrt{\left(\frac{\partial f}{\partial a}\right)^2 \sigma_a{}^2 + \left(\frac{\partial f}{\partial b}\right)^2 \sigma_b{}^2 + \left(\frac{\partial f}{\partial c}\right)^2 \sigma_c{}^2} + \ldots$$

Figure 13

SAMPLING

An estimated mean sample count rate of μ_n and standard deviation σ_n implies that

(1) there is approximately a 68% probability that the true mean count rate lies in the interval $\mu_n \pm \sigma_n$;

(2) there is approximately a 95% probability that the true mean count rate lies in the interval $\mu_n \pm 2\sigma_n$;

(3) there is approximately a 99% probability that the true mean count rate lies in the interval $\mu_n \pm 2.54\sigma_n$;

(4) there is approximately a 99.9% probability that the true mean count rate lies in the interval $\mu_n \pm 3\sigma_n$.

Figure 14

In our previous example we calculated

$$r_n = 62$$

$$\sigma_n = 4.6$$

$$2\sigma_n = 9.2$$

This implies there is approximately a 68% chance the true mean count rate lies between 57.4 and 66.6.

There is approximately a 95% chance the true mean count rate is between 52.8 and 71.2.

Figure 15

CHOOSING THE LENGTH OF A COUNT

A preliminary short count revealed approximately 30 c/min. An earlier background count of 60 minutes resulted in a count rate of 25 c/min. Utilizing the 95% confidence limit, calculate the required length of a count so that the true net count rate will be within 10% of the sample net count rate.

10% of 30 = 3 c/min

Using the 95% confidence limit,

$$2\sigma = 3$$

$$\sigma = 1.5$$

$$1.5 = \sqrt{\frac{30}{t_s} + \frac{25}{60}}$$

$$t_s = 16.4 \text{ min}$$

Figure 16*

*Reference: H. Cember, Introduction to Health Physics. (Used with permission of Pergamon Press, Inc.)

OPTIMUM COUNTING TIME

Optimum counting time for the sample and the background is given by

$$\frac{t_{bkg}}{t_s} = \sqrt{\frac{r_{bkg}}{r_s}}$$

Figure 17

OPTIMUM COUNTING TIME

Example: Only one hour is available to count a sample and background. What is the optimum time division between sample and background counting times if the approximate rates for the sample plus background and background alone are 1,000 and 20 counts/min, respectively?

$$\frac{t_{bkg}}{t_s} = \sqrt{\frac{20}{1,000}} = 0.14$$

Since the total time is 60 minutes:

$$t_{bkg} + t_s = 60$$

then

$$.14\ t_s + t_s = 60$$

$$.t_s = 53 \text{ min.}$$

$$t_{bkg} = 7 \text{ min.}$$

Similarly, for count rates of 60 and 20, the approximate times are

$$t_s = 38 \text{ min.}$$

$$t_{bkg} = 22 \text{ min.}$$

The deviations may be calculated as in Figure 9.

Figure 18*

*Reference: W. J. Price, Nuclear Radiation Detection. (Used with permission of McGraw-Hill Book Company.

NULL HYPOTHESIS

Example:

A sample of drinking water counted for 10 minutes resulted in 600 counts. A 30 minute background count resulted in 1,500 counts. At the 95% confidence limit is there any activity in the water?

$$r_n = \frac{600}{10} - \frac{1500}{30} = 60\text{-}50 = 10 \text{ c/min}$$

$$\sigma_n = \sqrt{\frac{60}{10} + \frac{50}{30}} = 2.8 \text{ c/min}$$

$$\frac{r_n}{\sigma_n} = \frac{10}{2.8} = 3.5$$

Since $\frac{r_n}{\sigma_n}$ is greater than 2, that is, 2.8 is greater than 2 standard deviations, there is less than a 5% probability that the net c/min is a result of statistics. Therefore, the null hypothesis is rejected and activity is assumed to be present in the sample.

Figure 19*

*Reference: H. Cember, Introduction to Health Physics. (Used with permission of Pergamon Press, Inc.)

Example (cont'd)

If, on the other hand, the drinking water resulted in 530 counts in 10 minutes, then:

$$r_n = \frac{530}{10} - \frac{1500}{30} = 53\text{-}50 = 3 \text{ c/min}$$

$$\sigma_n = \sqrt{\frac{53}{10} + \frac{50}{30}} = 2.6 \text{ c/min}$$

$$\frac{r_n}{\sigma_n} = \frac{3}{2.6} = 1.15$$

3 c/min represents only 1.15 standard from 0. Because it is less than 2 standard deviations, thus within the 95% confidence limit, we accept the null hypothesis.

Figure 20

CHI-SQUARE

Run	X	$\frac{(X_{obs} - X_{exp})^2}{X_{exp}}$
1	5,364	1.0267
2	5,319	.1557
3	5,329	.2831
4	5,211	1.1887
5	5,340	.4669
6	5,295	.0042
7	5,231	.6647
8	5,193	1.7896
9	5,303	.0305
10	5,318	.1450
Σ	52,903	5.7551

$$\chi^2 = \sum_i \frac{[(\text{observed-expected})_i]^2}{(\text{expected})_i}$$

$$\chi^2 = 5.7551$$

$$\upsilon = 9$$

$$P \approx .25$$

NOTE: For this case the average count $\bar{X}$ is substituted for X_{exp}.

Figure 21

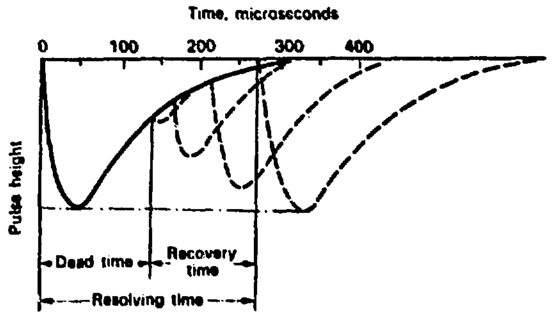
PERCENTILE VALUES (χ^2_p) for THE CHI-SQUARE DISTRIBUTION with ν degrees of freedom (shaded area = p)

χ^2_p

ν	$\chi^2_{.995}$	$\chi^2_{.99}$	$\chi^2_{.975}$	$\chi^2_{.95}$	$\chi^2_{.90}$	$\chi^2_{.75}$	$\chi^2_{.50}$	$\chi^2_{.25}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$
1	7.88	6.63	5.02	3.84	2.71	1.32	.455	.102	.0158	.0039	.0010	.0002	.0000
2	10.6	9.21	7.38	5.99	4.61	2.77	1.39	.575	.211	.103	.0506	.0201	.0100
3	12.8	11.3	9.35	7.81	6.25	4.11	2.37	1.21	.584	.352	.216	.115	.072
4	14.9	13.3	11.1	9.49	7.78	5.39	3.36	1.92	1.06	.711	.484	.297	.207
5	16.7	15.1	12.8	11.1	9.24	6.63	4.35	2.67	1.61	1.15	.831	.554	.412
6	18.5	16.8	14.4	12.6	10.6	7.84	5.35	3.45	2.20	1.64	1.24	.872	.676
7	20.3	18.5	16.0	14.1	12.0	9.04	6.35	4.25	2.83	2.17	1.69	1.24	.989
8	22.0	20.1	17.5	15.5	13.4	10.2	7.34	5.07	3.49	2.73	2.18	1.65	1.34
9	23.6	21.7	19.0	16.9	14.7	11.4	8.34	5.90	4.17	3.33	2.70	2.09	1.73
10	25.2	23.2	20.5	18.3	16.0	12.5	9.34	6.74	4.87	3.94	3.25	2.56	2.16
11	26.8	24.7	21.9	19.7	17.3	13.7	10.3	7.58	5.58	4.57	3.82	3.05	2.60
12	28.3	26.2	23.3	21.0	18.5	14.8	11.3	8.44	6.30	5.23	4.40	3.57	3.07
13	29.8	27.7	24.7	22.4	19.8	16.0	12.3	9.30	7.04	5.89	5.01	4.11	3.57
14	31.3	29.1	26.1	23.7	21.1	17.1	13.3	10.2	7.79	6.57	5.63	4.66	4.07
15	32.8	30.6	27.5	25.0	22.3	18.2	14.3	11.0	8.55	7.26	6.26	5.23	4.60
16	34.3	32.0	28.8	26.3	23.5	19.4	15.3	11.9	9.31	7.96	6.91	5.81	5.14
17	35.7	33.4	30.2	27.6	24.8	20.5	16.3	12.8	10.1	8.67	7.56	6.41	5.70
18	37.2	34.8	31.5	28.9	26.0	21.6	17.3	13.7	10.9	9.39	8.23	7.01	6.26
19	38.6	36.2	32.9	30.1	27.2	22.7	18.3	14.6	11.7	10.1	8.91	7.63	6.84
20	40.0	37.6	34.2	31.4	28.4	23.8	19.3	15.5	12.4	10.9	9.59	8.26	7.43
21	41.4	38.9	35.5	32.7	29.6	24.9	20.3	16.3	13.2	11.6	10.3	8.90	8.03
22	42.8	40.3	36.8	33.9	30.8	26.0	21.3	17.2	14.0	12.3	11.0	9.54	8.64
23	44.2	41.6	38.1	35.2	32.0	27.1	22.3	18.1	14.8	13.1	11.7	10.2	9.26
24	45.6	43.0	39.4	36.4	33.2	28.2	23.3	19.0	15.7	13.8	12.4	10.9	9.89
25	46.9	44.3	40.6	37.7	34.4	29.3	24.3	19.9	16.5	14.6	13.1	11.5	10.5
26	48.3	45.6	41.9	38.9	35.6	30.4	25.3	20.8	17.3	15.4	13.8	12.2	11.2
27	49.6	47.0	43.2	40.1	36.7	31.5	26.3	21.7	18.1	16.2	14.6	12.9	11.8
28	51.0	48.3	44.5	41.3	37.9	32.6	27.3	22.7	18.9	16.9	15.3	13.6	12.5
29	52.3	49.6	45.7	42.6	39.1	33.7	28.3	23.6	19.8	17.7	16.0	14.3	13.1
30	53.7	50.9	47.0	43.8	40.3	34.8	29.3	24.5	20.6	18.5	16.8	15.0	13.8
40	66.8	63.7	59.3	55.8	51.8	45.6	39.3	33.7	29.1	26.5	24.4	22.2	20.7
50	79.5	76.2	71.4	67.5	63.2	56.3	49.3	42.9	37.7	34.8	32.4	29.7	28.0
60	92.0	88.4	83.3	79.1	74.4	67.0	59.3	52.3	46.5	43.2	40.5	37.5	35.5
70	104.2	100.4	95.0	90.5	85.5	77.6	69.3	61.7	55.3	51.7	48.8	45.4	43.3
80	116.3	112.3	106.6	101.9	96.6	88.1	79.3	71.1	64.3	60.4	57.2	53.5	51.2
90	128.3	124.1	118.1	113.1	107.6	98.6	89.3	80.6	73.3	69.1	65.6	61.8	59.2
100	140.2	135.8	129.6	124.3	118.5	109.1	99.3	90.1	82.4	77.9	74.2	70.1	67.3

Figure 22

RESOLVING TIME



Relationship among dead time, recovery time, and resolving time.

Figure 23*

*Reference: H. Cember, Introduction to Health Physics. (Used with permission of Pergamon Press, Inc.)

RESOLVING TIME CORRECTION

$$R = \frac{R_o}{1 - R_o\tau}$$

Calculation of resolving time by the two source method:

$$\tau = \frac{R_1 + R_2 - R_{12} - R_b}{R_{12}^2 - R_1^2 - R_2^2}$$

Figure 24*

*Reference: H. Cember, Introduction to Health Physics. (Used with permission of Pergamon Press, Inc.)

THE DECAYING SOURCE

Let A_o be the count rate at t_o

t_1 be the time the count was started

t_2 be the time the count was finished

$\frac{dN_c}{dt}$ = the count rate at any time t

then

$$\frac{dN_c}{dt} = A_o e^{-\lambda t}$$

$$N_c = \int_{t_1}^{t_2} A_o e^{-\lambda t} dt = -\frac{1}{\lambda} A_o e^{-\lambda t} \Big]_{t_1}^{t_2}$$

$$N_c = \frac{A_o}{\lambda} (e^{-\lambda t_1} - e^{-\lambda t_2})$$

$$A_o = \frac{\lambda N}{(e^{-\lambda t_1} - e^{-\lambda t_2})}$$

Figure 25

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