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SUBJECT: ORACLE CODE FOR A GENERAL TWO-REGION, TWO-GROUP SPHERICAL HOMOGENEOUS REACTOR CALCULATION

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ORACLE CODE FOR A GENERAL TWO-REGION, TWO-GROUP SPHERICAL HOMOGENEOUS REACTOR CALCULATION*

SUMMARY

A general code has been written for the ORACLE for the solution of a spherical, homogeneous, two-region reactor by the two-group method. The reactor is general in that the core and blanket compositions are entirely arbitrary. The calculation can accommodate two fuel components plus seven other nuclear components in the blanket; two fuel components, poisons, moderator, and four other nuclear components in the core. The reactor may or may not contain a core tank separating the two regions. The only restriction in the composition is the requirement that the ratios of the concentration of one of the core fuel components to the concentration of the other core components (except moderator) be independent of core fuel concentration. The volume fraction of core moderator is assumed to be independent of core fuel concentration. For this calculation, the reactor dimensions and blanket composition are fixed; hence, the core composition is the variable for which a solution is sought.

The results which may be obtained from the calculation are:

1. For a steady-state system
   a. Critical core concentration
   b. Volume integral of the fast and thermal flux in the core and blanket

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* ORACLE Problem Number 191. This code supersedes the code described in CF-54-7-38, Oracle Code for Two-Group, Two-Region Homogeneous Reactor Calculation, by T. B. Fowler and R. A. Willoughby.

c. Neutron Balance, i.e., all neutron absorptions and leakages

d. Flux plot from the center of the core to the pressure vessel boundary

2. For a non-steady-state system

a. Reactor period

b. Effective multiplication constant

c. Mean lifetime of prompt neutrons

d. Temperature coefficient of reactivity.

INTRODUCTION

A machine code of this type is extremely useful in that consistent and accurate results may be obtained with a minimum of time and labor. This is realized especially in survey type calculations where, from a practical point of view, it is necessary to minimize the ratio of time spent in obtaining results to the time spent in analyzing them. A two-region, two-group, steady-state reactor calculation requires six to eight hours using a desk computer, whereas the ORACLE performs the same calculation in approximately fifteen seconds. The non-steady-state calculation, requiring approximately five minutes of ORACLE computing time, would be prohibitively long if done by hand.

The purpose of this memorandum is to enable one, who is unfamiliar with the ORACLE, to use the code for this calculation. Generally, the only labor involved is the preparation of the input parameter paper tape. With this tape and the code tape, the calculation can be performed by following the machine operating instructions. The code tape, as well as a printed copy of the code, is available in the author's box in the computer room, Building 4500.

This memorandum consists of three parts and an appendix. The first part lists the output answers that are obtained from the calculation, the second part lists the input parameters necessary for the calculation with instructions for preparing the input parameter tape, the third part gives the complete machine operating instructions, and the appendix shows the complete derivation of the two-region, two-group equations used, a description of the method of calculation, and certain restrictions in the calculation.

I. OUTPUT ANSWERS

All or certain parts (see Page 13) of the following answers are punched on the answer tape for each reactor calculation.

1. Case Number
2. P_T - total neutron production
3. L_T - total neutron losses
4. Σ_{2C}^{F1} - thermal macroscopic absorption cross section of core fuel component 1, cm^{-1}
5. Φ_{1C} - volume integral of fast flux in the core
6. Φ_{2C} - volume integral of thermal flux in the core
7. Φ_{1B} - volume integral of fast flux in the blanket
8. Φ_{2B} - volume integral of thermal flux in the blanket
9. A_{2C}^{F1} - thermal neutron absorption in core fuel component 1
10. A_{2C}^{F2} - thermal neutron absorption in core fuel component 2
11. A_{2C}^{3} - thermal neutron absorption in core component 3
12. A_{2C}^{4} - thermal neutron absorption in core component 4
13. A_{2C}^{5} - thermal neutron absorption in core component 5
14. A_{2C}^{6} - thermal neutron absorption in core component 6
15. A_{2C}^{P} - thermal neutron absorption in core poisons
16. A_{2C}^{M} - thermal neutron absorption in core moderator
17. A_{1C} - fast neutron absorption in core

18. A_{2B}^{F1} - thermal neutron absorption in blanket fuel component 1

19. A_{2B}^{F2} - thermal neutron absorption in blanket fuel component 2

20. A_{2B}^{3} - thermal neutron absorption in blanket component 3

21. A_{2B}^{4} - thermal neutron absorption in blanket component 4

22. A_{2B}^{5} - thermal neutron absorption in blanket component 5

23. A_{2B}^{6} - thermal neutron absorption in blanket component 6

24. A_{2B}^{7} - thermal neutron absorption in blanket component 7

25. A_{2B}^{8} - thermal neutron absorption in blanket component 8

26. A_{2B}^{9} - thermal neutron absorption in blanket component 9

27. A_{1B} - fast neutron absorption in blanket

28. A_{SH} - neutron absorption in core tank

29. L_1 - fast neutron leakage

30. L_2 - thermal neutron leakage

31. $\phi_{2C}(0)$ - thermal core flux at $r = 0$

32. $\phi_{2C}(a/4)$ - thermal core flux at $r = a/4$

33. $\phi_{2C}(a/2)$ - thermal core flux at $r = a/2$

34. $\phi_{2C}(3a/4)$ - thermal core flux at $r = 3a/4$

35. $\phi_{2C}(a)$ - thermal core flux at $r = a$

36. $\phi_{1C}(0)$ - fast core flux at $r = 0$

603 006

37. $\phi_{1C}(a/4)$ - fast core flux at $r = a/4$

38. $\phi_{1C}(a/2)$ - fast core flux at $r = a/2$

39. $\phi_{1C}(3a/4)$ - fast core flux at $r = 3a/4$

40. $\phi_{1C}(a)$ - fast core flux at $r = a$

41. $\phi_{2B}(b+T_B/4)$ - thermal blanket flux at $r = (b+T_B/4)$

where r is the radial distance from the center of the core and a is the core radius, and where b is the core radius plus the core tank shell thickness, and T_B is the blanket thickness.

42. $\phi_{2B}(b+T_B/2)$ - thermal blanket flux at $r = (b+T_B/2)$

43. $\phi_{2B}(b+3T_B/4)$ - thermal blanket flux at $r = (b+3T_B/4)$

44. $\phi_{2B}(R)$ - thermal blanket flux at $r = R$

where R is the pressure vessel radius

45. $\phi_{1B}(b+T_B/4)$ - fast blanket flux at $r = (b+T_B/4)$

46. $\phi_{1B}(b+T_B/2)$ - fast blanket flux at $r = (b+T_B/2)$

47. $\phi_{1B}(b+3T_B/4)$ - fast blanket flux at $r = (b+3T_B/4)$

48. $\phi_{1B}(R)$ - fast blanket flux at $r = R$

49. $\frac{1}{m} = \frac{\ell}{k_e - 1}$ - reactor period of non-steady-state system, sec.

50. k_e - effective multiplication constant of non-steady-state system

51. ℓ - mean lifetime of prompt neutrons, sec.

52. $\partial k/\partial T$ - temperature coefficient of reactivity.

The first output number, the case number, corresponds to the first number on the input parameter tape. The second and third output numbers are check numbers and should be equal, to at least five figures, for a valid calculation. The critical core fuel concentration is obtained from output item 4. Output items 5 through 48 are normalized[1,2] as shown in the appendix, and the method of obtaining the last four output items is also explained in the appendix. Answers 2 through 48 are for a steady-state system and answers 49 through 52 are for a non-steady-state reactor.

Each of the output numbers, except the first, is a decimal floating point number of the form:

```
0        0
F.XXXXXXFyy
```

The first digit is either 0 or F, depending upon whether the number is positive or negative, respectively; the next six digits are decimal fraction; the eighth digit is 0 or F, representing the sign of the exponent of ten; and the last two digits are the exponent of ten.

II. INPUT PARAMETERS

The following is a list of the input parameters for the calculation with the machine memory cell occupied by each number. This list of memory cells is used in determining the "loading instruction word" (input item 2) as explained below.

Memory Cell Address of	Input Number		
	1.	Case identification number	
	2.	Loading instruction word	
381	3.	a	- core radius, cm.
2	4.	t	- core tank thickness, cm.
3	5.	T_B	- blanket thickness, cm.
4	6.	e	- extrapolation distance, cm.
5	7.	p_B	- resonance escape probability for the blanket
6	8.	τ_B	- Fermi age for the blanket, cm^2
7	9.	D_{1B}	- fast diffusion coefficient for the blanket, cm.
8	10.	D_{2B}	- thermal diffusion coefficient for the blanket, cm.
9	11.	Σ_{2B}^{F1}	- thermal macroscopic absorption cross section of blanket fuel component 1, cm^{-1}
A	12.	Σ_{2B}^{F2}	- thermal macroscopic absorption cross section of blanket fuel component 2, cm^{-1}
B	13.	Σ_{2B}^{3}	- thermal macroscopic absorption cross section of blanket component 3, cm^{-1}
C	14.	Σ_{2B}^{4}	- thermal macroscopic absorption cross section of blanket component 4, cm^{-1}
D	15.	Σ_{2B}^{5}	- thermal macroscopic absorption cross section of blanket component 5, cm^{-1}

Memory Cell Address of			Input Number
E	16.	Σ_{2B}^{6}	- thermal macroscopic absorption cross section of blanket component 6, cm^{-1}
F	17.	Σ_{2B}^{7}	- thermal macroscopic absorption cross section of blanket component 7, cm^{-1}
390	18.	Σ_{2B}^{8}	- thermal macroscopic absorption cross section of blanket component 8, cm^{-1}
1	19.	Σ_{2B}^{9}	- thermal macroscopic absorption cross section of blanket component 9, cm^{-1}
2	20.	τ_C	- Fermi age for the core, cm^2
3	21.	D_{1C}	- fast diffusion coefficient for the core, cm.
4	22.	D_{2C}	- thermal diffusion coefficient for the core, cm.
5	23.	p_C	- resonance escape probability for the core
6	24.	$\frac{\Sigma_{2C}^{F2}}{\Sigma_{2C}^{F1}}$	- ratio of thermal macroscopic absorption cross section of core fuel component 2 to thermal macroscopic absorption cross section of core fuel component 1
7	25.	$\frac{\Sigma_{2C}^{3}}{\Sigma_{2C}^{F1}}$	- ratio of thermal macroscopic absorption cross section of core component 3 to thermal macroscopic absorption cross section of core fuel component 1
8	26.	$\frac{\Sigma_{2C}^{4}}{\Sigma_{2C}^{F1}}$	- ratio of thermal macroscopic absorption cross section of core component 4 to thermal macroscopic absorption cross section of core fuel component 1
9	27.	$\frac{\Sigma_{2C}^{5}}{\Sigma_{2C}^{F1}}$	- ratio of thermal macroscopic absorption cross section of core component 5 to thermal macroscopic absorption cross section of core fuel component 1
A	28.	$\frac{\Sigma_{2C}^{6}}{\Sigma_{2C}^{F1}}$	- ratio of thermal macroscopic absorption cross section of core component 6 to thermal macroscopic absorption cross section of core fuel component 1

Memory Cell Address of		Input Number
B	29. $\frac{\Sigma_{fC}^{F1}}{\Sigma_{2C}^{F1}}$	- ratio of thermal macroscopic fission cross section of core fuel component 1 to thermal macroscopic absorption cross section of core fuel component 1
C	30. $\frac{\Sigma_{fC}^{F2}}{\Sigma_{2C}^{F1}}$	- ratio of thermal macroscopic fission cross section of core fuel component 2 to thermal macroscopic absorption cross section of core fuel component 1
D	31. f_{pC}	- core poison fraction, i.e., $\Sigma_{2C}^{P} = f_{pC}\left[\Sigma_{fC}^{F1} + \Sigma_{fC}^{F2}\right]$
E	32. η_{B}^{F1}	- number of fission neutrons produced per thermal neutron absorption in blanket fuel component 1
F	33. η_{B}^{F2}	- number of fission neutrons produced per thermal neutron absorption in blanket fuel component 2
3A0	34. η_{C}^{F1}	- number of fission neutrons produced per thermal neutron absorption in core fuel component 1
1	35. η_{C}^{F2}	- number of fission neutrons produced per thermal neutron absorption in core fuel component 2
2	36. D_{SH}^{*}	- diffusion coefficient for the core tank, cm.
3	37. Σ_{a}^{SH}	- macroscopic absorption cross section of core tank, cm^{-1}
4	38. Σ_{2C}^{M}	- thermal macroscopic absorption cross section of core moderator, cm^{-1}
5	39. V_{1C}	- velocity of fast neutrons in the core, cm/sec.
6	40. V_{2C}	- velocity of thermal neutrons in the core, cm/sec.

* See restriction 6 in the appendix

603 010

Memory Cell Address of			Input Number
7	41.	V_{1B}	- velocity of fast neutrons in the blanket, cm/sec.
8	42.	V_{2B}	- velocity of thermal neutrons in the blanket, cm/sec.
9	43.	ΔT	- assumed temperature change from steady-state to non-steady-state system, ^{o}C
A	44.	$\dfrac{\sigma_{2C}^{F1}(T+\Delta T)}{\sigma_{2C}^{F1}(T)}$	- ratio of thermal microscopic absorption cross section of core fuel component 1 at $(T+\Delta T)^{o}C$ to thermal microscopic absorption cross section of core fuel component 1 at $T^{o}C$
B	45.	$\dfrac{\sigma_{2C}^{F2}(T+\Delta T)}{\sigma_{2C}^{F2}(T)}$	- ratio of thermal microscopic absorption cross section of core fuel component 2 at $(T+\Delta T)^{o}C$ to thermal microscopic absorption cross section of core fuel component 2 at $T^{o}C$

The machine may be made to do two different types of calculations, depending upon what is desired. One calculation is the steady-state calculation alone, and the other is the steady-state plus the non-steady-state calculation.

Consider first only the steady-state calculation. For this calculation, one may require that the output numbers 1 through 30, or the output numbers 1 through 48, be punched on the answer tape, depending upon the first two digits of the case identification number (first number on the input parameter tape). In this calculation, the first two digits of the case identification number are 00, for answers 1 through 30, or 0F, for answers 1 through 47, followed by eight arbitrary hexidecimal digits 0-F. For the steady-state calculations, only the input parameters 1 through 38 are used and the input numbers 39 through 45 may be zero or left out entirely; however, all of the input numbers 1 through 38 need not be punched on the input parameter tape for each case if several cases are to be run consecutively. The second input number (loading instruction word) determines the number of input numbers to be used for each case that are different from the preceeding case. This number consists of ten digits and is of this form:

00 aaa 00 ZZZ

where aaa is the memory cell address at the first input number to be loaded, and ZZZ is the memory cell address of the last input number to be loaded. Obviously, the numbers that are punched on the input tape for each case must be in consecutive order. Each of the input numbers except the first two are ten-digit, decimal, floating numbers (see Page 9) and all of the input numbers are punched on the input tape, separated by a single space and have a single space following the last number of each case.

Suppose, for example, that a series of three steady-state reactors are to be calculated with the answers 1 through 30 desired for the first two cases and the answers 1 through 48 desired for the third case. Suppose further that the second reactor differs from the first only in item 5 (blanket thickness) and that case 3 differs from case 2 in items 5 and 11. The input parameter tape would be prepared as follows:

For case 1, the case identification number would be punched as 00, followed by eight arbitrary digits; the loading instruction word would be 00 381 00 3A4; and the complete set of input numbers 3 through 38 would follow, with all numbers including the first two separated by a single space. (It is desirable to separate each case on the input tape by several inches of the tape "leader" or zeros.)

For case 2, the case identification number would be 00 for the first two digits, the loading instruction word would be 00 383 00 383, and this would be followed by input item 5 only. Again, each of these three numbers would be separated by a single space. Case 3 would have OF for the first two digits of the case identification number, 00 383 00 389 for the loading instruction word, with input numbers 5 through 11 following.

The steady-state plus non-steady-state calculation differs from the steady-state calculation in that two sets of input numbers are required for each reactor calculation, and one may obtain answers 1 through 30 followed by 49 through 52, or answers 1 through 52, depending again upon the first two digits of the case identification number. For this calculation, FO for the first two digits of the case identification number produces answers 1 through 30 followed by 49 through 52; whereas FF produces answers 1 through 52.

In this calculation it is assumed that a reactor which is operating in a steady state is suddenly made unstable by an instantaneous change in temperature (see appendix for further explanation of this calculation). Each reactor calculation then requires a set of input numbers corresponding to a steady-state system at temperature $T^{o}C$, followed by another set of input numbers corresponding to the same reactor but at temperature $(T+\Delta T)^{o}C$. The case identification numbers for both sets of input numbers must be identical.

For example, suppose one reactor is to be calculated and it is desired to obtain all the output answers 1 through 52, and suppose it is assumed for this case that, from steady state, the core temperature will suddenly change from $T^{o}C$ to $(T+\Delta T)^{o}C$. The input tape will need the case identification number with FF for the first two digits, the loading instruction word, and the complete set of input numbers 1 through 38 corresponding to the steady-state temperature $T^{o}C$. This set of input numbers will be followed (on the same tape) by another set of numbers consisting of the same case identification number, 00 392 00 3AB, for the loading word, and the input 20 through 38 plus 39 through 45 corresponding to temperature $(T+\Delta T)^{o}C$. Both of these sets of input numbers are punched on the input tape as explained above.

III. MACHINE OPERATING INSTRUCTIONS

The step-by-step machine operating procedure is listed below, followed by a simplified magnetic tape flow chart showing the position of the magnetic tapes and contents of the fast memory at each step in the calculation. Instruction for re-starting the calculation in case of a machine error that cannot be corrected are also given.

Procedure

1. Load first part of code tape by console order - 90:3EA. This loads the first part of the code into the fast memory to double space, storing first word in memory cell 3EA.

2. Without removing code tape from the reader, put the break point switch up and give console order - 43:157. This console order transfers machine control to the left of memory cell 157, at which point the machine writes onto magnetic tape the complete memory beginning with cell 000, loads the second part of the code tape into the fast memory storing first word in memory cell 170, writes onto magnetic tape cells 170-2EF, reads from magnetic tape onto the fast memory the first part of the code beginning with cell 000, sums the complete memory stopping at a break point at the right of cell 33A with sum in Q_L.

3. At this point the contents of Q_L should read A5935992E3. If this is the case, put the input tape under the reader and start the calculation by setting the operate switch to "stop" and back to continuous. If the sum in Q_L does not read correctly, begin over with step 1.

4. At this point, for the steady-state calculation, the machine will read in the input numbers for the first case, compute the reactor and punch out the answers, returning to the "break point stop" as in step 2, with the sum again in Q_L. The calculation for each succeeding case may be continued as in step 3. For the non-steady-state calculation, the machine will read in the first set of input numbers, calculate, then read in the following set of numbers and compute before punching the answers.

Magnetic Tape Flow Chart - Drive Zero

↓	Part I	↓	Part II	↓	Storage	↓
(a)	x = 1024 words	(b)	y = 384 words	(c)	Z = 128 words	(d)
	x^1 = 4 blocks		y^1 = 2 blocks		Z^1 = 1 block	

Beginning the calculation, the code is put under the reader and the console orders 90:3EA
43:157 given

The machine does the following: (tape at a)

1. WF:000 - 1024 words (tape at b)
2. 90:170 -
3. WF:170 - 384 words (tape at c)
4. HB:(x^1+y^1) = 6 blocks (tape at a)
5. RF:000 - 1024 words (tape at b)
6. HF:y^1 = 2 blocks (tape at c)
7. RF:380 - 37 input numbers (tape at d)
8. Sum memory and stop on break point at $33A^{II}$ with sum in Q_L
9. Input tape under reader, the machine is re-started. At this point the machine will read in the first set of input numbers and will compute for approximately 15 seconds.

663 017

10. HB:Z^1 = 1 block (tape at c)

11. WF:380 = 128 words (tape at d)

S.S. | At this point, for steady-state calculation, control goes to step 15. Otherwise, continue with step 12.

N.S.S. → 12. For the steady-state plus non-steady-state calculation, at this point the second set of input numbers is read in and the machine computes for approximately five minutes.

13. HB:Z^1 = 1 block (tape at c)

14. RF:380 - 128 words (tape at d)

S.S. → 15. HB:(y^1+Z^1) = 3 blocks (tape at b)

16. RF:170 - 384 words (tape at c)

17. At this point, machine computes for approximately 3 seconds and punches out the answers, then control goes to step 4.

If the calculation stops illegitimately and cannot be continued, e.g., if an error occurs in a computed number, simply position the magnetic tape to point c (see magnetic tape diagram), put the input tape (for the case on which the code stopped) back under the reader, and give console order 43:323. This re-starts the calculation beginning with step 4 above.

The legitimate stops in the code are as follows:

1. Cell 03A - attempted division by zero.
2. Cell 03B - computed number out of range.
3. Cell 133 - sinh, cosh out of range.
4. Cell 234 - attempted square root of negative number.

APPENDIX

In the following equations the items marked by two ** are output answers.

A. General non-steady-state two-group, two-region diffusion equations[3] are:

Core:

$$-D_{1C}\nabla^2\phi_{1C}(r,t) + \Sigma_{1C}\phi_{1C}(r,t) - \eta_C\Sigma^F_{2C}\phi_{2C}(r,t) = -\frac{1}{v_{1C}}\frac{\partial}{\partial t}\phi_{1C}(r,t)$$

$$-D_{2C}\nabla^2\phi_{2C}(r,t) + \Sigma_{2C}\phi_{2C}(r,t) - p_C\Sigma_{1C}\phi_{1C}(r,t) = -\frac{1}{v_{2C}}\frac{\partial}{\partial t}\phi_{2C}(r,t) \qquad (1)$$

Blanket:

$$-D_{1B}\nabla^2\phi_{1B}(r,t) + \Sigma_{1B}\phi_{1B}(r,t) - \eta_B\Sigma^F_{2B}\phi_{2B}(r,t) = -\frac{1}{v_{1B}}\frac{\partial}{\partial t}\phi_{1B}(r,t)$$

$$-D_{2B}\nabla^2\phi_{2B}(r,t) + \Sigma_{2B}\phi_{2B}(r,t) - p_B\Sigma_{1B}\phi_{1B}(r,t) = -\frac{1}{v_{2B}}\frac{\partial}{\partial t}\phi_{2B}(r,t)$$

where $\Sigma_{1C} = \frac{D_{1C}}{\tau_C}$ = fast macroscopic removal cross section in core

Σ^F_{2C} = thermal macroscopic absorption cross section of fuel in core

Σ_{2C} = total macroscopic absorption cross section in core

$\Sigma_{1B} = \frac{D_{1B}}{\tau_B}$ = fast macroscopic removal cross section in blanket

Σ^F_{2B} = thermal macroscopic absorption cross section of fuel in blanket

Σ_{2B} = total macroscopic absorption cross section in blanket

Assuming the reactor is on a stable period,

$$\phi_i(r,t) = \phi_i(r,o)\, e^{\frac{\Delta k_e}{\ell} t} \equiv \phi_i(r)\, e^{mt}$$

603 019

where

$$\Delta k_e = k_e - 1 = \text{excess reactivity}$$

$$m = \frac{\Delta k_e}{\ell} = \text{reciprocal reactor period.}$$

Then it follows that

$$\frac{1}{v_i}\frac{\partial}{\partial t}\phi_i(r,t) = \frac{m}{v_i}\phi_i(r)\,e^{mt},\quad i = 1C,\ 2C,\ 1B,\ 2B$$

If the assumption is made that an instantaneous change is made in the system giving rise to Δk_e, then at time zero equation (1) becomes

$$\begin{aligned}
-D_{1C}\nabla^2\phi_{1C} + (\Sigma_{1C}+K_{1C})\phi \quad - \eta_C\Sigma^F_{2C}\phi_{2C} &= 0\\
-D_{2C}\nabla^2\phi_{2C} + (\Sigma_{2C}+K_{2C})\phi_{2C} - p_C\Sigma_{1C}\phi_{1C} &= 0\\
-D_{1B}\nabla^2\phi_{1B} + (\Sigma_{1B}+K_{1B})\phi_{1B} - \eta_B\Sigma^F_{2B}\phi_{2B} &= 0\\
-D_{2B}\nabla^2\phi_{2B} + (\Sigma_{2B}+K_{2B})\phi_{2B} - p_B\Sigma_{1B}\phi_{1B} &= 0
\end{aligned} \qquad (2)$$

where

$$K_{1C} = \frac{m}{v_{1C}}$$

$$K_{2C} = \frac{m}{v_{2C}}$$

$$K_{1B} = \frac{m}{v_{1B}}$$

$$K_{2B} = \frac{m}{v_{2B}}$$

$$\phi_i = \phi_i(r),\ i = 1C,\ 2C,\ 1B,\ 2B$$

Note that for a steady-state system, $m = \frac{\Delta k_e}{\ell} = 0$

B. Solution of (2)

1. Blanket

We assume*

$$\phi_{1B} = S_1 E^1 Z + S_2 F^1 W$$
$$\phi_{2B} = E^1 Z + F^1 W \qquad (3)$$

where E^1 and F^1 are arbitrary constants and S_1 and S_2 are coefficients to be determined.

We require that

$$\nabla^2 Z - \kappa_1^2 Z = 0$$
$$\nabla^2 W - \kappa_2^2 W = 0 \qquad (4)$$

Using (3) and (4) in $(2)_B$ we obtain

$$\left[-D_{1B}E^1\kappa_1^2 S_1 + (\Sigma_{1B}+K_{1B})E^1 S_1 - \eta_B \Sigma^F_{2B}E^1\right] Z + \left[-D_{1B}F^1\kappa_2^2 S_2 + (\Sigma_{1B}+K_{1B}) F^1 S_2 - \eta_B \Sigma^F_{2B} F^1\right] W = 0$$

$$\left[-D_{2B}E^1\kappa_1^2 + (\Sigma_{2B}+K_{2B})E^1 - p_B S_1 \Sigma_{1B}E^1\right] Z + \left[-D_{2B}F^1\kappa_2^2 + (\Sigma_{2B}+K_{2B}) F^1 - p_B \Sigma_{1B} F^1 S_2\right] W = 0$$

and since the above equations must hold for all values of Z and W, it follows that

$$S_1 = \frac{\eta_B \Sigma^F_{2B}}{(\Sigma_{1B}+K_{1B}) - D_{1B}\kappa_1^2} = \frac{(\Sigma_{2B}+K_{2B}) - D_{2B}\kappa_1^2}{p_B \Sigma_{1B}}$$
$$S_1 = \frac{\eta_B \Sigma^F_{2B}}{(\Sigma_{1B}+K_{1B}) - D_{1B}\kappa_2^2} = \frac{(\Sigma_{2B}+K_{2B}) - D_{2B}\kappa_2^2}{p_B \Sigma_{1B}} \qquad (5)$$

603 021

*Note that the form of equations (3) and (4) is taken so that the calculation will hold for the case of a pure reflector, i.e., no fuel in the blanket and we take for S_1 and S_2

$$S_1 = \frac{D_{2B}}{p_B \Sigma_{1B}} \left[\frac{\Sigma_{2B}+K_{2B}}{D_{2B}} - \mathcal{H}_1^2 \right]$$

$$S_2 = \frac{D_{2B}}{p_B \Sigma_{1B}} \left[\frac{\Sigma_{2B}+K_{2B}}{D_{2B}} - \mathcal{H}_2^2 \right]$$

From (5) we write

$$\left[\frac{\Sigma_{2B}+K_{2B}}{D_{2B}} - \mathcal{H}_i^2 \right] \left[\frac{\Sigma_{1B}+K_{1B}}{D_{1B}} - \mathcal{H}_i^2 \right] = \frac{\eta_B \Sigma_{2B}^F p_B \Sigma_{1B}}{D_{1B} D_{2B}}, \quad i = 1, 2$$

and solving for $\mathcal{H}_1^2$ and $\mathcal{H}_2^2$ we get

$$\mathcal{H}_1^2 = \frac{1}{2} \left[\frac{\Sigma_{2B}+K_{2B}}{D_{2B}} + \frac{\Sigma_{1B}+K_{1B}}{D_{1B}} \right] + \sqrt{\left\{ \frac{1}{2} \left[\frac{\Sigma_{2B}+K_{2B}}{D_{2B}} + \frac{\Sigma_{1B}+K_{1B}}{D_{1B}} \right] \right\}^2 - \left(\frac{\Sigma_{2B}+K_{2B}}{D_{2B}} \right) \left(\frac{\Sigma_{1B}+K_{1B}}{D_{1B}} \right) + \frac{\eta_B \Sigma_{2B}^F p_B \Sigma_{1B}}{D_{1B} D_{2B}}}$$

$$\mathcal{H}_2^2 = \frac{1}{2} \left[\frac{\Sigma_{2B}+K_{2B}}{D_{2B}} + \frac{\Sigma_{1B}+K_{1B}}{D_{1B}} \right] - \sqrt{\left\{ \frac{1}{2} \left[\frac{\Sigma_{2B}+K_{2B}}{D_{2B}} + \frac{\Sigma_{1B}+K_{1B}}{D_{1B}} \right] \right\}^2 - \left(\frac{\Sigma_{2B}+K_{2B}}{D_{2B}} \right) \left(\frac{\Sigma_{1B}+K_{1B}}{D_{1B}} \right) + \frac{\eta_B \Sigma_{2B}^F p_B \Sigma_{1B}}{D_{1B} D_{2B}}}$$

where, for this particular calculation (with reference to the input numbers)

$$\eta_B = \frac{\eta_B^{F1} \Sigma_{2B}^{F1} + \eta_B^{F2} \Sigma_{2B}^{F2}}{\Sigma_{2B}^{F1} + \Sigma_{2B}^{F2}}$$

$$\Sigma_{2B}^F = \Sigma_{2B}^{F1} + \Sigma_{2B}^{F2}$$

$$\Sigma_{2B} = \Sigma_{2B}^{F1} + \Sigma_{2B}^{F2} + \Sigma_{2B}^3 + \Sigma_{2B}^4 + \Sigma_{2B}^5 + \Sigma_{2B}^6 + \Sigma_{2B}^7 + \Sigma_{2B}^8 + \Sigma_{2B}^9$$

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It should be noted that for the case where there is no fuel in the blanket, and

$$\frac{\Sigma_{2B}+K_{2B}}{D_{2B}} < \frac{\Sigma_{1B}+K_{1B}}{D_{1B}} ,$$

then

$$\mathcal{H}_1^2 = \frac{\Sigma_{1B}+K_{1B}}{D_{1B}} , \quad \mathcal{H}_2^2 = \frac{\Sigma_{2B}+K_{2B}}{D_{2B}} , \text{ and } S_2 = 0.$$

When

$$\frac{\Sigma_{2B}+K_{2B}}{D_{2B}} > \frac{\Sigma_{1B}+K_{1B}}{D_{1B}} ,$$

then

$$\mathcal{H}_1^2 = \frac{\Sigma_{2B}+K_{2B}}{D_{2B}} , \quad \mathcal{H}_2^2 = \frac{\Sigma_{1B}+K_{1B}}{D_{1B}} , \text{ and } S_1 = 0.$$

2. Core

For the core equations we assume

$$\begin{aligned} \phi_{1C} &= A^1 X + C^1 Y \\ \phi_{2C} &= s_1 A^1 X + s_2 C^1 Y \end{aligned} \tag{6}$$

where A^1, C^1, s_1, and s_2 have the same significance as in (3) and we require that X and Y satisfy

$$\begin{aligned} \nabla^2 X + \mu^2 X &= 0 \\ \nabla^2 Y - \nu^2 Y &= 0 \end{aligned} \tag{7}$$

using (6) and (7) in $(2)_A$, we obtain

$$\left[D_{1C}A^1\mu^2 + (\Sigma_{1C} + K_{1C})\,A^1 - \eta_C\Sigma^F_{2C}s_1A^1\right]X + \left[-D_{1C}C^1\nu^2 + (\Sigma_{1C}+K_{1C})\,C^1 - \eta_C\Sigma^F_{2C}s_2C^1\right]Y = 0$$

$$\left[D_{2C}s_1A^1\mu^2 + (\Sigma_{2C}+K_{2C})s_1A^1 - p_C\Sigma_{1C}A^1\right]X + \left[-D_{2C}s_2C^1\nu^2 + (\Sigma_{2C}+K_{2C})s_2C^1 - p_C\Sigma_{1C}C^1\right]Y = 0$$

It follows that

$$s_1 = \frac{D_{1C}\mu^2 + (\Sigma_{1C}+K_{1C})}{\eta_C\Sigma^F_{2C}} = \frac{p_C\Sigma_{1C}}{D_{2C}\mu^2 + (\Sigma_{2C}+K_{2C})}$$

$$s_2 = \frac{-D_{1C}\nu^2 + (\Sigma_{1C}+K_{1C})}{\eta_C\Sigma^F_{2C}} = \frac{p_C\Sigma_{1C}}{-D_{2C}\nu^2 + (\Sigma_{2C}+K_{2C})}$$

and hence,

$$\mu^2 = -\frac{1}{2}\left[\frac{\Sigma_{1C}+K_{1C}}{D_{1C}} + \frac{\Sigma_{2C}+K_{2C}}{D_{2C}}\right] + \sqrt{\left\{\frac{1}{2}\left[\frac{\Sigma_{1C}+K_{1C}}{D_{1C}} + \frac{\Sigma_{2C}+K_{2C}}{D_{2C}}\right]\right\}^2 + \frac{\eta_C\Sigma^F_{2C}\,p_C\Sigma_{1C}}{D_{1C}D_{2C}} - \left(\frac{\Sigma_{1C}+K_{1C}}{D_{1C}}\right)\left(\frac{\Sigma_{2C}+K_{2C}}{D_{1C}}\right)}$$

$$\nu^2 = +\frac{1}{2}\left[\frac{\Sigma_{1C}+K_{1C}}{D_{1C}} \quad \frac{\Sigma_{2C}+K_{2C}}{D_{2C}}\right] + \sqrt{\left\{\frac{1}{2}\left[\frac{\Sigma_{1C}+K_{1C}}{D_{1C}} \quad \frac{\Sigma_{2C}+K_{2C}}{D_{2C}}\right]\right\}^2 + \frac{\eta_C\Sigma^F_{2C}p_C\Sigma_{1C}}{D_{1C}D_{2C}} - \left(\frac{\Sigma_{1C}+K_{1C}}{D_{1C}}\right)\left(\frac{\Sigma_{2C}+K_{2C}}{D_{2C}}\right)}$$

Because of the fact that Σ^F_{2C} and hence Σ_{2C} is the unknown variable in this calculation and since for a two-region sphere

$$0 < \mu < \frac{\pi}{a}$$

it is most convenient to write the above equations as

$$\left(\frac{\Sigma_{2C}}{D_{2C}} + \mu^2\right) = \frac{\frac{K_{2C}}{D_{2C}}\left[\mu^2 + \left(\frac{\Sigma_{1C}+K_{1C}}{D_{1C}}\right)\right] + \frac{\eta_C^1 p_C \Sigma_{1C}}{D_{1C}}\left[\mu^2 + \frac{\Sigma_{2C}^M}{D_{2C}}\right]}{\frac{\eta_C^1 p_C \Sigma_{1C}}{D_{1C}} - \left[\mu^2 + \left(\frac{\Sigma_{1C}+K_{1C}}{D_{1C}}\right)\right]} \tag{8}$$

$$\nu^2 = \left(\frac{\Sigma_{2C}}{D_{2C}} + \mu^2\right) + \left(\frac{\Sigma_{1C}+K_{1C}}{D_{1C}}\right) + \frac{K_{2C}}{D_{2C}}$$

where for this calculation

$$\eta_C^1 = \left[\eta_C^{F1} + \eta_C^{F2}\frac{\Sigma_{2C}^{F2}}{\Sigma_{2C}^{F1}}\right]\left[\frac{1}{1 + \frac{\Sigma_{2C}^{F2}}{\Sigma_{2C}^{F1}} + \frac{\Sigma_{2C}^{3}}{\Sigma_{2C}^{F1}} + \frac{\Sigma_{2C}^{4}}{\Sigma_{2C}^{F1}} + \frac{\Sigma_{2C}^{5}}{\Sigma_{2C}^{F1}} + \frac{\Sigma_{2C}^{6}}{\Sigma_{2C}^{F1}} + \frac{\Sigma_{2C}^{p}}{\Sigma_{2C}^{F1}}}\right]$$

= an effective η_C of everything in the core except moderator, i.e., considering all core components except moderator as fuel.

and where

$$\frac{\Sigma_{2C}^{p}}{\Sigma_{2C}^{F1}} = f_{pC}\left[\frac{\Sigma_{fC}^{F1}}{\Sigma_{2C}^{F1}} + \frac{\Sigma_{fC}^{F2}}{\Sigma_{2C}^{F1}}\right]$$

and

$$\Sigma_{2C} = \Sigma_{2C}^{F1} + \Sigma_{2C}^{F2} + \Sigma_{2C}^{3} + \Sigma_{2C}^{4} + \Sigma_{2C}^{5} + \Sigma_{2C}^{6} + \Sigma_{2C}^{p} + \Sigma_{2C}^{M}$$

s_1 and s_2 are taken as

$$s_1 = \frac{\frac{\Sigma_{1C}p_C}{D_{2C}}}{\left[\frac{\Sigma_{2C}}{D_{2C}} + \mu^2\right] + \frac{K_{2C}}{D_{2C}}}\ , \quad s_2 = -\frac{\frac{\Sigma_{1C}p_C}{D_{2C}}}{\frac{\Sigma_{1C}+K_{1C}}{D_{1C}} + \mu^2} \tag{9}$$

From equations (4) and (7) we write

$$Z(r) = E_1 \frac{\sinh \mathcal{H}_1 r}{r} + F_1 \frac{\cosh \mathcal{H}_1 r}{r}$$

$$W(r) = E_2 \frac{\sinh \mathcal{H}_2 r}{r} + F_2 \frac{\cosh \mathcal{H}_2 r}{r}$$

$$X(r) = A_1 \frac{\sin \mu r}{r} + C_1 \frac{\cos \mu r}{r}$$

$$Y(r) = A_2 \frac{\sinh \nu r}{r} + C_2 \frac{\cosh \nu r}{r}$$

where E_1, E_2, F_1, F_2, A_1, A_2, C_1, and C_2 are arbitrary constants.

Applying the extrapolation boundary condition

$$Z(\tilde{R}) = W(\tilde{R}) = 0$$

and requiring that

$$X(r),\ Y(r) \neq \infty$$

requires

$$F_1 = -E_1 \frac{\sinh \mathcal{H}_1 \tilde{R}}{\cosh \mathcal{H}_1 \tilde{R}}$$

$$F_2 = -E_2 \frac{\sinh \mathcal{H}_2 \tilde{R}}{\cosh \mathcal{H}_2 \tilde{R}}$$

$$C_1 = C_2 = 0$$

hence

$$Z(r) = \frac{E^*}{r} \sinh \mathcal{H}_1 (\tilde{R}-r)$$

$$W(r) = \frac{C^*}{r} \sinh \mathcal{H}_2 (\tilde{R}-r)$$

$$X(r) = A_1 \frac{\sinh \mu r}{r}$$

$$Y(r) = A_2 \frac{\sin \nu r}{r}$$

and from (3) and (6) we have

$$\phi_{1B} = S_1 E \frac{\sinh \mathcal{H}_1(\tilde{R}-r)}{r} + S_2 F \frac{\sinh \mathcal{H}_2(\tilde{R}-r)}{r}$$

$$\phi_{2B} = E \frac{\sinh \mathcal{H}_1(\tilde{R}-r)}{r} + F \frac{\sinh \mathcal{H}_2(\tilde{R}-r)}{r}$$

(10)

$$\phi_{1C} = A \frac{\sin \mu r}{r} + C \frac{\sinh \nu r}{r}$$

$$\phi_{2C} = s_1 A \frac{\sin \mu r}{r} + s_2 C \frac{\sinh \nu r}{r}$$

Applying the thin shell interface boundary conditions[4]

$$\phi_{1C}(a) = \phi_{1S}(a)$$

$$\phi_{1B}(b) = \phi_{1S}(b)$$

$$\phi_{2C}(a) = \phi_{2S}(a)$$

$$\phi_{2B}(b) = \phi_{2S}(b)$$

$$-D_{1C}\phi^1_{1C}(a) = -D_{1S}\phi^1_{1S}(a)$$

$$-D_{2C}\phi^1_{2C}(a) = -D_{2S}\phi^1_{2S}(a)$$

$$-D_{1B}\phi^1_{1B}(b) = -D_{1S}\phi^1_{1S}(b)$$

$$-D_{2B}\phi^1_{2B}(b) = -D_{2S}\phi^1_{2S}(b)$$

where a = core radius

b = core radius plus shell thickness

D_{1S} = fast diffusion coefficient for shell

D_{2S} = thermal diffusion coefficient for shell

ϕ_{1S} = fast flux in shell

ϕ_{2S} = thermal flux in shell

ϕ^1_i = derivitive of flux, i = 1C, 1B, 2C, 2B, 1S, and 2S.

Then

$$a\phi_{1C}(a) = a\phi_{1B}(b) - bt\,\frac{D_{1B}}{D_{SH}}\,\phi^1_{1B}(b)$$

$$a^2 D_{1C}\phi^1_{1C}(a) = b^2 D_{1B}\phi^1_{1B}(b)$$

$$a\phi_{2C}(a) = a\phi_{2B}(b) - bt\,\frac{D_{2B}}{D_{SH}}\,\phi^1_{2B}(b) \qquad (11)$$

$$a^2 D_{2C}\phi^1_{2C}(a) = b^2 D_{2B}\phi^1_{2B}(b) - tb^2\Sigma_a^{SH}$$

where t = shell thickness

$D_{SH} = D_{1S} = D_{2S}$

Then using (10) in (11) one can obtain

$$A\sin\mu a + C\sinh\nu a + ES_1\left[-a/b - \frac{D_{1B}t}{D_{SH}b}m_1\right]\sinh\mathcal{H}_1T + FS_2\left[-a/b - \frac{D_{1B}t}{D_{SH}b}m_2\right]\sinh\mathcal{H}_2T = 0$$

$$As_1\sin\mu a + Cs_2\sinh\nu a + E\left[-a/b - \frac{D_{2B}t}{D_{SH}b}m_1\right]\sinh\mathcal{H}_1T + F\left[-a/b - \frac{D_{2B}t}{D_{SH}b}m_2\right]\sinh\mathcal{H}_2T = 0$$

$$Ag_1\sin\mu a + Cg_2\sinh\nu a + E\frac{D_{1B}}{D_{1C}}S_1m_1\sinh\mathcal{H}_1T + F\frac{D_{1B}}{D_{1C}}S_2m_2\sinh\mathcal{H}_2T = 0 \qquad (12)$$

$$As_1g_1\sin\mu a + Cs_2g_2\sinh\nu a + E\left[\frac{D_{2B}}{D_{2C}}m_1 + \frac{tb\Sigma_a^{SH}}{D_{2C}}\right]\sinh\mathcal{H}_1T + F\left[\frac{D_{2B}}{D_{2C}}m_2 + \frac{tb\Sigma_a^{SH}}{D_{2C}}\right]\sinh\mathcal{H}_2T = 0$$

where $m_1 = \mathcal{H}_1 b \coth \mathcal{H}_1 T + 1$

$m_2 = \mathcal{H}_2 b \coth \mathcal{H}_2 T + 1$

$g_1 = a\mu \cot a\mu - 1$

$g_2 = a\nu \coth a\nu - 1$

$T = (\tilde{R} - b)$

and

A, C, E, and F are unknown arbitrary constants.

For a non-trivial solution of the homogeneous set (12), it is necessary and sufficient that the determinant of the coefficient vanish, hence we write as the critical equation

$$(\sin \mu a)(\sinh \nu a)(\sinh \mathcal{H}_1 T)(\sinh \mathcal{H}_2 T) \begin{vmatrix} 1 & 1 & S_1\left[-a/b - \frac{D_{1B}t}{D_{SH}b} m_1\right] & S_2\left[-a/b - \frac{D_{1B}t}{D_{SH}b} m_2\right] \\ s_1 & s_2 & \left[-a/b - \frac{D_{2B}t}{D_{SH}b} m_1\right] & \left[-a/b - \frac{D_{2B}t}{D_{SH}b} m_2\right] \\ g_1 & g_2 & S_1 m_1 \frac{D_{1B}}{D_{1C}} & S_2 m_2 \frac{D_{1B}}{D_{1C}} \\ s_1 g_1 & s_2 g_2 & \left[\frac{D_{2B}}{D_{2C}} m_1 + \frac{tb\Sigma_a^{SH}}{D_{2C}}\right] & \left[\frac{D_{2B}}{D_{2C}} m_2 + \frac{tb\Sigma_a^{SH}}{D_{2C}}\right] \end{vmatrix} = 0$$

calling the value of the critical determinant Δ, we can write (using a_{ij}'s for the determinant elements)

$$\Delta = \left[\left(\frac{a_{11}}{a_{21}} - \frac{a_{12}}{a_{22}}\right)\left(\frac{a_{32}}{a_{22}} - \frac{a_{33}}{a_{23}}\right)\left(\frac{a_{43}}{a_{23}} - \frac{a_{44}}{a_{24}}\right) + \left(\frac{a_{12}}{a_{22}} - \frac{a_{13}}{a_{23}}\right)\left(\frac{a_{33}}{a_{23}} - \frac{a_{34}}{a_{24}}\right)\left(\frac{a_{41}}{a_{21}} - \frac{a_{42}}{a_{22}}\right) + \left(\frac{a_{13}}{a_{23}} - \frac{a_{14}}{a_{24}}\right)\left(\frac{a_{42}}{a_{22}} - \frac{a_{43}}{a_{23}}\right)\left(\frac{a_{31}}{a_{21}} - \frac{a_{32}}{a_{22}}\right)\right]$$
$$- \left[\left(\frac{a_{41}}{a_{21}} - \frac{a_{42}}{a_{22}}\right)\left(\frac{a_{32}}{a_{22}} - \frac{a_{33}}{a_{23}}\right)\left(\frac{a_{13}}{a_{23}} - \frac{a_{14}}{a_{24}}\right) + \left(\frac{a_{31}}{a_{21}} - \frac{a_{32}}{a_{22}}\right)\left(\frac{a_{12}}{a_{22}} - \frac{a_{13}}{a_{23}}\right)\left(\frac{a_{43}}{a_{23}} - \frac{a_{44}}{a_{24}}\right) + \left(\frac{a_{11}}{a_{21}} - \frac{a_{12}}{a_{22}}\right)\left(\frac{a_{33}}{a_{23}} - \frac{a_{34}}{a_{24}}\right)\left(\frac{a_{42}}{a_{22}} - \frac{a_{43}}{a_{23}}\right)\right] \quad (13)$$

where all factors outside the determinant have been discarded.

Knowing the value of , for a particular reactor, which makes $\Delta = 0$, we write

$$\Sigma_{2C} \equiv \left[\left(\frac{\Sigma_{2C}}{D_{2C}} + \mu^2\right) - \mu^2\right] D_{2C}$$

$$\overset{**}{\Sigma}{}^{F1}_{2C} = \frac{\Sigma_{2C} - \Sigma^{M}_{2C}}{1 + \frac{\Sigma^{F2}_{2C}}{\Sigma^{F1}_{2C}} + \frac{\Sigma^{3}_{2C}}{\Sigma^{F1}_{2C}} + \frac{\Sigma^{4}_{2C}}{\Sigma^{F1}_{2C}} + \frac{\Sigma^{5}_{2C}}{\Sigma^{F1}_{2C}} + \frac{\Sigma^{6}_{2C}}{\Sigma^{F1}_{2C}} + \frac{\Sigma^{P}_{2C}}{\Sigma^{F1}_{2C}}}$$

$$\Sigma^{i}_{2C} \equiv \Sigma^{F1}_{2C}\left(\frac{\Sigma^{i}_{2C}}{\Sigma^{F1}_{2C}}\right), \qquad i = F_2, 3, 4, 5, 6, P$$

It is obvious that the homogeneous set (12) cannot be solved for all four of the constants A, C, E, and F; therefore, we divide (12) by A and solve for $\frac{C}{A}$, $\frac{E}{A}$ and $\frac{F}{A}$; hence

$$\frac{C}{A} = \frac{\sin\mu a}{\sinh\nu a}\,\frac{|X|}{|Z|}$$

$$\frac{E}{A} = \frac{\sin\mu a}{\sinh \mathcal{P}_1 T}\,\frac{|Y|}{|Z|}$$

and

$$\frac{F}{A} = \frac{-\left[a_{21}\sin\mu a + a_{22}\,C/A\,\sinh\nu a + a_{23}\,E/A\,\sinh \mathcal{P}_1 T\right]}{a_{24}\sinh \mathcal{P}_2 T}$$

where

$$|X| = \begin{vmatrix} a_{11} & a_{13} & a_{14} \\ a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \end{vmatrix}, \quad |Y| = \begin{vmatrix} a_{12} & a_{11} & a_{14} \\ a_{22} & a_{21} & a_{24} \\ a_{32} & a_{31} & a_{34} \end{vmatrix}, \quad |Z| = \begin{vmatrix} a_{13} & a_{12} & a_{14} \\ a_{23} & a_{22} & a_{24} \\ a_{33} & a_{32} & a_{34} \end{vmatrix}$$

and a_{ij} are given in the critical determinant.

With $\frac{C}{A}$, $\frac{E}{A}$, and $\frac{F}{A}$ known, we can now write the neutron balance as

$$^{**}A^{i}_{2B} = \overset{I}{\underset{I}{\phi}}_{2B} \Sigma^{i}_{2B}, \quad i = F_1, F_2, 3, 4, 5, 6, 7, 8, 9$$

$$^{**}A_{1B} = \overset{I}{\underset{I}{\phi}}_{1B} \Sigma_{1B} (1-p_B)$$

$$^{**}A^{i}_{2C} = \overset{I}{\underset{I}{\phi}}_{2C} \Sigma^{i}_{2C}, \quad i = F_1, F_2, 3, 4, 5, 6, P, M$$

$$^{**}A_{1C} = \overset{I}{\underset{I}{\phi}}_{1C} \Sigma_{1C} (1-p_C)$$

$$^{**}A_{SH} = 4\pi b^2 t \Sigma^{SH}_{a} \phi_{2B}(b) = 4\pi b \, t \Sigma^{SH}_{a} \left[\frac{E}{A} \sinh \mathcal{H}_1 T + \frac{F}{A} \sinh \mathcal{H}_2 T\right]$$

$$^{**}L_1 = -4\pi R^2 D_{1B} \nabla \phi_{1B}(R) = 4\pi D_{1B} \Big\{ S_1 \frac{E}{A}\left[R \mathcal{H}_1 \cosh \mathcal{H}_1(\tilde{R}-R) + \sinh \mathcal{H}_2(\tilde{R}-R)\right]$$

$$+ S_2 \frac{F}{A}\left[R \mathcal{H}_2 \cosh \mathcal{H}_2(\tilde{R}-R) + \sinh \mathcal{H}_2(\tilde{R}-R)\right]\Big\}$$

$$^{**}L_2 = -4\pi R^2 D_{2B} \nabla \phi_{2B}(R) = 4\pi D_{2B} \Big\{ \frac{E}{A}\left[R \mathcal{H}_1 \cosh \mathcal{H}_1(\tilde{R}-R) + \sinh \mathcal{H}_1(\tilde{R}-R)\right]$$

$$+ \frac{F}{A}\left[R \mathcal{H}_2 \cosh \mathcal{H}_2(R-R) + \sinh \mathcal{H}_2(\tilde{R}-R)\right]\Big\}$$

where

$$^{**}\Phi_{2B} = \int_b^R \phi_{2B} 4\pi r^2 dr = 4\pi \left\{ \frac{E}{\mathcal{H}_1^2 A} \left[m_1 \sinh \mathcal{H}_1 T - \mathcal{H}_1 R \cosh \mathcal{H}_1 (\tilde{R}-R) - \sinh \mathcal{H}_1 (\tilde{R}-R) \right] \right.$$

$$\left. + \frac{F}{A\mathcal{H}_2^2} \left[m_2 \sinh \mathcal{H}_2 T - \mathcal{H}_2 R \cosh \mathcal{H}_2 (\tilde{R}-R) - \sinh \mathcal{H}_2 (\tilde{R}-R) \right] \right\}$$

$$^{**}\Phi_{1B} = \int_b^R \phi_{1B} 4\pi r^2 dr = 4\pi \left\{ \frac{S_1 E}{A\mathcal{H}_1^2} \left[m_1 \sinh \mathcal{H}_1 T - \mathcal{H}_1 R \cosh \mathcal{H}_1 (\tilde{R}-R) - \sinh \mathcal{H}_1 (\tilde{R}-R) \right] \right.$$

$$\left. + \frac{S_2 F}{A\mathcal{H}_2^2} \left[m_2 \sinh \mathcal{H}_2 T - \mathcal{H}_2 R \cosh \mathcal{H}_2 (\tilde{R}-R) - \sinh \mathcal{H}_2 (\tilde{R}-R) \right] \right\}$$

$$^{**}\Phi_{2C} = \int_0^a \phi_{2C} 4\pi r^2 dr = 4\pi \left\{ \frac{g_2 C}{A\nu^2} \sinh \nu a - \frac{g_1 A}{\mu^2 A} \sin \mu a \right\}$$

$$^{**}\Phi_{1C} = \int_0^a \phi_{1C} 4\pi r^2 dr = 4\pi \left\{ \frac{s_2 g_2 C}{A\nu^2} \sinh \nu a - \frac{s_1 g_1 A}{\mu^2 A} \sin \mu a \right\}$$

and ϕ_{1B}, ϕ_{2B}, ϕ_{1C}, ϕ_{2C} are as given in (10)

The total neutron production is given by

$$^{**}P_T = \eta_C^{F1} A_{2C}^{F1} + \eta_C^{F2} A_{2C}^{F2} + \eta_B^{F1} A_{2B}^{F1} + \eta_B^{F2} A_{2B}^{F2} ,$$

and the total neutron losses is given by

$$^{**}L_T = \sum A_{2B}^{i} + \sum A_{2C}^{i} + A_{1B} + A_{1C} + L_1 + L_2, \quad i = \text{reactor components}$$

The flux plot** is computed from (10), where A, C, E, and F take on the values A/A, C/A, E/A, and F/A. It will be noted that all of the above normalized answers[1,2] are equal to the actual answers divided by the constant A. The constant A may be evaluated if the core or blanket power is specified. For example, if the core power is specified, then

$$A = \frac{f\, P_C}{\sum_{fC} \phi_{2C}}$$

where

P_C = core power, kw

f = 3.38×10^{13} fissions/kw-sec

Σ_{fC} = macroscopic fission cross section of fuel in the core

ϕ_{2C} is as given above

and the actual absorption, leakages, and fluxes may be calculated by multiplying those output answers by A.

C. Method of Calculation

The method of calculation is given, briefly, as follows:

1. Steady-State System

With a particular set of input parameters, m is set equal to zero and the blanket functions are computed. Then, using the core dependent equations, the value of μ is found which makes $\Delta < 5\times10^{-5}$. With this value of μ, the calculation is completed producing either of the two sets of steady-state answers.

2. Non-Steady-State Calculation[5]

For this calculation, the steady-state system is first computed as above using the steady-state input parameters (corresponding to the steady-state temperature, $T^{o}C$. Then with the critical core fuel concentration of the steady-state system known, and using the non-steady-state input parameters (corresponding to $(T+\Delta T)^{o}C$), the value of

$$\frac{{}^{**}l}{m} = \frac{\ell}{\Delta k_e}$$

is found which gives this fuel concentration.* Then setting m = 0, the value of η_C^1 is found which makes the critical core fuel concentration of the non-steady-state system the same as that for the steady-state system. The multiplication is then taken to be

$$^{**}k_e = \frac{\eta_C^1(SS)}{\eta_C^1(NSS)} .$$

With k_e known, $\Delta k_e = k_e - 1$, $^{**}\ell = \frac{\Delta k_e}{m}$, and $\frac{^{**}\partial k_e}{\partial T} = \frac{\Delta k_e}{\Delta T}$ are computed.

In the answers that are produced for this calculation, items 2 through 48 are steady-state answers, and items 49 through 52 are for the non-steady-state system.

D. Restrictions in the Calculation:

1. All computed numbers must be in the range $10^{-38} < X < 10^{38}$.

2. For the steady-state calculation and the non-steady-state calculation for which m > 0, μ must be in the range $\frac{\pi}{64a} \leq \mu < \frac{\pi}{a}$.

3. For the non-steady-state calculation for which m < 0, μ must be in the range $\frac{\pi}{2a} \leq \mu < \frac{\pi}{a}$.

4. For the non-steady-state calculation, $\eta_C^1(NSS)$ must lie in the range $1.5 \leq \eta_C^1(NSS) < 2.5$.

---

* In particular the macroscopic absorption cross section of core fuel is used where

$$\Sigma_{ac}^{F}(T+\Delta T) = \Sigma_{ac}^{F1}(T)\left[\frac{\sigma_{ac}^{F1}(T+\Delta T)}{\sigma_{ac}^{F1}(T)}\right] + \Sigma_{ac}^{F2}(T)\left[\frac{\sigma_{ac}^{F2}(T+\Delta T)}{\sigma_{ac}^{F2}(T)}\right]$$

5. Generally speaking, the code will not compute reactors whose core radius, a, is less than approximately 15 cm; however, this restriction is highly dependent upon the core and blanket compositions as well as the other input parameters such as τ^1's, D^1's, etc.; hence, the above figure will vary considerably with each particular case.

6. In the case where a reactor has no shell, input item 36 (D_{SH}) must have a value, generally 1.

7. For the case of a non-steady-state calculation requiring a negative m, the machine will stop on $\sqrt{-\mathcal{H}_2^2}$ in the case where there is no fuel in the blanket and

$$\left|\frac{K_{2B}}{D_{2B}}\right| > \left|\frac{\Sigma_{2B}}{D_{2B}}\right| \quad \text{(the foregoing is true if } m \leqq -100\text{).}$$

Restriction 1 results from the floating point sub-routine used in the code. The lower limit in restriction 2 is for convenience only and will not be exceeded for any practical case. It follows, from equation (9), that μ and m, when $m < 0$, could have values such that $s_1 \rightarrow \infty$, hence restriction 3 assures that

$$\left(\frac{\Sigma_{2C}}{D_{2C}} + \mu^2\right) > \frac{K_{2C}}{D_{2C}} .$$

The limits imposed on η_C^1(NSS) are more or less arbitrary and can easily be changed; however, for most practical cases, the limits shown above will not be exceeded.

The fact that there is a lower limit on the core radius is not surprising when one considers the denominator in equation $(8)_a$, keeping in mind that μ is always taken as some number (between $\frac{\pi}{64}$ and π)

divided by a. If a is such that

$$\left(\mu^2 + \frac{\Sigma_{1C}+K_{1C}}{D_{1C}}\right) \rightarrow \frac{\eta^1_C P_C \Sigma_{1C}}{D_{1C}} \quad \text{, then} \left(\frac{\Sigma_{2C}}{D_{2C}} + \mu^2\right) \rightarrow \infty .$$

Actually, in the case where a is too small, the calculation stops with the computation of sinh νa and/or cosh νa, where νa is such that the function exceeds 10^{38} (restriction 1). Restriction 6 is actually not a restriction but is necessary because of the fact that $D_{SH}b$ is used as a divisor in the calculation; however, in the case where there is no shell, D_{SH} can have any finite value without affecting the results of the calculation, as long as t = 0.

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1. S. Visner, Nuclear Claculations for Homogeneous Reactors Producing U^{233}, ORNL-CF-51-10-110, October 22, 1951.

2. M. Tobias, P.N. Haubenreich and R. E. Aven, Conversion in a Two-Region Reactor, ORNL-CF-53-2-134, February 16, 1953.

3. S. Glasstone and M.C. Edlund, The Elements of Nuclear Reactor Theory, New York, D. Van Nostrand, 1952

4. M. Tobias, A. Thin Shell Approximation for Two-Group, Two-Region Spherical Reactor Calculation, ORNL-CF-54-6-135, June 16, 1954.

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OAK RIDGE NATIONAL LABORATORY

DATE: October 7, 1955

TO: Distribution

SUBJECT: Supplement to ORNL-CF-55-9-133

FROM: T. B. Fowler

As shown in the footnote on page 35, $\sum_{2C}^{F}(T+\Delta T)$ (for a non-steady-state system) is calculated by correcting $\sum_{2C}^{F}(T)$ for the change in the microscopic cross section only; i.e., the actual concentration of fuel, N^F, is unchanged. For a homogeneous system in which the density of the core and/or blanket solutions are affected the fuel concentration is also a function of temperature. This effect may be considered by writing input items 44 and 45 (page 13) as

$$\left[\frac{\sigma_{2C}^{F1}(T+\Delta T)}{\sigma_{2C}^{F1}(T)}\right] \quad \frac{\rho^{M}(T+\Delta T)}{\rho^{M}(T)}$$

and

$$\left[\frac{{}_{2C}^{F2}(T+\Delta T)}{{}_{2C}^{F2}(T)}\right] \quad \frac{\rho^{M}(T+\Delta T)}{\rho^{M}(T)}$$

respectively, where $\rho^M(T+\Delta T)$ and $\rho^M(T)$ are the densities of the moderator in grams per cm^3.

The left hand side of the last equation on page 22 should be S_2.

The first sentence on page 31 should read: "Knowing the value of μ, for a particular reactor, which makes D=0, we write".

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All subscripts in the footnote on page 35 should be 2C.

It has been noted that for some reactors, the value of the critical determinant cannot be made less than 5×10^{-5}; i.e., the change in μ required to change Δ beyond some particular value is so small as to be insignificant and the machine continues to compute without completing the calculation. In this case the machine should be stopped and the word F64189374C, representing 5×10^{-4}, or the word F951EB851F, representing 5×10^{-3}, should be inserted in memory cell 329 and the machine re-started with the order on which it was stopped. Another alternative is to insert one of the above words in memory cell 329 immediately after the first part of the code has been loaded into the memory.

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