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MURA-LWJ-KMT-3

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# A MARK I b BETATRON DESIGN

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An example of a 20 sector Mark I b FFAG betatron is described, showing that for a circumference factor of four, reasonable $\sigma$ values result. The production of the required magnetic fields is discussed, pointing up the problem of high current densities in the backwound pole face copper.

The results of MURA-LWJ-KMT-2 are used to calculate possible parameters for an FFAG betatron below.

Starting with $N = 20$, $\gamma = 1\frac{2}{3}$, $M = 5$ and <u>no straight sections</u>, the following parameters were determined from the expressions of II and III.*

$\beta_1 = 45°$

$\beta_2 = 27°$

$\alpha = 16.91°$

$\tan\alpha = .304$

$\theta_1 = 11.18°$

$\theta_2 = 6.82°$

$$\frac{\beta_1 + \beta_2}{\beta_1 - \beta_2} = 4.00$$

$$C = \frac{R}{P} = 3.925$$

$\sigma_x = 117° \cong \frac{2\pi}{3}$ $\qquad \nu_x \cong 6\frac{2}{3}$

$\sigma_z = 61.5° \cong \frac{\pi}{3}$ $\qquad \nu_z \cong 3\frac{1}{3}$

$MC + 1 = 20.6$ $\qquad \frac{P_i}{P_o} = \left(\frac{R_i}{R_o}\right)^{MC+1}$

For a betatron operating with a maximum guide field

* MURA-LWJ-KMT-2

of 10,000 gauss, three sets of numbers were found corresponding to 100, 50, and 25 Mev maximum energy. The subscript $i$ refers to injection values, $o$ refers to maximum energy values.

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|---|---|---|---|---|
| $K.E._o$ | 100 | 50 | 25 | Mev |
| $K.E._i$ | 50 | 35 | 25 | Kev |
| $H_o$ | 10,000 | 10,000 | 10,000 | gauss |
| $H_i$ | 33 | 33 | 33 | gauss |
| $R_o$ | 130 | 65 | 32.5 | cm. |
| $R_i$ | 98 | 49 | 24.5 | cm. |
| $\rho_o$ | 33 | 16.5 | 8.25 | cm. |
| $\rho_i$ | 25 | 12.5 | 6.25 | cm. |
| $\Delta R = R_o - R_i$ | 32 | 16 | 8 | cm. |
| $S_{10}$ | 25.5 | 12.75 | 6.37 | cm. |
| $S_{20}$ | 15.5 | 7.75 | 3.87 | cm. |

For a betatron accelerating electrons at the velocity of light at constant radius R, the flux change through the orbit, $\Delta\Phi$ , required to give an electron on energy E is given by

$$\Delta\Phi = \frac{2\pi R}{300} E,$$

where R is meters, E is in Mev, and $\Delta\Phi$ is in webers. The above 100 Mev and 25 Mev designs could operate with a 25% duty cycle if used with the cores of the existing 320 and 80 Mev Illinois Betatrons. Straight sections were introduced of lenth, l, such that

$$\frac{S_1 + S_2 + 2l}{S_1 + S_2} = 41/31$$

in the previous designs.

For the same values of R, the arc lengths $S_1$ and $S_2$ were considerably shortened.

From the expressions of section IV in MURA-LWJ-KMT-2 the following parameters were found.

$M_e = 5$

$M = \frac{31}{41} \times 5 = 3.78$

$C = \frac{41}{31} \times 3.925 = 5.2$

$\delta = \frac{l}{\rho} = .200$

$\tan \alpha = .304$

$\beta_1 = 45°$

$\beta_2 = 27°$

$\sigma_x = 107.6° \cong \frac{5}{8}\pi$ $\qquad \nu_x \cong 6$

$\sigma_z = 101° \cong \frac{\pi}{2}$ $\qquad \nu_z \cong 5$

This rather large straight section had the effect of bringing the $\sigma_s$ closer together, as the larger circumference factor would indicate. Since the addition of the straight sections would require a smaller radius of curvature for the same R, maximum fields of 13,000 gauss would be required to reach the energies indicated.

<u>Pole Contours and Backwindings</u>

In the two dimensional approximation a potential function which gives a field of the form

$$H = H_o \left(\frac{R}{R_o}\right)^{MC}$$

is (reference 1)

$$V(R, \phi) = \frac{H_o R_o}{MC+1} \left(\frac{R}{R_o}\right)^{MC+1} \sin(MC+1)\phi$$

For vertical displacement from the median plan

$$z << R,$$

$$\phi = \frac{z}{R},$$

$$V(R, z) = \frac{H_o R_o}{MC+1} \left(\frac{R}{R_o}\right)^{MC+1} \sin(MC+1)\frac{z}{R}$$

If the field fall-off is to be accomplished entirely with iron contour with no current backwinding, the gap spacing becomes very large, since

$$\frac{z}{z_0} \cong \frac{H_0}{H}$$

The field inside the large gap regions would become very distorted at the ends of sectors if the gap spacing is comparable to the spacing between sectors, and distorted within the sectors if the spacing is comparable to the sector lengths. Although such "end effects" may not have a strong effect on the $\sigma_s$ , this must receive more theoretical and experimental attention if and when a design is seriously considered.

Since $H = \nabla V$ and $\oint \vec{H} \cdot \vec{ds} = I$ , backwound currents distributed in a thin layer on a pole surface will produce any desired potential function $V$ in the gap if

$$(\nabla V)_s = \frac{dI}{ds} ,$$

where . is along the pole surface. Current backwindings, in thin layers or not, can be considered to change the potential of the iron by an amount

$$\Delta V = \delta I ,$$

where $\Delta I$ is the amount of current crossed. To drop the field to zero,

$$\Delta V = V_0 = I ;$$

i.e. the entire current must be backwound.

The backwinding can be accomplished simply in two ways. One may use discrete increments of current separated by exponential pole pieces (as mentioned in reference 1), or flat (parallel) iron poles with current distribution given by

$$\frac{dI}{dx} = (\nabla V)_x .$$

For $(MC+1)\frac{z}{R} << \frac{\pi}{2}$, $I_0 = V_0 = H_0 Z_0$

$$V = H_0 Z \left(\frac{R}{R_0}\right)^{MC} = I_0 \frac{Z}{Z_0}\left(\frac{R}{R_0}\right)^{MC}$$

The field may be controlled by iron pole contour from the region of highest field (at $R_0$) to a radius R, where the field has fallen a factor of 3 to 10, and then the pole surfaces remain parallel and current back-wound for $R < R_1$. If $z$ is the pole spacing,

$$\frac{Z_1}{Z_0} = \left(\frac{R_0}{R_1}\right)^{MC}, \quad R_0 > R > R_1;$$

$$\frac{dI}{dx} = I_0 \frac{MC}{R_1}\left(\frac{R}{R_1}\right)^{(MC-1)} \quad R \leq R_1$$

$\int_0^{R_1} \frac{dI}{dR} dR = I_0$, as should be the case.

At $R = R_1$,

$$\frac{dI}{dx} = \frac{I_0}{R_1/MC},$$

so that the current density is the same as if all the current were distributed over

$$\Delta R = \frac{R_1}{MC}.$$

<u>Magnet Parameters</u>

For the design figures given in above for a 100 Mev 20 sector Betatron, the following magnet parameters are given:

$$R_0 = 130 \text{ cm}$$
$$M = 5$$
$$C = 3.925$$
$$H_0 = 10{,}000 \text{ gauss}$$
$$Z_0 = 0.25 \text{ cm (vertical semiaperture) at } R = R_0$$

$$Z_1 = 2.5 \text{ cm at } R = R_1$$

$$\left(\frac{R_0}{R_1}\right)^{MC} = \frac{Z_1}{Z_0} = 10 \quad , \quad MC = 19.6$$

$$\left(\frac{R_0}{R_1}\right)^{19.6} = 10, \quad \frac{R_0}{R_1} = 1.125$$

$$\therefore R_1 = 115 \text{ cm.}$$

Assuming negligible iron reluctance,

$$I_0 = .8\, H_0 Z_0 ,$$

where $I_0$ is in ampere-turns, $Z_0$ in centimeters, and $H_0$ in gauss. For the above,

$I_0 = 2000$ ampere turns (above the median plane).

If backwinding is used on parallel pole surfaces beginning at $R = R_1$, the initial backwound current density,

$$\frac{dI}{dR} = \frac{I_0 MC}{R_1} = \frac{2000}{115}(19.6) = 340 \frac{\text{amp.-turns}}{\text{cm.}}$$

If the copper layer is to be thin, a significant cooling problem arises. Copper could be imbedded in the iron, requiring some shimming.

The total flux per unit length carried by the return leg of the magnet is given by

$$\Phi = \int_0^{R_0} H\, dR = \frac{H_0 R_0}{MC+1} .$$

The thickness of the return leg must therefore be

$$\frac{R_0}{MC+1}$$

if it is to carry a maximum field of $H_0$. For the above parameters,

$$\frac{R_0}{MC+1} = 6.3 \text{ cm.}$$

Conclusions

The Mark Ib appears to be theoretically and structurally the simplest of the FFAG accelerator types. We have attempted to supplement the simplest linear theory given in MURA-KRS-6 and to show feasable numbers for a low energy application or test of the principles. The chief design problem encountered is the high current density of backwound copper. Certain problems have been neglected in the theoretical treatment of the problem. Among these are non-linearities (quadratic non-linearities appear to be no more than about 20% for 1 cm amplitude of oscillation), effects of the smoothing of edge effects into the straight sections and magnets due to finite vertical aperture and the change in M value due to the equilibrium orbit making an angle $\alpha$ to the gradient dH/dR at the ends of sectors.