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THE TEMPERATURE COEFFICIENTS OF NRX AND ZEEP

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BY

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<u>PART A</u>: Temperature Coefficients of NRX

<u>PART B</u>: Temperature Coefficient of a Zeep Lattice

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SUMMARY

Experimental measurements of the uniform temperature coefficients of the NRX reactor, for both moderator and rod-assembly temperature changes, give the following results:

For Variation of Moderator Temperature (°C)

$$\frac{2\alpha^2}{h}\frac{dh_w}{dt} \times 10^7 = 0.52 - 5.0 \times \frac{10^3}{\rho_m}\frac{d\rho_m}{dt}$$

For Variation of Rod-Assembly Temperature (°C)

$$\frac{2\alpha^2}{h}\frac{dh_w}{dt} \times 10^7 = 1.53 + 1.08 \times \frac{10^3}{\rho_w}\frac{d\rho_w}{dt}$$

Combining these two results, the Overall Uniform Temperature Coefficient is given by:

$$\frac{2\alpha^2}{h}\frac{dh_w}{dt} \times 10^7 = 2.05 - 5.0 \times \frac{10^3}{\rho_m}\frac{d\rho_m}{dt} + 1.08 \times \frac{10^3}{\rho_w}\frac{d\rho_w}{dt} \quad \text{(Expt.)}$$

The calculated result, based on Lattice Theory and other data is:

$$\frac{2\alpha^2}{h}\frac{dh_w}{dt} \times 10^7 = 2.0 - 5.9 \times \frac{10^3}{\rho_m}\frac{d\rho_m}{dt} + 1.08 \times \frac{10^3}{\rho_w}\frac{d\rho_w}{dt} \quad \text{(Calc.)}$$

In both of these expressions, the coefficient of $\frac{1}{\rho_w}\frac{d\rho_w}{dt}$ has been chosen to agree with measured data on the critical height with and without cooling water in the rod-assemblies. The agreement

of Calculated and Experimental values of the uniform temperature coefficient is better than the accuracy of the data used in the calculations would warrant.

In Part B experimental measurements of the uniform temperature coefficient of a Zeep lattice are compared with the calculated values. There is good agreement between calculations and experiment. For lattices of this type, with relatively large values of κ^2, the leakage terms dominate the contributions to the temperature coefficient. Large corrections are required for changes of radial and vertical extrapolation lengths.

PART A (Temperature Coefficients of NRX)

PAGE

PART B

TEMPERATURE COEFFICIENT OF A ZEEP LATTICE

LIST OF SYMBOLS

ρ_m = moderator density

ρ_w = cooling water density

h_D = excess height (cm) of polymer level above weir-box reading

$k = k_\infty$ = reproduction factor of lattice

${L_s}^2$ = slowing down area of lattice

L^2 = thermal diffusion area of lattice

χ^2 = two-group Laplacian

η = number of fast neutrons produced per neutron captured in natural uranium

p = resonance escape probability

ϵ = fast fission multiplication factor

$f = f_u$ = thermal utilization in the uranium

$\sigma_t(m)$ = thermal transport cross-section of moderator

$\sigma_t(w)$ = " " " " of cooling water

$\frac{\Delta\sigma_a}{\sigma_a}$ = fractional change of absorption cross-section with neutron energy, assuming all absorption cross-sections vary in the same way with temperature

${L_m}^2$ = thermal diffusion area of moderator

f_m = thermal utilization in the moderator

h_w = weir-box height

δ = extrapolated height minus actual height of core

R = the effective radius of the core

R_i = actual radius of core

r = the effective radius minus the actual radius of core

List of Symbols (cont'd)

α_{al} = thermal expansion coefficient aluminum (linear)

L_r = 'effective' diffusion length in side reflector

L_c = diffusion length in graphite

h = $h_w + h_D + \delta$ = extrapolated height of core

$\alpha^2 = \frac{\pi^2}{h^2}$ = vertical Laplacian

$\beta^2 = \frac{2.4^2}{R^2}$ = radial Laplacian of simple cylindrical core

h_1 = height of heavy water in Zeep, measured from lower end of uranium rods

PART A

THE UNIFORM TEMPERATURE COEFFICIENTS OF NRX

1 INTRODUCTION

During the initial series of start-up experiments on NRX, the uniform temperature coefficients for both the moderator and the rod-assemblies were measured. The experiments are described briefly in TPI-51 but the results have not been reported in detail. Since the experimental results are of interest, they are reported here, and compared with the results expected, based on simple lattice theory analysis and other data.

The measurements were part of a series carried out by the physicists and engineers in collaboration with the operations branch. Credit for the success of these particular experiments is due to the careful planning of Drs. D. G. Hurst and W. H. Watson, and the execution of these plans by the Operations Group.

The methods of analysis and calculation used here have been given in various reports. Rumsey and Volkoff (MT-221) have calculated the 'levelling-effect', and a recent analysis of the temperature coefficient of graphite piles by Codd (RM/2) uses similar methods. Hurst and Pressesky (memorandum to G. C. Laurence, Oct. 26, 1950, File No. B3205-2-1) have given an analysis of the NRX data, and Stewart (memorandum to G. C. Laurence, May 16, 1952) has discussed the temperature coefficient of a similar Zeep lattice and pointed out a discrepancy between the calculated and measured values.

After the completion of these calculations, D. W. Hone pointed out that the lattice constants used for these calculations differ from those currently in use, particularly in the value of f_m, the fraction of the neutrons absorbed in the moderator. However, the calculated temperature coefficient is insensitive to changes in f_m and the results would not be changed appreciably by this revision of the lattice constants.

2 EXPERIMENTAL RESULTS

(2.a) Reactivity Changes with Moderator Temperature

The effect of variation of moderator temperature was measured on Feb. 5, 1948, over a range of moderator temperatures from 44°F to 100°F. The heavy water was heated in the storage tanks, then the calandria was filled and the pile power raised to about 4 kw. Variations of weir-box setting and a control rod were used to maintain the pile power near this level throughout the experiment. The observations of weir-box setting refer to intervals when the control rod was completely withdrawn and $dP/dt = 0$. For such readings the pile is very close to critical, and these readings have been used to indicate the variation of critical height as the moderator temperature decreased. The temperature readings plotted in Figure 1 are those measured at the inlet to the polymer heat exchanger. The initial cooling rate was ~2°F/hr. This cooling rate was increased somewhat below 95°F by using cooling water on the polymer heat exchanger. From 95°F to 82°F the temperature drop through the heat exchanger was less than 2°F, between 82°F and 65°F this temperature drop exceeded 2°F and was as high as 5°F between 80°F and 75°F.

The use of the inlet temperature to the polymer cooler as representative of the mean polymer temperature in the calandria is certainly good for those periods when the temperature drop through the heat exchanger was small. The experimental data of Figure 1 lie on a smooth curve and suggest that the difference between the mean temperature in the calandria and that measured at the inlet to the polymer heat exchanger agree within a fraction of a degree Fahrenheit at all times. Some small corrections have been made to the experimental data. For some experimental points, the polymer level in the calandria had not reached equilibrium with the weir-box setting and appropriate corrections have been made. Where the correction to the weir-box reading exceeded 0.05 cm, the experimental points are circled.

A further small correction is required due to the change in viscosity of heavy water with temperature and the consequent change with temperature of the pressure drop through the pipe connecting the weir-box to the calandria. J. Haddow of the Plant Design Branch has calculated this correction, and shown that it is approximately linear over the moderator temperature range used. If h_D is the excess height of the polymer in the calandria above the weir-box level, he has found that

$$\frac{dh_D}{dt} = -5.1 \times 10^{-3} \quad (\mathrm{cm}/^{\circ}\mathrm{F}),$$

when the flow over the weir-box is 5 gallons/min. This correction has not been applied to the data of Figure 1, but has been used in the analysis (see section 7, page 29).

The subsequent analysis shows that there is reasonable agreement between the results of Figure 1 and the expected changes with temperature. Although minor corrections have been made to the experimental results it should be mentioned here that other possible errors might have affected the results. In particular, there was no direct evidence that the flow over the weir-box remained constant during the moderator temperature experiment. The flow into the calandria was kept constant, but it is possible that leakage through the dump valves changed with moderator temperature, thus varying the flow over the weir-box and the value of h_D. The consistency of other measurements of critical height suggest, however, that such leakage was probably too small to cause appreciable error.

A preliminary experiment was carried out on Feb. 3, 1948, over a smaller temperature range. Since the results were similar but somewhat less accurate than in this experiment, they have not been included here.

(2.b) Reactivity Changes with Rod-Assembly Temperature

The effect of uniform variation of rod-assembly temperature was measured by circulating heated water through the X-rod assemblies, and measuring critical heights as described in (2.a). Minor corrections were required for a slow drift of moderator temperature during this experiment.

Heated water was supplied to the top header of NRX at a flow rate of approximately 30 g.p.m. This water was distributed uniformly over the X-rods, passed through the pile, and then recirculated. Temperature readings were taken of the water entering the top header, and leaving the pile. The delay time between the two temperature points was approximately thirty minutes.

Critical height measurements were started with the system at the maximum temperature, after steady operation for several hours. Under these conditions the inlet temperature was 119.5°F and the outlet temperature about 109°F. After an accurate measurement of the critical height it was intended to reduce the inlet water temperature in steps of about 15°F, and after each step hold the inlet temperature constant for periods of about 1 hour while new critical height measurements were made. During the actual experiment, the variation of inlet water temperature with time was a poor approximation to the desired variation.

Analysis of the inlet and outlet water temperatures as functions of time gave a mean delay between the two temperature points of about thirty minutes. This agrees with estimates based on flow rate (~ 30 g.p.m.) and the volume of water in the piping. At those times when the inlet and outlet temperatures were relatively steady, their difference is given approximately by $T_D = 0.173\ (Tin - 65^{\circ}F)$. This temperature drop through the system is probably explained by heat exchange with the cooling air flow around the rods. The pile power of about four kilowatts used for this experiment was sufficient to raise the water temperature only $0.8^{\circ}F$.

From a comparison of the critical height measurements with inlet water temperature, it can be shown that the effective temperature of the water in the core is directly related to the inlet water temperature about twelve minutes previous to the critical height measurement. Changes of reactivity associated with abrupt changes of the inlet water temperature give this delay time unambiguously.

The correction to apply to the inlet water temperature to allow for cooling during transit to the core has to be estimated. Assuming a linear temperature drop with time gives 12/30 or 40% of the total drop between the two temperature points. Since the cooling air flow is such that the cooling rate is probably higher after leaving the core, only 30% of the total temperature drop has been subtracted from the inlet temperature to get the effective temperature in the core. An error in this estimate will introduce only a minor error in the results.

For readings taken below 58°F, the temperature of the recirculated water was reduced by injecting cold water from the 40 lb. water supply into the circuit. These readings are probably less accurate than those above 58°F. The point at 37.5°F was taken with 40 lb. cooling water at the conclusion of the experiment, the effective temperature in this case being taken as that of the outlet water. The corrected experimental results are shown in Figure 2.

Weir-box readings have been corrected,

(a) To a moderator temperature of 70°F. During the experiment the moderator temperature drifted slowly over a range of 65°F to 74°F.

(b) To allow for the relaxation time of the polymer level after adjustment to a new weir-box setting. The measurements where a correction of .05 cm or larger has been made are indicated by open circles in Figure 2.

The effective temperature used in Figure 2 has been obtained by subtracting 0.3 x 0.173 (Tin - 65°F) from the inlet temperature measured 12 minutes prior to the critical height measurement.

Points enclosed in a square refer to measurements made soon after a sudden drop in inlet water temperature, and have a relatively large error associated with the estimated effective temperature. All the points plotted are those taken with the control rod completely withdrawn and $dP/dt = 0$.

3 DISCUSSION OF EXPERIMENTAL RESULTS

The shape of the curve giving the variation of weir-box height with moderator temperature, Figure 1, indicates the strong dependence of the moderator temperature coefficient on the density of the moderator. On the other hand, the relatively small curvature in Figure 2 does not permit any accurate measurement of the effect of density changes of the water coolant. Measurements by Sargent et al. (MP 235, MP 246) and Hurst (TecPI-42) of the reactivity of an NRX type lattice, with and without cooling water, can be used to evaluate accurately the effect of density changes of the cooling water.

The relatively large value of the Rod-Assembly temperature coefficient was at first rather puzzling, but has been explained as arising from the effect of the water annulus in modifying the effective temperature of the neutrons in the uranium. Since a neutron traversing the water annulus has roughly a 50% chance of making a scattering collision, the effective neutron temperature in the uranium will be some mean temperature between that of the water, and that of the moderator. Since this differential temperature effect is difficult to predict quantitatively, we compare the overall temperature coefficient, determined by the sum of the rod-assembly and moderator coefficients, with that expected on simple theory.

4 THE CALCULATED UNIFORM TEMPERATURE COEFFICIENT

To 'calculate' $\frac{d\mathcal{X}^2}{dt}$ we start from the two group equation for a reacting lattice

$$(1 + \mathcal{X}^2 L_s^2)\ (1 + \mathcal{X}^2 L^2) = k = \eta\,\epsilon\,p\,f$$

From this relation we have

$$\Delta\mathcal{X}^2 \left\{ \frac{L_s^2}{1 + \mathcal{X}^2 L_s^2} + \frac{L^2}{1 + \mathcal{X}^2 L^2} \right\} = \frac{\Delta\eta}{\eta} + \frac{\Delta\epsilon}{\epsilon} + \frac{\Delta p}{p} + \frac{\Delta f}{f} - \frac{\Delta L^2}{L^2}\ x$$

$$\left\{ \frac{\mathcal{X}^2 L^2}{1 + \mathcal{X}^2 L^2} \right\} - \frac{\Delta L_s^2}{L_s^2} \left\{ \frac{\mathcal{X}^2 L_s^2}{1 + \mathcal{X}^2 L_s^2} \right\} \qquad (1)$$

To evaluate $\Delta\mathcal{X}^2$ for a small uniform temperature change, we require to know the temperature dependence of the various contributing parameters. Only ϵ (the fast fission factor) is considered independent of the temperature.

(4.a) Eta-Effect

A temperature coefficient for η of Natural Uranium was first suggested by Fermi (CP-455) to explain a serious discrepancy between the measured and calculated temperature coefficients of graphite moderated natural uranium piles. Subsequent cross-section measurements substantiate this hypothesis, but the best value for the temperature coefficient of η is believed to come from the analysis of reactor experiments. The data are summarized by J. Chernik, BNL-1344. For these calculations we choose the value

$$\frac{\Delta\eta}{\eta} = - 6 \times 10^{-5} \, \Delta T \; (^{\circ}C)$$

(4.b) Resonance Escape

A change $\frac{\Delta p}{p}$ with temperature arises both from the Doppler broadening of the resonance levels of U238 and the changes in moderator density with temperature. Experimental data on Doppler broadening have been obtained from the analysis of 'swing' experiments using heated uranium. At the Chalk River Conference, Jan. 1953, Fenning quoted recent Harwell experiments as giving $\frac{dk_\infty}{k_\infty} = - 1.6 \times 10^{-5}/^{\circ}C$. This value applies to a graphite pile with an assumed value for $p \doteq 0.9$.

To get the temperature dependence of p, we assume that p is given by $p = e^{-Z}$ with $Z = \frac{N_u}{N_m}\frac{A_u}{A_m}\frac{1}{\sigma_m a}\left\{\int \frac{\sigma dE}{E}\right\}_{eff.}$

where N_u = no. atoms uranium/cc., A_u = area U in unit lattice cell.

N_m = no. atoms moderator/cc., A_m = area moderator in unit lattice cell.

σ_m = scattering cross-section of moderator

a = average logarithmic energy loss per collision for moderator

$I = \left\{\int \frac{\sigma dE}{E}\right\}_{eff}$ = effective resonance absorption integral

Then $\frac{\Delta p}{p} = Z\left\{\frac{\Delta\rho_m}{\rho_m} - \frac{\Delta I}{I}\right\}$

From Harwell Data $\frac{dk_\infty}{k_\infty} = \frac{dp}{p} = -1.6 \times 10^{-5}/^\circ C$, $(p = 0.9)$

we get $\frac{\Delta I}{I} = + 1.5 \times 10^{-4}/^\circ C$

Applying this to NRX, assuming $p = 0.915$, $Z = .089$

we find $\frac{\Delta p}{p} = .089 \frac{\Delta\rho_m}{\rho_m} - 1.33 \times 10^{-5}\ \Delta T\ (^\circ C)$

This simple expression for $\frac{\Delta p}{p}$ neglects the effect of the cooling water, which would be expected to reduce the coefficient of $\Delta\rho_m/\rho_m$ and to add a term proportional to $\Delta\rho_w/\rho_w$. For the present we neglect the density effect of the water in changing $\Delta p/p$.

(4.c) Levelling Effect and Thermal Utilization

If the thermal neutron capture cross-sections all have the same variation with neutron energy, and the flux distribution and densities of the lattice materials remain unchanged, then $\Delta f/f$ would be zero for uniform temperature changes. It is customary to assume that all capture cross-sections have a $1/v$ dependence on neutron velocity for this calculation, although the existence of the Eta-effect implies either a different temperature variation of fission and capture cross-sections of natural uranium, or a variation of ν, the number of neutrons per fission, with neutron energy. Thus the calculated temperature dependent variation of f arises from the 'levelling-effect', the name used to describe the flattening of the flux distribution across a unit cell as the capture cross-sections decrease with increasing neutron temperature. The changes in moderator and cooling water density in the lattice also affect the thermal utilization. In appendix 1 the method used to evaluate $\Delta f/f$ for the NRX lattice is described.

If we assume that all the capture cross-sections have the same variation with neutron energy, then we have (appendix 1).

$$\Delta f_u/f_u = -2.092\times10^{-2}\ \Delta\rho_m/\rho_m - .1645\times10^{-2}\ \Delta\sigma_t(m)/\sigma_t(m)$$

$$-4.61\times10^{-2}\ \Delta\rho_w/\rho_w - 1.906\times10^{-2}\ \Delta\sigma_t(w)\sigma_t(w)$$

$$-3.378\times10^{-2}\ \Delta\sigma_a/\sigma_a$$

(4.d) **Fast Leakage**

The term $\frac{\Delta L_s^2}{L_s^2}\left(\frac{\chi^2 L_s^2}{1+\chi^2 L_s^2}\right)$ represents the contribution due to the change in the leakage current of 'fast' neutrons.

We set $\frac{\Delta L_s^2}{L_s^2} = -\frac{2\Delta\rho_m}{\rho_m}$ since the slowing down area is inversely proportional to the square of the density of the slowing down medium. This neglects the effect of the cooling water density changes on the slowing down area and the fact that there is a slight reduction of the slowing down area with increasing temperature of thermal neutrons. With

$L_s^2 = 114 \quad \chi^2 = 4.21\times10^{-4}\ \text{cm.}^{-2} \quad (h_c = 242\ \text{cm.})$.

then

$$-\frac{\Delta L_s^2}{L_s^2}\left(\frac{\chi^2 L_s^2}{1+\chi^2 L_s^2}\right) = 9.16\times10^{-2}\ \frac{\Delta\rho_m}{\rho_m}$$

It should be remembered that this contribution is proportional to $\aleph^2$.

(e) Thermal Leakage

The term $-\frac{\Delta L^2}{L^2}\left(\frac{\aleph^2 L^2}{1+\aleph^2 L^2}\right)$ is the contribution due to the change in the leakage current of thermal neutrons.

To evaluate $\Delta L^2/L^2$ we use the approximate relation

$$L^2 = L_m{}^2 f_m$$

$$\frac{\Delta L^2}{L^2} = \frac{\Delta L_m{}^2}{L_m{}^2} + \frac{\Delta f_m}{f_m} = -2\frac{\Delta\rho_m}{\rho_m} - \frac{\Delta\sigma_a(m)}{\sigma_a(m)} - \frac{\Delta\sigma_t(m)}{\sigma_t(m)} + \frac{\Delta f_m}{f_m}$$

In appendix 1 we have shown that, assuming all capture cross-sections have the same temperature variation,

$$\begin{aligned}\Delta f_m/f_m = 1.0646\ \Delta\rho_m/\rho_m &+ .0837\ \Delta\sigma_t(m)/\sigma_t(m)\\ + .1756\ \Delta\rho_w/\rho_w &+ .2035\ \Delta\sigma_t(w)/\sigma_t(w)\\ + .4025\ \Delta\sigma_a/\sigma_a &\end{aligned}$$

Using this equation and $L^2 = 156\ \text{cm}^2$ $\aleph^2 = 4.21 \times 10^{-4}\ \text{cm}^2$
($h_c = 242$ cm)

we get $\frac{\aleph^2 L^2}{1+\aleph^2 L^2} = 6.16 \times 10^{-2}$

$$\begin{aligned}-\frac{\Delta L^2}{L^2}\left(\frac{\aleph^2 L^2}{1+\aleph^2 L^2}\right) = 5.76\times10^{-2}\ \Delta\rho_m/\rho_m &+ 5.64\times10^{-2}\Delta\sigma_t(m)/\sigma_t(m)\\ - 1.08 \times 10^{-2}\ \Delta\rho_w/\rho_w &- 1.25\times10^{-2}\ \Delta\sigma_t(w)/\sigma_t(w)\\ + 3.68 \times 10^{-2}\ \Delta\sigma_a/\sigma_a &\end{aligned}$$

(4.f) Collected Data {sections (4a) to (4e)}

Using equation (1) page 15, and the following values for pile parameters

$$L_s^2 = 114 \text{ cm}^2 \qquad L^2 = 156 \text{ cm}^2 \qquad \aleph^2 = 4.21 \times 10^{-4} \text{ cm}^{-2} \quad (h = 242 \text{ cm})$$

$$\left\{ \frac{L_s^2}{1+\aleph^2 L_s^2} + \frac{L^2}{1+\aleph^2 L^2} \right\} = 255.2 \text{ cm}^2$$

we get the contributions to $\Delta\aleph^2$ for a small temperature change $\Delta T(^{\circ}C)$ given on the following page.

5 CHANGES OF THERMAL TRANSPORT CROSS-SECTIONS WITH TEMPERATURE

The collected data in section (4.f) show that the reactivity change due to the change of absorption cross-sections with temperature is small since the contributions from thermal utilization and thermal leakage for this particular lattice are of nearly equal magnitude but of opposite sign. The contributions arising through temperature variation of the transport cross-sections must be considered. These contributions are evaluated in this section and the results have been inserted in the last line of the following tabulation (page 22). The terms in ΔT have been left separate to show the importance of the contributions due to the assumed variation of transport cross-sections

Contributions to $10^7\Delta\mathcal{B}^2$ (Equation 2)

Eta Effect $-2.35\Delta T$

Resonance Escape $-0.52\Delta T \quad +3.49\text{x}10^3\frac{\Delta\rho_m}{\rho_m}$

Thermal Utilization $-1.32\text{x}10^3\frac{\Delta\sigma_a}{\sigma_a} -0.820\text{x}10^3\frac{\Delta\rho_m}{\rho_m} -0.064\text{x}10^3\frac{\Delta\sigma_t(m)}{\sigma_t(m)} -1.81\text{x}10^3\frac{\Delta\rho_w}{\rho_w} -0.747\text{x}10^3\frac{\Delta\sigma_t(w)}{\sigma_t(w)}$

Fast Leakage $+3.59\ \text{x}10^3\frac{\Delta\rho_m}{\rho_m}$

Thermal Leakage $+1.44\text{x}10^3\frac{\Delta\sigma_a}{\sigma_a} +2.26\text{x}10^3\frac{\Delta\rho_m}{\rho_m} +2.21\ \text{x}10^3\frac{\Delta\sigma_t(m)}{\sigma_t(m)} -0.42\text{x}10^3\frac{\Delta\rho_w}{\rho_w} -0.49\text{x}10^3\frac{\Delta\sigma_t(w)}{\sigma_t(w)}$

$$\text{Total} = -2.87\Delta T+0.12\text{x}10^3\frac{\Delta\sigma_a}{\sigma_a} +8.52\text{x}10^3\frac{\Delta\rho_m}{\rho_m} +2.15\text{x}10^3\frac{\Delta\sigma_t(m)}{\sigma_t(m)} -2.23\text{x}10^3\frac{\Delta\rho_w}{\rho_w} -1.24\text{x}10^3\frac{\Delta\sigma_t(w)}{\sigma_t(w)}$$

$$= -2.87\Delta T-0.21\Delta T \quad +8.52\text{x}10^3\frac{\Delta\rho_m}{\rho_m} -0.99\Delta T \quad -2.23\text{x}10^3\frac{\Delta\rho_w}{\rho_w} +1.40\Delta T$$

with neutron energy, and the consequent uncertainty in the calculated result arising from lack of experimental data to justify the assumptions.

Collecting terms in the last line of Equation 2, page 22, we have

$$10^7 \Delta \mathcal{K}^2 = -2.67\ \Delta T(^{\circ}C) + 8.52 \times 10^3\ \Delta\rho_m/\rho_m - 2.23 \times 10^3 \Delta\rho_w/\rho_w \qquad (3)$$

There are no experimental data to give the variation of the transport cross-sections of heavy water $\sigma_t(m)$, and of light water $\sigma_t(w)$, with temperature. As an approximation we use the measured data for the total cross-sections, and hope that the errors involved due to changes of angular distribution with energy are not too large.

Near 2500 metres/sec. neutron velocity ($T = 293^{\circ}K$) the scattering cross-section variation with energy is approximately given by the following expressions:

$$\sigma_s(m)\ (\text{barns}) = 10.5 + \frac{0.63}{E^{0.5}}$$

E = neutron energy in electron volts

$$\sigma_s(w)\ (\text{barns}) = 43.2 + \frac{3.0}{E^{0.6}}$$

These expressions are derived from published curves of the variation of scattering cross-sections with neutron energy

THIS NUMBER -0.135 IS BASED ON THE 2200 mps σ_s AND NOT ON 2500 mps
IF ONE USES 2500 mps σ THEN THIS SHOULD BE $-0.125 \frac{\Delta E}{E}$

whence

$\frac{\Delta\sigma_t}{\sigma_t} = -\frac{.315}{\sigma_t}\frac{dE}{E^{3/2}}$ ∴ FORM USED HERE IS ONLY APPROXIMATE.

$$\frac{\Delta\sigma_t(m)}{\sigma_t(m)} = -0.135 \frac{\Delta E}{E} = -0.46 \times 10^{-3} \Delta T(^\circ C)$$

$$\frac{\Delta\sigma_t(w)}{\sigma_t(w)} = -0.33 \frac{\Delta E}{E} = -1.13 \times 10^{-3} \Delta T(^\circ C)$$

Assuming $\Delta E/E = \Delta T/T$ and $T = 293^\circ K$

Since the absorption cross-sections are assumed to follow the 1/v low, we take

$$\frac{\Delta\sigma_a}{\sigma_a} = -\frac{1}{2}\frac{\Delta T}{T} = -1.71 \times 10^{-3} \Delta T \ (^\circ C) \qquad (T = 293^\circ K)$$

These relations were used to obtain the last line of Equation 2, page 22.

6 DENSITY CHANGES OF THE NRX COOLING WATER

In calculating the effect of density changes of the NRX cooling water, we have neglected possible contributions due to the variation of p and L_s^2 with cooling water density. For this reason the calculation is considered likely to be in error and it is preferable to use experimental data on measurements of the reactivity of the NRX lattice, with and without cooling water, to determine the effects of changes in cooling water density.

From the exponential experiments of Sargent et al (loc. cit.), Clayton (CRE-400A) has calculated the change in $\mathcal{K}^2$ for the NRX lattice with and without cooling water to be 1.37×10^{-4} cm^{-2}.

In G.P.I. 7, experimental results on the NRX pile gave the following weir-box settings:

No cooling water	168.3 cm
With cooling water	225.3 cm

Using an extrapolation length of 20 cms to get the critical heights this gives

$$\Delta(\pi^2/h^2) = 1.14 \times 10^{-4}\ \mathrm{cm}^{-2}$$

$$\text{or}\quad \Delta\mathcal{K}^2 = 1.08\ \Delta(\pi^2/h^2) = 1.23 \times 10^{-4}\ \mathrm{cm}^{-2} \quad \left\{\begin{matrix}\text{See Sec.7,} \\ \text{page32}\end{matrix}\right\}$$

This is in satisfactory agreement with the measurement deduced from the exponential experiments and has a much smaller probable error.

In Appendix 1 we have shown that for small variations of water density

$$\frac{\Delta f_u}{f_u} = -4.61 \times 10^{-2}\ \Delta\rho_w/\rho_w$$

$$\frac{\Delta f_m}{f_m} = 0.1756\ \Delta\rho_w/\rho_w$$

Since we wish, however, to estimate the effect of variation of the density to zero (no water), for comparison with the experimental value, we require more accurate expressions. Using the nomenclature of Appendix 1 it can be shown that

$$\frac{\Delta f_u}{f_u} = -f_u\left\{4.249 \times 10^{-2}\,\frac{\Delta\rho_w}{\rho_w'} + 8.22\times10^{-3}\,\frac{\Delta\rho_w}{\rho_w'}\left(\frac{\rho_w}{\rho_w'}\right)\right\}$$

$$\frac{\Delta f_m}{f_m} = \frac{\Delta f_u}{f_u} + 4.76 \times 10^{-3}\,\frac{\Delta\rho_w}{\rho_w'}\left\{\frac{1}{R_m' + S_m' + B_{wm}'(\rho/\rho')}\right\}$$

Where primes are used to denote initial values, $\rho_w' = 1$

Substituting in Equation 1, page 15, we get

$$\Delta\varkappa^2\left\{\frac{L_s^{\,2}}{1+\varkappa^2 L_s^{\,2}} + \frac{L^2}{1+\varkappa^2 L^2}\right\} = \frac{\Delta f_u}{f_u} - \frac{\Delta f_m}{f_m}\left(\frac{\varkappa^2 L^2}{1+\varkappa^2 L^2}\right)$$

Approximate integration of this equation over the variation of ρ_w from $\rho_w=\rho_w'$ to $\rho_w=0$ gives the change in $\varkappa^2$ due to the water in the cooling annuli as $2.4 \times 10^{-4}\,\mathrm{cm}^{-2}$.

Thus the effect of the cooling water, calculated without contributions due to changes in p and $L_s^{\,2}$, is larger by a factor of 2 than the experimental value.

To make the calculated expression relating $\Delta\varkappa^2$ to $\Delta\rho_w/\rho_w$ agree with the experimental data on the effect of removing the cooling water, we add a "correction term" proportional to $\Delta\rho_w/\rho_w'$ as follows

$$\Delta\varkappa^2\left\{\frac{L_s^{\,2}}{1-\varkappa^2 L_s^{\,2}} + \frac{L^2}{1+\varkappa^2 L^2}\right\} = \frac{\Delta f_u}{f_u} - \frac{\Delta L^2}{L^2}\left\{\frac{\varkappa^2 L^2}{1+\varkappa^2 L^2}\right\} + 2.7 \times 10^{-2}\,\frac{\Delta\rho_w}{\rho_w}$$

This correction term, the last term of the right hand side, has been chosen to make the expression give the correct experimental value (1.23×10^{-4} cm^{-2}) when the water density changes from ρ_w' to zero.

Assuming that this correction term is required due to the simplified calculation used for evaluating contributions from p and L_s^2, we now assume that the effect of the simple calculation has been to transfer this contribution from $\Delta\rho_w/\rho_w$ to $\Delta\rho_m/\rho_m$. Correction for this is then as follows

Simplified Calculation (Equation 3, page 23)

$$10^7 \Delta \mathfrak{B}^2 = -2.67 \; \Delta T(^\circ C) + 8.52 \times 10^3 \; \Delta\rho_m/\rho_m - 2.23 \times 10^3 \; \Delta\rho_w/\rho_w$$

Correction Terms $\qquad - 1.06 \times 10^3 \; \Delta\rho_m/\rho_m + 1.06 \times 10^3 \; \Delta\rho_w/\rho_w$

or

$$10^7 \Delta \mathfrak{B}^2 = -2.67 \; \Delta T(^\circ C) + 7.46 \times 10^3 \; \Delta\rho_m/\rho_m - 1.17 \times 10^3 \; \Delta\rho_w/\rho_w \qquad (4)$$

7 THE VARIATION OF RADIAL AND VERTICAL EXTRAPOLATION DISTANCE WITH TEMPERATURE

In the previous sections, lattice calculations and other data have been used to predict the value of $d\mathcal{K}^2/dT$ for the NRX lattice. We consider now the relation of $d\mathcal{K}^2/dT$ to the variation of Weir-Box height with Temperature which has been determined experimentally.

The NRX side reflector is graphite, extending well above the heavy water level used for these experiments, and its temperature remained relatively constant throughout the experiments. Since a neutron entering the reflector material will reach thermal equilibrium with the reflector very early in its life time, we can consider the nuclear properties of the reflector as unchanged during the temperature changes of the experiment.

The top reflector of NRX is composed of the forest of rod-assemblies projecting above the heavy water level; the bottom reflector is made up of the aluminum base of the calandria and the lower thermal shield. We shall treat these reflectors as well as having fixed nuclear properties with little justification except that their effect is very small.

To estimate the effects of the changing extrapolation lengths, we treat the NRX as a simple cylindrical core, neglecting the effect of the central thimble, and assume the extrapolation lengths are small compared with the core dimensions. With these simplifying assumptions the Laplacian $\aleph^2$ of the NRX core may be written as

$$\aleph^2 = \frac{\pi^2}{h^2} + \frac{2.4^2}{R^2} = \alpha^2+\beta^2$$

where h = extrapolated height

R = the 'effective' radius

now $h = h_W + h_D + \delta$

where h_W = the weir box height

h_D = the excess height of polymer above the weir box

δ = the extrapolated height minus the actual height of core

and $R = R_c + r$

where R_c = actual radius of the core

r = effective radius minus the actual radius

The temperature variations of h_D, δ, R_c and r are as follows:

$$\frac{1}{R_c}\frac{dR_c}{dT} = \text{thermal expansion coefficient of aluminum}$$

$$= \alpha_{al} = 2.5 \times 10^{-5}/^{o}C$$

$$\frac{dh_D}{dT} = -9.2 \times 10^{-3}/^{o}C$$ (see page 10)

$$\frac{1}{\delta}\frac{d\delta}{dT} = -\frac{1}{\rho_m}\frac{d\rho_m}{dT} - \frac{1}{\sigma_t(m)}\frac{d\sigma_t(m)}{dT}$$

This relation is derived from the assumption that

$$\delta \doteq \frac{\text{transport mean free path in core}}{\text{transport mean free path in reflector}} \times \begin{matrix}\text{diffusion length in}\\ \text{reflector}\end{matrix} \{\text{TL-5, pg. 6}\}$$

and the assumption that the nuclear properties of the reflector do not change with temperature.

$$\frac{1}{r}\frac{dr}{dT} = -\frac{1}{\rho_m}\frac{d\rho_m}{dT} - \frac{1}{\sigma_t(m)}\frac{d\sigma_t(m)}{dT} + L_r^2\alpha^2\frac{1}{h}\frac{dh}{dT}$$

This relation is derived assuming

$$r \doteq \frac{\text{transport mean free path in core}}{\text{transport mean free path in reflector}} \times L_r$$

with

L_r = 'effective' diffusion length in the side reflector.

For L_r^2 we use the approximate relation

$$\frac{1}{L_r^2} = \frac{\pi^2}{h^2} + \frac{1}{L_c^2} = \alpha^2 + \frac{1}{L_c^2}$$

with

L_c = diffusion length in graphite, assumed to remain unchanged for variations of temperature in the core.

Using these relations we have:

$$
\begin{aligned}
\frac{d\mathcal{B}^2}{dT} = & - \frac{2\alpha^2}{h}\left\{1 + \frac{r}{R}\,\beta^2 L_r{}^2\right\}\frac{dh_W}{dT} && \text{Weir Box Variation} \\
& - \frac{2\alpha^2}{h}\left\{1 + \frac{r}{R}\,\beta^2 L_r{}^2\right\}\frac{dh_D}{dT} && \text{Polymer Head Variation} \\
& - \frac{2\alpha^2}{h}\left\{1 + \frac{r}{R}\,\beta^2 L_r{}^2\right\}\delta\left\{- \frac{1}{\rho_m}\frac{d\rho_m}{dT} - \frac{1}{\sigma_t(m)}\frac{d\sigma_t(m)}{dT}\right\} && \left\{\text{Variation of Vertical Extrapolation Length}\right. \\
& - \frac{2\beta^2}{R}\,R_c\,\alpha_{al} && \text{Variation of Core Diameter} \\
& - \frac{2\beta^2}{R}\,r\left\{- \frac{1}{\rho_m}\frac{d\rho_m}{dT} - \frac{1}{\sigma_t(m)}\frac{d\sigma_t(m)}{dT}\right\} && \left\{\text{Variation of Radial Extrapolation Length}\right.
\end{aligned}
$$

The term in brackets $\left\{1 + \frac{r}{R}\,\beta^2 L_r{}^2\right\}$ arises from the assumed change in radial extrapolation length with variation of Weir Box Height. The first term on the right hand side is the major term, the remaining terms being small correction terms which we now evaluate.

The neutron density distribution for NRX with the reactor in approximately the state used for temperature coefficient measurements is given by Sargent (CRP-460). Relevant data are

h = W. B. Reading + 21.8 cm.

244.4 = 226.6 + 21.8

Extrapolation length below Calandria Floor = 8.1 cm.

" " above Heavy Water Surface = 10.2 cm.

i.e. δ = 18.3 cm.

Radial Distribution extrapolates to zero at

$R = 161.4$ cm.

Internal Radius of Calandria = 133.4 cm. = R_c

Effective Core Radius = 128.2 cm.

In choosing $R_c = 133.4$ we neglect the small difference between the internal radius of the calandria and the effective core radius.

We use $r = 28$ cm.

also $\alpha^2 = 1.65 \times 10^{-4} \text{cm.}^{-2}$

$\beta^2 = 2.5 \times 10^{-4} \text{cm.}^{-2}$

We use here the β^2 given by Sargent, rather than that corresponding to a simple core without the central thimble. (For the correction terms the difference is not important).

$$L_r^2 = 1.8 \times 10^3 \text{cm.}^2 \quad \left\{ \begin{array}{l} \text{from } \alpha^2 = 1.65 \times 10^{-4} \text{cm.}^2 \\ \qquad L_c^2 = 2.5 \times 10^3 \text{ cm.}^2 \end{array} \right\}$$

These values give

$$\left(1 + \frac{r}{R} \beta^2 L_r^2\right) = 1.08$$

Substituting the calculated value for $\frac{d\mathcal{R}^2}{dT}$ {Equation 4} we have:

$$
\begin{aligned}
10^7 \frac{d\varkappa^2}{dT} &= -2.67 + 7.46 \times 10^3 \frac{1}{\rho_m} \frac{d\rho_m}{dT} - 1.17 \times 10^3 \frac{1}{\rho_w} \frac{d\rho_w}{dT} \\
&= -1.08 \times 10^7 \times \frac{2\alpha^2}{h} \frac{dh_w}{dT} && \text{Weir Box Variation} \\
&\quad +0.13 && \text{Polymer Head Variation} \\
&\quad -0.12 + 0.26 \times 10^3 \frac{1}{\rho_m} \frac{d\rho_m}{dT} && \text{Variation Vertical Extrapolation Length} \\
&\quad -0.10 && \text{Variation Core Diameter} \\
&\quad -0.40 + 0.87 \times 10^3 \frac{1}{\rho_m} \frac{d\rho_m}{dT} && \text{Variation Radial Extrapolation Length}
\end{aligned}
$$

Or

$$
10^7 \times \frac{2\pi^2}{h^3} \frac{dh_w}{dT} = 2.0 - 5.9 \times 10^3 \frac{1}{\rho_m} \frac{d\rho_m}{dT} + 1.08 \times 10^3 \frac{1}{\rho_w} \frac{d\rho_w}{dT} \qquad (5)
$$

It is apparent that the above terms, although each is small compared with $d\varkappa^2/dT$, must be included if we wish agreement to within 20% to be significant in comparing calculated values of $d\varkappa^2/dT$ with experimental data on variation of Weir-Box height with temperature. Equation (5) is the final result of our calculations which we now compare with the experimental data.

8 ANALYSIS OF EXPERIMENTAL RESULTS

The previous analysis indicates that the experimental data should be related to temperature and density changes by equations as follows:

For Variation of Moderator Temperature

$$10^7 \frac{2\pi^2}{h^3} \frac{dh_w}{dT} = C_1 \frac{1}{\rho_m} \frac{d\rho_m}{dT} + C_2$$

For Variation of Rod Assembly Temperature

$$10^7 \frac{2\pi^2}{h^3} \frac{dh_w}{dT} = C_3 \frac{1}{\rho_w} \frac{d\rho_w}{dT} + C_4$$

The smoothed curves of Figures 1 and 2 have been used to obtain $\frac{dh_w}{dT} \times \frac{2\pi^2}{h^3}$, with the approximation that $h = h_w + \Delta$ and treating Δ as a constant equal to 22 cm. To obtain the constants C_1 and C_2 we have plotted $\frac{2\pi^2}{h^3} \frac{dh_w}{dT}$ against $\frac{1}{\rho_m} \frac{d\rho_m}{dT}$. The constants C_3 and C_4 have been evaluated similarly. The numerical data used are given in Table 1, for variation of Moderator Temperature, and in Table 2, for variation of Rod-Assembly Temperature.

The data of Tables 1 and 2 are plotted in Figures 3 and 4. Since any realistic estimate of experimental errors is difficult, the vertical lines through the experimental points have been drawn to indicate the effect of an error of 0.01 cm in measurement of the difference in Weir-Box readings over the 10°F temperature interval used for analysing the data.

TABLE 1

Reactivity Changes with Variation of Moderator Temperature

Moderator Temp(^{o}F)	Critical Height	$10^7\Delta(\pi^2/h^2)$	$\frac{10^7\Delta(\pi^2/h^2)}{\Delta T(^{o}F)}$	$\frac{10^4}{\rho_m}\frac{d\rho_m}{dT}(^{o}F)$
45	241.18	.984		
			- .197	+0.27
50	241.25	0		
			- .380	-0.14
60	241.52	-3.80		
			- .583	-0.60
70	241.94	-9.63		
			- .794	-0.998
80	242.51	-17.57		
			- .935	-1.34
90	243.19	-26.92		
			-1.123	-1.655
100	244.01	-38.15		

Data for ρ_m as a function of temperature are taken from a paper by Tsing. Lien Chang and Lü - Ho Tung (Nature 163, p 737, May 1949)

TABLE 2

Reactivity Changes with Variation of Rod Assembly Temperature

Rod Assembly Temp($^\circ$F)	Critical Height(cm)	$10^7\Delta(\pi^2/h^2)$	$\frac{10^7\Delta(\pi^2/h^2)}{\Delta T(^\circ F)}$	$\frac{10^4}{\rho_w}\frac{\Delta\rho_w}{\Delta T(^\circ F)}$
40	242.66	+ 7.82		
			- .782	- .275
50	243.23	0		
			- .754	- .690
60	243.78	- 7.54		
			- .707	- 1.05
70	244.30	-14.61		
			- .713	- 1.365
80	244.83	-21.74		
			- .707	- 1.650
90	245.36	-28.81		
			- .672	- 1.910
100	245.86	-35.53		
			- .617	- 2.145
110	246.33	-41.70		

Data for variation of ρ_w with temperature are taken from the International Critical Tables.

The equation of the straight line of Figure 3 is

$$10^7 \frac{2\pi^2}{h^3} \frac{dh_w}{dT(^oF)} = +0.29 - 5.0 \times 10^3 \frac{1}{\rho_m} \frac{d\rho_m}{dT(^oF)} \qquad (6)$$

Giving the moderator temperature coefficient in the desired form.

In Figure 4 two straight lines are drawn, the full line equation being

$$10^7 \frac{2\pi^2}{h^3} \frac{dh_w}{dT(^oF)} = + 0.80 + 0.75 \times 10^3 \frac{1}{\rho_w} \frac{d\rho_w}{dT(^oF)} \qquad (7)$$

The equation of the dotted line is

$$10^7 \frac{2\pi^2}{h^3} \frac{dh_w}{dT(^oF)} = + 0.85 + 1.08 \times 10^3 \frac{1}{\rho_w} \frac{d\rho_w}{dT(^oF)} \qquad (8)$$

The coefficient of $\frac{1}{\rho_w} \frac{d\rho_w}{dT}$ has been chosen in this case to agree with that deduced from other experimental data. It can be seen from inspection of Figure 4 that either equation is a reasonable fit of the experimental data so we choose Equation 8 with the 'correct' coefficient of $\frac{1}{\rho_w} \frac{d\rho_w}{dT}$.

Combining equations (6) and (8) we get the experimental overall Uniform Temperature Coefficient.

$$10^7 \frac{2\pi^2}{h^3} \frac{dh_w}{dT(^oC)} = 2.05 - 5.0 \times 10^3 \frac{1}{\rho_m} \frac{d\rho_m}{dT(^oC)} + 1.08 \times 10^3 \frac{1}{\rho_w} \frac{d\rho_w}{dT(^oC)}$$

This is to be compared with the predicted relation (Eqn.5, page 33)

$$10^7 \frac{2\pi^2}{h^3} \frac{dh_w}{dT(^oC)} = 2.0 - 5.9 \times 10^3 \frac{1}{\rho_m} \frac{d\rho_m}{dT(^oC)} + 1.08 \times 10^3 \frac{1}{\rho_w} \frac{d\rho_w}{dT(^oC)}$$

9 DISCUSSION

Comparison of the experimental and calculated results shows that the agreement is reasonable, and better than one might expect in view of the uncertainties in the calculation. The relatively large contributions included in the calculations due to the estimated variation of transport mean free paths in water and polymer with neutron energy render the agreement of the constant coefficients somewhat fortuitous. The assumption of 1/v cross-sections in the lattice calculations is also questionable, since the near cancellation of the levelling and leakage effects makes the results very sensitive to departures of the uranium cross-section from the 1/v law.

The discrepancy between the calculated and experimental values of the coefficient of $\frac{1}{\rho_m}\frac{d\rho_m}{dT}$ may be due to an error in assessing the contribution via the resonance escape variation. Evidence from lattice experiments, presented at the Jan., 1953, conference here, suggested that the effective resonance integral should be only 80% of that used in these calculations. {Effective Res. Integral = 7.3(1 + 2.67 S/M) barns rather than 9.25 (1 + 2.67 S/M) barns.} The correspondingly smaller values of (1-p) would result in reducing the calculated coefficient of $\frac{1}{\rho_m} \frac{d\rho_m}{dT}$ from 5.9 to 5.2, in excellent agreement with the experiments.

It should be noted that in the calculated value a generous allowance has been made for effects of the cooling water on the coefficient of $\frac{1}{\rho_m} \frac{d\rho_m}{dT}$, giving a marked reduction from the simple calculation neglecting this effect.

The differential relationship used to analyse the experimental results will not give exact results when applied over the finite temperature range of the experiment. However, the fractional changes in κ^2, σ, ρ_m ρ_w and absolute temperature are relatively small and it seems unlikely that a large error Has been introduced for this reason, particularly since the temperature contributions to the moderator temperature coefficient are small compared with the density contributions.

10 CONCLUSION

There is good agreement between the measured and calculated temperature coefficient of the NRX reactor. Further , the separate dependence of reactivity on moderator density and neutron temperature both have the expected values. Since discrepancies of 20 or 30% might have been attributed to lack of exact data for use in calculations, and second order effects in the analysis of data taken over a relatively large temperature range, the good agreement should be treated with this reservation. Corrections of the order of 20% of the measured $\Delta(\pi^2/h^2)$ have been made for changes in radial extrapolation length with temperature.

Part B

The Uniform Temperature Coefficient of a Zeep Lattice

11 INTRODUCTION

The relation between critical height and temperature has been measured for a standard Zeep lattice (RDP-54), containing 85 rods in a 7.75 inch hexagonal arrangement. J. D. Stewart (memorandum to G. C. Laurence, May 16, 1952) has pointed out that there is a considerable discrepancy between the experiment and a simple theoretical estimate. In this section a more detailed analysis of this data is given, using the method of the report. It is shown that reasonable agreement between theory and experiment exists, when corrections are applied for the effect of the reflectors.

In the experiment only the uniform temperature coefficient has been measured, and the experimental data do not embrace the temperature of maximum density of heavy water. For these reasons, the separation of effects due to moderator density changes from the other contributions is not accurate, using the simple method applied to the NRX data. It has been necessary to calculate the uniform temperature coefficient of the lattice at four different temperatures (20°C, 35°C, 55°C, 80°C) for comparison with the experimental data.

12 THE CALCULATED UNIFORM TEMPERATURE COEFFICIENT

The calculated values of $d\mathcal{K}^2/dT$ have been obtained for the four temperatures using the method of the Report. The results are given below:

at 20°C

$$10^7\left(\frac{d\mathcal{K}^2}{dT}\right) = -7.13 + 10.17 \times \frac{10^3}{\rho_m}\frac{d\rho_m}{dT}$$

at 35°C

$$10^7\left(\frac{d\mathcal{K}^2}{dT}\right) = -6.85 + 9.93 \times \frac{10^3}{\rho_m}\frac{d\rho_m}{dT}$$

at 55°C

$$10^7\left(\frac{d\mathcal{K}^2}{dT}\right) = -6.58 + 9.71 \times \frac{10^3}{\rho_m}\frac{d\rho_m}{dT}$$

at 80°C

$$10^7\left(\frac{d\mathcal{K}^2}{dT}\right) = -6.13 + 9.36 \times \frac{10^3}{\rho_m}\frac{d\rho_m}{dT}$$

The relatively rapid variation with temperature of the 'constants' in the first and second terms of the R.H.S. of the equations precludes the accurate analysis of the experimental data into temperature and density contributions using a linear plot.

The data used for the calculation of $\frac{d\mathcal{K}^2}{dT}$ at the various temperatures are given in Table 3. The pile parameters at 20°C are those given by J.D. Stewart (loc.cit.); those at the other temperatures have been calculated using the same assumptions as in the report.

Table 3

Pile Parameters

	20°C	35°C	55°C	80°C
$\aleph^2 (cm^{-2})$	$7.017 x 10^{-4}$	$6.845 x 10^{-4}$	$6.615 x 10^{-4}$	$6.292 x 10^{-4}$
$L_s^2 (cm^2)$	111.6	112.33	114.05	117.07
L^2	185.2	188.95	194.40	201.7
R_m	$1.03 x 10^{-2}$	$1.0198 x 10^{-2}$	$1.0027 x 10^{-2}$	$.9774 x 10^{-2}$
S_m	$0.47 x 10^{-2}$	$.4617 x 10^{-2}$	$.4241 x 10^{-2}$	$.386 x 10^{-2}$
R_{al}	$0.62 x 10^{-2}$	$.6159 x 10^{-2}$	$.6104 x 10^{-2}$	$.6035 x 10^{-2}$
f_u	0.9792	.97946	.98003	.98071
G_o	1.30	1.291	1.280	1.265
p	.9372	.9368	.9361	.9352

Capture cross-sections have been assumed to follow the 1/v law, moderator transport cross-section has been assumed to vary as the scattering cross-section of heavy water (see relation, page 23). Values of $\aleph^2$ at the various temperatures have been computed using the measured change of critical height with temperature, and assuming the variation of vertical and radial extrapolation lengths as indicated in the following section. A measured value of $\aleph^2$ ($\aleph^2 = 6.90 \times 10^{-4} cm^{-2}$) at 31°C has been used to normalize the data.

Table 4 gives the separate values for the contributions to $d\chi^2/dT$ arising from the Eta-effect, the change in the resonance trap and thermal utilization and the change in the fast and thermal neutron 'leakage'. For this lattice it can be seen that the major contributions to the temperature coefficient arise from the change in 'leakage' terms with temperature and moderator density. Changes of reactivity due to changes in absorption and transport cross-sections have been related to temperature changes using the relations given in the report. The value of the 'constant' term in the relations giving $d\chi^2/dT$ is particularly sensitive to the assumed variation of moderator transport cross-section with energy, and would change by $\sim$20% if it were assumed that this cross-section stayed constant with changing neutron energy.

Table 4

Contributions to$(10^7 d\mathfrak{K}^2/dT)$

	20°C		35°C		55°C		80°C	
	ΔT	$10^3 \Delta\rho_m/\rho_m$	ΔT	$10^3 \Delta\rho_m/\rho_m$	ΔT	$10^3 \Delta\rho_m/\rho_m$	ΔT	$10^3 \Delta\rho_m/\rho_m$
Eta-effect	-2.244		-2.208		-2.156		-2.083	
Resonance Escape	-0.364	+2.424	-0.360	+2.400	-0.355	+2.368	-0.351	+2.326
Thermal Utilization	+0.640	-0.722	+0.583	-0.701	+0.497	-0.652	+0.412	-0.596
Fast Leakage		+5.430		+5.255		+5.042		+4.763
Thermal Leakage	-5.161	+3.037	-4.862	+2.979	-4.569	+2.952	-4.105	+2.870
Total = $10^7 \frac{d\mathfrak{K}^2}{dT}$	$-7.13+10.17\times10^3\frac{1}{\rho_m}\frac{d\rho_m}{dT}$		$-6.85+9.93\times\frac{10^3}{\rho_m}\frac{d\rho_m}{dT}$		$-6.58+9.71\times\frac{10^3}{\rho_m}\frac{d\rho_m}{dT}$		$-6.13+9.36\times\frac{10^3}{\rho_m}\frac{d\rho_m}{dT}$	

13 THE VARIATION OF RADIAL AND VERTICAL EXTRAPOLATION LENGTHS WITH TEMPERATURE

We consider now the relation of $d\mathcal{B}^2/dT$ to the variation of the critical height of the polymer with temperature which was measured in the experiment. The treatment used follows the same pattern as that used for the analysis of the NRX data.

The Zeep side reflector is graphite, extending well above the heavy water level used for these experiments, and the measured temperature change during the experiment was small (40°C. was measured at the inner boundary of the reflector when the lattice was at 80 °C). We thus consider the neutron capture cross-section of the side reflector independent of the lattice temperature.

The Zeep bottom reflector is made up of the curved base of the tank, containing a maximum of eight inches of heavy water below the ends of the uranium rods, with graphite below the tank. Since most of the vertical extrapolation length is related to this lower reflector, we have estimated percentage changes in the vertical extrapolation length as if it were an infinite heavy water reflector, with its temperature varying with that of the lattice (Case A), and alternatively as if it were of graphite for which no temperature change occurred during the experiment (Case B). The latter assumption gives good agreement between theory and experimental measurements.

We write $\mathcal{B}^2 = \frac{\pi^2}{h^2} + \frac{2.4^2}{R^2}$

h = extrapolated height
R = the 'effective' radius

$h = h_1 + \delta$

where h_1 = heavy water level, measured from bottom of uranium rods
δ = the extrapolated height minus the actual height of the core

$R = R_c + r$

R_c = actual radius of core
r = effective radius minus the actual radius

The temperature dependence assumed for δ, R_c, r are as follows:

$$\frac{1}{R_c}\frac{dR_c}{dT} = \text{thermal expansion coefficient of aluminum}$$

$$= \alpha_{al} = 2.5 \times 10^{-5}/^{\circ}C.$$

$$\left\{\begin{array}{ll} \frac{1}{\delta}\frac{d\delta}{dT} = -\frac{1}{\rho_m}\frac{d\rho_m}{dT} - \frac{1}{2\sigma_t(m)}\frac{d\sigma_t(m)}{dT} - \frac{1}{2\sigma_c(m)}\frac{d\sigma_c(m)}{dT} & \text{Case A} \\ \text{or} & \\ \frac{1}{\delta}\frac{d\delta}{dT} = -\frac{1}{\rho_m}\frac{d\rho_m}{dT} - \frac{1}{\sigma_t(m)}\frac{d\sigma_t(m)}{dT} & \text{Case B} \end{array}\right.$$

Case A is for the polymer reflector assumed to follow the core temperature change

Case B is for the graphite reflector (at base) assumed to have constant temperature

$$\frac{1}{r}\frac{dr}{dT} = -\frac{1}{\rho_m}\frac{d\rho_m}{dT} - \frac{1}{\sigma_t(m)}\frac{d\sigma_t(m)}{dT} + L_r^2\alpha^2\frac{1}{h}\frac{dh}{dT}$$

and $\frac{1}{L_r^2} = \alpha^2 + \frac{1}{L_c^2}$

with L_c = diffusion length in graphite

These relations are derived from the assumption that r and δ equal

$$\frac{\text{transport mean free path in core}}{\text{transport mean free path in reflector}} \times \text{diffusion length in the reflector}$$

It should be emphasized again that these relations for the reflectors are only approximate and are based on the assumption of a side reflector of the same height as the extrapolated height for the core with $h_r \ll R$. This model is only a poor approximation to the Zeep reflector and the consequent corrections deduced from the model will have a relatively large error associated with them.

Table 5 gives the parameters used in evaluating the reflector corrections at the various temperatures chosen for calculation. The values of δ and R at the bottom of Table 5 are experimental values at 31°C, deduced from the vertical and radial flux distributions taken at this temperature (See Report RDP-54). The value chosen for R_c, 102.87 cm, is the internal radius of the Zeep tank. The actual core radius, computed from the number of cells (85) and the pitch (7.75") is 95.3 cm. Thus our choice of $R_c = 102.87$ cm neglects the effect of the thin polymer reflector around the core; this will lead to an underestimate of the effect of the side reflector but greatly simplifies the calculation.

In a similar temperature coefficient measurement using an NRU type lattice, measurements were made of the flux distribution at both 25°C and 58°C (RDP-55) but the errors involved in measuring R and δ at the two temperatures are such that the changes in r and δ cannot be determined from the measurements with any precision.

Table 5

	20°C	35°C	55°C	80°C
h_1 (exptl)	138.2	141.3	145.6	152.4
δ (cm)	33.03	33.68	34.66	36.02
h (cm)	171.23	174.98	180.26	188.42
$\alpha^2 (cm^{-2})$	3.366×10^{-4}	3.2235×10^{-4}	3.0374×10^{-4}	2.780×10^{-4}
R_c (cm)	102.84	102.88	102.93	103.00
r (cm)	22.97	23.44	24.16	25.28
R (cm)	125.81	126.32	127.09	128.28
$\beta^2 (cm^{-2})$	3.6512×10^{-4}	3.6217×10^{-4}	3.578×10^{-4}	3.5119×10^{-4}
$\gamma^2 (cm^{-2})$	7.017×10^{-4}	6.845×10^{-4}	6.615×10^{-4}	6.292×10^{-4}
L_r^2	1.357×10^3	1.384×10^3	1.421×10^3	1.475×10^3

Calculated from $L_c^2 = 2.5 \times 10^3$

$$\text{Experimental Values at } 31^{\circ}\text{C} \left\{ \begin{array}{l} \delta = 33.5 \\ \left. \begin{array}{l} r = 23.3 \\ R_c = 102.87 \end{array} \right\} R = 126.17 \end{array} \right.$$

Differentiation of the relation $\aleph^2 = \frac{\pi^2}{h^2} + \frac{2.4^2}{R^2} = \alpha^2+\beta^2$ for Case A (a polymer reflector varying with core temperature responsible for δ) gives:

$$\begin{aligned}
\frac{d\aleph^2}{dT} = &-\frac{2\alpha^2}{h}\left\{1 + \frac{r}{R}\,\beta^2 L_r^{\,2}\right\}\frac{dh_1}{dT} \\
&-\frac{2\alpha^2}{h}\left\{1 + \frac{r}{R}\,\beta^2 L_r^{\,2}\right\}\delta\left\{-\frac{1}{\rho_m}\frac{d\rho_m}{dT} - \frac{1}{2\sigma_c(m)}\frac{d\sigma_c(m)}{dT} - \frac{1}{2\sigma_t(m)}\frac{d\sigma_t(m)}{dT}\right\} \\
&-\frac{2\beta^2}{R}\,R_c\,\alpha_{al} \\
&-\frac{2\beta^2}{R}\,r\left\{-\frac{1}{\rho_m}\frac{d\rho_m}{dT} - \frac{1}{\sigma_t(m)}\frac{d\sigma_t(m)}{dT}\right\}
\end{aligned}$$

for Case B (a graphite reflector of fixed temperature responsible for δ)

$$\begin{aligned}
\frac{d\aleph^2}{dT} = &-\frac{2\alpha^2}{h}\left\{1 + \frac{r}{R}\,\beta^2 L_r^{\,2}\right\}\frac{dh_1}{dT} \\
&-\frac{2\alpha^2}{h}\left\{1 + \frac{r}{R}\,\beta^2 L_r^{\,2}\right\}\delta\left\{-\frac{1}{\rho_m}\frac{d\rho_m}{dT} - \frac{1}{\sigma_t(m)}\frac{d\sigma_t(m)}{dT}\right\} \\
&-\frac{2\beta^2}{R}\,R_c\,\alpha_{al} \\
&-\frac{2\beta^2}{R}\,r\left\{-\frac{1}{\rho_m}\frac{d\rho_m}{dT} - \frac{1}{\sigma_t(m)}\frac{d\sigma_t(m)}{dT}\right\}
\end{aligned}$$

The factor $\left\{1 + \frac{r}{R}\beta^2 L_r^2\right\}$ arises from the assumption that the side reflector acts as a reflector with the same height as the extrapolated height h of the core. We have already determined calculated values for $d\chi^2/dT$ at the chosen temperatures. dh_1/dT is given by experiment. We require values for the second, third and fourth terms on the R.H.S. of these equations to compare the calculated dh_1/dT with the experimental values.

Table 6 gives the calculated values for these correction terms where we have labelled the various terms as follows:

δ correction = second term on R.H.S. of the equations x 10^7

R_c correction = third term on R.H.S. of the equations x 10^7

r correction = fourth term on R.H.S. of the equations x 10^7

Inspection of Table 6 shows that the R_c correction due to thermal expansion of the Zeep tank is very small. The other corrections are relatively large, however, and the difference between Case A and Case B reflects the importance of the assumption regarding the effective temperature of the reflector during the experiment.

Table 6

Corrections for $10^7 d\mathscr{R}^2/dT$

	20°C		35°C		55°C		80°C	
$\left(1+\frac{r}{R}\beta^2 L_r^2\right)$	1.0905		1.093		1.097		1.102	
	ΔT	$10^3\Delta\rho_m/\rho_m$	ΔT	$10^3\Delta\rho_m/\rho_m$	ΔT	$10^3\Delta\rho_m/\rho_m$	ΔT	$10^3\Delta\rho_m/\rho_m$
δ correction, Case A	-1.53	1.42	-1.39	1.36	-1.23	1.28	-1.05	1.17
Case B	-0.65	1.42	-0.59	1.36	-0.525	1.28	-0.45	1.17
R_c correction	-0.15		-0.15		-0.145		-0.14	
r correction	-0.61	1.33	-0.59	1.34	-0.56	1.36	-0.53	1.38
Total Case A	-2.29	2.75	-2.13	2.70	-1.93	2.64	-1.72	2.55
Case B	-1.41	2.75	-1.33	2.70	-1.23	2.64	-1.12	2.55

14 THE PREDICTED VARIATION OF CORE HEIGHT WITH TEMPERATURE

The predicted variation of core height, h_1, with temperature is obtained using the following relation:

$$10^7 \frac{2\alpha^2}{h} \frac{dh_1}{dT} = \left\{ -10^7 \frac{d\aleph^2}{dT} + \delta \text{ correction} + R_c \text{correction} + \text{r correction} \right\} \div \left\{ 1 + \frac{r}{R} \beta^2 k_r^2 \right\}$$

Using the values given in TABLES 4 and 6 we have:

at 20°C

$$10^7 \frac{2\alpha^2}{h} \frac{dh_1}{dT} = 4.44 - 6.81 \times \frac{10^3}{\rho_m} \frac{d\rho_m}{dT}$$ Case A

$$10^7 \frac{2\alpha^2}{h} \frac{dh_1}{dT} = 5.25 - 6.81 \times \frac{10^3}{\rho_m} \frac{d\rho_m}{dT}$$ Case B

at 35°C

$$10^7 \frac{2\alpha^2}{h} \frac{dh_1}{dT} = 4.32 - 6.62 \times \frac{10^3}{\rho_m} \frac{d\rho_m}{dT}$$ Case A

$$10^7 \frac{2\alpha^2}{h} \frac{dh_1}{dT} = 5.05 - 6.62 \times \frac{10^3}{\rho_m} \frac{d\rho_m}{dT}$$ Case B

at 55°C

$$10^7 \frac{2\alpha^2}{h} \frac{dh_1}{dT} = 4.24 - 6.44 \times \frac{10^3}{\rho_m} \frac{d\rho_m}{dT}$$ Case A

$$10^7 \frac{2\alpha^2}{h} \frac{dh_1}{dT} = 4.88 - 6.44 \times \frac{10^3}{\rho_m} \frac{d\rho_m}{dT}$$ Case B

at 80°C

$$10^7 \frac{2\alpha^2}{h} \frac{dh_1}{dT} = 4.00 - 6.18 \times \frac{10^3}{\rho_m} \frac{d\rho_m}{dT}$$ Case A

$$10^7 \frac{2\alpha^2}{h} \frac{dh_1}{dT} = 4.56 - 6.18 \times \frac{10^3}{\rho_m} \frac{d\rho_m}{dT}$$ Case B

In Table 7 we give the final values for $10^7 \frac{2\alpha^2}{h} \frac{dh_1}{dT}$ for the temperatures used, with the appropriate values of $\frac{1}{\rho_m} \frac{d\rho_m}{dT}$. These values are plotted in Figure 5 where they are compared with the values deduced from the experimental data as described in the next section.

If r and δ are both assumed to be independent of core height and temperature, then $\frac{2\alpha^2}{h} \frac{dh_1}{dT} = \frac{-d\aleph^2}{dT}$. Thus comparison of the second and third rows in Table 7 give a direct measure of the effect of the calculated variations of extrapolation lengths with core height and temperature. Case A gives a 35% reduction and Case B a 30% reduction of the value of $\frac{2\alpha^2}{h} \frac{dh_1}{dT}$ below $-\frac{d\aleph^2}{dT}$.

Table 7

		20°C	35°C	55°C	80°C
$\frac{10^3}{\rho_m} \frac{d\rho_m}{dT}$		-0.1305	-0.298	-0.463	-0.617
$10^7 \frac{d\aleph^2}{dT}$		-8.46	-9.81	-11.08	-11.91
$10^7 \frac{2\alpha^2}{h} \frac{dh_1}{dT}$	Case A	5.33	6.29	7.22	7.81
	Case B	6.14	7.02	7.86	8.37

15 ANALYSIS OF THE EXPERIMENTAL RESULTS

The data given in Table 8, columnsl and 2, are the experimental results of the measurement of the relation between h_1 and temperature for a Zeep lattice (RDP-54). The lattice consisted of 85 Zeep rods in a hexagonal pattern with 7.75 inch spacing. The experimental accuracy of the data is given as $\pm 0.2^\circ C$ for the temperature data, and ± 0.01 cm for the relative measurements of h_1. No data are available to give experimentally the variation of the radial Laplacian or vertical extrapolation lengths with temperature.

To determine dh_1/dT from the experimental data, graphical methods have been used to 'smooth' the experimental data. The values for h_1 and dh_1/dT, determined in this way, are given in Columns 3, 4 and 5, Table 8. The values of dh_1/dT ($\sim 0.25 cm/^\circ C$) show that the temperature errors ($\pm 0.2^\circ C$) are the major errors in the experimental data, the errors in measurement of h_1 (± 0.01 cm) are only 1/5 as important. A least squares fit to the experimental data (D. Hone, RDP 54), $h_1 = h_o + 0.190(T-25) + 9.7 \times 10^{-4}(T-25)^2$ gives values for dh_1/dT agreeing within 1% of those given in Table 8. The experimental error associated with the value of dh_1/dT is such that the experimental values may be in error by several percent, particularly in the temperature ranges 25°C to 35°C, and 65°C to 80°C.

Column 7, Table 8, gives the experimental values of

$\frac{2\alpha^2}{h}\frac{dh_1}{dT}$. The values of δ used to determine h are given in column 6. Since $\frac{2\alpha^2}{h} = \frac{2\pi^2}{h^3}$, the calculated variation of δ with h is by no means negligible in influencing the values in column 7.

Table 8

(1) T(°C)	(2) h_1(cm)	(3) T(°C)	(4) h_1(cm)	(5) dh_1/dT (cm/°C)	(6) δ(cm)	(7) $10^7 \frac{2\alpha^2}{h}\frac{dh_1}{dT}$	(8) $-\frac{10^3}{\rho_m}\frac{d\rho_m}{dT}$
80	152.40	80	152.40	.294	36.02	8.66	6.17
70.3	149.62	75	150.96	.284	35.74	8.63	5.88
66.3	148.60	70	149.56	.275	35.47	8.58	5.59
62.8	147.65	65	148.21	.267	35.20	8.53	5.26
58.3	146.35	60	146.90	.257	34.93	8.45	4.94
53.7	145.32	55	145.64	.248	34.66	8.34	4.62
48.7	144.12	50	144.43	.238	34.41	8.20	4.26
44.6	143.13	45	143.26	.228	34.17	8.05	3.86
40.0	142.13	40	142.15	.218	33.93	7.90	3.43
36.0	141.33	35	141.08	.208	33.69	7.71	2.98
32.7	140.60	30	140.06	.199	33.46	7.51	2.48
30.6	140.15	25	139.09	.189	33.24	7.30	1.93
27.2	139.50	20	138.17	.179	33.03	7.03	1.30
25.7	139.24						

16 COMPARISON OF CALCULATED AND EXPERIMENTAL TEMPERATURE COEFFICIENTS

In figure 5, Curve 1, we have plotted the experimental values of $\frac{2\alpha^2}{h} \frac{dh_1}{dT} \times 10^7$ against $\frac{1}{\rho_m} \frac{d\rho_m}{dT}$. The corresponding temperatures are indicated along the horizontal scale. Curve 2 is drawn through the calculated values of $(-\frac{d\mathcal{B}^2}{dT} \times 10^7)$ and should agree with the experimental values _if_ the radial and vertical extrapolation lengths remained constant during the experiment. The calculated values of $d\mathcal{B}^2/dT$ given here are somewhat smaller than those found by J. D. Stewart (loc.cit) since levelling effects have been included, but they are much larger than the experimental values of $\frac{2\alpha^2}{h} \frac{dh_1}{dT}$.

Curves 3 and 4 are drawn through the values of $\frac{2\alpha^2}{h} \frac{dh_1}{dT}$ calculated with allowances for the expected variation of the extrapolation lengths with temperature (case A and case B). The agreement between theory and experiment is better, and is probably as good as can be expected in view of our lack of knowledge of the variation of the transport mean free path with moderator temperature. Curve 4 (Case B) is closest to the experimental curve.

Curve 5 gives the experimental values for $\frac{2\alpha^2}{h} \frac{dh_1}{dT}$ calculated on the assumption of no change in the vertical extrapolation length with changing temperature. The variation with temperature of this curve agrees better with the calculated curves and suggests that Case A overestimates the change of vertical extrapolation length with temperature.

17 CONCLUSIONS

There is no real discrepancy between the experimental data on reactivity changes with temperature and the 'theoretical' estimates. For lattices of the Zeep type, with relatively large values of χ^2, the leakage terms dominate the various contributions to the temperature coefficient. Corrections for the change of radial and vertical extrapolation lengths with temperature become important in comparing experimental data with the calculated reactivity changes with temperature.

APPENDIX 1

Evaluation of $\Delta f_u/f_u$ and $\Delta L^2/L^2$ for the NRX Lattice

If for an infinite lattice

f_u = fraction of thermal neutrons absorbed in Uranium
f_{al} = " " " " " in Aluminum
f_m = " " " " " in Moderator
f_w = " " " " " in Water

Then using the notation of CRE 400 A we have

$$\left.\begin{aligned} R_m &= \frac{N_m\ \sigma_a(m)\ V_m}{N_u\ \sigma_a(u)\ V_u}\ G_o(\kappa_u a) \\ R_{al} &= \frac{N_{al}\ \sigma_a(al)\ V_{al}}{N_u\ \sigma_a(u)\ V_u}\ G_o(\kappa_u a) \\ R_w &= \frac{N_w\ \sigma_a(w)\ V_w}{N_u\ \sigma_a(u)\ V_u}\ G_o(\kappa_u a) \end{aligned}\right\}\ \text{Relative Absorption Terms}$$

$$\left.\begin{aligned} B_{wm} &= \frac{\kappa_w{}^2 t^2\ N_m \sigma_a(m)\ V_m}{N_w\ \sigma_a(w)\ V_w} \\ B_{w\ al} &= \frac{\kappa_w{}^2 t^2\ N_{al}\sigma_a(al)\ V_{al}}{N_w\sigma_a(w)\ V_w} \end{aligned}\right\}\ \text{Blocking Terms}$$

$$S_m = (1 + R_{al} + R_w)\chi(\kappa_m c, \kappa_m b)$$

$$S_w = 1/2\ \kappa_w{}^2 t^2$$

And

$$\frac{f_m}{f_u} = R_m + S_m + B_{wm}$$

$$\frac{f_{al}}{f_u} = R_{al} + B_{w\ al}$$

$$\frac{f_w}{f_u} = R_w + S_w$$

$$f_m + f_{al} + f_w + f_u = 1$$

For the initial conditions we choose the following values for the NRX Lattice (CRE-400 A):

$$\left.\begin{array}{ll} R'_m = 1.4672 \times 10^{-2} & B'_{wm} = 4.72 \times 10^{-3} \\ R'_{al} = 3.7762 \times 10^{-2} & B'_{wal} = 1.214 \times 10^{-2} \\ R'_w = 2.5588 \times 10^{-2} & \\ & S'_m = 1.81 \times 10^{-3} \\ \chi = 1.7 \times 10^{-3} & S'_w = 4.11 \times 10^{-3} \end{array}\right\} \text{The primes refer to initial conditions}$$

$$\frac{f_m}{f_u} = .0212 \qquad \frac{f_{al}}{f_u} = .0499 \qquad \frac{f_w}{f_u} = .0297 \qquad f_u = 0.90843$$

We also will use

$$L_s^2 = 114 \text{ cm}^2 \qquad \gamma^2 = 4.21 \times 10^{-4} \qquad \left\{\begin{array}{l} \text{W.Box} = 220 \text{ cm} \\ h_c = 242 \text{ cm} \end{array}\right\}$$

$$L^2 = 156 \text{ cm}^2$$

$$\frac{L_s^2}{1+\gamma^2 L_s^2} = 108.8 \qquad \frac{\gamma^2 L_s^2}{1+\gamma^2 L_s^2} = 4.58 \times 10^{-2}$$

$$\frac{L^2}{1+\gamma^2 L^2} = 146.4 \qquad \frac{\gamma^2 L^2}{1+\gamma^2 L^2} = 6.16 \times 10^{-2}$$

To evaluate $\Delta L^2/L^2$ we use the simple relation (approx)

$$L^2 = L_m^2 \, f_m$$

Whence $\frac{\Delta L^2}{L^2} = \frac{\Delta L_m^{\;2}}{L_m^{\;2}} + \frac{\Delta f_m}{f_m} = -\frac{2\Delta\rho_m}{\rho_m} - \frac{\Delta\sigma_a(m)}{\sigma_a(m)} - \frac{\Delta\sigma_s(m)}{\sigma_s(m)} + \frac{\Delta f_m}{f_m}$

We require then to evaluate $\Delta f_u/f_u$ and $\Delta f_m/f_m$ for small changes in the various absorption and scattering cross-sections, and densities of the lattice materials.

We will calculate the differential coefficients which are valid for small changes in cross-sections and densities of the materials of the lattice. For simplicity we neglect the linear expansion coefficients of the uranium and aluminum. We consider first the case where all the absorption cross-sections change in the same way, thus a ratio of absorption cross-sections may be considered constant.

We have then

$$\Delta R_m = R_m\,(\Delta\rho_m/\rho_m + \Delta\,G_o/G_o)$$

$$\Delta R_{al} = R_{al}(\qquad\qquad + \Delta\,G_o/G_o)$$

$$\Delta R_w = R_w\,(\Delta\rho_w/\rho_w + \Delta\,G_o/G_o)$$

$$\Delta S_m = S_m\left\{\frac{\Delta R_{al} + \Delta R_w}{1 + R_{al} + R_w} + \frac{\Delta\chi}{\chi}\right\}$$

$$\Delta S_w = S_w\left\{\frac{2\Delta\rho_w}{\rho_w} + \frac{\Delta\sigma_a(w)}{\sigma_a(w)} + \frac{\Delta\sigma_t(w)}{\sigma_t(w)}\right\}$$

$$\Delta B_{wm} = B_{wm}\left\{\frac{\Delta\rho_w}{\rho_w} + \frac{\Delta\sigma_t(w)}{\sigma_t(w)} + \frac{\Delta\rho_m}{\rho_m} + \frac{\Delta\sigma_a(m)}{\sigma_a(m)}\right\}$$

$$\Delta B_{wal} = B_{wal}\left\{\frac{\Delta\rho_w}{\rho_w} + \frac{\Delta\sigma_t(w)}{\sigma_t(w)} + \frac{\Delta\sigma_a(al)}{\sigma_a(al)}\right\}$$

For $\Delta G_o/G_o$ we use the approximate relation $G_o = 1 + \frac{\gamma_o^2 a^2}{8}$

Whence $\frac{\Delta G_o}{G_o} = \frac{1}{G_o}\frac{\gamma_o^2 a^2}{8}\frac{\Delta\sigma_a(u)}{\sigma_a(u)} = 0.184\,\frac{\Delta\sigma_a(u)}{\sigma_a(u)} \quad \left\{G_o = 1.205\right\}$

For $\frac{\Delta\chi}{\chi}$ we use the approximate relation

$$\frac{\Delta\chi}{\chi} = \frac{\Delta\gamma_m^2}{\gamma_m^2} = \frac{2\Delta\rho_m}{\rho_m} + \frac{\Delta\sigma_a(m)}{\sigma_a(m)} + \frac{\Delta\sigma_t(m)}{\sigma_t(m)}$$

derived from

$$\chi(\gamma_m c, \gamma_m b) = \chi(y,x) = \frac{x^2}{2}\left\{\frac{x^2}{x^2-y^2}\log\frac{x}{y} - \frac{3}{4} + \frac{y^2}{4x^2}\right\} \quad \left\{\begin{matrix}\text{CRE-400 A}\\ \text{p 19}\end{matrix}\right\}$$

Now

$$\Delta\left(\frac{1}{f_u}\right) = \Delta(R_m + R_{al} + R_w + S_m + S_w + B_{wm} + B_{wal})$$

and $\Delta\left(\frac{f_m}{f_u}\right) = \Delta(R_m + S_m + B_{wm})$

also $\frac{\Delta f_u}{f_u} = -f_u\,\Delta\left(\frac{1}{f_u}\right)$

$$\frac{\Delta f_m}{f_m} = \frac{f_u}{f_m}\,\Delta\left(\frac{f_m}{f_u}\right) + \frac{\Delta f_u}{f_u}$$

Using these relations, and substituting for the R's, S's, and B's, etc., we get:

$$\frac{\Delta f_u}{f_u} = -2.092 \times 10^{-2} \Delta\rho_m/\rho_m -.1645 \times 10^{-2} \Delta\sigma_t(m)/\sigma_t(m)$$
$$-4.61 \times 10^{-2} \Delta\rho_w/\rho_w -1.906 \times 10^{-2} \Delta\sigma_t(w)/\sigma_t(w)$$
$$-3.378 \times 10^{-2} \Delta\sigma_a/\sigma_a$$

$$\frac{\Delta f_m}{f_m} = 1.0646 \Delta\rho_m/\rho_m + .0837 \Delta\sigma_t(m)/\sigma_t (m)$$
$$+ .1756 \Delta\rho_w/\rho_w + .2035 \Delta\sigma_t (w)/\sigma_t (w)$$
$$+ .40253 \Delta\sigma_a/\sigma_a$$

Where $\Delta\sigma_a/\sigma_a$ is the change in the absorption cross-sections, assumed to be the same for all the absorption cross-sections.

The fact that the coefficients of $\Delta\sigma_a/\sigma_a$ and $\Delta\sigma_t(w)/\sigma_t(w)$ are of the same order of magnitude shows immediately that the variation of the transport cross-sections with neutron energy is of importance in estimating the overall temperature coefficient.

If the absorption cross-sections do not have the same variation with neutron energy, the expressions for $\Delta f_u/f_u$ and $\Delta f_m/f_m$ can be obtained following the same procedure as before.

They become

$$\frac{\Delta f_u}{f_u} = -2.092 \times 10^{-2}\ \Delta\rho_m/\rho_m - .1645 \times 10^{-2}\Delta\sigma_t(m)/\sigma_t(m)$$

$$-4.610 \times 10^{-2}\ \Delta\rho_w/\rho_w - 1.906 \times 10^{-2}\Delta\sigma_t(w)/\sigma_t(w)$$

$$+7.102 \times 10^{-2} \left\{\frac{\Delta\sigma_a(u)}{\sigma_a(u)} - \frac{\Delta G_o}{G_o}\right\}$$

$$-1.927 \times 10^{-2}\ \Delta\sigma_a(m)/\sigma_a(m) - 4.542 \times 10^{-2}\ \Delta\sigma_a(al)/\sigma_a(al)$$

$$-2.704 \times 10^{-2}\ \Delta\sigma_a(w)/\sigma_a(w)$$

$$\frac{\Delta f_m}{f_m} = 1.0646\ \Delta\rho_m/\rho_m + .0837\ \Delta\sigma_t(m)/\sigma_t(m)$$

$$+.1756\ \Delta\rho_w/\rho_w + .2035\ \Delta\sigma_t(w)/\sigma_t(w)$$

$$-.6262 \left\{\frac{\Delta\sigma_a(u)}{\sigma_a(u)} - \frac{\Delta G_o}{G_o}\right\}$$

$$+.9808\ \Delta\sigma_a(m)/\sigma_a(m) + .0424\ \Delta\sigma_a(al)/\sigma_a(al)$$

$$+.0250\ \Delta\sigma_a(w)/\sigma_a(w)$$

Comparing these more general expressions with the previous ones, it can be seen that the coefficients of $\Delta\sigma_a/\sigma_a$ are the difference of relatively large coefficients associated with the absorption cross-sections of the uranium, aluminum, water and moderator. Consequently any variation of the uranium absorption cross-section from the 1/v law can be expected to have a large effect in altering the calculated contributions of the changes in absorption coefficients with temperature to the overall temperature coefficients.

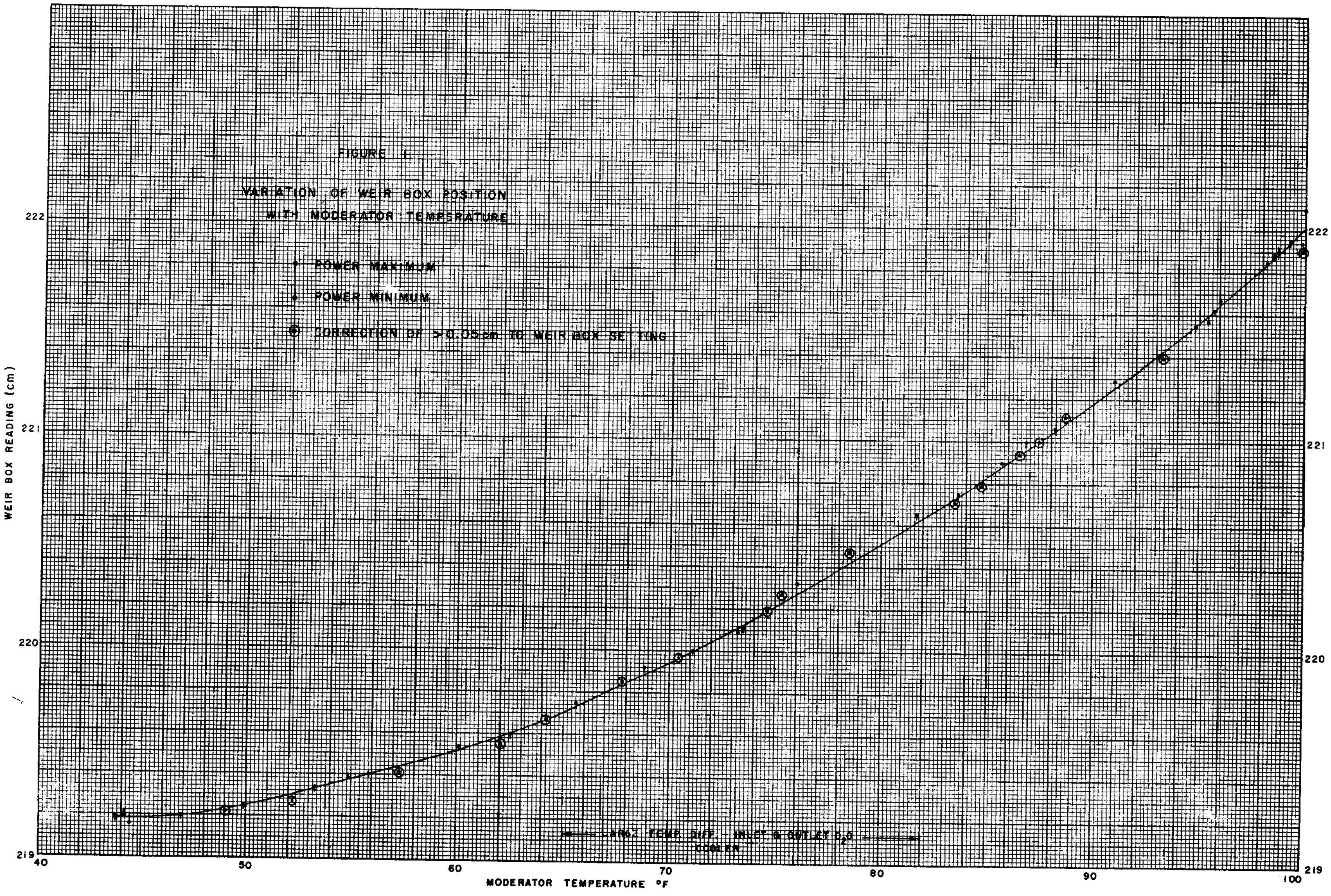

FIGURE 1

VARIATION OF WEIR BOX POSITION WITH MODERATOR TEMPERATURE

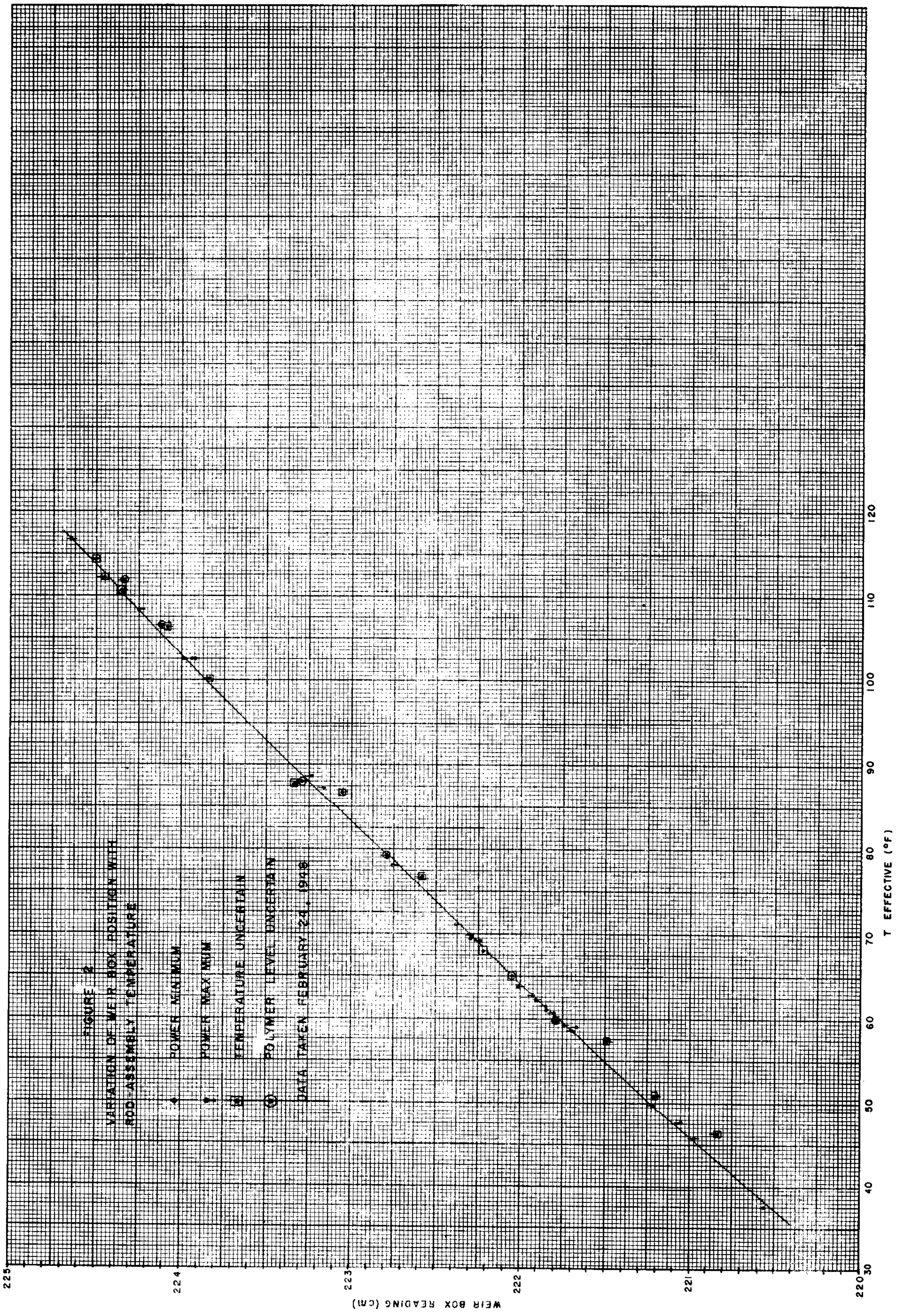

FIGURE 2

VARIATION OF WEIR BOX POSITION WITH ROD-ASSEMBLY TEMPERATURE

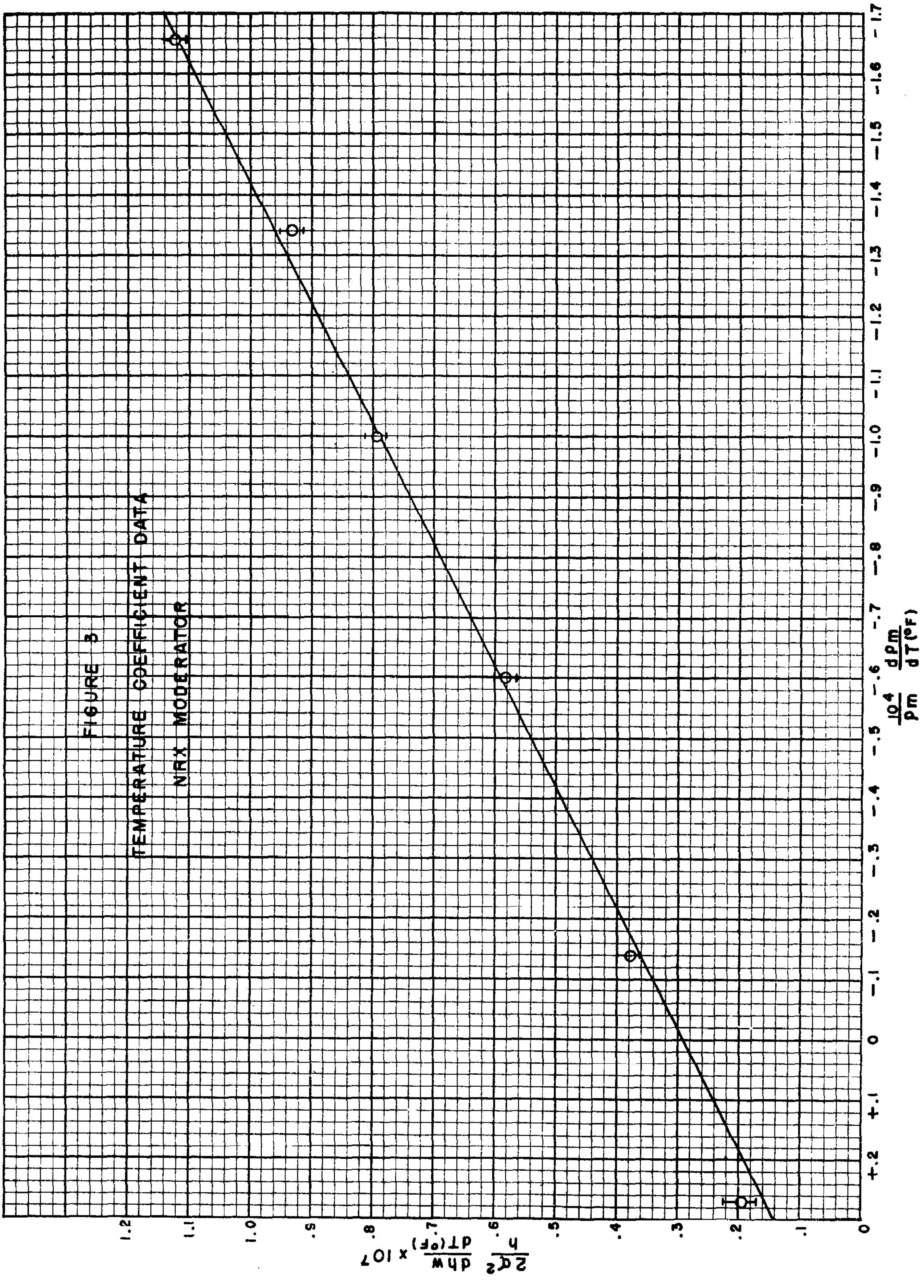

FIGURE 3

TEMPERATURE COEFFICIENT DATA

NRX MODERATOR

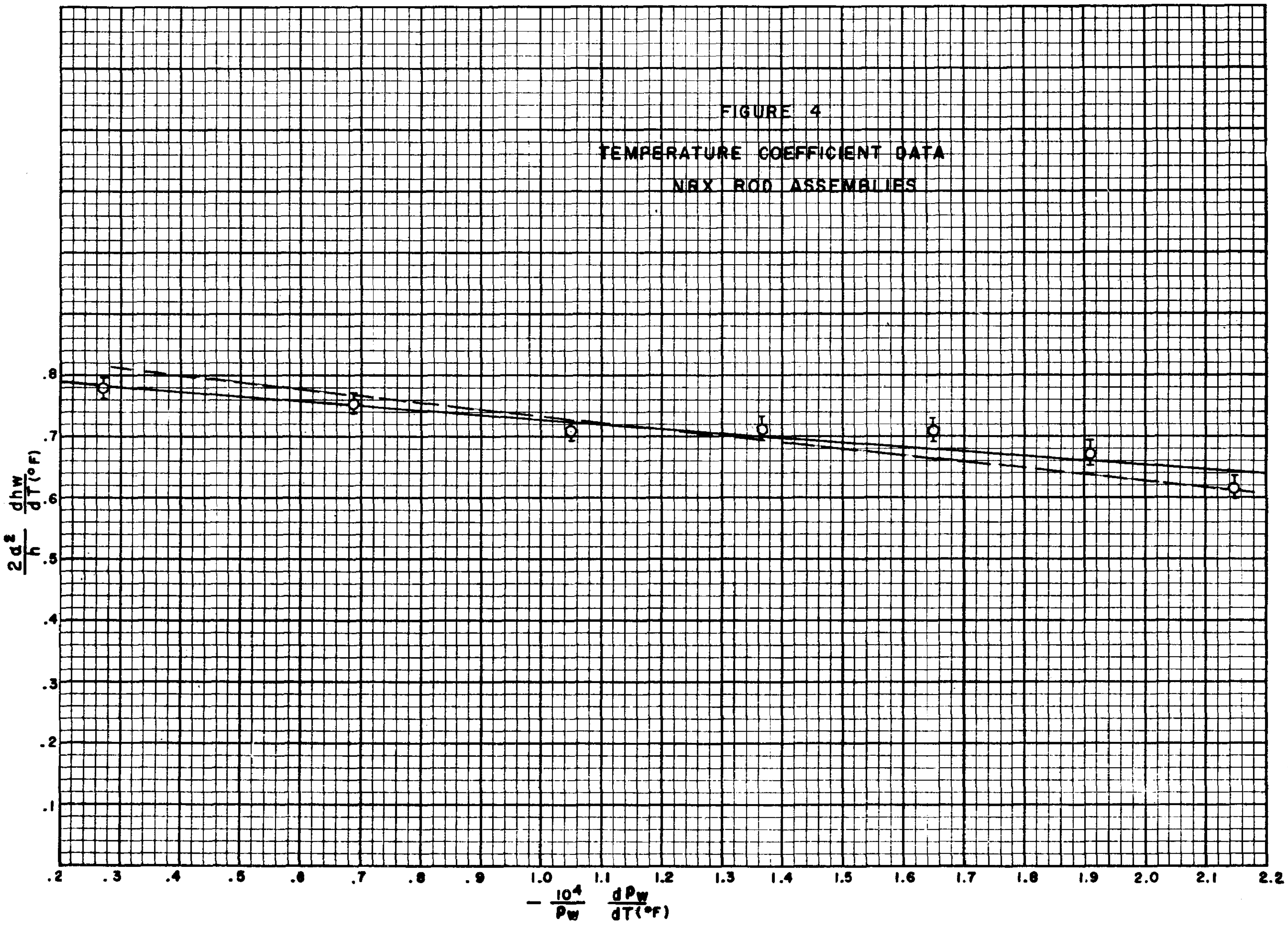

FIGURE 4

TEMPERATURE COEFFICIENT DATA

NRX ROD ASSEMBLIES

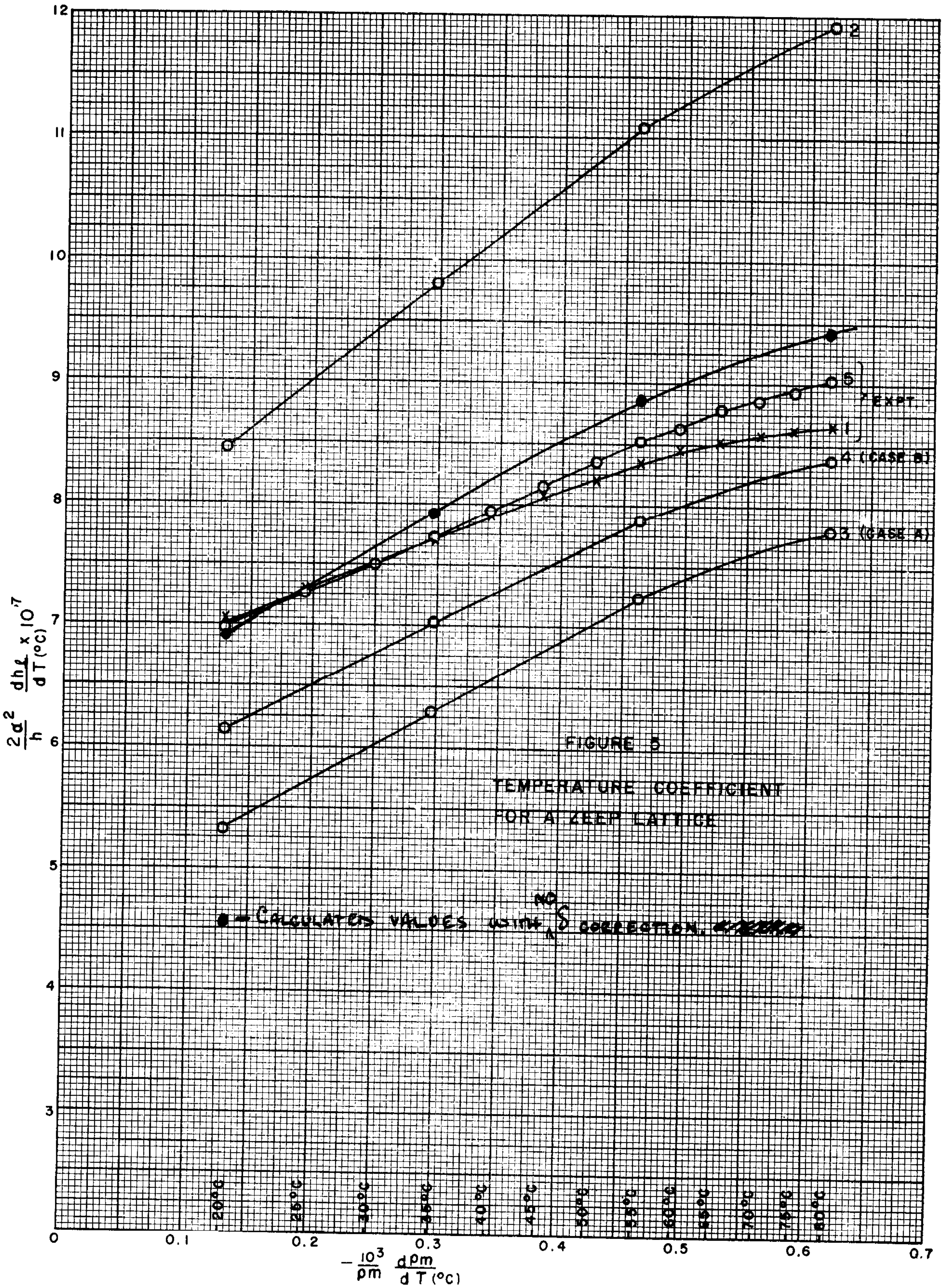

FIGURE 5

TEMPERATURE COEFFICIENT FOR A ZEEP LATTICE

● – CALCULATED VALUES WITH NO δ CORRECTION.