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PROGRESS REPORT
MAY, 1955 – DECEMBER, 1956

ATOMICS INTERNATIONAL
A DIVISION OF NORTH AMERICAN AVIATION, INC.

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EDITED BY:
M. CARL WALSKE

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ABSTRACT

NUCLEAR MEASUREMENTS

The disadvantage factors have been measured for one-inch-diameter thorium rods inserted in a lattice of one-inch-diameter 0.9-per cent enriched uranium rods in graphite. Similar measurements were made for rods of thorium alloyed to 2, 3, and 4 per cent with uranium-235.

NEUTRON THERMALIZATION

The elementary "age" theory of the slowing down of neutrons has been improved by estimating the energy at which slowing-down ceases and at which the neutron may be considered to make the transition to a thermal distribution. This has been accomplished by means of a comparison with slowing-down theories which take into account the thermal motion of the moderator. The comparison must be made with respect to a single parameter; this may be either the migration area of thermal neutrons, the migration area to absorption, or the mean number of collisions before absorption. A straightforward comparison theory used at first differed from the elementary theory only in that the mean energy loss was made proportional to E-2kT, rather than to E; later, however, the Wilkins theory of slowing-down was applied. Using this theory, and basing the comparison on the mean number of collisions before absorption, the transition energy was found to be five times thermal.

In connection with the application of Wilkins' theory, Wilkins' differential equation has been solved explicitly to the second order in the absorption parameter Δ. In addition, this equation has been programmed for solution on the IBM 704.

The treatment of resonance escape probability in the elementary slowing-down theory has also been improved by the use of Wilkins' equation. The effect for a sharp resonance has been calculated generally, and the effect for a broad resonance not actually in the thermal range has been estimated in the WKB approximation. The result of the latter calculation is that the usual factor E^{-1} in the resonance integral must be replaced by $(E-2kT)^{-1}$, which is in agreement with the first comparison theory of the first paragraph above. The result for a sharp resonance is the same, provided that the resonance is not in the thermal range.

I. NUCLEAR MEASUREMENTS

(W. W. Brown, D. H. Martin)

Thermal-neutron distributions have been measured in one-inch-diameter thorium rods inserted in a lattice of one-inch-diameter 0.9 per cent enriched uranium rods in graphite. The uranium lattice consisted of a 5-foot-high square array of 36 cells at a spacing of 7 inches, with the rod axes vertical. The system was mounted on top of the water boiler reactor thermal column. A thorium rod, consisting of two 6-inch-long slugs which were inserted as a pair between the uranium slugs in one of the rods, was located near the center of the array.

Inserts in the thorium slugs and, at the same vertical level in the assembly, inserts in the graphite were provided for holding foils for activation measurements. From the activation measurements with bare and cadmium-covered dysprosium foils along the cell radius, the relative average thermal-neutron flux in the thorium, in the graphite, and at the thorium-graphite interface can be obtained. Measurements were made using rods of thorium, and thorium alloyed to 2, 3, and 4 per cent with uranium-235. The values of the disadvantage factor F, the ratio of the flux at the interface to the average flux in the rod, are 1.10, 1.33, 1.45, and 1.53, respectively. The statistical errors in these values are estimated to be between 1 and 2 per cent.

II. NEUTRON THERMALIZATION

(E. R. Cohen, M. N. Moore, E. U. Vaughan)

A. INTRODUCTION

In the elementary (Fermi "age") theory of neutron slowing-down, the thermal motion of the moderator is neglected. The neutrons thus collide with stationary nuclei and hence always lose energy in a collision. When the neutron has reached thermal energies, on the other hand, the neutrons are considered to have a stationary velocity distribution. Simplified theories have usually neglected the transition region where one velocity distribution changes into the other. "Age" theory usually assumes that the distribution which is appropriate to the slowing down of neutrons in a moderator whose motion is negligible can be extended from high energies down to a transition energy, at which point the distribution changes abruptly to the Maxwellian distribution of thermal neutrons. The Fermi theory itself provides no means to determine this transition energy; for this purpose, there is required a theory capable of following the transition in more detail.

In such an improved theory, the transition energy would be replaced by a transition region, which would be taken partly from the region of slowing-down and partly from the thermal region of the more elementary theory. The distribution of the thermal neutrons is consequently non-Maxwellian; hence the thermal cross-sections should properly be averages over a non-Maxwellian distribution. Moreover, the epithermal neutrons would no longer have the usual slowing-down distribution, and, as a result, the effective resonance integral and corresponding resonance escape probability of an absorption resonance in the epithermal range require modification.

In the case of neutron slowing-down in a heavy moderator, where the scattering cross-section is independent of energy, Wilkins[1] and Hurwitz[2] have shown that the distribution is approximately a solution of the second-order differential equation

$$x N''(x) + (2x^2 - 1) N'(x) + (4x - \Delta) N(x) = 0 ,$$

in which $x = \left(\frac{E}{kT}\right)^{1/2}$ is a measure of the neutron velocity, $N(x)\,dx$ is the probability that the neutron velocity lies between x and $x + dx$, and the single quantity Δ represents the physical properties of the moderator. If the slowing-down power is represented by $\xi \sigma_s$, and the absorption cross-section by $\sigma_a(x)$, then we have

$$\Delta = \frac{4x \ \sigma_a(x)}{\xi \ \sigma_s} .$$

This expression is constant in the case of $1/v$ absorption.

Wilkins has begun the treatment of this equation (for the case of constant Δ) by giving the power series solution (suitable for small x) and the asymptotic expansion (for large x). He has also joined these two solutions by numerical integration for a few values of Δ. Further work on the neutron distribution, including applications to the problems mentioned above, are reported below.

B. PRELIMINARY STUDY BASED ON APPROXIMATE NEUTRON DISTRIBUTION

The mean logarithmic neutron energy loss per collision in the slowing-down region is $\xi = \frac{2}{m}$, where m is the moderator mass. For neutrons approaching thermal energies, however, a better estimate is

$$\xi_{eff} = \xi \left(1 - \frac{2kT}{E}\right) = \xi \left(1 - \frac{2}{x^2}\right) .$$

This results[1] in a spectrum N(x) represented by $\frac{1}{x^2 - 2}$, rather than by $\frac{1}{x^2}$ as in the age theory; this conclusion is confirmed by Wilkins' asymptotic expression. A simple way to improve the usual theory is to use this more accurate spectrum in the epithermal region, joining it to the thermal neutrons by requiring continuity of N(x) at the joint, as well as continuity of the slowing down density, given by

$$q(x) = (2kT)^{1/2} \frac{\xi \sigma_s \Delta}{4} \int_0^x N(x')dx' .$$

As a further improvement, we may consider the neutron temperature T_n as an adjustable parameter, rather than forcing it to equal the moderator temperature T. This represents the fact, mentioned above, that the thermal neutrons do not actually achieve the Maxwellian distribution of temperature T which the collisions tend to bring about; the reason for this is that absorption removes the neutrons before they reach the Maxwellian distribution. A further condition is required to fix the new parameter; this has been taken as the agreement of the ratio of the moments of order minus two and zero with the corresponding ratio calculated (to first order in Δ) from the power series solution of Wilkins' equation.

Several criteria have been used to determine the age of thermal neutrons. All require some extension of the preceding theory — which applies to an infinitely-extended moderator — to make it applicable to a finite system. The extension is not difficult if diffusion theory is applicable, as it is sufficient to consider the Fourier components of the spatial distribution.

The first method used[3] represents the spatial distribution by a buckling. In the elementary theory, the energy at which slowing-down ceases is fixed by equating the non-leakage probability in this theory (which is a function of the desired cutoff energy) to the total absorption probability calculated from the improved distribution. Cutoff energies determined, as is this one, by the "age to absorption," have been designated generically by E_{eff}. The calculation has actually been performed in the limit of zero buckling.

A different method[1,3,4] is to evaluate the second moment of the spatial distribution of thermal neutrons, utilizing the fact that the nth moment of a distribution is related to the nth derivative with respect to frequency of the Fourier transform of the distribution. This requires a somewhat more elaborate procedure for including spatial dependence than the preceding method. Moreover, it has a different physical significance, since many neutrons are absorbed before becoming thermal, and consequently the mean migration area to absorption is smaller than the mean migration area of thermal neutrons. The age of thermal neutrons can now be obtained as the difference between the migration area and the thermal diffusion area, which is known to be L^2, where L is the thermal diffusion length. The age so found determines a corresponding cutoff energy. Cutoff energies determined in this way from the migration area of thermal neutrons are generically designated by E_m.

The absorption age cutoff energy E_{eff} can be estimated[1,4] in the same way as E_m, using now the total neutron density rather than the density of thermal neutrons only. This is justified, since the absorption rate is given by $v\sigma_a N$, which is proportional to N when σ_a follows the 1/v-law. The results are close to those of the first estimate of E_{eff}; consequently, the latter need not be further reported.

The principle results of general interest are the values of T_n, E_m, and E_{eff}. It is found that, for $0 < \Delta < 0.8$, the expression

$$T_n = T_o (1 + 0.30\Delta)$$

is reasonably accurate. Difficulties encountered in attempting to confirm this relationship experimentally have brought out the deficiencies in the assumed distribution — as exemplified by the fact that there is a discontinuity of slope at the point where the Maxwellian and slowing-down distributions join.

The cutoff energies E_m and E_{eff}, though becoming logarithmically infinite as $\Delta \rightarrow 0$, vary slowly over a wide range of values of Δ. Since these energies enter logarithmically into the age, and also in view of other deficiencies of the age theory, accurate values are not required. In the range $0.02 < \Delta < 1.0$, adequate estimates are given by $E_m = 4kT$ and $E_{eff} = 16kT$. It appears that E_{eff}, rather than E_m, should be suitable to reactor calculations; its value, however, is surprisingly large.

C. IMPROVED THEORY BASED ON WILKINS' DISTRIBUTION

In order to improve the theory presented above, Wilkins' differential equation has been studied carefully.[5] The difficulty of its solution lies in the fact that the coefficients of the power-series solution are determined by a three-term recursion formula. However, in the case $\Delta = 0$, this reduces to a two-term formula. Hence, if the solution is expanded in powers of Δ, the power-series expansions of the coefficients of successive powers of Δ (which are, of course, functions of x) are relatively easy to determine. Since the difficulty, nevertheless, increases rapidly for higher powers of Δ, this method has been explicitly carried out only to order Δ^2.

As an expansion to order Δ^2 is inadequate for large values of Δ, this analytical work has been supplemented by numerical work for several values of Δ. The method used was that of Wilkins, mentioned in Section A above; the power series for small x was joined to the asymptotic series for large x by integration. In the present work, this was done on a Nordsieck Differential Analyzer. The resulting neutron spectra are presented in Fig. 1.

It is a consequence of the theory of Wilkins' equation that the solution for small x is proportional to x^2 (like the Maxwellian $4\pi^{-1/2}x^2 e^{-x^2}$), while for large x it is proportional to $C(\Delta)/x^2$ (in agreement with age theory). An important problem is the determination of the coefficient $C(\Delta)$ in the asymptotic expression. As a result of the work on expansion in powers of Δ, it is found that, to order Δ^2,

$$C(\Delta) = \frac{\Delta}{2}(1 + 0.798873\Delta + 0.286606\Delta^2).$$

Moreover, the omitted higher terms are all positive; consequently, this estimate is a lower bound for all Δ. A somewhat more intricate argument leads to an

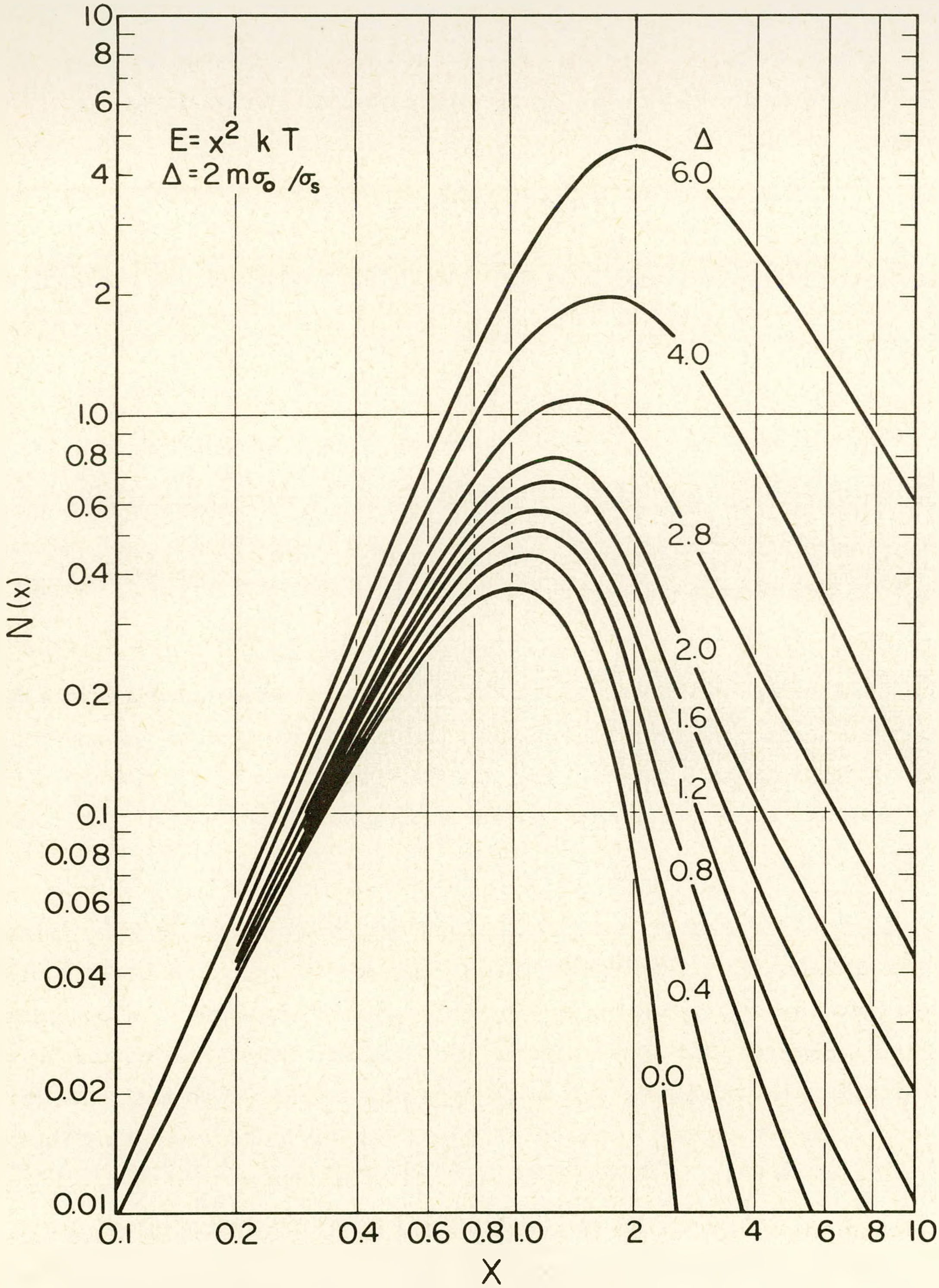

Fig. 1. Neutron Velocity Spectrum

upper bound in the form

$$C(\Delta) \leq \frac{\Delta}{2} \left[\frac{1 + 0.440129\Delta}{1 - 0.358744\Delta}\right]$$

The mechanical integrations mentioned above lead to estimates of $C(\Delta)$ for several values of Δ larger than those for which expressions in powers of Δ are adequate. In this way, $C(\Delta)$ is fairly well determined for $\Delta < 10$.

It was felt that more accurate information on N(x) and $C(\Delta)$ was required than could be obtained from a differential analyzer. Consequently, a program has been developed for the calculation of these functions on the IBM 704. The results for $C(\Delta)$ are shown in Fig. 2, which presents a curve of $\gamma(\Delta) = \Delta^{-1} \ln(2C/\Delta)$ as a function of Δ, together with the upper and lower bounds found for small Δ.

The approximations of Section B above can be tested[5] by means of this improved theory. In particular, the asymptotic distribution $\frac{C(\Delta)}{x^2-2}$ is found to be an improvement upon the distribution $\frac{C(\Delta)}{x^2}$. A further improvement suggested by Nelkin[2] is still more accurate, but is probably less convenient to work with, since it cannot be interpreted in terms of an effective mean logarithmic energy loss ξ_{eff}.

On the other hand, the concept of an effective neutron temperature is less satisfactory. In Fig. 3 are presented curves of the slowing-down density q(x) as functions of x for various values of Δ. These curves were obtained by integration of the distributions N(x), already discussed. Since the abscissa is logarithmic, a change of scale of x (corresponding, in view of the definition of x, to a change of neutron temperature) is represented by a horizontal displacement of the curves. The fact that no curve for $\Delta \neq 0$ can be brought into coincidence with that for $\Delta = 0$ by such a displacement indicates the inadequacy of representing distributions for $\Delta \neq 0$ by means of a Maxwellian with altered temperature.

The treatment of the age of thermal neutrons by means of Wilkins' distribution[5] has required still another definition of the cutoff energy E_{eff} corresponding to the "absorption age." Since the theory has been developed for an infinite moderator, the study of spatial distributions has been replaced by the study of collision

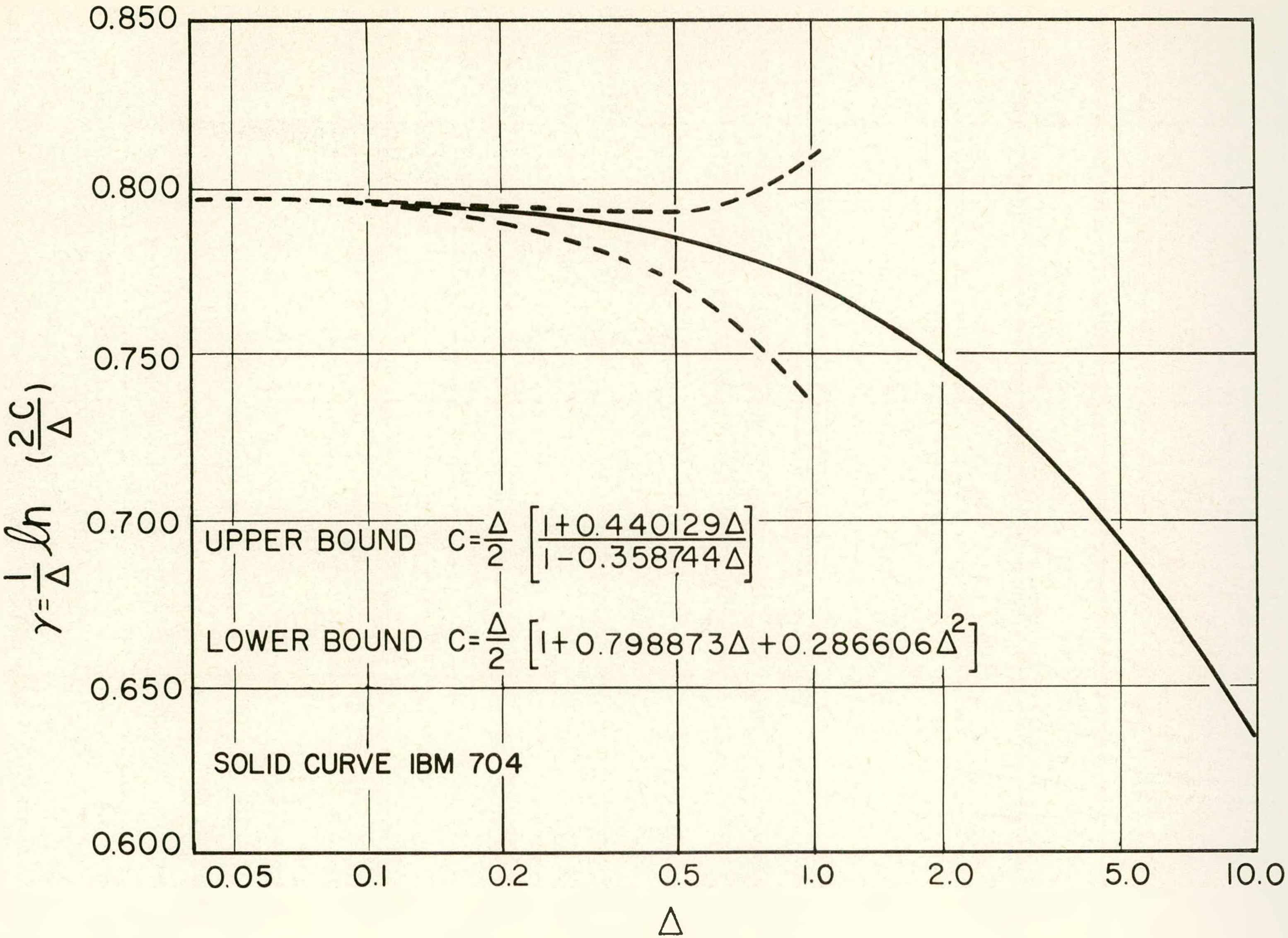

Fig. 2. Asymptotic Neutron Density

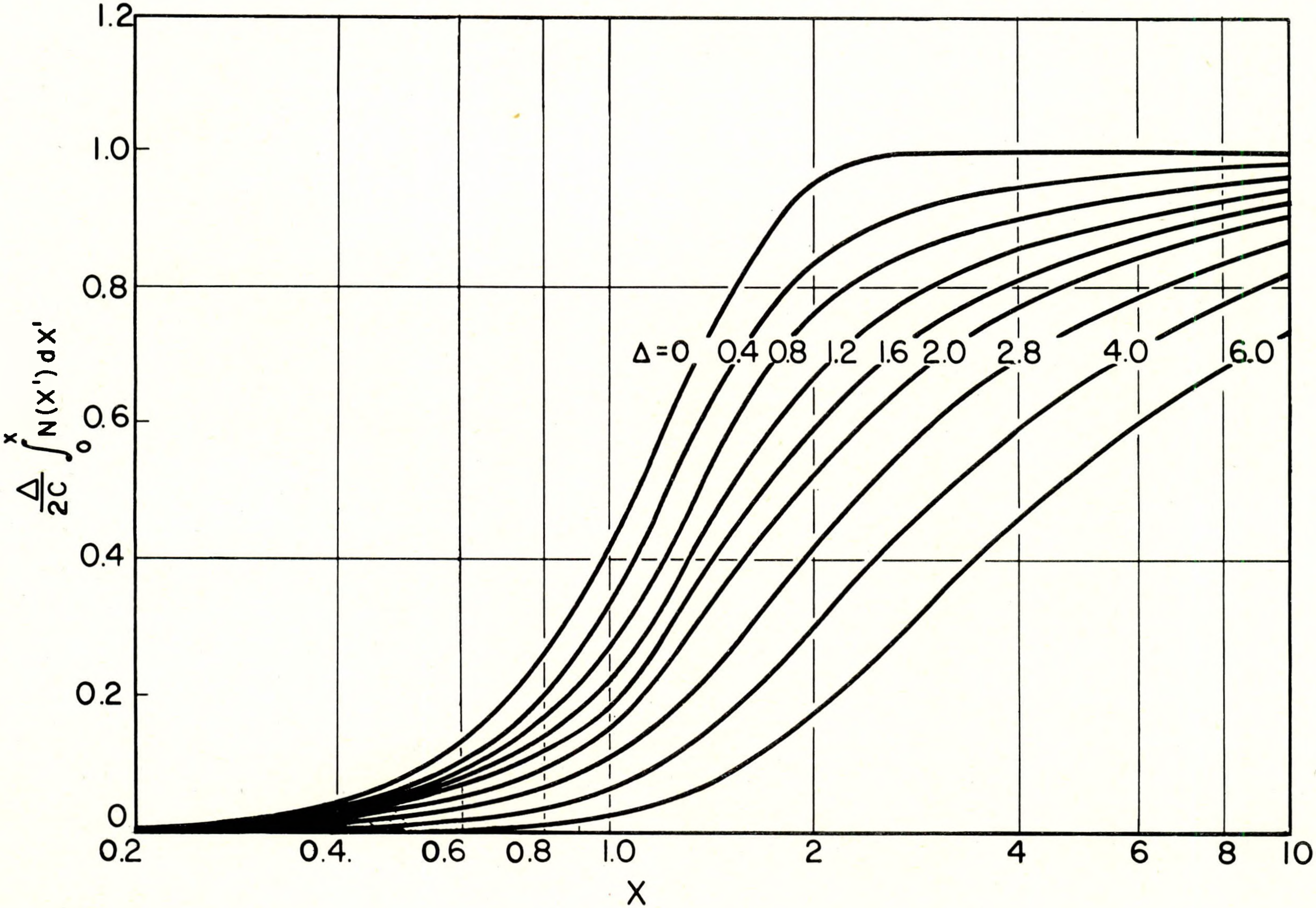

Fig. 3. Neutron Slowing-Down Density

rates. The connection with age is provided by the fact that the migration area in the diffusion approximation is proportional (for constant scattering cross-section) to the number of scatters, the constant of proportionality being $(3\sigma_s^2)^{-1}$. The mean number of scatters before absorption is merely the ratio of the total scattering rate to the total absorption rate. The latter is the asymptotic value of the slowing down density q(x) for large x, and is related to $C(\Delta)$, which is already known to the second order in Δ. The former can be evaluated by similar methods to the same order, and the ratio then gives the migration area to absorption to this order. Since the first term in $C(\Delta)$ is $\Delta/2$, the first term in the migration area is of order Δ^{-1}. This term turns out to be just the diffusion area L^2 of thermal neutrons; the age of thermal neutrons is then just the sum of the remaining two terms. When expressed in terms of a cutoff energy, the result is given by

$$E_{eff} = 5.17kT \; e^{-1.13\Delta} \approx 5.2kT \; .$$

This appears to be a better estimate than the value 16kT found at an earlier stage of the investigation.

D. ABSORPTION BY AN EPITHERMAL RESONANCE

Absorption of neutrons by a resonance in the epithermal energy range is larger than indicated by the age theory, since this theory underestimates the neutron density in the region considered. Moreover, the usual definition of resonance escape probability (as the probability that a neutron of energy barely above the resonance will escape capture in the resonance) requires revision. In fact, the neutron density will, in contrast to the familiar case of resonance in the slowing-down region, be reduced in the region just *above* the resonance. Physically, this arises from the absorption in the resonance of neutrons which would, in its absence, have been *speeded up* past the resonance. The effect of the resonance is best represented by the ratio of the absorption at energies below resonance to the absorption that would have occurred there if there had been no resonance, the asymptotic value of N(x) at large x being the same in both cases. This ratio is accordingly taken as the resonance escape probability, denoted by p.

The problem can be solved for a narrow resonance, since the solution below the resonance is merely a multiple of that with no resonance, and the solution above resonance is a linear combination of the same solution with a second solution of Wilkins' equation; i.e., one which for large x behaves like $x^2 e^{-x^2}$, rather than like x^{-2}, as does the usual solution. The distribution must be continuous at the resonance, but its first derivative has a discontinuity which is determined by the strength of the resonance. In order to fit these boundary conditions, it is necessary to determine the second solution; this has been done, as was the first, by mechanical extension of the asymptotic expression to lower energies by means of a differential analyzer. The results are presented in Figs. 4 and 5, which show, respectively, typical neutron spectra and the factor by which to multiply the usual resonance integral in order to obtain the effective resonance integral for this case.

These results hold for resonances which are narrow, but not too narrow. Although the width should be small compared to unity (on the x scale), it must be large compared to the mean velocity change $\frac{1}{2} \xi_{eff} x$ in a single collision; otherwise, the approximations leading to Wilkins' differential equation will fail.

The most perspicuous result of this calculation is that, unless the resonance is in or very near to the thermal range, the effective resonance integral differs from the usual resonance integral by the factor $\xi / \xi_{eff} = x^2/(x^2 - 2)$. This agrees with the increase of the neutron density by this factor, as discussed in B, above. Essentially the same result can be obtained for a broad resonance by solving the problem in the WKB approximation. To show this, we introduce the new dependent variable

$$\nu(x) = x^{-\frac{1}{2}} e^{\frac{x^2}{2}} N(x),$$

which satisfies the equation

$$\nu'' = \left(x^2 - 4 + \frac{\Delta}{x} + \frac{3}{4x^2}\right) \nu ,$$

and therefore has the approximate solutions

$$\nu \approx \left(x^2 - 4 + \frac{\Delta}{x} + \frac{3}{4x^2}\right)^{-1/4} \exp\left\{ \pm \int^x dx' \sqrt{x'^2 - 4 + \frac{\Delta}{x'} + \frac{3}{4x'^2}} \right\}$$

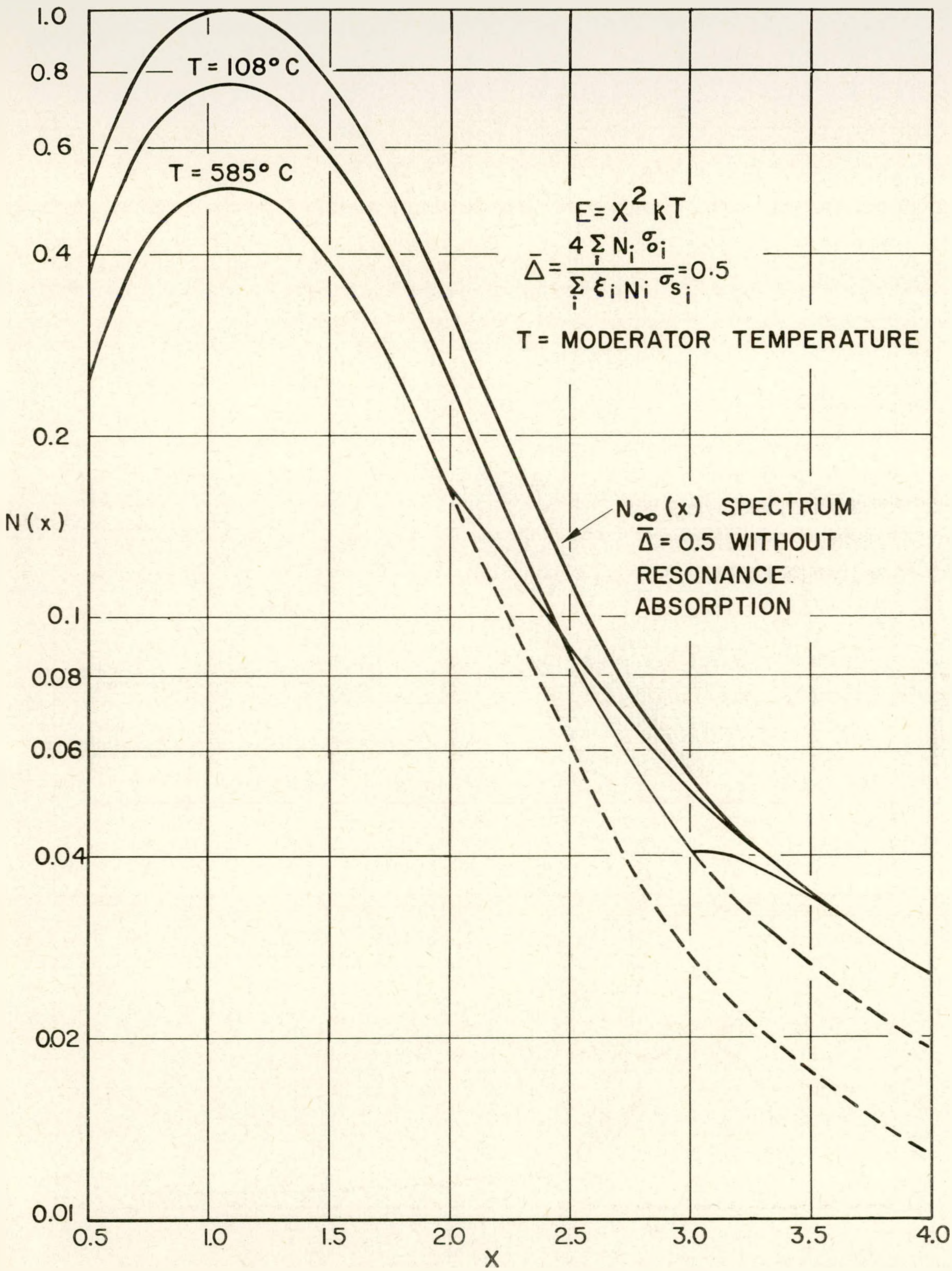

Fig. 4. Neutron Velocity Spectrum in a Heavy Moderator With an Absorption Resonance

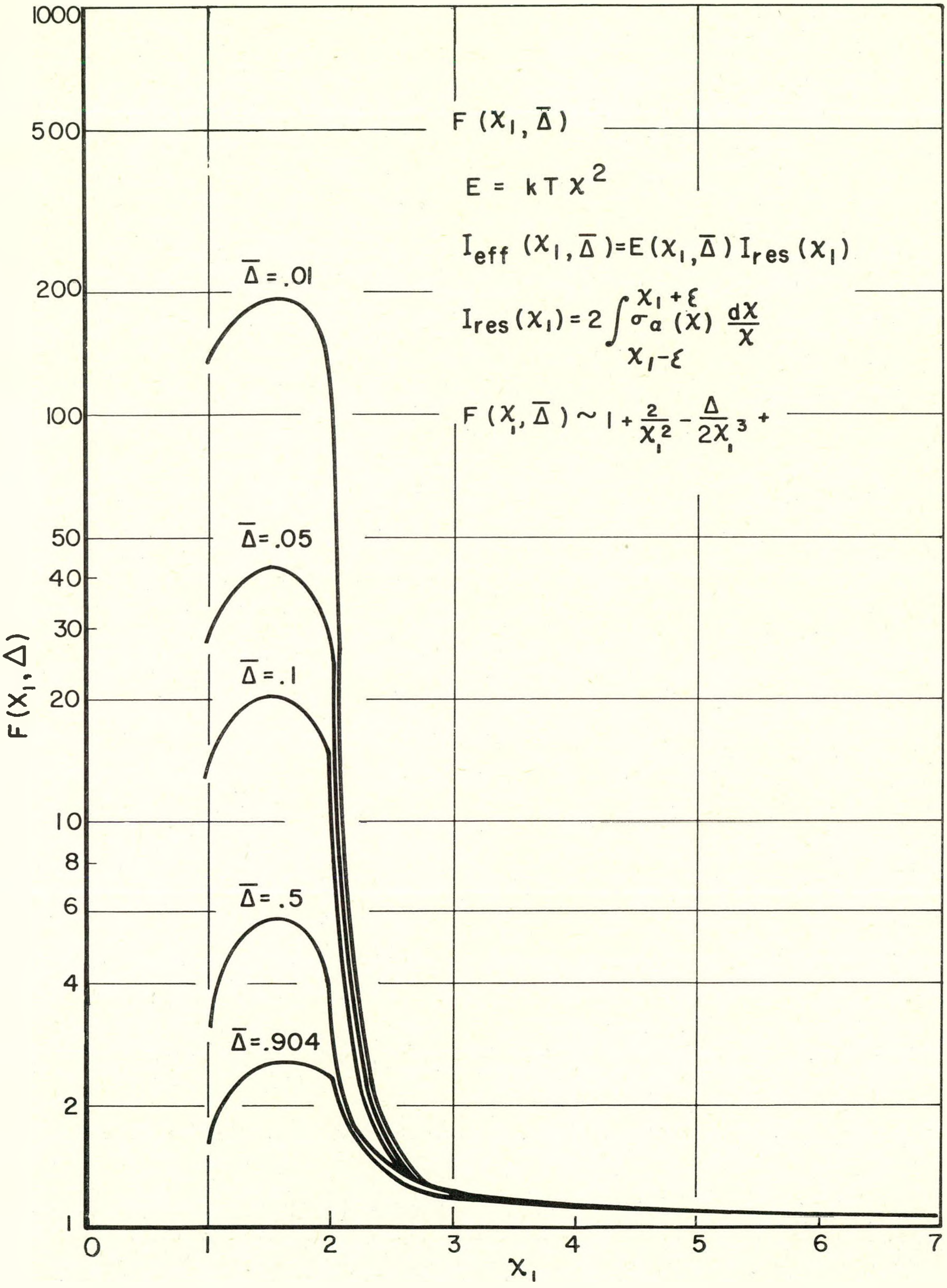

Fig. 5. The Resonance Integral Enhancement Factor

The largest positive zero of the radicand is $x = r_2 < 2$. For larger values of x the radical is real, and the proper choice of sign is the one which gives an increasing exponential. Consequently

$$N(x) \approx x^{-2} \left[1 - \frac{4}{x^2} + \frac{\Delta}{x^3} + \frac{3}{4x^4}\right]^{-1/4} \exp\left\{\int_x^{\infty} \frac{dx}{x} \left[x^2 - 2 - \sqrt{x^4 - 4x^2 + \Delta x + \frac{3}{4}}\right]\right\}$$

is the required solution, as is confirmed by the fact that it is proportional to x^{-2} for sufficiently large x.

If there is a resonance in the region $x > 2$ which makes a contribution $\sigma_r(x)$ to the absorption cross-section, then the result is unchanged, except that Δ is replaced by $\Delta + 2mx\, \sigma_r/\sigma_s$. The definition of resonance escape probability given above then yields

$$-\ln p \approx \int \frac{dx}{x} \left\{\sqrt{P(x,\Delta) + 2mx^2\, \sigma_r/\sigma_s} - \sqrt{P(x,\Delta)}\right\},$$

where $P(x,\Delta) = x^4 - 4x^2 + \Delta x + 3/4$. This integral may be extended indifferently over the resonance region or the entire x-axis.

If now we let

$$\sigma_r/\sigma_s \ll P(x,\Delta)/(2mx^2),$$

then expansion of the first radical yields approximately

$$-\ln p \approx \int \frac{dx}{x} \frac{m\sigma_r}{\sigma_s} \frac{x^2}{\sqrt{P(x,\Delta)}}$$

For large x, $P(x,\Delta) \approx x^4 - 4x^2$, and the above relations may be written

$$\sigma_r/\sigma_s << (x^2 - 4)/(2m),$$

$$-\ln p \approx \int \frac{dE}{E} \frac{\sigma_r}{\xi \sigma_s} \frac{1}{1 - 2/x^2},$$

where the relation $\xi \approx 2/m$ has been employed. The last result is, as anticipated, the usual resonance integral modified by the factor $(1 - 2/x^2)^{-1}$ in the integrand.

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