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BNL-1627

Subject Category: PHYSICS

[redacted]

UNITED STATES ATOMIC ENERGY COMMISSION

BUCKLING OF LIGHT WATER LATTICES (.600" DIAMETER RODS, 1.3% AND 1.15% 25)

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November 5, 1953

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ATOMIC ENERGY COMMISSION · UNITED STATES OF AMERICA

UNCLASSIFIED

Work performed under Contract No. AT(30-2)-Gen-16.

Date Declassified: October 27, 1955.

Printed in USA, Price 20 cents. Available from the Office of Technical Services, Department of Commerce, Washington 25, D. C.

BROOKHAVEN NATIONAL LABORATORY

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INTRODUCTION:

The lattices reported on here are the first of those we are studying jointly with WAPD. At Brookhaven we are measuring critical masses of clean, symmetric subcritical assemblies. In addition to studies more directly related to PWR design, Westinghouse is taking to critical a representative set of lattices in the geometry we are using.

The criticality measurements we make are of three kinds. Some lattices are more conveniently measured one way than another; therefore we do not use each method in every case. Although some measurements remain to be done on some of the lattices, results seem to be in final enough form for dissemination now.

CRITICALITY MEASUREMENTS:

Method 1:

The first procedure, and that used most often, is described in our report on measurements with .750" diameter rods of 1% enrichment.[1] An approximately cylindrical lattice is placed in a water tank located over a thermal column on top of the Brookhaven pile. We measure the vertical relaxation length L of the fundamental component of the thermal neutron flux excited by thermal column neutrons in this subcritical water-uranium assembly. The measurements are interpreted in terms of the critical size equation

$$B^2 = \left(\frac{2.4048}{R + \lambda}\right)^2 - \frac{1}{L^2} \qquad (1)$$

We define R as the loaded radius, and λ as the constant reflector savings. (1) may be rewritten as

$$L = \left\{\left(\frac{2.4048}{R + \lambda}\right)^2 - B^2\right\}^{-1/2} \qquad (2)$$

1. BNL Log No. C-6701

We measure L as a function of R, and find the best values of B^2 and λ which fit the calculated functional dependence of L and R in (2) to the measured data. The criterion these best values must satisfy is that they must give

$$\Sigma\left(\frac{L_{obs}-L_{calc}}{L_{obs}}\right)^2 \quad \text{its minimum value.} \qquad (3)$$

Quite large ranges of R are used, and there had been criticism in the past because of our assumption that λ is unaffected by such large changes in loading radius. Principally because of this criticism, we have increased the number of values of R at which L is measured, taking points at smaller intervals, and extending the range upward to a k_{eff} of about .97 in each case. This has made possible separate determinations of B^2 and λ from measurements made far from critical and relatively near critical, and has also led to improved accuracy of the results obtained by combining all measurements.

Still, such procedures can not definitely be used to establish the validity of the method. They can at best show self-consistency of the results they lead to. For this reason, we considered it essential that criticality measurements also be made in other ways.

Method 2:

The second procedure we have used is that of the standard exponential experiment, with B^2 and λ being determined from plots of the measured radial and axial variation of the thermal neutron flux. Far enough from the source and the boundaries, the flux should have the form

$$\phi = A\, J_o\left(\frac{2.4048}{R+\lambda}\, r\right) \sinh\left(\frac{z_o - z}{L}\right) \qquad (4)$$

$R+\lambda$ and L are determined by least squares fitting of the measured flux to (4), and B^2 is given directly by (1).

In earlier buckling measurements[1] we tried getting radial traverses by exposing foils at the centers of lattice cells. The small sizes of the assemblies provided, at most, five points on a single radius, and this number we found too small for accurate buckling measurements.

Lately we have found that quite adequate radial flux plots can be measured if the foils used are exposed in fuel rods. The number of points available on a single radius is then about doubled, and excellent accuracy can be obtained.

Method 3:

This is the method of the almost critical assembly. We use a separate facility, with a second water tank in which to place the lattice being measured. Uranium is loaded in the presence of a Po-Be source and suitable flux monitors and safety rod trip circuits until a k_{eff} of about .99 is reached. At this point the source is removed, and the lattice partly unloaded in steps.

Because of spontaneous fissions, the flux density in the core has the form

$$\phi = \sum_{j,k=1}^{\infty} C_{j,k} \sin \frac{j\pi z}{h} J_o\left(\frac{\xi_k r}{R+\lambda}\right) \quad (5)$$

with

$$C_{j,k} = \frac{12\left\{1 - (-1)^j\right\} Q}{\pi j \lambda_t \xi_k J_1(\xi_k) \left\{\left(\frac{\xi_k}{R+\lambda}\right)^2 + \left(\frac{k\pi}{h}\right)^2 - B^2\right\}} \quad (6)$$

Q = source of spontaneous fission neutrons
h = height of the assembly
ξ_k = successive zeroes of $J_o(u)$
λ_t = transport mfp

For the nearly critical lattices we have used, only the fundamental mode j,k = 1 is important,[2] so that

$$\phi = \frac{\text{constant} \cdot \sin \frac{\pi z}{h} J_o\left(\frac{\xi_1 r}{R+\lambda}\right)}{\left(\frac{\xi_1{}^2}{R+\lambda}\right)^2 + \left(\frac{\pi}{h}\right)^2 - B^2} \quad (7)$$

But

$$B^2 = \left(\frac{\xi_1}{R_c+\lambda}\right)^2 + \left(\frac{\pi}{h}\right)^2 \quad (8)$$

(R_c the critical loaded radius) for a critical finite cylinder. Thus

$$\phi = \frac{\text{constant} \cdot \sin \frac{\pi z}{h} J_o\left(\frac{\xi_1 r}{R+\lambda}\right)}{\left(\frac{\xi_1}{R+\lambda}\right)^2 - \left(\frac{\xi_1}{R_c+\lambda}\right)^2} \quad (9)$$

The thermal flux is measured at a fixed point on the central axis. Thus a plot of $1/\phi$ vs. $\left(\frac{1}{R_c+\lambda}\right)^2$ is a straight line intersecting the axis at

$$\frac{1}{(R+\lambda)^2} = \frac{1}{(R_c+\lambda)^2} \quad (10)$$

Of course, the procedure is essentially that of the critical assembly. But since the lattices are not loaded all the way, the analytical justification of extrapolating the flux plots to critical must be made.

2. The fluxes measured had ~.01% harmonic content.

FOIL COUNTING METHODS:

Flux measurements for methods 1 and 2 are made with indium foils, about five mils thick. Each foil is counted in each of six counters, three times on each side. At the end of each set of foil activity measurements, the combined total of saturated activities for the separate counters are used to intercalibrate the counters. Thus measurements are not dependent on assuming constant intercalibration factors.

The procedure of counting each foil in several counters makes it possible to find and eliminate faulty counts caused by faulty electronics.

Foils for a given measurement are matched to within .1% in sensitivity; thus no weight corrections are required.

Each foil is counted to at least 10,000 total counts (or at most 1% mean statistical error). The overall mean counting error is then about 1% per flux point.

EXPERIMENTAL METHODS:

(1) RELAXATION LENGTHS

All relaxation length measurements except those for the 1:1, 1.15% enrichment lattice have been made with .2202" diameter foils located along a line in the center of a lattice cell. Foils were oriented vertically, so as to equalize fluxes on the two faces.

The foil holders used are small lucite rods, with holes milled in to contain the foils. Tight fitting lucite plugs fit in the holes over the foils, keeping them secure.

The 1:1 volume ratio is so tight packed that there is not enough room between fuel rods to insert the foil holder rod. For the 1:1, 1.3% enrichment lattice, a fuel channel was therefore left empty, and the foil rod was inserted in this position. The value of the omitted fuel rod was then determined, and used to correct the actual loading to an effective one. Thus, as pointed out in our earlier note on relaxation length measurements with 1.3% enrichment rods, these effective loadings are not necessarily integral numbers of rods.

The same lattice with 1.15% enriched rods was measured somewhat differently. The foils used to get fluxes for relaxation lengths were exposed in a fuel rod cut roughly into 8 cm. sections. The foil diameters were the same as those of the fuel rods (.600"); otherwise all techniques were the same as discussed above.

The axial flux variation is a sinh function, as stated in (4). For small L, we can approximate

$$\sinh \frac{z_o - z}{L} \cong \frac{1}{2} e^{\frac{z_o - z}{L}}$$

and L can be obtained from a simple exponential fit to the measured flux values. For large L, this procedure is not adequate; the precise expression must be used.

We make end effect corrections as follows. z_0 is obtained at high loading from a many-point sinh fit to measured data, and this value is used throughout a set of L measurements. Each measured flux in a relaxation length determination is corrected by a factor

$$\frac{e^{\frac{z_0 - z}{L_0}}}{\sinh \frac{z_0 - z}{L_0}}$$

with L_0 an approximate value of L. The corrected flux values are fitted to an exponential; the fit provides a second approximation L_1 to L. New correction factors based on L_1 are applied to the flux values, and the calculation is recycled. This procedure is repeated until it converges.

This method of obtaining L is considerably simpler than fitting a sinh function to each set of flux values.

(2) RADIAL TRAVERSES

As stated before, the foils used for radial traverses are placed in fuel rods. Two such split rods are used. One is used for foils exposed at successive positions on a radius; the other rod contains a monitor foil.

Traverses are made along two separate radii, as shown in figure 1. In measurements with the 1:1, 1.15% lattice, traverses were made along both radii at two separate heights; this was an attempt to discover if the reflector savings is independent of the height in the lattice.

(3) SPONTANEOUS FISSION APPROACHES

The lattices are loaded up to a k_{eff} of about .99, with appropriate flux monitors, safety rod, and trip circuits, and with flux levels maintained by multiplication of neutrons from a Po-Be source. At this point the source is removed from the lattice, and taken away from the vicinity. A small BF_3 counter is inserted at the central fuel channel position, and count rates are recorded for several smaller loaded radii.

Since count rates are generally fairly small (less than 100cpm), counting for several hours is necessary to provide adequate statistics.

RESULTS:

Method 1:

Relaxation lengths measured with 1.3% enriched rods are listed in a previous memorandum to you.[3] Relaxation lengths for lattices with 1.15% rods are listed in tables I - IV. Volume ratios of 4:1, 3:1, 2:1, and 1.5:1 have been measured by this method.

Values of buckling and reflector savings resulting from least squares fits of measured L's to (2) are given in tables V, VI, VII, and VIII, and are plotted in figures 2 and 3. For comparison, the figures also include results obtained earlier with .750" diameter rods, 1% enriched.[1]

3. BNL Log No. C-7471

Method 2:

So far values of B^2 and λ have been obtained by this method for two assemblies, having volume ratios 1.5:1 and 1:1, with 1.15% enrichment. These results are included in tables VII and VIII and figures 2 and 3.

The 1:1 lattice at 1.15% was measured only by this method. The 1.5:1 lattice at 1.15% was measured by both methods 1 and 2.

Method 3:

Only one spontaneous fission approach has been completed so far. The critical loading only can be obtained by this method; B^2 and λ can not be found separately.

The critical approach curve for the 2:1 lattice is shown in figure 4. Of course, the curve depends on the value of λ assumed. We show four such curves corresponding to values of λ between 0 and 7.5 cm. The value of $\frac{1}{(R_c+\lambda)^2}$ indicated by the approach curve depends on the value of λ assumed, but the value of R_c indicated is quite independent of the λ chosen. The indicated critical number of rods for $\lambda = 7.5$ is 471. The critical number for $\lambda = 0$ is 472. Thus for a reasonable value of λ (method 1 gives 7.10 cm.), the critical loading is 471 rods.

Using the values of B^2 and λ determined by method 1, we find a predicted critical size of 467 rods. The difference amounts to about .9% in B^2; This is the same order of magnitude as the random errors in the buckling measurements themselves.

CONCLUSIONS:

From tables V, VI, VII, and VIII it can be seen that the values of B^2 and λ obtained by method 1 at high and low loadings agree quite well,[4] and that further these two sets of values are in agreement to within experimental error with those values obtained from all loadings. We conclude that method 1 gives self-consistent values of B^2 and λ.

In the one case so far in which measurement has been made by methods 1 and 2, B^2 and λ agree to within experimental error.

In the one case so far in which measurement has been made both by methods 1 and 3, the predicted critical sizes agree to within experimental error.

We have been informed[5] that in those cases in which critical assemblies have already been made at WAPD with the 1.3% enriched rods we used, the critical sizes found agree with our predicted values to within our experimental error.

4. Except for the 1:1, 1.3% enrichment lattice. Here the special method used (because of close rod spacings - see earlier in this report) apparently introduced some systematic errors.

5. Private communication from R. Creagan.

We conclude that our methods 1, 2, and 3 of determining criticality agree to within their individual probable errors; it seems reasonable to suppose that the values of B^2 and λ we reported earlier[1] for .750" diameter, 1.027% enriched rods are essentially correct (although the accuracy with which they were measured was not as good as we are now obtaining).

It is interesting to note, as originally pointed out by K. Puechl, that two-group theory predicts that as the loading radius of the assemblies we use increases, λ should decrease. This feature has been explored theoretically by J. Chernick.[6] From his results it can be seen that the expected decrease in λ should be about .2 cm. over the loading ranges we have used.

If this effect were real, its result for the 1.5:1, 1.15% lattice would be to make B^2 as determined by method 1 about 2.5% smaller than that found by method 2. λ as found by method 1 would be about .2 cm. larger than that found by method 2. Actually, the two sets of measurements agree to within their experimental errors, and what small difference exists is in the opposite direction.

6. A summary of the Univac calculations is given in BNL Log No. C-7368

TABLE I

BUCKLING DATA

Enrichment 1.15%

4:1 Volume Ratio

No. Rods	L	No. Rods	L
313	42.153	245	26.048
307	40.305	235	24.602
301	37.554	223	22.964
295	36.126	211	21.563
289	34.910	199	20.402
283	32.781	187	19.278
277	31.558	175	17.888
271	30.441	163	17.044
265	29.171	151	16.115
259	27.975	139	15.314
253	27.473	127	14.304

Loadings	$(B^2 + \sigma B^2) \times 10^{-4}$ cm^2	$\lambda + \sigma\lambda$ cm.
High	35.973 ± .525	6.457 ± .226
Low	35.955 ± .369	6.453 ± .104
All	36.034 ± .158	6.436 ± .061

TABLE II

BUCKLING DATA

Enrichment 1.15% 3:1 Volume Ratio

No. Rods	L	No. Rods	L
295	49.230	223	24.682
289	45.429	211	22.745
283	41.012	199	21.240
277	37.920	187	19.656
271	34.855	175	18.133
265	33.972	163	17.755
259	32.269	151	15.878
253	30.766	139	15.136
247	29.231	127	13.999
241	27.458	115	13.263
235	26.935	103	12.303

Loadings	$(B^2 + \sigma B^2) \times 10^{-4}$ cm.^{2}	$\lambda + \sigma\lambda$ cm.
High	46.819 ± .811	6.759 ± .248
Low	47.518 ± .955	6.662 ± .178
All	47.120 ± .326	6.676 ± .095

TABLE III

BUCKLING DATA

Enrichment 1.15%

2:1 Volume Ratio

No. Rods	L	No. Rods	L
343	38.731	277	24.768
337	34.529	265	23.004
331	33.669	253	21.737
325	32.084	241	20.356
319	30.328	229	19.633
313	29.929	217	18.287
307	29.464	205	17.521
301	28.194	193	16.635
295	27.572	181	15.849
289	26.613	169	14.795
283	25.548	157	14.108

Loadings	$(B^2 + \sigma B^2) \times 10^{-4}$ cm.^{2}	$+\sigma$ cm.
High	47.534 ± 1.097	7.291 ± .297
Low	48.153 ± .633	7.106 ± .119
All	48.215 ± .308	7.104 ± .248

TABLE IV

BUCKLING DATA

Enrichment 1.15% 1.5:1 Volume Ratio

No. Rods	L	No. Rods	L
439	31.057	307	19.706
427	30.127	295	18.927
415	28.597	283	18.216
403	27.156	271	17.533
391	25.881	259	17.037
379	25.001	247	16.196
367	23.896	235	15.794
355	22.717	223	15.144
343	21.923	211	14.673
331	21.194	199	14.025
319	20.478	187	13.664

Loadings	$(B^2 + \sigma B^2) \times 10^{-4} cm.^2$	$+ \sigma$ cm.
High	40.148 ± .552	7.523 ± .164
Low	40.140 ± .681	7.512 ± .124
All	40.229 ± .301	7.499 ± .080

TABLE V

Bucklings of lattices with 1.3% enriched, .600" diameter rods.
(B^2 in $cm^{-2} \times 10^{-4}$)

Volume Ratios	High 11 Loadings	Low 11 Loadings	All Loadings
1:1	31.08 ± 1.24	34.71 ± 1.18	32.11 ± .54
1.5:1	53.52 ± 1.25	52.31 ± .78	51.87 ± .50
2:1	60.88 ± .78	60.58 ± 1.50	61.08 ± .32
3:1	61.16 ± .43	58.84 ± .94	60.99 ± .26
4:1	49.82 ± .75	49.17 ± .69	50.28 ± .27

TABLE VI

Reflector Savings of lattices with 1.3% enriched, .600" diameter rods.
(λ in cm.)

Volume Ratios	High 11 Loadings	Low 11 Loadings	All Loadings
1:1	8.13 ± .27	7.60 ± .16	7.94 ± .10
1.5:1	7.02 ± .32	7.38 ± .16	7.44 ± .10
2:1	7.08 ± .16	7.11 ± .21	7.04 ± .06
3:1	6.67 ± .09	6.94 ± .12	6.70 ± .05
4:1	6.80 ± .21	6.82 ± .12	6.64 ± .07

TABLE VII

Bucklings of lattices with 1.15% enriched, .600" diameter rods.
(B^2 in $cm^{-2} \times 10^{-4}$)

Volume Ratios	Method 1: High 11 Loadings	Method 1: Low 11 Loadings	Method 1: All Loadings	Method 2
1:1				21.19 ± .23*
1.5:1	40.15 ± .55	40.14 ± .68	40.23 ± .30	39.87 ± .56
2:1	47.53 ± 1.10	48.15 ± .63	48.22 ± .31	
3:1	46.82 ± .81	47.52 ± .96	47.12 ± .33	
4:1	35.97 ± .53	35.96 ± .37	36.03 ± .16	

* In all cases probable errors are derived from statistical fluctuations only.

TABLE VIII

Reflector Savings of lattices with 1.15% enriched, .600" diameter rods.
(λ in cm.)

Volume Ratios	Method 1: High 11 Loadings	Method 1: Low 11 Loadings	Method 1: All Loadings	Method 2
1:1				8.15 ± .13
1.5:1	7.52 ± .16	7.51 ± .12	7.50 ± .08	7.62 ± .18
2:1	7.29 ± .30	7.11 ± .12	7.10 ± .25	
3:1	6.76 ± .25	6.66 ± .18	6.68 ± .10	
4:1	6.46 ± .06	6.46 ± .23	6.45 ± .10	

PATTERN OF RADIAL TRAVERSES

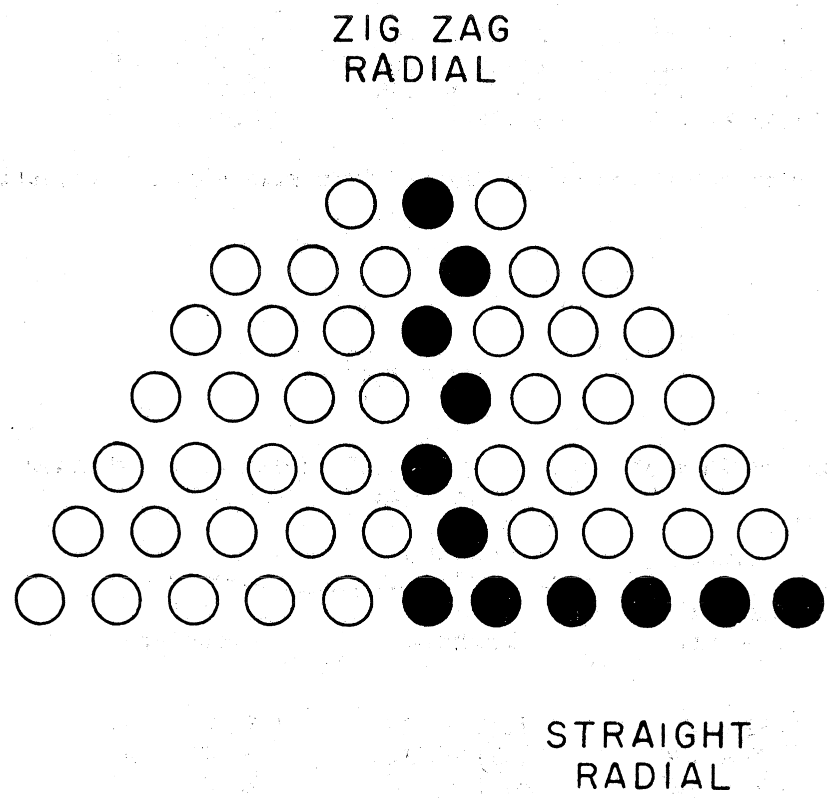
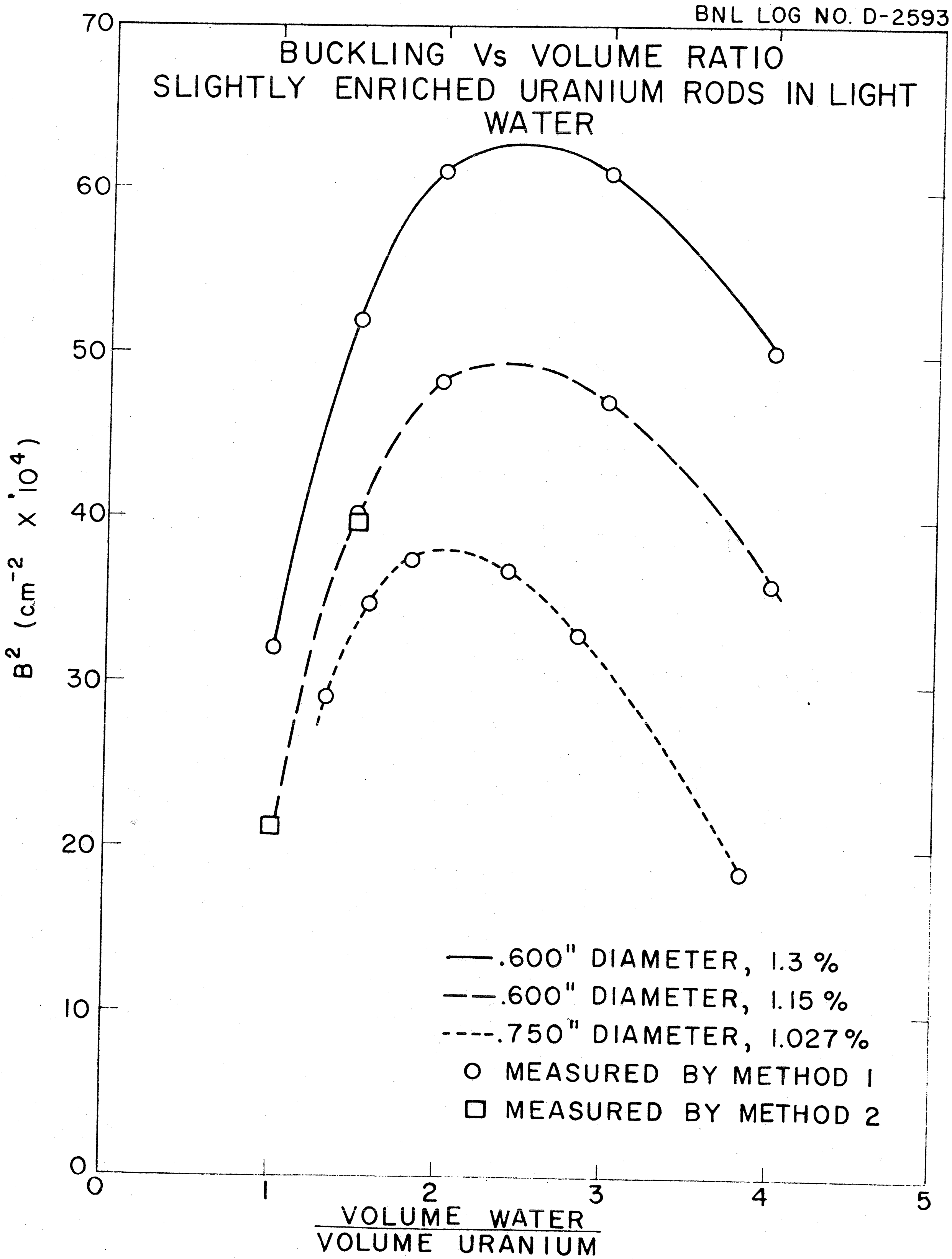


FIG. 1



BNL LOG NO. D-2593
BUCKLING Vs VOLUME RATIO
SLIGHTLY ENRICHED URANIUM RODS IN LIGHT WATER
B^2 (cm^{-2} x 10^4)
70
60
50
40
30
20
10
0
0
1
2
3
4
5
VOLUME WATER / VOLUME URANIUM
.600" DIAMETER, 1.3 %
.600" DIAMETER, 1.15 %
.750" DIAMETER, 1.027 %
MEASURED BY METHOD 1
MEASURED BY METHOD 2

FIG. 2

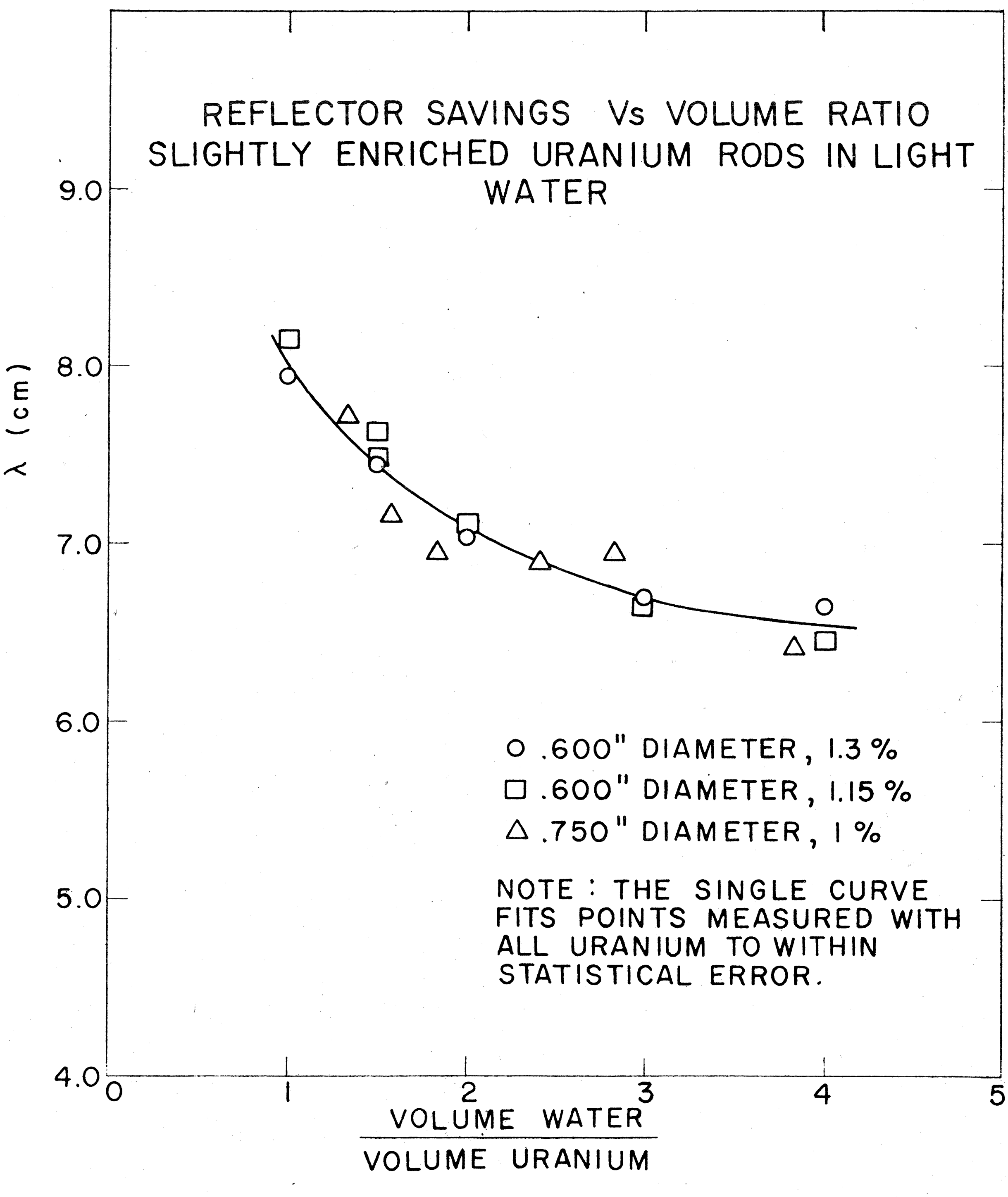


FIG. 3

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