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STRONG-FOCUSING COCKROFT-WALTON ACCELERATOR

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STRONG-FOCUSING COCKCROFT-WALTON ACCELERATOR

John M. Wilcox

December, 1955

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ABSTRACT

The properties of a strong-focusing Cockcroft-Walton accelerator have been calculated. Permanent quadrupole magnets would be installed in each drift tube. It appears that the space-charge repulsion can be overcome so that the machine can accelerate a proton beam of 50 milliamperes or more.

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INTRODUCTION

At the suggestion of James D. Gow, the properties of a Cockcroft-Walton accelerator tube employing permanent strong-focusing magnets have been investigated. Blewett, Christofilos, and Vash,[1] working at Brookhaven National Laboratory, have developed a technique for producing a permanent quadrupole field in small cylindrical magnets by pulsing an extremely large current through suitably shaped electrodes inside the cylinder. In May of 1955 they had magnetized a 1-inch-long cylinder of Indox, 1 inch i.d. and 2.5 inches o.d., to produce a field gradient of 1035 gauss/cm. Such magnets should be ideally suited for insertion in the drift tubes of a Cockcroft-Walton machine. Their focusing is weak compared to that in alternating-gradient synchrotrons, but the resulting small angular divergence of the beam is very desirable.

J. P. Blewett[2] has suggested the use of a succession of electric or magnetic quadrupole lenses to overcome the gap defocusing in a linear accelerator, and a treatment of this problem by means of an impulse-approximation method has been made by Johnston[3] for the Minnesota linac. This impulse method has been applied to the problem described here.

The injection system for the Bevatron consists of a PIG ion source, a Cockcroft-Walton tube to 460 kev, a proton "buncher" to increase the acceptance of the linac, and a linear accelerator to 10 Mev. The present Cockcroft-Walton is limited by the space-charge repulsion of the beam to a beam current of a few milliamperes. If this repulsion can be overcome by strong focusing, the large increase in the beam through the Cockcroft-Walton that would be achieved should be reflected in a proportional increase in the Bevatron output current.

PROPOSED MACHINE

The present proposal is to place a permanent quadrupole magnet in the middle of each drift tube of a Cockcroft-Walton machine, which is otherwise quite similar to the one presently in use at the Bevatron. This would save a considerable amount of engineering and drafting, since many of the existing drawings could be used in the new machine. The specifications would be (see Fig. 1):

Cockcroft-Walton Accelerator

L = length of sections = repeat length = 2.75 inches

ℓ = length of quadrupole magnets = 1.5 inches

R = radius of electrostatic cylinders = 0.75 inches

number of sections = 20

injection energy = 60 kv

final energy = 460 kv

length of tube = 55 inches

k = gradient of quadrupole magnets = 700 gauss/cm

I = proton current = 50 milliamperes

r_0 = initial radius of injected beam = 0.5 inch

r_f = final radius of beam = approx. 3/16 inch

quadrupole sequence: N S N S N S

BEAM ORBIT CALCULATIONS

The beam orbit calculation proceeds in the following manner. There are three forces acting on a beam proton, (1) the space-charge repulsion, (2) the strong-focusing forces, and (3) the electrostatic focusing forces at the gaps. The latter become negligible about halfway down the tube. The total radial momentum received by a proton in passing through one drift tube is lumped into an impulse, assumed to be applied at the center of the tube. The resultant trajectory is a series of straight lines with breaks at the impulse points. This method is of course approximate, but it has the advantage that the calculations can be handled in a finite amount of time and many geometries can be investgated. The greatest error found by the Minnesota group with this method was 15%.

The slope of the trajectory is described as dr/dn, where n is the ordinal number of each drift tube (Z = nL), and the change in slope occurring at an impulse point is Δ dr/dn. Since the impulse received from each of the above forces is proportional to the radial displacement r, we can define a deflection

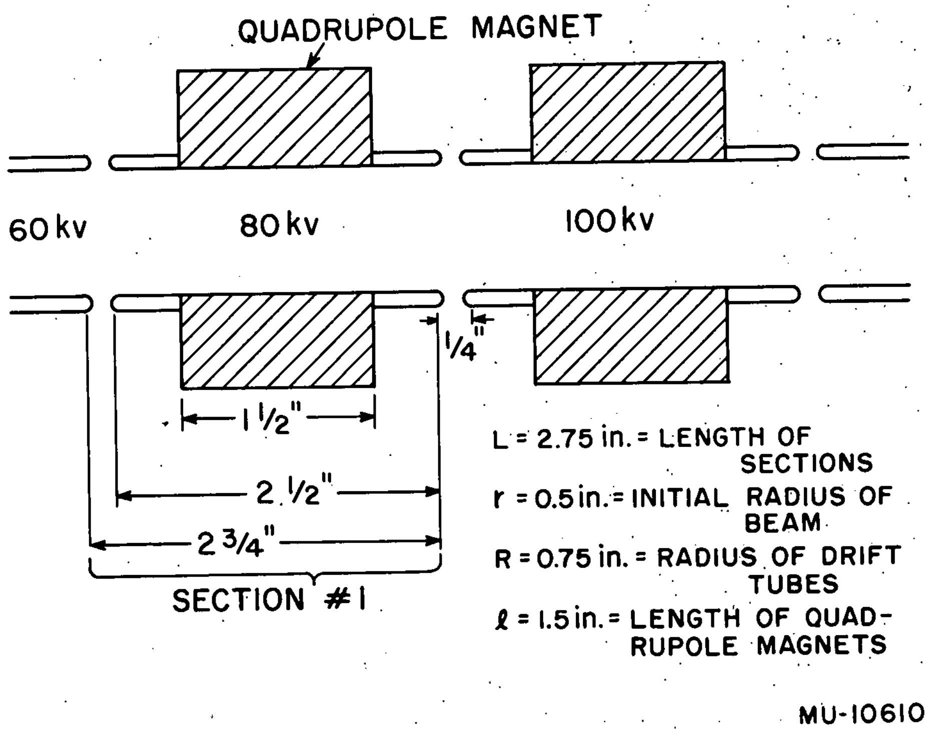

Fig. 1. Drift tube geometry.

constant D such that

$$D_s = \left(\frac{1}{r}\ \Delta\, dr/dn\right)_{\text{space charge}}$$ change of slope due to space-charge repulsion (1)

$$D_q = \left(\frac{1}{r}\ \Delta\, dr/dn\right)_{\text{quadrupole lens}}$$ change of slope due to strong-focusing magnets (2)

$$D_{es} = \left(\frac{1}{r}\ \Delta\, dr/dn\right)_{\substack{\text{electrostatic}\\ \text{lens}}}$$ change of slope due to electrostatic focusing (3)

Note that a negative value of D implies focusing and a positive value implies defocusing.

In some cases we can calculate D by noting that it is related to the focal length F of a lens element by

$$\frac{L}{F} = D = \frac{1}{r}\ \Delta\, dr/dn, \tag{4}$$

where L is the drift tube repeat length.

The value of D_s is derived in Appendix A for a beam with cylindrical symmetry as

$$D_s = \frac{L^2}{r_n^2}\ \frac{I}{v^3}\ \frac{e}{2\pi k_0 m}\ \Delta n, \tag{5}$$

where L = drift tube repeat length,

r_n = beam radius,

I = beam current,

v = beam velocity,

e = charge of beam particles,

m = mass of beam particles,

k_0 = permittivity of free space.

After the beam has passed through the first quadrupole lens, it will no longer have a circular cross section but will have a larger radius in the x direction and a smaller radius in the y direction (see Fig. 2). This effect remains and increases as the beam goes down the tube. For this calculation the maximum and

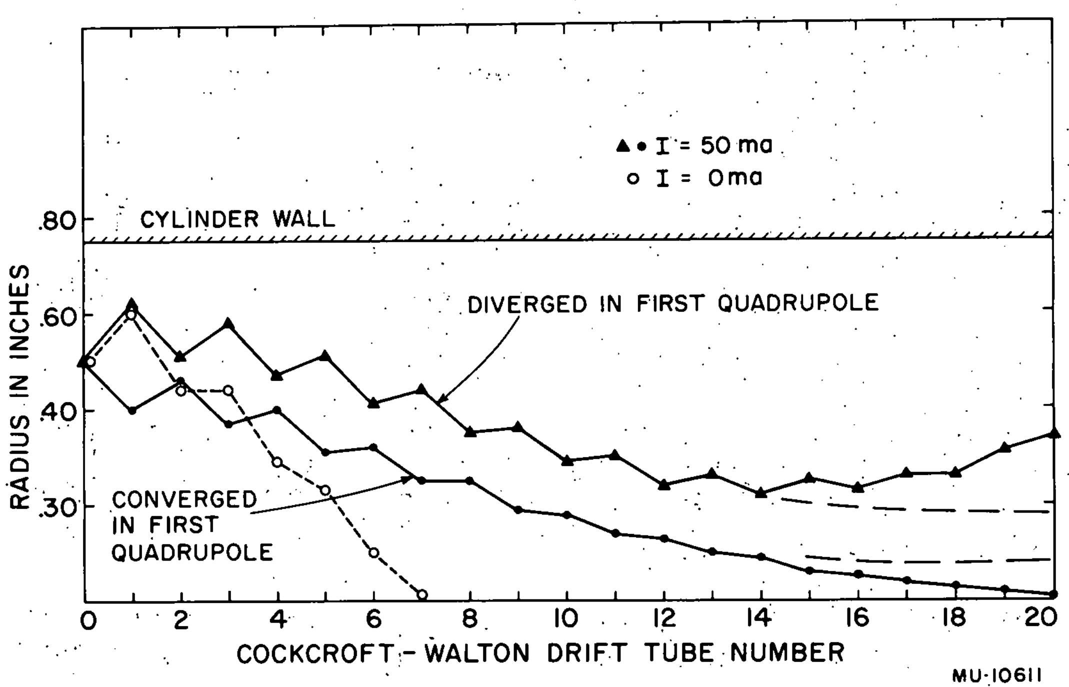

Fig. 2. Beam trajectory for 50 ma current.

minimum radii have been averaged to give the radius inserted in Eq. (5). As long as these two radii are not greatly different this should be a good approximation, but near the end of the tube it tends to break down. Of course the space-charge force becomes nonlinear as soon as the beam departs from cylindrical symmetry anyway.

The value of D_q is derived in Appendix B as

$$D_q = \frac{L\,k^{1/2}}{p^{1/2}} \sin \frac{k^{1/2}\ell}{p^{1/2}} \simeq \frac{k\ell L}{p}, \tag{6}$$

where L = drift tube repeat length,

ℓ = length of quadrupole magnet,

k = field gradient of quadrupole magnet,

p = momentum of beam particle (gauss-cm).

The value of D_{es} is derived in Appendix C, by use of Eq. (4) between D and the focal length of a cylindrical electrostatic lens. Since the lens strengths involved are weaker than those plotted in graphs such as given by Terman[4] or Zworykin,[5] the focal length had to be calculated with the aid of some numerical theory developed by Zworykin.

Note that as a proton goes down the tube the space-charge repulsion D_s decreases as $1/V^{3/2}$ while the strong focusing D_q decreases as $1/V^{1/2}$, where V is the voltage. However, as the beam is focused, the space-charge repulsion increases as $1/r^2$, so that in fact D_s and D_q may remain roughly proportional. As mentioned above, the electrostatic focusing D_{es} becomes negligible after the first few drift tubes.

We begin the numerical calculations by assuming an initial radius and slope, and calculating the effects of the deflections. At each deflection point, the new slope is obtained from

$$(dr/dn)_{n+} = (dr/dn)_{n-} + D_n\, r_n, \tag{7}$$

and the new radius is calculated from the old radius and the slope by

$$r_{n+1} = r_n + (dr/dn)_{n+}\,\Delta n; \tag{8}$$

and since $\Delta n = 1$ between drift tubes,

$$r_{n+1} = r_n + (dr/dn)_{n+}.$$

The values of D_n used in Eq. (7) is the algebraic sum of D_s, D_q, and D_{es}.

RESULTS AND DISCUSSION

A trajectory plotted in this manner is shown in Fig. 2. When the injected beam comes into the first quadrupole magnet, one section of it is diverged and the sections at right angles are converged. This distortion of the beam is due to a failure of the space-charge approximation used, which overestimates the diverging effect on the initially divergent beam and underestimates the diverging effect on the initially convergent beam. The dashed lines in Fig. 2 indicate the general trend of the actual forces on the beam. In practice the beam will have to be adjusted experimentally by changing the divergence or convergence of the injected beam from the ion source, and by having the final quadrupole magnets electromagnets so that their strengths can be adjusted.

The quadrupole lenses produce relatively large local changes in the angular deflection of the beam. Therefore, as suggested by Johnston,[3] the first lens is made half-strength in order to decrease the initial enlargement of a parallel injected beam.

Figure 2 shows the trajectory for an initially parallel beam of 50 ma, and Fig. 3 is the same for a beam of 75 ma. The open circles in Fig. 2 indicate the trajectory for a zero-current beam, i.e., no space-charge repulsion. The qualitative behavior of the system can be inferred from these figures. A considerable variation in the injected beam current can still be focused, but a change in ion-source current from pulse to pulse will be reflected in a change of the size and divergence of the output beam. Thus the ion source should be induced to give a uniform output from pulse to pulse, and also each individual pulse should be steady, i.e., have a rectangular waveform output. Then by adjusting the divergence or convergence of the injected beam and trimming the final electromagnet quadrupoles one can maximize the output beam. It is apparent from the figures that an acceptance angle of a few degrees can be tolerated.

The divergence of a 50-ma beam in the absence of the strong-focusing lenses is shown in Fig. 4.

Further calculations are proceeding on the above points.

It is planned to use the titanium-discharge ion source developed by Ruby and Gow.[6] The mass spectrum from this source includes a considerable fraction of heavy ions, including those of titanium. Equation (5) for the space-charge force includes the factors

$$\frac{1}{mv^3} \sim \frac{1}{1/2\, m\, v^2 \cdot v} \sim \frac{1}{V\sqrt{\frac{2V}{m}}} \qquad \frac{\sqrt{m}}{V^{3/2}} ,$$

where V is the voltage. Thus since all particles will have the same energy V, the heavy particles will contribute to the space-charge repulsion as $\sqrt{m}$, i.e., one milliampere of titanium ions would contribute as much repulsion as 6.9 milliamperes of protons. Thus the use of some form of ion trap in the injected beam will be desirable.

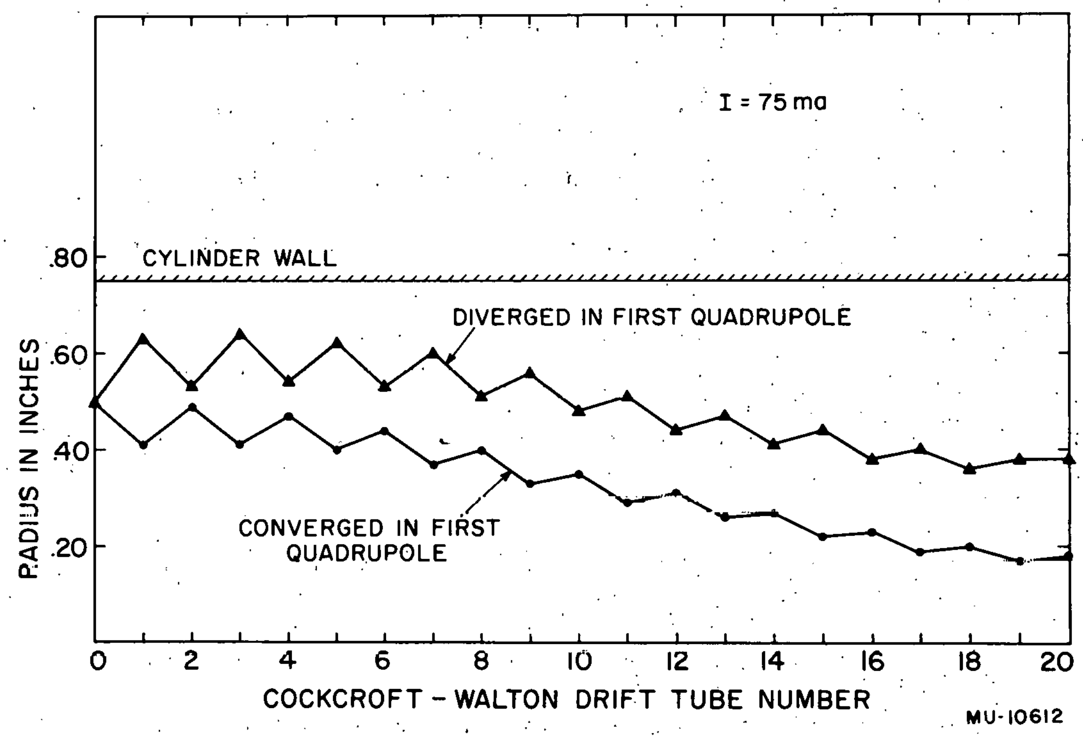

Fig. 3. Beam trajectory for 75 ma current.

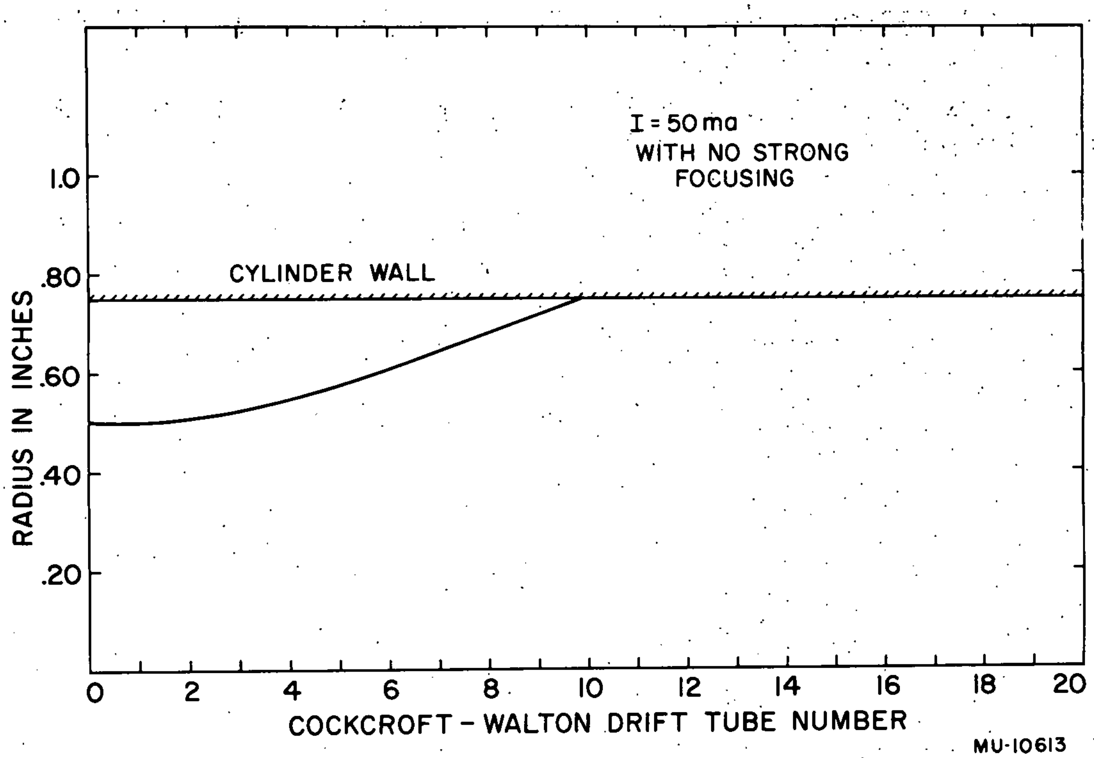

Fig. 4. Beam trajectory without strong focusing.

Breakdowns caused by an electron avalanche traveling in the reverse direction in the tube should be greatly inhibited by the bending action of the quadrupole magnets.

The effect of random errors has been discussed by Smith and Gluckstern.[7] They consider the following types of imperfections:

(a) displacement and tilt of the axis of zero field,

(b) angular misalignment of the transverse axes,

(c) fluctuations of lens strengths about computed values.

Smith and Gluckstern emphasize that extreme care must be taken in the mechanical alignment of the system in order to realize the maximum beam output.

This work was done under the auspices of the U. S. Atomic Energy Commission.

APPENDIX

A. SPACE-CHARGE REPULSION CONSTANT

The radial impulse received by a beam particle from space-charge repulsion while traveling a length L in the Z direction is

$$\Delta p_r = e\, E_r\, \Delta t, \qquad \text{(a)}$$

where $$\Delta t = \frac{\Delta Z}{v} = \frac{L \Delta n}{v}, \qquad \text{(b)}$$

and from Poisson's equation for cylindrical symmetry

$$E_r = -\frac{\partial V}{\partial r} = \frac{\rho r}{2K_0}; \qquad \text{(c)}$$

where ρ is the charge density and K_0 is the permittivity of free space.

$$\text{Now} \quad \Delta\left(\frac{dr}{dn}\right) = \Delta\left(L\,\frac{dr}{dZ}\right)$$

$$= \Delta\left(L\,\frac{dr}{dt}\,\frac{dt}{dZ}\right)$$

$$= \frac{L}{v}\,\Delta\left(\frac{dr}{dt}\right)$$

$$= \frac{L}{v}\,\frac{\Delta p_r}{m} \qquad \text{(d)}$$

Now D_s is defined as $\frac{1}{r}\,\Delta\left(\frac{dr}{dn}\right)_{\text{space charge}}$, and, substituting in Eqs. (a), (b), (c), and (d), we have

$$D_s = \frac{1}{r}\,\Delta\left(\frac{dr}{dn}\right) = \frac{L^2}{v^2}\,\frac{e\rho}{2K_0 m}\,\Delta n. \qquad \text{(e)}$$

Assuming a uniform charge distribution, we have

$$\rho = \frac{I}{\pi r_n^2 v} \qquad \text{(f)}$$

and we have Eq. (5)

$$D_s = \frac{L^2}{r_n^2} \frac{I}{v^3} \frac{e}{2\pi K_0 m} \Delta n. \tag{5}$$

As a general check on the calculations and on the impulse approximation, the spreading of an initially parallel proton beam of 10 ma at 100 kv was calculated, and found to agree within a few percent with the analytic results of E. R. Harrison.[8] (See Fig. 5.)

B. ALTERNATING-GRADIENT DEFLECTION CONSTANT

From the definition of the deflection constant D we have

D = L/f, where f is the focal length of a lens, and L is the repeat length.

Courant et al.[9] give, for the focal length of a quadrupole lens,

$$f = \frac{1}{K^{1/2} \sin (K^{1/2} \ell)}, \tag{15}$$

where

$$K = \frac{1}{BR} \frac{d B_x}{d y}, \tag{16}$$

ℓ = length of quadrupole magnet,

p = BR; R is radius of curvature in field B,

B_x = ky for the quadrupole magnet.

$$\therefore \quad K = \frac{k}{p} \quad \text{and } f = \frac{p^{1/2}}{k^{1/2} \sin\left(\frac{k^{1/2}\ell}{p^{1/2}}\right)} .$$

Thus

$$D_q = \frac{L}{f} = \frac{L\, k^{1/2}}{p^{1/2}} \sin \frac{k^{1/2}\ell}{p^{1/2}} \simeq \frac{k \ell L}{p} .$$

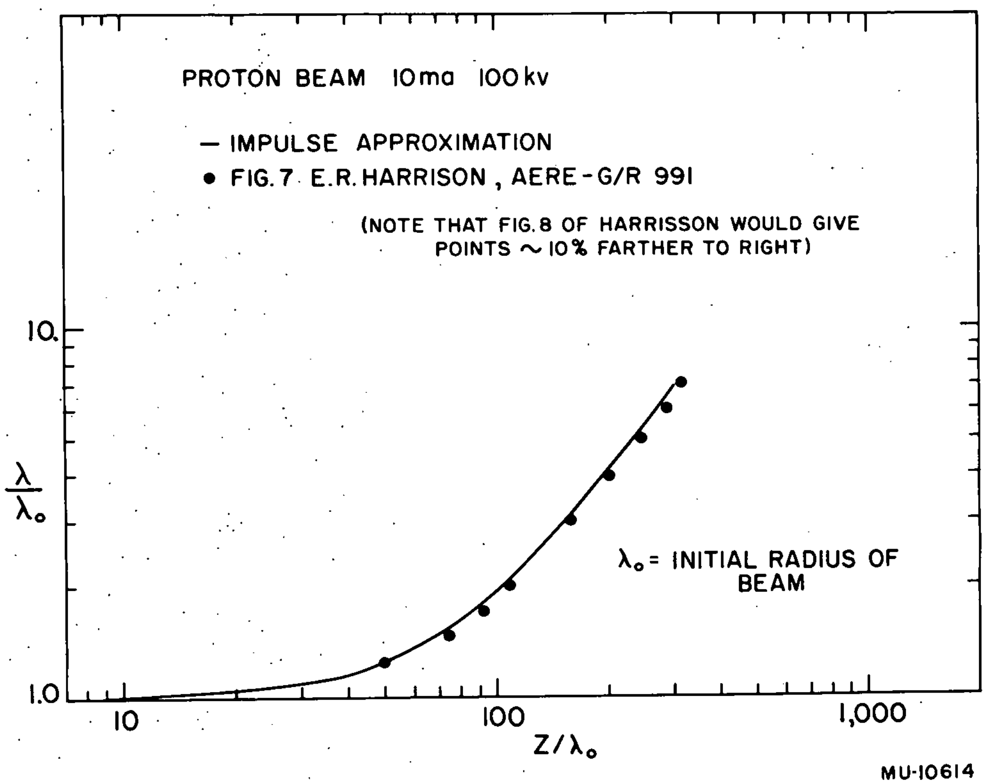

Fig. 5. Beam divergence caused by space charge.

Alternatively, D_q can be derived in the manner of Appendix A. The radial impulse received by a beam particle in passing through a quadrupole magnet of length ℓ is

$$\Delta p_y = F_y \Delta t = \frac{B_x e v}{c} \Delta t, \qquad \text{(a)}$$

with $B_x = k y$ and $\Delta t = \frac{\ell}{v}$,

where z is measured down the tube, and x and y are orthogonal.

we have $\Delta p_y = \frac{k y e \ell}{c}$, (b)

and as developed in Appendix A,

$$\Delta \left(\frac{dy}{dn}\right) = \frac{L}{v} \frac{\Delta p_y}{m} . \qquad \text{(c)}$$

Substituting, we obtain

$$D_q = \frac{1}{y} \Delta \left(\frac{dy}{dn}\right) = \frac{k e \ell L}{m v c} . \qquad \text{(d)}$$

Now Force $= \frac{B e v}{c} = \frac{m v^2}{r}$,

and $p = B r = m v \frac{c}{e}$.

Then $D_q = \frac{k \ell L}{m v \frac{c}{e}} = \frac{k \ell L}{p}$,

which agrees with the previous results.

C. ELECTROSTATIC FOCUSING CONSTANT

The deflection constant D is again calculated from the relationship $D = L/f$, where f is the focal length of an equal-diameter cylindrical lens. This focal length is plotted by Terman[4] and Zworykin,[5] but not for the weak lenses encountered here. Therefore it has been computed with the aid of some theory developed by Zworykin.[5] We deal with a small value of V_{object}/V_{image} so that the lens is weak and can be considered a thin lens. For this case, Formula 13.35 on page 437 of Zworykin gives

$$\frac{1}{f} = \frac{3}{16} \left(\frac{V_{ob}}{V_{im}}\right)^{1/4} \int_a^b \left(\frac{V'}{V}\right)^2 \, dZ, \qquad \text{(a)}$$

where the integral is taken over the gap. Also on page 379, Fig. 11.10 (b), Zworykin has plotted the voltage V and derivative V' over the gap between two equal-diameter cylinders. Thus a simple numerical integration can be done to yield the required focal lengths, which are shown in Fig. 6. Some focal lengths were computed by this method in the range included in the graph in Terman[4] and found to agree.

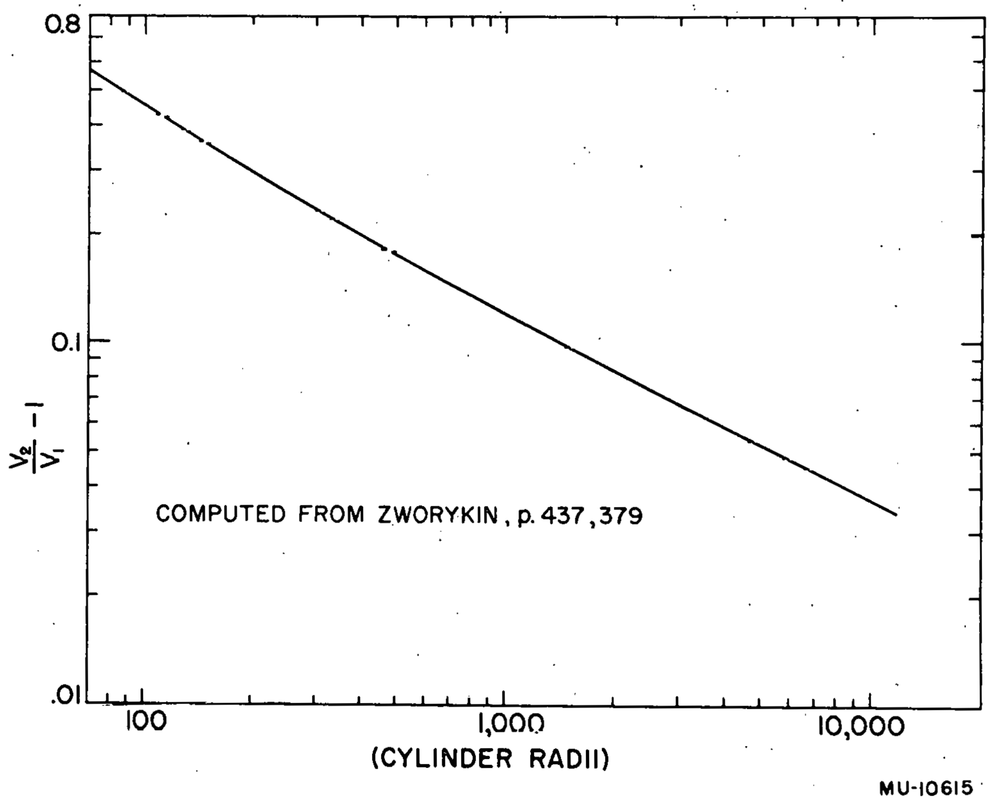

Fig. 6. Focal length of equal-diameter cylindrical electric lenses.

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