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THE REACTION $\pi^- p \to \pi^- \pi^- \pi^+ p$ AT 25 AND 40 GeV/c

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ABSTRACT

The reaction $\pi^- p \to \pi^- \pi^- \pi^+ p$ has been measured at 25 and 40 GeV/c at the Serpukhov Proton Accelerator. The production cross-section as a function of beam energy, momentum transfer, and 3π mass is presented, and results of the partial-wave analysis of the 3π system are discussed.

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1. INTRODUCTION

During the period 1970-72 the CERN-IHEP Boson Spectrometer has been operating in a 25-40 GeV/c negative beam of the Serpukhov Proton Accelerator. Data on the inclusive reaction $\pi^- p \to X^- p$ at 25 and 40 GeV/c have been published[1,2]. Here we present results about the $\pi^-\pi^-\pi^+$ decay mode of the X^- system, for low X^- masses.

The events are measured in the spectrometer[3], identified by a kinematic fit, and analysed by the partial-wave method[4]. Detailed results about production and decay of the A systems (i.e. A_1, A_2, and A_3), which are strongly produced, are presented in a separate paper[5]. Here we discuss a few general features of the reaction $\pi^- p \to \pi^-\pi^-\pi^+ p$ at high energies.

i) energy and momentum-transfer dependence of the cross-section as a function of the 3π effective mass;

ii) partial-wave decomposition of the 3π system.

2. SELECTION AND IDENTIFICATION OF THE EVENTS

The trigger (see Ref. 1) demands an incident beam particle interacting inside the liquid hydrogen target, thereby producing a slow recoil proton, which is detected in the proton telescope. In the off-line analysis, events are selected that satisfy the following criteria:

i) an incident particle in the beam hodoscopes;

ii) a proton, momentum analysed by time of flight and measured by the two wire spark chambers of the proton telescope; the proton track has to intersect with the beam track inside the liquid hydrogen target ("proton vertex");

iii) three tracks in the magnetic spectrometer, originating from the proton vertex; at least two of them have to be measured in the spark chambers behind the magnet and are thus momentum analysed;

iv) the χ^2 probability obtained by fitting the hypothesis $\pi^- p \to \pi^-\pi^-\pi^+ p$ to the data (using the constraints of energy- and momentum-conservation) exceeds 0.02.

The number of events thus selected for the different runs at 25 and 40 GeV/c are listed in Table 1.

In the selected sample the background from events $\pi^- p \to \pi^- K^- K^+ p$ and $\pi^- p \to \pi^- \bar{p}pp$ does not exceed 1%; the background from reactions with missing neutral particles is less than 2% [3].

The 3π mass resolution in the A_1-A_2 region is of the order of $\sigma \doteq 10$ MeV and 15 MeV at 25 GeV/c and 40 GeV/c, respectively.

3. PARTIAL-WAVE METHOD

The partial-wave method is used to extract the physical information from the data, taking into account the acceptance and inefficiencies of the apparatus.

As we are interested in the production and the decay of the 3π system, we write the cross-section W for the reaction $\pi^- p \to \pi^- \pi^- \pi^+ p$, as a product of the density matrix ρ, which describes the production of the 3π system and the probability MM^* that the 3π system decays into three π mesons:

$$W(stm_{3\pi}s_1s_2\alpha\beta\gamma) = \sum_{a,b} M_a \, \rho_{ab} \, M_b^* \quad (1)$$

i) s, t, and $m_{3\pi}$ are the production variables of the 3π system (s is the total centre-of-mass energy squared; t the four-momentum transfer squared to the proton, $m_{3\pi}$ the mass of the 3π system).

ii) $s_1, s_2, \alpha, \beta, \gamma$ are the decay variables of the 3π system ($s_1 s_2$ the coordinates on the Dalitz plot, which completely specify the internal orientation of the three pions; α, β, γ, the Euler angles which specify their external orientation relative to fixed space axes).

iii) $\rho_{ab}(stm_{3\pi})$ ($a \equiv J^P M \ell S$) is the density matrix for the production of 3π systems with spin, parity, and polarization $J^P M$, which decay via the ℓ-wave into $(\pi^+\pi^-)$ systems with spin S. Its real diagonal elements ρ_{aa} are the production probability of the state $|a\rangle$, and its complex non-diagonal elements ρ_{ab} measure the interference between the two states $|a\rangle$ and $|b\rangle$ [$|\rho_{ab}|$ being the intensity of the inteference, arg (ρ_{ab}) the relative phase between the two states].

iv) $M_a(s_1 s_2 \alpha\beta\gamma)$ is the amplitude, symmetric under interchange of the two negative pions, for the ℓ-wave decay of the $J^P M$ state into a $(\pi^+\pi^-)$ system with spin S. The explicit form used is

$$M_a(s_1 s_2 \alpha\beta\gamma) = \sum_{m\mu} (JM|\ell S m\mu)\ p_1^\ell Y_\ell^m(\theta_1\phi_1)\ A_S(s_1)\ q_1^S Y_S^\mu(\bar{\theta}_1\bar{\phi}_1) + (1 \leftrightarrow 2), \quad (2)$$

where: $\theta_1\phi_1 p_1$ are the angles and momentum of $(\pi^+\pi_2^-)$ in the 3π frame; $\bar{\theta}_1\bar{\phi}_1 q_1$ are the angles and momentum of the π^+ in the $\pi^+\pi_2^-$ frame and $A_S(s_1)$ is the dipion propagator (see Table 2).

The dipion systems of spin S are parametrized by the known $\pi^+\pi^-$ resonances of spin S = 0, 1, 2. Higher spin states, as well as interactions between the two negative pions, are neglected. To reduce the number of free parameters of our model we make further assumptions*):

i) partial waves with the same spin, parity, and polarization $J^P M$, but different ℓS-decay modes are coherent;

ii) partial waves with the same spin and parity J^P have the same polarization.

Assumption (i) has been tested for several waves [e.g. $0^-(S/P)$**), $1^+(S/P)$, $2^+(D/P)$, $2^-(S/P)$] in several $m_{3\pi}$ intervals, and found to be correct within the statistical sensitivity of the data. Assumption (ii) has not been tested: it is, however, not very relevant to our data, as we find essentially only one M-value for each spin-parity state.

The maximum likelihood method is used to fit expression (1) for the different s, t, and $m_{3\pi}$ bins to the data, in order to obtain the unknown parameters, taking into account the positivity conditions on the density matrix as well as the acceptance and inefficiencies of the spectrometer.

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*) Mathematically we can express these assumptions by the formula

$$\rho_{ab} = C_i\ \tilde{\rho}_{cd}\ C_j^* ,$$

where: $a \equiv J_1^{P_1} M_1 S_1 \ell_1$, $b \equiv J_2^{P_2} M_2 S_2 \ell_2$

$c \equiv J_1^{P_1} M_1$, $d \equiv J_2^{P_2} M_2$

$i \equiv J_1^{P_1}\ S_1 \ell_1$, $j \equiv J_2^{P_2}\ S_2 \ell_2$.

The C's are complex constants independent of the decay variables of the 3π system.

The following effects have been corrected for:

i) geometrical acceptance of the proton telescope;

ii) geometrical acceptance of the magnetic spectrometer;

iii) efficiencies of the spark chambers and the track-finding program;

iv) absorption of pions in the hydrogen target, and the decay of pions in the magnetic spectrometer.

By maximizing the logarithm of the likelihood function we obtain the best estimates for the unknown parameters of our model and the acceptance for the s, t, and $m_{3\pi}$ interval investigated. Figure 1 shows the acceptance of the magnetic spectrometer as a function of 3π mass at 25 and 40 GeV/c.

4. RESULTS CONCERNING THE REACTION $\pi^- p \to \pi^- \pi^- \pi^+ p$

4.1 Effective mass spectra

Figures 2a and 2b show the 3π mass spectra corrected for the acceptance of the magnetic spectrometer for the two momentum transfer intervals $0.04 < |t| < 0.17$ $(\text{GeV}/c)^2$ and $0.17 < |t| < 0.33$ $(\text{GeV}/c)^2$ measured at 40 GeV/c. In the low t-interval, we find two broad structures known as

$$A_1:\ m \approx 1150\ \text{MeV}\ ,\quad \Gamma \approx 300\ \text{MeV}\qquad (J^P = 1^+)$$

$$A_3:\ m \approx 1650\ \text{MeV}\ ,\quad \Gamma \approx 350\ \text{MeV}\qquad (J^P = 2^-)\ .$$

In the high t-interval there is, in addition, evidence for production of the

$$A_2:\ m \approx 1310\ \text{MeV}\ ,\quad \Gamma \approx 100\ \text{MeV}\qquad (J^P = 2^+)\ .$$

The $\pi^+\pi^-$ mass spectrum of Fig. 2c is dominated by the ρ meson (m = 769 MeV, Γ = 138 MeV) and shows evidence for production of the f meson (m = 1265 MeV, Γ = 154 MeV). Demanding that at least one of the $\pi^+\pi^-$ mass combinations is in the ρ band (665 MeV < $m_{\pi^+\pi^-}$ < 865 MeV) yields the shaded spectra of Fig. 2. No significant structure is seen in the $\pi^-\pi^-$ mass spectrum of Fig. 2d.

4.2 The cross-section of the reaction $\pi^- p \to \pi^- \pi^- \pi^+ p$ as a function of p_{inc}, t, and $m_{3\pi}$

In the limited momentum transfer interval accepted by the apparatus (see Table 1), the differential cross-section is well described by the exponential

$$\frac{d\sigma}{dt} \propto \exp\,(bt)\ .$$

Figure 3 and Table 3 show the dependence of the slope parameter b on 3π mass at 25 and 40 GeV/c. The parameter b decreases smoothly from a value of 15 $(\mathrm{GeV}/c)^{-2}$ at $m_{3\pi} < 1$ GeV, to a value of about 5 $(\mathrm{GeV}/c)^{-2}$ at 2 GeV*). Within the errors, the values found at 25 GeV/c and 40 GeV/c are the same.

Assuming that the exp (bt) dependence can be extrapolated, we calculate the integrated cross-sections, which we display in Table 3. Within the systematic errors, the cross-sections are the same at 25 and at 40 GeV/c. Figure 4 compares the integrated cross-section for different mass bins found in this experiment with results from bubble chamber experiments[6]. For $p_{inc} < 10$ GeV/c, the overlap between low 3π masses and N^* resonances in the $p\pi$ system is large; above 10 GeV/c this effect becomes smaller and the cross-sections decrease only slightly with p_{inc}. Fitting the expression

$$\sigma(m_{3\pi}, p_{inc}) \propto p_{inc}^{\,n(m_{3\pi})}$$

to the data yields the values for n listed in Table 3. The values are close to $n = -0.25 \pm 0.02$, found for elastic scattering $\pi^- p \to \pi^- p$ between 7 and 40 GeV/c [7].

5. RESULTS OF THE PARTIAL-WAVE ANALYSIS OF THE 3π SYSTEM

Here we summarize the main features of the 3π system as obtained by the partial-wave analysis. Details about production and decay of the A systems will be discussed in a separate paper[5]

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*) A similar variation of b with $m_{3\pi}$ has already been observed at lower energies by J. Bartsch et al., Physics Letters 27 B, 336 (1968).

i) The number of partial waves of the 3π system that are significantly different from zero is relatively small. Below a 3π mass of 1.5 GeV these are*,**)

$$J^PM^\eta = 0^-0^+(S \to \varepsilon\pi,\ P \to \rho\pi)\ ,\quad 1^+0^+ \text{ and } 1^+1^+(P \to \varepsilon\pi,\ S \to \rho\pi)\ ,$$

$$2^-0^+(P \to \rho\pi)\ ,\ 2^+1^+(D \to \rho\pi)\ ;$$

above $m_{3\pi} = 1.5$ GeV these are, in addition,

$$2^-0^+(S \to f\pi)\ ,\quad 3^+0^+(D \to \rho\pi,\ P \to f\pi)\ ,\quad 2^+1^+(P \to f\pi)\ .$$

The mass dependence of the main partial waves is shown in Fig. 5. The states $1^+(S)$, $2^+(D)$, and $2^-(S)$ show strong structures, known as A_1, A_2, and A_3. All other waves are either only weakly produced or show no significant structure. One notices that all states have the same naturality $[P(-1)^J]$ as the incident pion with the exception of the A_2 resonance in the 2^+ wave.

ii) Only states which are produced by natural parity exchange***) in the high-energy limit are produced strongly, the upper limit for states produced by unnatural parity exchange being a few per cent[3].

iii) In the Gottfried-Jackson (t-channel helicity) system the different partial waves are mainly produced with the smallest third component of spin, allowed by production via natural parity exchange (i.e. $M = 0$ for the series 0^-, 1^+, 2^-, and $|M| = 1$ for the series 1^-, 2^+, 3^- ...).

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*) We use 3π states $|J^PM^\eta\rangle \propto |J^PM\rangle + \eta P P_\pi (-)^{J+M} |J^P\ -M\rangle$, which are eigenstates of $|J_z|$ (referred to t-channel axes, eigenvalue M) and of reflection in the production plane (eigenvalue ηP_π). The letters S, P, D refer to $\ell = 0,1,2$.

**) In the region below $m_{3\pi} = 1.4$ GeV we have investigated systematically possible contributions from all partial waves with $J \leq 3$, any allowed M^η, with decay modes into $\varepsilon\pi$, $\rho\pi$, $f\pi$ with all allowed values of $\ell \leq 3$[3].

***) If the target proton is unpolarized and the polarization of the recoil proton not measured, parity conservation requires the absence of density matrix elements between states of opposite η (i.e. opposite eigenvalues under reflection in the production plane). A t-channel exchange of a particle X or a Regge pole with natural (unnatural) parity $\sigma = P_X(-)^{J_X} = +1(-1)$ contributes at high energies only to the production of states with $\eta = \sigma$. The $\eta = -\sigma$ amplitudes vanish for $M = 0$ independent of energy, for $M \neq 0$ they are supressed by a factor $\propto (\cos\Theta_t)^{-1} \propto s^{-1}$ relative to the $\eta = \sigma$ amplitudes. Thus identifying $\eta = +1(-1)$ with natural (unnatural) parity exchange is either exact or quite accurate in this experiment (Ref. 8).

iv) The interference between all pairs of partial waves is found to be essentially maximal, suggesting that one of the four possible amplitudes (proton spin-flip, $\eta = \pm 1$; proton non-flip, $\eta = \pm 1$) dominates. This circumstance makes it possible to determine the relative phase between different partial waves with reasonable precision.

REFERENCES AND FOOTNOTES

1) Y.M. Antipov et al., Missing-mass spectra observed in $\pi^- p \rightarrow pX^-$ at 25 and 40 GeV/c, Phys. Letters 40 B, 147 (1972).

2) Y.M. Antipov et al., Differential cross-sections and scaling in the reaction $\pi^- p \rightarrow X^- p$ at 25 and 40 GeV/c, submitted to Nuclear Phys. B. (1973).

3) R. Klanner, Analysis of the reaction $\pi^- p \rightarrow \pi^- \pi^- \pi^+$ at 25 and 40 GeV/c, CERN NP Internal Report 73-9 (1973), and Thesis at University of Munich (1973).

4a) The program to perform the partial-wave analysis was developed at the University of Illinois by G. Ascoli, D.V. Brockway, L. Eisenstein, M.L. Ioffredo, J.D. Hansen, U.E. Kruse and P.F. Schultz. In the following publications the authors applied the method to data of the reaction $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$.

b) G. Ascoli et al., Phys. Rev. Letters 26, 929 (1971).

c) D.V. Brockway, Study of the three-pion final state interactions in the reaction $\pi^- p \rightarrow \pi^- \pi^- \pi^- p^+$ at 5 and 7.5 GeV/c, University of Illinois Report COO-1195-197 (1970).

d) G. Ascoli et al., Summary of the experimental situation regarding A_1, A_2 and A_3 production, submitted to the 16th Int. Conf. on High-Energy Physics, Batavia 6-13 September 1972.

e) G. Ascoli et al., Spin Parity analysis of the A_3, Phys. Rev. D7, 669 (1973).

5) Y.M. Antipov et al., A_1, A_2 and A_3 production in $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$ at 25 and 40 GeV/c, submitted to Nuclear Phys. B (1973).

6) The information about the bubble chamber data comes from, K. Paler, Nuclear Phys. B18, 211 (1970) and Reference 4e) above.

7) Y.M. Antipov et al., Elastic scattering of $\pi^- p$, $K^- p$, and $\bar{p}p$ at 25 and 40 GeV/c, submitted to Nuclear Phys. B, (1973).

8) K. Buchner, G. Dehm, G. Göbel, H. Hupe, F. Wagner, W. Wittek and G. Wolf, Nuclear Phys. B34, 293 (1971) and references therein.
W. Wittek, Untersuchung der Reaktionen $K^+ n \rightarrow K^+ \pi^- p$ bei 3 GeV/c, $K^+ n \rightarrow K^{*0} p$ bei 2.3 und 3 GeV/c und $K^- p \rightarrow \overline{K^{*0}}$ im Primärimpul bereich von 2 bis 11 GeV/c, Thesis at University of Munich (1971).

9) Particle Data Group, Phys. Letters 33 B, (1970).

10) G. Grayer et al. Int. Conf. on Experimental Meson Spectroscopy, Philadelphia 1972, Eds. A.H. Rosenfeld and K-W. Lai (American Institute of Physics, New York, 1972), p. 5.

Table 1

Events used for the analysis

Running period	April 1971	June 1971	June 1971	June 1971	October 1971
Incident momentum [GeV/c]	40	25	25	40	40
Momentum transfer [$(GeV/c)^2$]	0.17-0.33	0.17-0.33	0.10-0.33	0.10-0.33	0.04-0.33
Number of raw triggers	549,800	446,800	410,050	288,550	349,000
Number of 3π events	15,200	15,800	15,550	10,000	15,030

Table 2

Parametrization of the $\pi^+\pi^-$-amplitude with spin S

$$A_S(s_1) = (m_S^2 - s_1 - i m_S \gamma_S)^{-1}$$

$$\gamma_S(s_1) = \Gamma_S \left(\frac{q_1}{q_0}\right)^{2S+1} \frac{m_S}{\sqrt{s_1}}$$

S	Name of resonance	m_S [MeV]	Γ_S [MeV]
0	ε	760	400
1	ρ	769	138
2	f	1265	154

The ε and ρ parameters are from fits to our data, the f^0 parameters are from the literature[9]. An altogether different parametrization of the S = 0 propagator, based on recent $\pi^-\pi^+$ scattering results[10], has also been tried, with no significant changes in any of our results.

Table 3

Slope b of differential cross-section $d\sigma/dt$, integrated cross-section σ and exponent n of the energy dependence for the reaction $\pi^- p \to \pi^- \pi^- \pi^+ p$ versus 3π mass. The systematic errors are given in parentheses.

	25 GeV/c		40 GeV/c		11-40 GeV/c
$m_{3\pi}$ [GeV]	b [GeV/c^{-2}]	σ [μb]	b [GeV/c^{-2}]	σ [μb]	n [$\sigma \propto p_{inc}^{n}$]
0.8-1.0	14.3 ± 0.8	36.6 ± 3.3 (3.7)	14.9 ± 0.6	37.3 ± 2.2 (3.7)	-0.31 ± 0.11
1.0-1.1	11.7 ± 0.5	51.6 ± 3.6 (5.2)	12.6 ± 0.5	50.9 ± 2.3 (5.2)	-0.29 ± 0.10
1.1-1.2	10.0 ± 0.4	68.4 ± 3.4 (7.0)	10.7 ± 0.4	64.5 ± 2.2 (6.5)	
1.2-1.3	8.3 ± 0.4	71.7 ± 3.1 (7.6)	8.5 ± 0.4	63.5 ± 2.1 (6.4)	-0.29 ± 0.10
1.3-1.4	7.3 ± 0.4	56.5 ± 2.6 (7.0)	6.1 ± 0.4	53.9 ± 2.1 (5.5)	
1.4-1.5	7.2 ± 0.6	35.6 ± 2.3 (5.0)	7.2 ± 0.5	30.6 ± 1.5 (3.1)	-0.42 ± 0.11
1.5-1.6	6.2 ± 0.6	35.3 ± 2.1 (5.2)	7.2 ± 0.5	33.2 ± 1.5 (3.4)	
1.6-1.8	6.6 ± 0.5	80.9 ± 3.8 (11.0)	7.2 ± 0.3	72.5 ± 2.1 (7.4)	-0.50 ± 0.11
1.8-2.0	5.1 ± 0.6	48.5 ± 3.2 (7.0)	5.6 ± 0.6	44.0 ± 2.1 (4.6	

Figure captions

Fig. 1 : Acceptance of the magnetic spectrometer for the reaction $\pi^- p \to \pi^- \pi^- \pi^+ p$.

Fig. 2 : Mass spectra measured at 40 GeV/c:

a) $m_{3\pi}$ in the interval $0.04 < |t| < 0.17$ $(GeV/c)^2$, corrected for acceptance,

b) $m_{3\pi}$ in the interval $0.17 < |t| < 0.33$ $(GeV/c)^2$, corrected for acceptance. Events with $m_{\pi^+\pi^-}$ in the ρ band ($0.665 < m_{\pi^+\pi^-} < 0.865$ GeV) are dashed.

c) $m_{\pi^+\pi^-}$ combined data, not corrected for acceptance.

d) $m_{\pi^-\pi^-}$ combined data, not corrected for acceptance.

Fig. 3 : Slope b of the differential cross-section $d\sigma/dt$ versus 3π mass at 25 and 40 GeV/c.

Fig. 4 : Integrated cross-section of the reaction $\pi^- p \to \pi^- \pi^- \pi^+ p$ as a function of incident momentum and 3π mass. Solid lines are the results of the fit described in the text.

Fig. 5 : Mass dependence of different partial waves at 40 GeV/c. The notation used for the 3π states is: $J^P \ell M^\eta$

J^P ... Spin and parity,

M ... 3^{rd} component of J in the Gottfried-Jackson system

ℓ ... angular momentum of $(\pi\pi)^0 - \pi$ decay, of the 3π system,

η ... eigenvalue of reflection operator in the production plane: +(-) natural (unnatural) parity exchange.