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PION-DEUTERON SCATTERING IN THE IMPULSE APPROXIMATION

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ABSTRACT

The pure scattering model of Fernbach, Green, and Watson is used to obtain impulse approximation expressions for the elastic, elastic plus inelastic (closure approximation), and charge exchange (closure approximation) π-D differential cross sections. The net interference where p-wave scattering is dominant is found to be constructive; the interference due to charge exchange scattering is always negative. The Coulomb effect gives rise to strong destructive interference in the π^+-D elastic component at 85 Mev (laboratory energy of incident pion) for angles $< 45^o$. The multiple scattering correction

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as calculated on Brueckner's model is approximately 10% of the free particle cross section and positive. However, a perturbation field-theoretic treatment of the $(\frac{3}{2}, \frac{3}{2})$ double scattering indicates that Brueckner's model is unreliable at low energies. The binding correction to the impulse approximation is calculated using a p-state interaction hamiltonian (pv coupling) and found to be -3.2 mb at 85 Mev. The forward peak in the elastic differential cross section is found appreciably reduced by destructive interference arising from elastic ''absorptive'' scattering. It is concluded that, at energies $\lesssim$ 100 Mev, absorption corrections are significant, with the multiple scattering correction becoming important at higher energies.

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I. INTRODUCTION

A new experiment on the scattering of positive pions by deuterons at 85 Mev, described in detail in the accompanying paper[1], has motivated the following study of pion-deuteron

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[1] K. Rogers and L. M. Lederman, Phys. Rev. (to be published), hereafter referred to as Paper I.

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scattering. In view of the present improved status of the phase shift analysis of pion-nucleon scattering[2], it is of

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[2] J. Orear, Phys. Rev. 100, 288 (1955).

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interest to carry through the method of approach adopted by Fernbach et al[3]: the problem is formulated in terms of the

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[3] Fernbach, Green, and Watson, Phys. Rev. 84, 1084 (1951), hereafter referred to as FGW.

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impulse approximation[4] with a phenomenological analysis being

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[4] G. Chew and G. Wick, Phys. Rev. 85, 636 (1952); G. Chew and M. L. Goldberger, Phys. Rev. 87, 778 (1952).

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made of the leading terms[5] in the expansion of the transition

[5] Eq. (10), ref. 4 (Chew and Wick); Eqs (13) and (21), ref. 4 (Chew and Goldberger).

operator for the complex system. In the following, the appellation ''usual'' impulse approximation is given to these leading terms which form a linear superposition of the two particle operators referring to the scattering of mesons by single free nucleons. As the work of FGW is essentially independent of any detailed assumptions about the individual scattered amplitudes, their conclusions regarding the use of further simplifying approximations constitute a valid basis for our treatment.

In Section II, the pure scattering model of FGW is used to derive expressions for the elastic, elastic plus inelastic, and charge exchange ($\pi^+ + D \rightarrow \pi^0 + 2p$) differential cross sections. As the elastic contribution was observed down to 20° in the laboratory, the Coulomb effect is included in the formalism. A general discussion of the properties of these cross sections is given.

The higher order terms[6] in the expansion of the transition

[6] Eq. (21) and discussion, ref. 4 (Chew and Goldberger).

operator give rise to the well known multiple scattering corrections and the binding or ''potential'' correction. In

Section III we discuss the first of these, the multiple scattering, from the standpoint of two adiabatic models. We find, using Brueckner's model for multiple scattering[7], that

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[7] K. A. Brueckner, Phys. Rev. 89, 834 (1953); Phys. Rev. 90, 715 (1953).

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the total cross section is approximately 10 % larger than the total free particle cross section at 85 Mev. Calculations are also presented which indicate large reductions in the total π-D cross section, as well as the elastic differential cross section, at energies in the neighborhood of the pion-nucleon resonance. A qualitative study is also made of the ($t = 3/2$, $j = 3/2$) multiple scattering at 85 Mev in the double scattering approximation via a p-state interaction hamiltonian with pseudovector coupling.

In Section IV the potential correction to the (3/2, 3/2) scattering is obtained from a perturbation field-theoretic treatment using the same p-state interaction hamiltonian; for 85-Mev pions, this correction is found to be -3.2 mb.

In Section V the effects of absorption are discussed and a modified FGW model including absorption developed in some detail. Using a simple phenomenological model for pion absorption, significant interference effects on the elastic differential cross section are found to result. For the Hulthén wave function a reduction in the elastic cross section

(E_{pion} = 85 Mev) of approximately 5% is computed.

Some multiple scattering formulae are collected in Appendix A, and their derivation, an extension to a spin dependent amplitude of Brueckner's model for multiple scattering, is sketched in Appendix B.

II. THE ''USUAL'' IMPULSE APPROXIMATION

In what follows, we take $\hbar = c = 1$; also, a', $\underset{\sim}{b}'$ refer to quantities measured in the pion-nucleon barycentric system with the corresponding quantities a, $\underset{\sim}{b}$ measured in the laboratory system.

Notation and Conventions

$\underset{\sim}{k}_0$, ω_0	vector momentum and energy respectively of the incident pion
$\underset{\sim}{k}$, ω	vector momentum and energy respectively of the scattered pion
θ	$\cos^{-1}(\underset{\sim}{k}_o \cdot \underset{\sim}{k}/k_o k)$
$v_\pi = k'/\omega'$	
M, μ	average nucleonic mass, average pion mass respectively
$E_{k'}$	total energy of nucleon in pion-nucleon barycentric system
$v_p = k'/E_{k'}$	
$\alpha = 1/137$	
$\underset{\sim}{T}$	meson isotopic spin operator[8] (with $T^2 = 2$)

[8] A convenient matrix representation of $\underset{\sim}{T}$ may be found in L. Schiff, Quantum Mechanics (McGraw-Hill Book Co., Inc., New York, 1949), page 144, Eq. (24.15).

$\underset{\sim}{K} = \underset{\sim}{k}_o - \underset{\sim}{k}$ recoil momentum of dinucleon system

$\eta^{\ell}_{tj} = \sin\delta^{\ell}_{tj} \;\exp i\delta^{\ell}_{tj}$ where δ^{ℓ}_{tj} is the phase shift for momentum k' with indices t, j, ℓ labelling the states of total isotopic spin, total angular momentum, and orbital angular momentum, respectively, of the pion-nucleon system.

$$\cos\theta' = \frac{(\mu^2+2M\omega_o+M^2)\cos\theta[1-(\mu/M)^2\sin^2\theta]^{\frac{1}{2}} - [\omega_o+(\mu^2/M \;\; (\omega_o+M)\sin^2\theta}{[(\omega_o+M)^2 - (k_o\cos\theta)^2]}$$

$$J(\theta) \equiv \frac{d(\cos\theta')}{d(\cos\theta)} =$$

$$\frac{(\mu^2+2M\omega_o+M^2)\{[\omega_o+(\mu^2/M)]\cos\theta+(\omega_o+M)[1+(\mu/M)^2\sin^2\theta]^{\frac{1}{2}}\}^2}{[1-(\mu/M)^2\sin^2\theta]^{\frac{1}{2}}[(\omega_o+M)^2 - (k_o\cos\theta)^2]^2}$$

E_t^{β} projection operator onto the state of total isotopic spin t for pion + nucleon (β)

$E_{3/2}^{\beta} = \frac{1}{3}(2 + \underset{\sim}{\tau}^{\beta} \cdot \underset{\sim}{T})$

$E_{\frac{1}{2}}^{\beta} = 1 - E_{3/2}^{\beta}$

$F_{j,\ell}^{\beta}$ projection operator onto the state of total angular momentum j, orbital angular momentum ℓ for pion + nucleon (β)

$$F^{\beta}_{\frac{3}{2},1} = (4\pi)^{-1}(2\cos\theta' + i\underset{\sim}{\sigma}^{\beta} \cdot \underset{\sim}{n}\sin\theta') \qquad \text{where } \underset{\sim}{n}\sin\theta' = (\underset{\sim}{k}'_{o} \times \underset{\sim}{k}')/k'^{2}$$

$$F^{\beta}_{\frac{1}{2},1} = (3/4\pi)P_{1}(\cos\theta') - F^{\beta}_{\frac{3}{2},1}$$

$$F^{\beta}_{\frac{1}{2},0} = 1/4\pi$$

$$\underset{\sim}{R}^{\beta} = \underset{\sim}{X} \pm \tfrac{1}{2}\underset{\sim}{r}\,, \qquad \text{where } |\underset{\sim}{r}| = |\underset{\sim}{R} - \underset{\sim}{R}^{2}|$$

$$f(\theta) \equiv g(\theta)e^{i(\underset{\sim}{k}_{o} - \underset{\sim}{k})\cdot\underset{\sim}{X}}$$

$$\Lambda^{1} = \frac{1}{4}(3 + \underset{\sim}{\sigma}^{1} \cdot \underset{\sim}{\sigma}^{2})$$

$$\Lambda^{o} = 1 - \Lambda^{1}$$

$$\frac{\rho_{D}}{\rho_{F}} = \frac{\int k^{2}dk\,\delta(E_{f} - \omega - K^{2}/4M)}{\int k^{2}dk\,\delta(E_{f} - \omega - K^{2}/2M)}$$

$$H(2K) \equiv \int d\underset{\sim}{r}\, |\Psi_{D}(\underset{\sim}{r})|^{2}\, e^{i\underset{\sim}{K}\cdot\underset{\sim}{r}}$$

$$g_{1}(\theta) = \langle T = 0,\ \pi^{\pm}\ |g(\theta)|\ T = 0, \pi^{\pm} \rangle$$

$$g_{2}(\theta) = \langle T = 1,\ \pi^{\pm}\ |g(\theta)|\ T = 0,\ \pi^{\pm} \rangle$$

$$g_{3}(\theta) = \langle T = 1,\ \pi^{o}\ |g(\theta)|\ T = 0,\ \pi^{\pm} \rangle$$

The ''nuclear'' scattered amplitude (in the laboratory system) for pions on free nucleons is given by

$$f_N(\theta) = [J(\theta)]^{\frac{1}{2}} \lambda' \sum_{t,j,\ell,\beta} \eta_{tj}^{\ell} E_t^{\beta} 4\pi F_{j,\ell}^{\beta} e^{i(\underset{\sim}{k}_o - \underset{\sim}{k})\cdot \underset{\sim}{R}^{\beta}} . \quad (2.1)$$

At energies $\gtrsim$ 30 Mev, though not so high that relativistic corrections need be made, the Coulomb effect may be adequately accounted for through the addition to $f_N(\theta)$ of the Coulomb amplitude in Born approximation,[9]

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[9] J. Ashkin and L. Smith, Technical Report No. 1, Carnegie Institute of Technology, February 2, 1953 (unpublished).

$$f_c(\theta) = - [J(\theta)]^{\frac{1}{2}} \lambda' \sum_{\beta} \frac{\alpha}{4\sin^2(\theta'/2)\cdot(v_\pi' + v_p')} (\tau_Z^{\beta}+1) T_Z e^{i(\underset{\sim}{k}_o - \underset{\sim}{k})\cdot \underset{\sim}{R}^{\beta}} . \quad (2.2)$$

Thus, the scattered amplitude for pions on free nucleons in the laboratory system takes the form

$$f(\theta) = [J(\theta)]^{\frac{1}{2}} \lambda' \sum_{\beta} \left\{ \sum_{t,j,\ell} \eta_{t,j}^{\ell} E_t^{\beta} 4\pi F_{j,\ell}^{\beta} - \frac{\alpha(\tau_Z^{\beta}+1) T_Z}{4\sin^2(\theta'/2)\cdot(v_\pi' + v_p')} \right\} e^{i(\underset{\sim}{k}_o - \underset{\sim}{k})\cdot \underset{\sim}{R}^{\beta}} . \quad (2.3)$$

For elastic scattering ($\pi^{\pm} + D \rightarrow \pi^{\pm} + D$) it follows that

$$\frac{d\sigma^E}{d\Omega}(\theta) = \frac{1}{3}\, \mathrm{Tr}\Lambda^1(\Psi_D(\underset{\sim}{r}'), g_1^+(\theta)\Psi_D(\underset{\sim}{r}'))\Lambda^1(\Psi_D(\underset{\sim}{r}), g_1(\theta)\Psi_D(\underset{\sim}{r})\ )\ \frac{\rho_D}{\rho_F}. \tag{2.4}$$

Evaluating the indicated trace and labelling the phase shifts in the conventional way,[10] we have

[10] G. C. Wick, Revs. Modern Phys. 27, 339 (1955).

$$\frac{d\sigma^E}{d\Omega}(\theta) = \frac{4}{27}\, \bar{\lambda}'^2 |H(K)|^2 J(\theta)\, \frac{\rho_D}{\rho_F}\, \{3\,|\eta_1 + 2\eta_3 - \frac{3\alpha}{4\sin^2(\theta'/2)\cdot(v_\pi' + v_p')}$$

$$+(\eta_{11}+2\eta_{13}+2\eta_{31}+4\eta_{33})\cos\theta'\,|^2 + 2\sin^2\theta'\,|\eta_{13}+2\eta_{33}-\eta_{11}-2\eta_{31}|^2\}. \tag{2.5}$$

We take for $\Psi_D(\underset{\sim}{r})$ the Hulthén wave function (neglecting D state admixture)

$$\Psi_D(\underset{\sim}{r}) = \left[\frac{\gamma\beta(\gamma + \beta)}{2\pi(\beta - \gamma)^2}\right]^{\frac{1}{2}} \frac{(e^{-\gamma r} - e^{-\beta r})}{r}$$

with $\beta = 7\gamma$,[11] with the result that

[11] Eq. (18), ref. 3.

$$H(K) = \frac{4\gamma\beta(\gamma+\beta)}{(\beta-\gamma)^2}\ \frac{1}{K}\left\{\tan^{-1}\left(\frac{K}{4\gamma}\right) + \tan^{-1}\left(\frac{K}{4\beta}\right) - 2\tan^{-1}\frac{K}{2(\gamma+\beta)}\right\}. \quad (2.6)$$

The marked effect of the destructive Coulomb interference at angles $< 45^\circ$ for positive 85-Mev pions may be noted in Figure 1 where $\sigma^E(\theta)_{\text{Nuclear}}$ and $\sigma^E(\theta)_{\text{Nuclear + Coulomb}}$ are plotted together for angles $\lesssim 45^\circ$ in the laboratory system.

For inelastic scattering ($\pi^+ + D \rightarrow \pi^\pm + n + p$) one notes that the possible final dinucleon states are ($S \equiv$ total spin of the n-p system, $T \equiv$ total isotopic spin of the n-p system) even parity, with $T = 0$, $S = 1$ or $T = 1$, $S = 0$, and odd parity, with $T = 1$, $S = 1$ or $T = 0$, $S = 0$. Thus, the differential cross section for the sum of elastic and inelastic scattering in the closure approximation[12] takes the form,

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[12] The closure approximation is discussed in detail in ref. 3, Section III; see also G. Chew and H. Lewis, Phys. Rev. 84, 779 (1951); M. Lax, Revs. Modern Phys. 23, 287 (1951), Section VII.

$$\frac{d\sigma^{E+I}}{d\Omega}(\theta) = \frac{1}{3}\,\mathrm{Tr}\left\{\sum_{i=1}^{2}\sum_{j=0}^{1}\left(\Psi_D(\underset{\sim}{r}'), g_i^+(\theta)\tfrac{1}{2}\left[\delta(\underset{\sim}{r}-\underset{\sim}{r}') + (-1)^{i+j}\,\delta(\underset{\sim}{r}+\underset{\sim}{r}')\right]\Lambda^j g_i(\theta)\Lambda^1\Psi_D(\underset{\sim}{r})\right)\right\}. \quad (2.7)$$

Following FGW we make the replacement in the closure approximation to the Elastic + Inelastic cross section,

$$H(2K) \rightarrow H'(2K) \equiv H(2K)(\rho_D/\rho_F) \quad . \qquad (2.8)$$

Hence, there results

$$\frac{d\sigma^{E+I}}{d\Omega}(\theta) = \frac{2}{9} \lambda'^2 J\{[1+H'(2K)][|\eta_1+2\eta_3 - \frac{3\alpha}{4\sin^2(\theta'/2)\cdot(v'_\pi+v'_p)}$$

$$+\cos\theta'(\eta_{11}+2\eta_{31}+2\eta_{13}+4\eta_{33})|^2 + \frac{2}{3}\sin^2\theta'|\eta_{13}+2\eta_{33}-\eta_{11}-2\eta_{31}|^2]$$

$$+[1-H'(2K)][|-\eta_1+\eta_3 - \frac{3\alpha}{4\sin^2(\theta'/2)\cdot(v'_\pi+v'_p)}$$

$$+\cos\theta'(-\eta_{11}+\eta_{31}-2\eta_{13}+2\eta_{33})|^2+ \frac{2}{3}\sin^2\theta'|\eta_{11}-\eta_{31}-\eta_{13}+\eta_{33}|^2]$$

$$+[1-H'(2K)]\frac{1}{3}\sin^2\theta'|\eta_{13}+2\eta_{33}-\eta_{11}-2\eta_{31}|^2$$

$$+[1+H'(2K)]\frac{1}{3}\sin^2\theta'|-\eta_{13}+\eta_{33}+\eta_{11}-\eta_{31}|^2\Big\} \quad . \qquad (2.9)$$

As the quantities, $\sin^2\theta'$, $[1-H'(2K)]$ are small for values of θ near zero, the forward peak in $\sigma^{E+I}(\theta)$ is almost entirely elastic. The comparison with experiment of the resulting curve for $E_{pion} = 85$ Mev may be found in Paper I. Expression (2.9) may be rewritten as

$$\sigma^{E+I}(\theta) = \sigma^{n}(\theta) + \sigma^{p}(\theta) + H'(2K)\ \{\ .\ .\ .\ \}\ . \tag{2.10}$$

= (sum of free particle differential scattering cross sections) + (interference terms).

Elastic scattering is largely responsible for the interference terms in $\sigma^{E+I}(\theta)$. Their net effect is positive (constructive) with a large positive contribution coming from scattering angles $< \theta_m$ (corresponding to $\theta' = \pi/2$) and a much smaller positive <u>or</u> negative one from $\theta > \theta_m$. As H(2K) is small compared to 1 for large momentum transfers or large scattering angles, $\sigma^{E+I}(\theta)$ differs only slightly from $(d\sigma^{n}/d\Omega) + (d\sigma^{p}/d\Omega)$ in the backward hemisphere at 85 Mev. (ρ_D/ρ_F) and H(2K) as functions of the scattering angle θ are plotted in Figure 3 and Figure 4 respectively for 85-Mev mesons. In Figure 2 $(d\sigma^{E+I}/d\Omega)$ has been plotted with and without inclusion of the Coulomb effect in order to emphasize the large destructive interference arising from it in π^{+}-D scattering.

We consider now the differential cross section for charge exchange π-D scattering. Noting that the states accessible to the diproton system are odd parity, S = 1, and even parity, S = 0, we may write the charge exchange differential cross section in the closure approximation as:

$$\frac{d\sigma^c}{d\Omega}(\theta) = \frac{1}{3}\,\mathrm{Tr}\sum_{j=0}^{1}\left(\Psi_D(\underset{\sim}{r}'),g_3^+(\theta)\tfrac{1}{2}[\delta(\underset{\sim}{r}-\underset{\sim}{r}')+(-1)^j\delta(\underset{\sim}{r}+\underset{\sim}{r}')]\Lambda^j g_3(\theta)\Lambda^1\Psi_D(\underset{\sim}{r})\right)$$

$$= J\lambda'^2 \quad \frac{1}{27}\left\{\,[1-H(2K)][6|\eta_1-\eta_3+(\eta_{11}-\eta_{31}+2\eta_{13}-2\eta_{33})\cos\theta'|^2\right.$$

$$+\ 4\sin^2\theta'|\eta_{13}-\eta_{33}-\eta_{11}+\eta_{31}|^2]$$

$$\left.+\ 2[1+H(2K)]\sin^2\theta'|\eta_{11}-\eta_{31}-\eta_{13}+\eta_{33}|^2\right\}\ . \qquad (2.11)$$

The charge exchange differential cross section (closure approximation) may also be written as

$$d\sigma^c/d\Omega = d\sigma_n^c/d\Omega\ -\ H(2K)\ \{\ .\ .\ .\ \}$$

where $(d\sigma_n{}^c/d\Omega)$ is the charge exchange differential cross section for positive pions on free neutrons (or negative pions on free protons). Here, the interference is always destructive. Note that at energies where p-wave scattering is dominant ($E_{pion} \gtrsim 70$ Mev) the net interference in $(d\sigma^c/d\Omega)$ + $(d\sigma^{E+I}/d\Omega)$ is <u>constructive</u>. In the closure approximation, $\sigma^c(\theta)$ clearly vanishes at $\theta = 0$ since that part of $\sigma^c(\theta)$ which arises from transitions to odd parity final states contains $[1 - H(2K)]$ as a factor while the part arising from transitions to even parity final states contains $\sin^2\theta'$ as a factor. As before, detailed comparison of (2.11) with experiment may be found in paper I.

III. THE MULTIPLE SCATTERING CORRECTION

We consider the multiple scattering correction to π-D scattering first on the fixed point source model due to Brueckner[7] where off-the-energy-shell scattering is neglected. The coupled equations for the scattered amplitude are solved in an approximation described in Appendix B. Expressions for the total cross section and the elastic differential cross section are exhibited in Appendix A. The resulting values of σ_{free}, σ_{exact}, as well as $\sigma_{double\ scattering}$, for 85-and 190-Mev pions are tabulated In Table I, where in both cases δ_{33}, δ_3, and δ_1 were taken from the Orear fit[2] to pion-nucleon scattering data.

At 85 Mev double scattering furnishes the principal contribution to the multiple scattering correction. While the quantitative results (see Table I) are sensitive to the 'small' phase shifts, δ_{31}, δ_{13}, and δ_{11}, an enhancement of the free particle cross section of approximately 10% by multiple scattering is indicated.

On the other hand, at pion kinetic energies in the neighborhood of the resonance energy for $(\frac{3}{2}, \frac{3}{2})$ scattering, multiple scattering reduces the total cross section well below the free particle cross section. At these energies, the multiple scattering correction to the elastic differential cross section calculated in the impulse approximation is also appreciable (see Table II).

We now consider qualitatively the $(\frac{3}{2}, \frac{3}{2})$ double scattering correction resulting from a field theoretic treatment in

Born approximation of an interaction hamiltonian similar to that of Chew.[13] Besides the expectation that such a treatment has

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[13] S. Gartenhaus, Phys. Rev. <u>100</u>, 900 (1955), Eq. (1).

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some validity at low energies, the procedure has the added advantage of specifying the behavior of the meson-nucleon scattering matrices off the energy shell.

We use the fixed point source interaction hamiltonian in the charge-symmetric pseudoscalar theory with pseudovector coupling.

$$H_I = (4\pi)^{\frac{1}{2}} \frac{f}{\mu} \sum_{\beta=1}^{2} \sum_{\alpha=1}^{3} \tau_\alpha^{(\beta)} \underset{\sim}{\sigma}^{(\beta)} \cdot \phi_\alpha(\underset{\sim}{R}^\beta) \tag{3.1}$$

Following Chew and Goldberger,[14] the elastic double scattering

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[14] Ref.4 (Chew and Goldberger), Eq. (29a).

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impulse approximation matrix element, $(\underset{\sim}{k}'|\mathfrak{M}_d|\underset{\sim}{k})$ is given by (see Figure 5 for the diagrammatic representation),

$$(\underset{\sim}{k}'|\mathfrak{M}_d|\underset{\sim}{k}) = \langle \pi^\pm, T=0 | (\Psi_D(\underset{\sim}{r}) e^{-i\frac{1}{2}(\underset{\sim}{k}+\underset{\sim}{k}')\cdot\underset{\sim}{r}} ,$$

$$\int (\underset{\sim}{k}'|t_1|\underset{\sim}{\eta}) \frac{e^{i\underset{\sim}{\eta}\cdot\underset{\sim}{r}}\, d\eta (2\pi)^{-3}}{\omega_k - \omega_\eta + i\epsilon}$$

$$(\underset{\sim}{\eta}|t_2|\underset{\sim}{k}) \Psi_D(\underset{\sim}{r})\ ) | \pi^\pm, T=0 \rangle + (\underset{\sim}{r} \to \underset{\sim}{r}, 1 \longleftrightarrow 2) . \tag{3.2}$$

Note the adiabatic approximation in (3.2).

Specializing to $(\frac{3}{2}, \frac{3}{2})$ scattering in the Born approximation, we make the replacement,[15]

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15 Aitken, Mahmoud, Henley, Ruderman, and Watson, Phys. Rev. 93, 1349 (1954), Eq. (30).

$$(\underset{\sim}{\eta}'|t_\beta|\underset{\sim}{\eta}) \rightarrow (\underset{\sim}{\eta}'|\upsilon^{(\beta)}_{\frac{3}{2},\frac{3}{2}}|\underset{\sim}{\eta}) =$$

$$\left(\frac{32}{3}\pi^2\frac{f^2}{\mu^2}\right)\frac{\eta'\eta}{(\omega_{\eta'}\omega_\eta)^{\frac{1}{2}}}[\omega_{\mathbf{k}}-\omega_{\eta'}-\omega_\eta]^{-1}\,E_{\frac{3}{2}}{}^{\beta}(\underset{\sim}{\eta}'|F_{\frac{3}{2}}{}^{\beta}|\underset{\sim}{\eta}) \tag{3.3}$$

where

$$(\underset{\sim}{\eta}'|F_{\frac{3}{2}}{}^{\beta}|\underset{\sim}{\eta}) = \frac{1}{4\pi\eta'\eta}\left(2\underset{\sim}{\eta}'\cdot\underset{\sim}{\eta} + i\underset{\sim}{\sigma}^{\beta}\cdot\underset{\sim}{\eta}\times\underset{\sim}{\eta}'\right). \tag{3.4}$$

The resolution of the denominator of $(\underset{\sim}{k}'|\mathfrak{M}_d|\underset{\sim}{k})$ into partial fractions,

$$\frac{1}{\omega_\eta^{\,3}}\,\frac{1}{(\omega_{\mathbf{k}}-\omega_\eta+i\epsilon)} = \left(\frac{1}{\omega_{\mathbf{k}}\omega_\eta^{\,3}} + \frac{1}{\omega_{\mathbf{k}}^{\,2}\omega_\eta^{\,2}} + \frac{1}{\omega_{\mathbf{k}}^{\,3}\omega_\eta}\right) + \frac{1}{\omega_{\mathbf{k}}^{\,3}(\omega_{\mathbf{k}}-\omega_\eta+i\epsilon)} \tag{3.5}$$

readily yields the separation of $(\underset{\sim}{k}'|\mathfrak{M}_d|\underset{\sim}{k})$ into contributions from real and virtual double scattering. Thus, we have

$$(\underset{\sim}{k}'|\mathfrak{M}_d|\underset{\sim}{k}) = (\underset{\sim}{k}'|\mathfrak{M}_d{}^{\text{real}}|\underset{\sim}{k}) + (\underset{\sim}{k}'|\mathfrak{M}_d{}^{\text{virtual}}|\underset{\sim}{k})$$

where

$$(\underset{\sim}{k}'|\mathfrak{M}_d^{\text{real}}|\underset{\sim}{k}) = (\frac{32}{3}\pi^2\frac{f^2}{\mu^2})^2\frac{k^2}{\omega_k^4}\frac{2}{9}(\Psi_D(\underset{\sim}{r})e^{-\frac{i}{2}(\underset{\sim}{k}+\underset{\sim}{k}')\cdot\underset{\sim}{r}},$$

$$\cdot\int e^{i\underset{\sim}{\eta}\cdot\underset{\sim}{r}}(\underset{\sim}{k}'|F_{\frac{3}{2}}^{(1)}|\underset{\sim}{\eta})(\underset{\sim}{\eta}|F_{\frac{3}{2}}^{(2)}|\underset{\sim}{k})\frac{\eta^2 d\underset{\sim}{\eta}(2\pi)^{-3}}{\omega_k-\omega_\eta+i\epsilon}\Psi_D(\underset{\sim}{r}))$$

$$+(1\longleftrightarrow 2,\ \underset{\sim}{r}\rightarrow -\underset{\sim}{r}) \qquad (3.6a)$$

and

$$(\underset{\sim}{k}'|\mathfrak{M}_d^{\text{virtual}}|\underset{\sim}{k}) = (\frac{32}{3}\pi^2\frac{f^2}{\mu^2})^2\frac{k^2}{\omega_k^2}\frac{2}{9}(\Psi_D(\underset{\sim}{r})e^{-\frac{i}{2}(\underset{\sim}{k}+\underset{\sim}{k}')\cdot\underset{\sim}{r}},$$

$$\cdot\int e^{i\underset{\sim}{\eta}\cdot\underset{\sim}{r}}(\underset{\sim}{k}'|F_{\frac{3}{2}}^{(1)}|\underset{\sim}{\eta})(\underset{\sim}{\eta}|F_{\frac{3}{2}}^{(2)}|\underset{\sim}{k})\eta^2 d\underset{\sim}{\eta}(2\pi)^{-3}$$

$$\cdot\left(\frac{1}{\omega_\eta^3}+\frac{1}{\omega_k\omega_\eta^2}+\frac{1}{\omega_k^2\omega_\eta}\right)\Psi_D(\underset{\sim}{r})) + (1\longleftrightarrow 2, \underset{\sim}{r}\rightarrow -\underset{\sim}{r}). \qquad (3.6b)$$

As $\text{Im}\,(\underset{\sim}{k}|\mathfrak{M}_d|\underset{\sim}{k}) = \text{Im}\,(\underset{\sim}{k}|\mathfrak{M}_d^{\text{real}}|\underset{\sim}{k})$, it follows that the correction to the total free particle cross section is[16]

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16 We make use of the ''optical theorem'', cf. B. Lippman and J. Schwinger, Phys. Rev. 79, 969 (1950), Eq.(1.75).

$$\Delta\sigma = -\ 2\omega_k \lambda\!\!\!{}^{-} \frac{1}{3}\ \mathrm{Sp}\ \{\Lambda^1 \mathrm{Im}\ (\underset{\sim}{k}|\mathfrak{M}_d^{\ \mathrm{real}}|\underset{\sim}{k})\}$$

$$= \frac{16}{27}\ \pi\lambda\!\!\!{}^{-2}\ (\frac{4}{3}\ \frac{f^2}{\mu^2}\ \frac{k^3}{\omega_k})^2\ \{\ \frac{14}{3} \langle (j_0(kr))^2 \rangle$$

$$+ \frac{22}{3} \langle (j_2(kr))^2 \rangle \} \quad . \qquad (3.7)$$

A second Born approximation to the phase shift analysis of the S-matrix for pion-nucleon scattering[17] yields $\delta_{33}^{(2)} = 4f^2k^3/3\mu^2\omega_k$;

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[17] G. F. Chew, Phys. Rev. 94, 1755 (1954), Eqs. (9) and (10).

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thus (3.7) is seen to be formally identical with that obtained from the Brueckner model in the Born approximation for $(\frac{3}{2}, \frac{3}{2})$ scattering only. It is easily shown, for a reasonable cut-off radius, that $(\underset{\sim}{k}'|\mathfrak{M}_d^{\ \mathrm{virtual}}|\underset{\sim}{k})$, the matrix element for off-the-energy-shell scattering, furnishes contributions to the double scattering correction to the elastic differential cross section at 85 Mev which are of the same order of magnitude as those furnished by $(\underset{\sim}{k}'|\mathfrak{M}_d^{\ \mathrm{real}}|\underset{\sim}{k})$. Thus, it is possible to conclude that the Brueckner model is not reliable at energies far below the resonance where the real scattering is weak and at 85 Mev in particular, and that the neglect of scattering off the energy shell is a bad approximation in this region.

IV. THE 'POTENTIAL' CORRECTION

In our systematic analysis of corrections to the ''usual'' impulse approximation, we turn now to consideration of those terms in the perturbation expansion of the transition matrix for the problem which form the 'potential' correction to the p-wave single scattering under the given interaction hamiltonian. These terms occur as 'interruptions' in the single scattering with virtual mesons exchanged by the two nucleons during the scattering. For simplicity we shall only be concerned with the 'potential' correction to the total cross section resulting from $(\frac{3}{2}, \frac{3}{2})$ single scattering, i.e. we apply the optical theorem to the imaginary part of the 'potential' correction to the scattering matrix in the forward direction. It should be noted that ladder corrections (Figure 6 C), i.e. the exchange of virtual mesons before or after the scattering takes place, are removed by using the 'correct' deuteron wave function. This follows from the fact that in the forward direction, (elastic scattering), ladder corrections are decoupled from the real scattering.

In terms of the Chew-Goldberger formalism, the impulse approximation matrix element of the 'potential' correction to $(\frac{3}{2}, \frac{3}{2})$ elastic single scattering in the forward direction is given by[18]

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[18] Ref. 4 (Chew and Goldberger), Eq. (24).

$$(\underset{\sim}{k}_0|\mathfrak{M}_p|\underset{\sim}{k}_0) = \Sigma_\beta \langle \pi^\pm, T=0|(\Psi_D(\underset{\sim}{r}), \int (\underset{\sim}{k}_0|\upsilon^{(\beta)}_{\frac{3}{2},\frac{3}{2}}|\underset{\sim}{\ell})$$

$$[(\underset{\sim}{\ell}|v_g|\underset{\sim}{\ell})(\underset{\sim}{\ell}|\upsilon^{(\beta)}_{\frac{3}{2},\frac{3}{2}}|\underset{\sim}{\ell}) - (\underset{\sim}{\ell}|\upsilon^{(\beta)}_{\frac{3}{2},\frac{3}{2}}|\underset{\sim}{k}_0)V(\underset{\sim}{r})]$$

$$\frac{d\underset{\sim}{\ell}\,(2\pi)^{-3}}{(\omega_0-\omega_\ell+i\epsilon)^2}\,\Psi_D(\underset{\sim}{r})\;)\;|T=0,\pi^\pm\rangle\,. \quad (4.1)$$

The term of order f^2 in $(\underset{\sim}{\ell}|v_g|\underset{\sim}{\ell})$ is (see Figures 6a and 6b)

$$(\underset{\sim}{\ell}|v_g^{(2)}|\underset{\sim}{\ell}) = -4\pi\left(\frac{f}{\mu}\right)^2 \underset{\sim}{\tau}^1\cdot\underset{\sim}{\tau}^2 \int \frac{\underset{\sim}{\sigma}^1\cdot\underset{\sim}{\eta}\,\underset{\sim}{\sigma}^2\cdot\underset{\sim}{\eta}\; e^{i\underset{\sim}{\eta}\cdot\underset{\sim}{r}}}{\omega_\eta(\omega_\eta+\omega_\ell-\omega_0)}\,\frac{d\underset{\sim}{\eta}}{(2\pi)^3}\,. \quad (4.2)$$

Note that the terms of order f^2 and f^4 of $(\underset{\sim}{k}_0|v_g|\underset{\sim}{k}_0)$ are just the familiar second and fourth order nucleon-nucleon potentials discussed in detail by Brueckner and Watson.[19]

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[19] K. A. Brueckner and K. M. Watson, Phys. Rev. 92, 1023 (1953), Eq. (60).

The potential $V(\underset{\sim}{r})$ is related to the binding energy of the deuteron, $-\epsilon_D$, by the Schrödinger equation,

$$[\, -\frac{\Delta}{M} + V(\underset{\sim}{r})\,]\; \Psi_D(\underset{\sim}{r}) = \epsilon_D \Psi_D(\underset{\sim}{r}). \tag{4.3}$$

In evaluating $\mathrm{Im}(\underset{\sim}{k}_0 | \mathfrak{M}_p | \underset{\sim}{k}_0)$, the following relations are useful:

$$\mathrm{Im}\left[\frac{1}{(\omega_0 - \omega_\ell + i\epsilon)^2}\right] = -\pi \frac{d}{d\omega_\ell} \delta(\omega_\ell - \omega_0) \quad , \tag{4.4a}$$

$$\left[\frac{d}{d\omega_\ell} (\ell | v_g^{(2)} | \ell)\right]_{\omega_\ell = \omega_0} = -\frac{2}{\pi} \left(\frac{f}{\mu}\right)^2 \underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2 \; \underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\nabla} \; \underset{\sim}{\sigma}^2 \cdot \underset{\sim}{\nabla} \; K_0(\mu r). \tag{4.4b}$$

Further, it seems a reasonable approximation at low energies ($\omega_k \sim \mu$) to replace $(k_0 | v_g | k_0)$ and $V(\underset{\sim}{r})$ in the final expression for $(\underset{\sim}{k}_0 | \mathfrak{M}_p | \underset{\sim}{k}_0)$ by a charge-independent static potential $V(\underset{\sim}{r}, \underset{\sim}{\sigma}, \underset{\sim}{\tau})$ which fits all the low energy data. One such ''omnibus'' potential, due to Feynman,[20] which fits high energy n-p data

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[20] Notes on High Energy Phenomena and Meson Theories by R. P. Feynman, 1951 (unpublished), p. 28.

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as well, is

$$V(\underset{\sim}{r}, \underset{\sim}{\sigma}, \underset{\sim}{\tau}) = \frac{1}{2}(1 + P_x)\,(-46.1 \text{ Mev}) \left[\frac{e^{-\mu_c r}}{\mu_c r} + .54\, S_{12} \frac{e^{-\mu_t r}}{\mu_t r}\right] \tag{4.5}$$

with $\mu_c^{-1} = 1.18 \times 10^{-13}$ cm, $\mu_t^{-1} = 1.69 \times 10^{-13}$ cm.

The potential correction to the total cross section (for $(\frac{3}{2}, \frac{3}{2})$ single scattering) is then,

$$\Delta\sigma_{tot} = -2\,\frac{\omega_0}{k_0}\,\frac{1}{3}\;\mathrm{Sp}\,[\,\mathrm{Im}(\underset{\sim}{k}_0|\mathfrak{M}_p|\underset{\sim}{k}_0)\Lambda^1\,]$$

$$= \frac{320}{81}\,\pi\,\lambda^2\,\delta_{33}^2\left[1 + \frac{3}{(k/\mu)^2}\right]\;\frac{\int|\Psi_D|^2 V_c(r)\,d\underset{\sim}{r}}{\hbar c\omega_0}$$

$$- \frac{1600}{243}\,\lambda^2\,\delta_{33}^2\;f^2\int|\Psi_D|^2\left[K_0(\mu r) - \frac{K_1(\mu r)}{\mu r}\right]d\underset{\sim}{r} \qquad (4.6)$$

where $V_c(r) = (-46.1\ \mathrm{Mev})\,(\mu_c r)^{-1}\,e^{-\mu_c r}$. We treat $\delta_{33} = 4f^2k^3/3\mu^2\omega$ phenomenologically, setting δ_{33} equal to its experimental value. In calculating $\int f^2\,[K_0(\mu r) - (\mu r)^{-1}K_1(\mu r)]\,|\Psi_D|^2\,d\underset{\sim}{r}$ we make use of the parameters used by Brueckner and Watson[19] to fit the low energy properties of the two nucleon system, namely $f^2 = .0855$ and the core radius, $r_0 = .3 \times 10^{-13}$ cm. (The procedure is equivalent to the assumption of two phenomenological coupling constants, the one energy-dependent (scattering interaction), and the other energy-independent (nucleon-nucleon interaction)). At 85 Mev, taking $\delta_{33} = .27$, we find $\Delta\sigma_{tot} = -3.5$ mb + .3 mb = -3.2 mb.

The principal sources of error in our treatment are: (1) the phenomenological fit of the Born approximation; (2) the omission of terms for which three mesons are simultaneously present in an intermediate state (see Figure 6d,e) (these terms increase in importance as the energy of the incident meson decreases); (3) neglect of the terms $\left[\frac{d}{d\omega_\ell}(\ell|v_g^{(4)}|\ell)\right]_{\omega_\ell=\omega_0}$ in the evaluation of (4.1).

V. THE EFFECTS OF ABSORPTION; A MODIFIED FGW MODEL

As the FGW model neglects mesic absorption, its contribution to the total π-D cross section must be separately calculated. A calculation of the cross section for pion absorption at 85 Mev from the inverse process is in good agreement with experiment.[1] One notes that the absorption cross section is a not inconsiderable fraction of the total cross section at this energy, indeed $\sigma_{abs} \gtrsim 10\%$ of $\sigma(\pi,D)$. Two questions now arise: (1) what is the reaction of the absorption (i.e. the ''absorptive'' scattering) on the scattering calculated on the FGW model? (2) How could one modify the FGW model, albeit crudely, to take into account more realistically the effects of absorption?

We shall discuss the ''absorptive'' scattering first. For simplicity, in estimating this effect, we consider only the elastic component. As the fit we obtain for the observed elastic and elastic plus inelastic cross sections is most sensitive to the details of the theory (e.g. the specification of the ''small'' pion-nucleon phase shifts) at angles $\lesssim$ 45° where the scattering is principally elestic, this is also a reasonable approximation to make. We utilize the model for pion absorption and production due to Brueckner and Watson.[21]

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[21] Brueckner, Serber, and Watson, Phys. Rev. 84, 258 (1951); K. M. Watson, Phys. Rev. 89, 575 (1953); N. Francis and K. M. Watson, Am. J. Phys. 21, 659 (1953).

This model comprehends absorption as a cooperative process, with the necleons being separated a distance of the order of $\hbar/(M\mu)^{\frac{1}{2}}c$ or less, in order to accept a momentum transfer of the order of $(M\mu)^{\frac{1}{2}}c$. Thus, this model furnishes a contribution to the scattering from the region where the adiabatic approximation to nucleonic motion breaks down, the region of $r < \hbar/p$ where $p \sim (M\mu)^{\frac{1}{2}}c$.

The implications of these statements are obvious. (1) In interpreting over intermediate nucleon relative momenta in the expression for the matrix element for absorptive scattering, given below, one must subtract out the contribution from low momenta, i.e. for $r > \hbar/(M\mu)^{\frac{1}{2}}c$. (2) This ''strong'' absorption model implies the breakdown of the impulse approximation for small internucleon separations; thus, when the nucleons are close together, the incident pion is either absorbed or ''absorptively'' scattered, with these being little or no possibility of scattering in the sense of the impulse approximation, i.e. as a two-body problem. We will pursue this point further when we discuss a modified FGW model.

In terms of the R-matrix formalism described in detail by Brueckner,[22] the transition amplitude for the elastic ''absorptive'' scattering of a meson from momentum $\underset{\sim}{q}$ to $\underset{\sim}{q}'$ in the π-D center of mass system is

$$\mathfrak{M}(\underset{\sim}{q}', \underset{\sim}{q}) \equiv \mathfrak{M}'(\underset{\sim}{q}', \underset{\sim}{q}) - \mathfrak{M}^{s}(\underset{\sim}{q}', \underset{\sim}{q}) \equiv$$

$$\int R^{+}(\underset{\sim}{q}', \underset{\sim}{p}) \frac{d\underset{\sim}{p}\,(2\pi)^{-3}}{E+i\delta-p^{2}/M} R(\underset{\sim}{q}, \underset{\sim}{p}) \,|\phi_{D}(\underset{\sim}{p})|^{2}$$

$$- \int R^{+}(\underset{\sim}{q}', 0) \frac{d\underset{\sim}{p}\,(2\pi)^{-3}}{E} R(\underset{\sim}{q}, 0) \,|\phi_{D}(\underset{\sim}{p})|^{2} \qquad (5.1)$$

where $R(\underset{\sim}{q}, \underset{\sim}{p})$ is the transition amplitude for absorption of meson $\underset{\sim}{q}$ by two nucleons (plane wave representation) leading to a final state of two nucleons with relative momentum $\underset{\sim}{p}$; $R^{+}(\underset{\sim}{q}', \underset{\sim}{p})$ is the corresponding amplitude for meson production. Note that

$$\phi_{D}(\underset{\sim}{p}) = \int \Psi_{D}(\underset{\sim}{r}) e^{-i\underset{\sim}{p}\cdot\underset{\sim}{r}} d\underset{\sim}{r}\,, \quad \text{and}$$

$$E = \omega_{q} + q^{2}/4M\,. \qquad (5.2)$$

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[22] K. A. Brueckner, Phys. Rev. 98, 769 (1955).

It is instructive to examine the consequences of a simple ansatz for $R(\underset{\sim}{q}, \underset{\sim}{p})$ which incorporates some of the observed features of pion absorption. We will assume for simplicity that R is independent of $|\underset{\sim}{p}|$ [23] and linear in

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[23] The assumption that the dependence of R on $\underset{\sim}{r}$ (the relative coordinate of the dinucleon system) may be approximated by a delta-function, is consistent with the choice of the Hulthen wave function for $\Psi_D(\underset{\sim}{r})$.

$\underset{\sim}{q}$ (p-wave absorption). The factor $(\underset{\sim}{p} \cdot \underset{\sim}{q})$ is suggested by the strong $\cos\theta$ dependence of the differential cross section for pion absorption.[24] The factors σ_i^{β}, τ_+^{β} (for π^+-D

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[24] Durbin, Loar, and Steinberger, Phys. Rev. 84, 581 (1951)

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absorption) induce transitions to the singlet final state (T = 1) required by p-wave absorption. The parameter G is fit to the observed total absorption cross section. Thus, we have[25]

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[25] Cf. L. Wolfenstein, Phys. Rev. 98, 766 (1955), the 'B'-term of the collision matrix M, Eq. (1).

$$R(\underset{\sim}{q}, \underset{\sim}{p}) \rightarrow G \sum_{\beta} (\underset{\sim}{\sigma}^{\beta} \cdot \underset{\sim}{q}) (\underset{\sim}{p} \cdot \underset{\sim}{q}) (pq)^{-1} \tau_{+}^{\beta} \qquad (5.3)$$

with

$$\sigma_a = -\frac{2}{v_r} \quad \mathrm{Im} \left\{\frac{1}{3} \mathrm{Sp}\, \Lambda^1 \langle T=0 | \mathfrak{M}(\underset{\sim}{q},\underset{\sim}{q}) | T=0 \rangle \right\}$$

$$= \frac{2}{9\pi} |G|^2 M \frac{q(2\omega_q M)}{(\omega_q + 2M)} p |\phi_D(p)|^2 \qquad (5.4)$$

where $p = (ME)^{\frac{1}{2}}$.

We calculate the ratio of the real to the imaginary part of $\mathfrak{M}(\underset{\sim}{q}', \underset{\sim}{q})$. (These give rise to opposite interference effects in the elastic scattering.) For the Hulthén wave function, a choice consistent with our model for R, $\phi_D(p) \propto [(p^2+\gamma^2)(p^2+\beta^2)]^{-1}$ and we have for $q/\mu = 1.13$ ($E_{pion} = 85$ Mev), $\mathrm{Re}\mathfrak{M}(\underset{\sim}{q}',\underset{\sim}{q}) / \mathrm{Im}\mathfrak{M}(\underset{\sim}{q}',\underset{\sim}{q}) \simeq 3.2$.

Since,

$$\mathfrak{M}(\underset{\sim}{q}',\underset{\sim}{q}) = (3/8)\sigma_a v_r q^{-2} \cos\theta'' \, \Sigma_{\beta\beta'} (\underset{\sim}{\sigma}^{\beta} \cdot \underset{\sim}{q}')(\underset{\sim}{\sigma}^{\beta'} \cdot \underset{\sim}{q}) \tau_{-}^{\beta} \tau_{+}^{\beta'} (-i+3.2) \qquad (5.5)$$

and

$$\langle \mathfrak{M} \rangle_{T=0,T=0} = (-i+3.2) \frac{3}{8} \sigma_a v_r \cos\theta'' (\underset{\sim}{\sigma}^1 - \underset{\sim}{\sigma}^2) \cdot \frac{\underset{\sim}{q}'}{q} (\underset{\sim}{\sigma}^1 - \underset{\sim}{\sigma}^2) \cdot \frac{\underset{\sim}{q}}{q}, \qquad (5.6)$$

where $\cos\theta'' = \underset{\sim}{q}^{-2} \underset{\sim}{q}' \cdot \underset{\sim}{q}$ and $v_r = q\left(\frac{1}{\omega_q} + \frac{1}{2M}\right)$, the ''absorptive'' elastic scattered amplitude in the laboratory system is then,

$$f_a(\theta) = -\frac{1}{2\pi} \langle \mathfrak{M} \rangle \left[\frac{J'(\theta)}{v_r} \frac{d\underset{\sim}{q}'}{dEd\Omega''} \right]^{\frac{1}{2}}$$

where $J'(\theta) = \frac{d\Omega''}{d\Omega}$.

From

$$\Delta(d\sigma^E/d\Omega)_{absorptive} = \frac{1}{3} \mathrm{Tr}\ \Lambda^1 g_1^{+}(\theta)(\rho_D/\rho_F)^{\frac{1}{2}}\ \Lambda^1 f_a(\theta) + h.c., \tag{5.7}$$

we have,

$$\Delta\left(\frac{d\sigma^E}{d\Omega}\right)_{abs} = \frac{16}{9}\ H(K)\ \lambda'\ (J\ \rho_D/\rho_F)^{\frac{1}{2}}\ \mathrm{Re}\Big[\ C\ \{\cos\theta''\big[\eta_1 + 2\eta_3 + (4\eta_{33}+\eta_{11}+2\eta_{13}+2\eta_{31})\cos\theta' - \frac{3a}{4\sin^2(\theta'/2).(v_\pi+v_p)}\big]^* - \sin\theta'\sin\theta''\ (2\eta_{33}+\eta_{13}-2\eta_{31}-\eta_{11})^*\ \}\Big] \tag{5.8}$$

with $C = -\frac{1}{2\pi}\ J'^{\frac{1}{2}}\ q\ (-i + 3.2)\ \frac{3}{8}\ \sigma_a\ \cos\theta''$.

It should be noted that the corrections discussed in Sections III and IV, proceeding from an adiabatic approximation, are apart from that discussed here. The absorptive scattering correction is sensitive to the choice of deuteron wave function as well as to the method of subtraction of the low momentum contribution; in particular, the correction decreases as $\mathrm{Re}\mathfrak{M}/\mathrm{Im}\mathfrak{M}$ decreases. As the major part of the p-wave absorption has been fit to the entire absorption cross section, one expects some exaggeration in the resulting angular dependence of this effect. One sees that ''absorptive'' scattering depresses the elastic scattering at small angles, and interferes constructively with it in the backward hemisphere. Thus, the inclusion of the effects of absorption even on the basis of the crude model described above, may be expected to improve the fit to the elastic differential cross section. We shall discuss the quantitative results briefly at the close of this section.

We consider now the modification in the FGW model entailed by the assumption that absorption is the dominant process at short distances ($r \lesssim \hbar/(M\mu)^{\frac{1}{2}} c$). This is simply to cut out of the domain of the impulse approximation the region about the origin (of $\underset{\sim}{r}$) of radius $r_0 = \hbar/(ME)^{\frac{1}{2}} c$. A simple argument may be given for this procedure, as follows: the inclusion of absorption in a pure scattering model is equivalent to the addition of an imaginary potential to the scattering interaction. As the absorption is both strong and localized, this potential will be strong and of short range. One would find the meson wave function considerably attenuated inside the range of this potential, i.e. the contribution to the form factor furnished by this region effectively vanishes. The importance of this region depends on two factors: (1) the behavior of the deuteron wave function at short distances or its momentum dependence for high momenta; (2) the size of the momentum transfer. The form factor $\int\int_0^\infty \psi_F^* \psi_D e^{i\underset{\sim}{K}\cdot\frac{1}{2}\underset{\sim}{r}} d\underset{\sim}{r}$ goes over to $\int\int_{r_o}^\infty \psi_F^* \psi_D e^{i\underset{\sim}{k}\cdot\frac{1}{2}\underset{\sim}{r}} d\underset{\sim}{r}$, with the reduction increasing with angle (i.e. with momentum transfer). Thus we have,

$$\tfrac{1}{2}[1 \pm H(2K)] \rightarrow \tfrac{1}{2}[1 \pm H(2K)] - \tfrac{1}{2}\left[\int\int_0^{r_o} |\psi_D|^2 (1 \pm e^{i\underset{\sim}{K}\cdot\underset{\sim}{r}}) d\underset{\sim}{r}\right],$$

and also,

$$|H(K)|^2 \rightarrow |H(K)|^2 \left\{ 1 - \frac{2\int\int_0^{r_o} e^{i\frac{1}{2}\underset{\sim}{K}\cdot\underset{\sim}{r}} |\psi_D|^2 d\underset{\sim}{r}}{H(K)} \right\} \quad (5.9)$$

At 85 Mev, a reduction in the elastic cross section of 1 mb is found resulting from ''absorptive'' scattering and the 'attenuation' effect. There is, thus, some indication that at medium energies (70 - 120 Mev) these effects may with multiple scattering depress the total π-D cross section below the total free particle cross section, a result for which there is experimental evidence.[26] The combined corrections

[26] Ashkin, Blaser, Feiner, Gorman, and Stern, Phys. Rev. 96, 1104 (1954).

are plotted in Figure 7.

The author is indebted to Professor R. Serber who suggested this problem to him and who gave constant guidance and encouragement throughout the work. The author is also indebted to Professor L. M. Lederman and Dr. K. C. Rogers for helpful discussions.

APPENDIX A

We present below the expression for the total cross section (neglecting absorption) derived from the Brueckner model. Scattering off the energy shell has been neglected; the adiabatic approximation has been assumed; the ''small'' phase shifts η_{11}, η_{13}, η_{31} have also been omitted.

$$\sigma_{tot} = 8\pi\lambda\!\!\!^{-2} \; \mathrm{Im} \Big\langle \frac{4k^2}{1-5\alpha^2\xi^2} \Big\{ \alpha - (\beta+\gamma)\alpha ik\, h_1^{(1)}(\chi) j_1(\chi) + \frac{11}{9}\alpha^2 ik^3\, h_2^{(1)}(\chi) j_2(\chi) + \frac{7}{9}\alpha^2 ik^3\, h_0^{(1)}(\chi) j_0(\chi) \Big\} + \Delta^{-1}\Big\{ (2\beta+\gamma) + (2\beta^2+4\beta\gamma+\gamma^2) ik\, h_0^{(1)}(\chi) j_0(\chi) - 4ik^3(\beta+\gamma)\alpha\, h_1^{(1)}(\chi) j_1(\chi) \Big\} \Big\rangle \qquad \text{(A.1)}$$

k = pion momentum in pion-nucleon barycentric system

$\alpha = \eta_{33}/3k^3$ $\qquad G = ik\, h_0^{(1)}(\chi)$

$\beta = (\eta_3 - \eta_1)/3k$ $\qquad \xi = (-ik^3/3)[\, h_0^{(1)}(\chi) + h_2^{(1)}(\chi)]$

$\gamma = \eta_1/k$ $\qquad \mu = -ik^2\, h_1^{(1)}(\chi)$

$\chi = kr$ $\qquad \langle A \rangle = \int |\Psi_D(\underset{\sim}{r})|^2 A \, d\underset{\sim}{r}$

$\Delta = 1 - (2\gamma^2+\beta^2+6\gamma\beta)\, G^2 + 2\mu^2\alpha(\beta+3\gamma)$

The elastic differential cross section derived from the Brueckner model is given by ($\underset{\sim}{k}_0$= momentum of incident pion, $\underset{\sim}{k}$ = momentum of scattered pion)

$$(d\sigma^E/d\Omega_{cm})_{\substack{\text{adiabatic}\\ \text{approx.}}} \simeq 4|a|^2 + (8/3)|b|^2 + 8\,\mathrm{Im}[ac^* + (2/3)\underset{\sim}{k}\cdot\underset{\sim}{k}_0\, ae^* + (2/3)\underset{\sim}{b}^*\cdot\underset{\sim}{d}] \quad \text{(A.2)}$$

$$a = \langle\, j_0(Z)\ \{[1 - 5\alpha^2\xi^2]^{-1}\, 4\alpha\underset{\sim}{k}\cdot\underset{\sim}{k}_0 + \Delta^{-1}(2\beta + \gamma)\ \}\,\rangle$$

$$\underset{\sim}{b} = \langle\, [1 - 5\alpha^2\xi^2]^{-1}\, 2\alpha\underset{\sim}{k}_0 \times \underset{\sim}{k}\ j_0(Z)\rangle$$

$$c \equiv \langle\, [1 - 5\alpha^2\xi^2]^{-1}\, (8/3)\underset{\sim}{k}\cdot\underset{\sim}{k}_0\alpha^2 k^3\ [2\, h_2^{(1)}(\chi) j_2(y) + h_0^{(1)}(\chi) j_0(y)]\,\rangle - \langle\, [1 - 5\alpha^2\xi^2]^{-1} 4(\beta + \gamma)\, \alpha\underset{\sim}{k}\cdot\underset{\sim}{\ell}(1/\ell)k^2\, h_1^{(1)}(\chi) j_1(y)\,\rangle + \langle\, \Delta^{-1}\ [(2\beta^2 + \gamma^2 + 4\beta\gamma)\, h_0^{(1)}(\chi) j_0(y) - k^2(\beta + \gamma) 4\alpha\underset{\sim}{k}_0\cdot\underset{\sim}{\ell}(1/\ell)\, h_1^{(1)}(\chi) j_1(y)]\,\rangle$$

$$\underset{\sim}{d} \equiv \langle\ -[1 - 5\alpha^2\xi^2]^{-1}\ \{\ (4/3)\underset{\sim}{k}_0 \times \underset{\sim}{k}\alpha^2 k^3 [\, h_2^{(1)}(\chi) j_2(y) + 2\, h_0^{(1)}(\chi) j_0(y)] + 2(\beta + \gamma)\alpha\underset{\sim}{k} \times \underset{\sim}{\ell}(1/\ell)k^2\, h_1^{(1)}(\chi) j_1(y)\} + \Delta^{-1}\ 2(\beta + \gamma)\alpha\underset{\sim}{k}_0 \times \underset{\sim}{\ell}(1/\ell)k^2\, h_1^{(1)}(\chi)\ j_1(y)\,\rangle$$

$$e \equiv \langle [1 - 5\alpha^2\xi^2]^{-1} (2/3)\alpha^2 k [h_0^{(1)}(\chi) j_0(y) - h_2^{(1)}(\chi) j_2(y)] \rangle$$

$$y = \ell r$$

$$Z = \tfrac{1}{2}|\underset{\sim}{k} - \underset{\sim}{k}_0| r$$

$$\underset{\sim}{\ell} = \tfrac{1}{2}(\underset{\sim}{k} + \underset{\sim}{k}_0)$$

Note that $(d\sigma^E/d\Omega_{lab}) = (d\sigma^E/d\Omega_{cm}) \cdot J(\theta_{lab})$, where $J(\theta_{lab}) = d\Omega_{cm}/d\Omega_{lab}$ and may be found in Section II.

APPENDIX B

The extension of Brueckner's formalism to a three phase shift, spin-dependent amplitude is indicated below. Following Brueckner, we write the solution to the wave equation outside the range of the scatterers,

$$\psi(\underset{\sim}{r}) = \psi_0(\underset{\sim}{r}) + (\mathcal{A} - i\underset{\sim}{A}\cdot\underset{\sim}{\nabla}) \frac{e^{ik|\underset{\sim}{r}-\underset{\sim}{r}_A|}}{|\underset{\sim}{r}-\underset{\sim}{r}_A|}$$

$$+ (\mathcal{B} - i\underset{\sim}{B}\cdot\underset{\sim}{\nabla}) \frac{e^{ik|\underset{\sim}{r}-\underset{\sim}{r}_B|}}{|\underset{\sim}{r}-\underset{\sim}{r}_B|} \qquad \text{(B.1)}$$

The amplitudes $\underset{\sim}{A}, \mathcal{A}$ are related to the incoming wave

$$\psi_0 = e^{i\underset{\sim}{k}_0\cdot\underset{\sim}{r}} t_0 \qquad \text{by}$$

$$\left.\begin{aligned} \underset{\sim}{A} &= -\frac{i}{3k^3}\eta_{33}T_A(2\underset{\sim}{\nabla} + i\underset{\sim}{\sigma}^A \times \underset{\sim}{\nabla})\psi_0 \\ \mathcal{A} &= \frac{1}{3k}(\eta_3 - \eta_1)T_A + \frac{1}{k}\eta_1 \end{aligned}\right\} \qquad \text{(B.2)}$$

with $T_A = \begin{pmatrix} 2 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix}$, $T_B = \begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}$, $T_A{}^2 = 3T_A$,

$T_A T_B T_A = T_A$, the rows and columns being labelled according to Brueckner. $\underset{\sim}{A}, \underset{\sim}{B}, \mathcal{A}, \mathcal{B}$ are given by the coupled equations:

$$\underset{\sim}{A} = -\frac{i}{3k^3}\eta_{33}\, T_A \,(2\underset{\sim}{\nabla} + i\underset{\sim}{\sigma}^A \times \underset{\sim}{\nabla})\left[\psi_0 + (\mathcal{B} - i\underset{\sim}{B}\cdot\underset{\sim}{\nabla})\frac{e^{ik|\underset{\sim}{r}-\underset{\sim}{r}_B|}}{|\underset{\sim}{r}-\underset{\sim}{r}_B|}\right]_{\underset{\sim}{r}=\underset{\sim}{r}_A}$$

$$\mathcal{A} = \left[\frac{1}{3k}(\eta_3 - \eta_1)\, T_A + \frac{1}{k}\eta_1\right]\left[\psi_0 + (\mathcal{B} - i\underset{\sim}{B}\cdot\underset{\sim}{\nabla})\frac{e^{ik|\underset{\sim}{r}-\underset{\sim}{r}_A|}}{|\underset{\sim}{r}-\underset{\sim}{r}_B|}\right]_{\underset{\sim}{r}=\underset{\sim}{r}_A} \qquad (B.3)$$

$$\underset{\sim}{B} = -\frac{i}{3k^3}\eta_{33}\, T_B \,(2\underset{\sim}{\nabla} + i\underset{\sim}{\sigma}^B \times \underset{\sim}{\nabla})\left[\psi_0 + (\mathcal{A} - i\underset{\sim}{A}\cdot\underset{\sim}{\nabla})\frac{e^{ik|\underset{\sim}{r}-\underset{\sim}{r}_A|}}{|\underset{\sim}{r}-\underset{\sim}{r}_A|}\right]_{\underset{\sim}{r}=\underset{\sim}{r}_B}$$

$$\mathcal{B} = \left[\frac{1}{3k}(\eta_3 - \eta_1)\, T_B + \frac{1}{k}\eta_1\right]\left[\psi_0 + (\mathcal{A} - i\underset{\sim}{A}\cdot\underset{\sim}{\nabla})\frac{e^{ik|\underset{\sim}{r}-\underset{\sim}{r}_A|}}{|\underset{\sim}{r}-\underset{\sim}{r}_A|}\right]_{\underset{\sim}{r}=\underset{\sim}{r}_B}$$

To first order in the propagator in the numerator and to second order in the denominator for $\underset{\sim}{A}$, $\mathcal{A}$, there results,

$$\underset{\sim}{A} = \underset{\sim}{S}_A - \alpha T_A\,(2 + i\underset{\sim}{\sigma}^A \times)\,(i\underset{\sim}{n}\mu q_B + \xi\underset{\sim}{S}_B + \underset{\sim}{n}\,\underset{\sim}{n}\cdot\zeta\underset{\sim}{S}_B)$$

$$+ \alpha^2 T_A T_B \xi^2\,[4\underset{\sim}{A} - \underset{\sim}{\sigma}^A \times (\underset{\sim}{\sigma}^B \times \underset{\sim}{A})\,] \qquad (B.4)$$

$$\mathcal{A} = q_A + (\beta T_A + \gamma)(G q_B - i\underset{\sim}{n}\cdot\mu\underset{\sim}{S}_B) + (\beta T_A + \gamma)\,[(\beta T_B + \gamma)G^2$$

$$- 2\alpha T_B \mu^2]\,\mathcal{A} \qquad (B.5)$$

$$\underset{\sim}{S}_A = \alpha T_A(2\underset{\sim}{k}_0 + i\underset{\sim}{\sigma}^A \times \underset{\sim}{k}_0)\ \psi_0\ (\underset{\sim}{r}_A)$$

$$q_A = (\beta T_A + \gamma)\quad \psi_0\ (\underset{\sim}{r}_A)$$

$$\underset{\sim}{n} = r^{-1}\ \underset{\sim}{r}$$

$$G = ik\ h_0^{(1)}(\chi)$$

$$\xi = (-ik^3/3)[\ h_0^{(1)}(\chi) + h_2^{(1)}(\chi)]$$

$$\zeta = ik^3\ h_2^{(1)}(\chi)$$

$$\mu = -\ ik^2\ h_1^{(1)}(\chi)$$

Under the indicated approximation,

$$\mathcal{a} \simeq [1 - (\beta T_A + \gamma)(\beta T_B + \gamma)G^2 + (\beta T_A + \gamma)\ T_B 2\alpha\mu^2]^{-1}$$

$$\{q_A + (\beta T_A + \gamma)(Gq_B - i\underset{\sim}{n}\cdot\mu\underset{\sim}{S}_B)\ \} \qquad \text{(B.6)}$$

The denominator in (B.6) is approximately rationalized after expansion of T_A, T_B, T_AT_B in terms of the basis system of four hypercomplex numbers, $\underset{\sim}{\tau}$, 1.

Noting that

$$\frac{1}{\chi + T_AT_By} = \frac{T_AT_B}{\chi + y} + \frac{1 - T_AT_B}{\chi}\ ,$$

with $(T_AT_B)^2 = T_AT_B$, as well as,

$$\frac{1}{\chi + \Lambda^1 y} = \frac{\Lambda^1}{\chi + y} + \frac{\Lambda^o}{\chi}\ ,$$

with $(\Lambda^1)^2 = \Lambda^1$, one easily solves (B.4) for $\underset{\sim}{A}$. Further, from the following relations, where f denotes the scattered amplitude,

$$f(\underset{\sim}{k} \leftarrow \underset{\sim}{k}_0) = \left\langle (\mathcal{a} + \underset{\sim}{A}\cdot\underset{\sim}{k})\, e^{-i\underset{\sim}{k}\cdot\underset{\sim}{r}_A} + A \longleftrightarrow B \right\rangle , \qquad (B.7)$$

$$\sigma_{tot} = 4\pi ƛ \ \mathrm{Im}\, \frac{1}{3}\, \mathrm{Sp}\, [f(\underset{\sim}{k}_0 \leftarrow \underset{\sim}{k}_0)_{11}\, \Lambda^1] \quad , \qquad (B.8)$$

and

$$(d\sigma^E/d\Omega) = \frac{1}{3}\, \mathrm{Sp}\, \Lambda^1 f(\underset{\sim}{k} \leftarrow \underset{\sim}{k}_0)^+{}_{11}\, \Lambda^1 f(\underset{\sim}{k} \leftarrow \underset{\sim}{k}_0)_{11} \quad , \qquad (B.9)$$

it is a straightforward, though tedious, calculation to arrive at the expressions presented in Appendix A.

TABLE CAPTIONS

I. Total cross sections on the Brueckner model at 85 and 190 Mev in the laboratory. At 85 Mev the phase shifts have been taken as $\delta_{33} = .27$, $\delta_3 = -.12$, $\delta_1 = .16$. At 190 Mev we have taken $\delta_{33} = \pi/2$, $\delta_3 = -.183$, $\delta_1 = .266$. $\sigma_{free} \equiv$ sum of the free particle cross sections; $\sigma_{exact} \equiv$ the result of the exact treatment of the multiple scattering.

II. The elastic π-D differential cross section calculated on the Brueckner model at 85 and 190 Mev in the laboratory. At 85 Mev the phase shifts have been taken as $\delta_{33} = .27$, $\delta_1 = .16$, $\delta_{13} = \delta_{31} = \delta_{11} = 0$. At 190 Mev the phase shifts used are $\delta_{33} = \pi/2$, $\delta_3 = -.183$, $\delta_1 = .266$, $\delta_{13} = \delta_{31} = \delta_{11} = 0$. The differential cross sections are given in the laboratory system.

TABLE I

k/μ	E_{lab} Mev	σ_{free} mb	$\sigma_{double\ scattering}$ mb	σ_{exact} mb
1.03[a]	85	54	61.5	63.3
1.03[b]	85	55.3	58	
1.66[a]	190	250		210

a $\delta_{13} = \delta_{31} = \delta_{11} = 0$

b $\delta_{13} = \delta_{31} = -.044, \quad \delta_{11} = 0$

TABLE II

k/μ	$\sigma^E(0)_{\text{impulse approx.}}$	$\sigma^E(0)_{\text{corrected}}$	$\sigma^E(\pi)_{\text{impulse approx.}}$	$\sigma^E(\pi)_{\text{corrected}}$
1.03	14.1 mb/sterad	13.1 mb/sterad[a]	1.63 mb/sterad[a]	1.67 mb/sterad
1.66	99.5 mb/sterad	69.5 mb/sterad	2.0 mb/sterad	.33 mb/sterad

[a] These values were computed in the double scattering approximation.

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FIGURE CAPTIONS

1. The effect of the Coulomb interaction on elastic π^+-D scattering at 85 Mev ($k'/\mu = 1.03$). Curve A: $(d\sigma^E/d\Omega)_{NUCLEAR}$ vs. laboratory angle θ. Curve B: $(d\sigma^E/d\Omega)_{NUCLEAR+COULOMB}$ vs. θ. The phase shifts used are $\delta_{33} = .27$, $\delta_{31} = \delta_{13} = -.044$, $\delta_{11} = 0$, $\delta_1 = .16$, and $\delta_3 = -.12$.

2. The effect of the Coulomb interaction on the (elastic + inelastic) differential cross section at 85 Mev ($k'/\mu = 1.03$). Curve A: $(d\sigma^{E+I}/d\Omega)_{NUCLEAR}$ vs. laboratory angle θ. Curve B: $(d\sigma^{E+I}/d\Omega)_{NUCLEAR+COULOMB}$ vs. θ. The phase shifts used are enumerated in the caption for Figure 1.

3. ρ_D/ρ_F vs. laboratory angle θ ($k'/\mu = 1.03$).

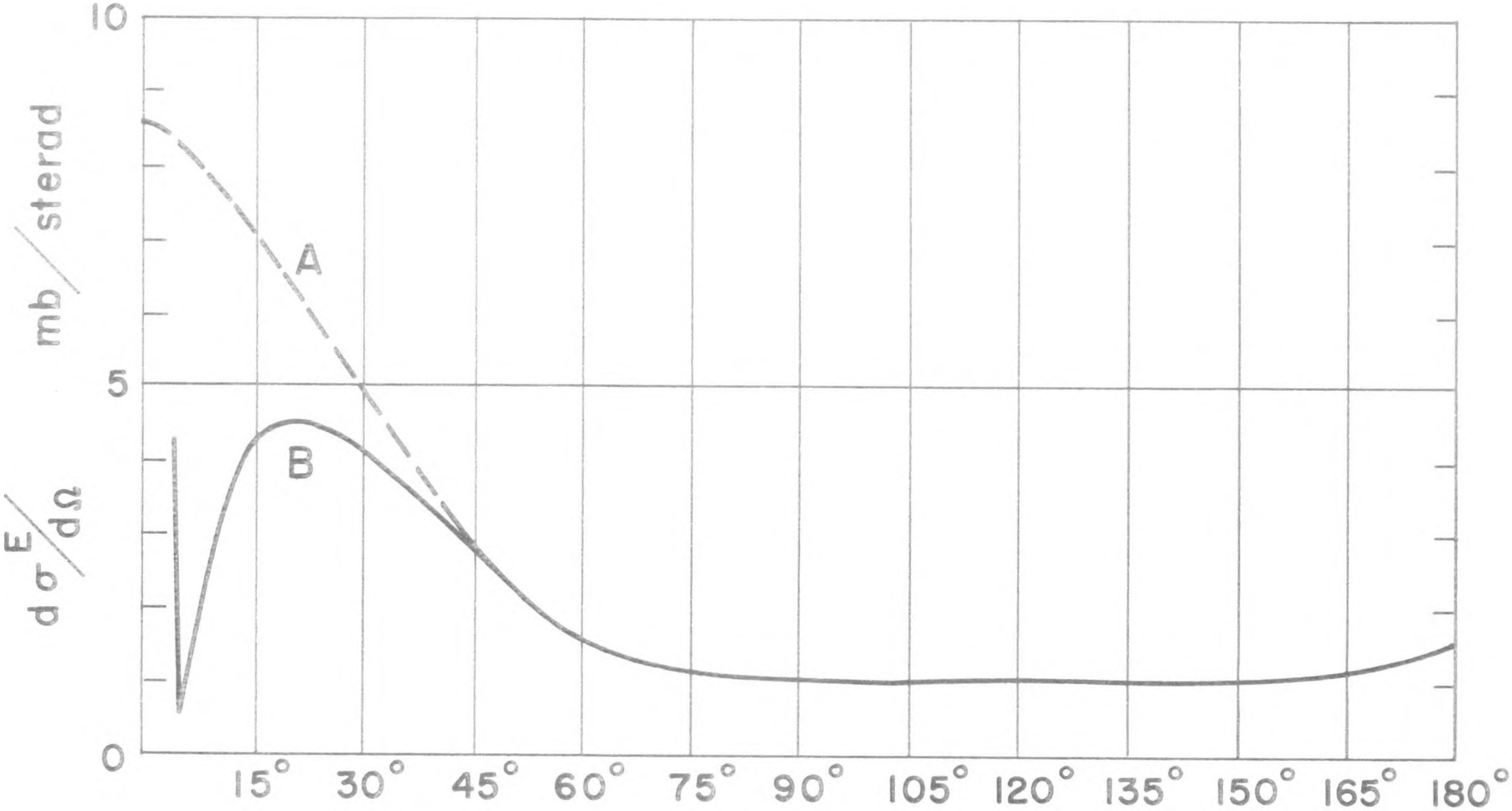
4. H(2K) vs. laboratory angle θ ($k'/\mu = 1.03$).

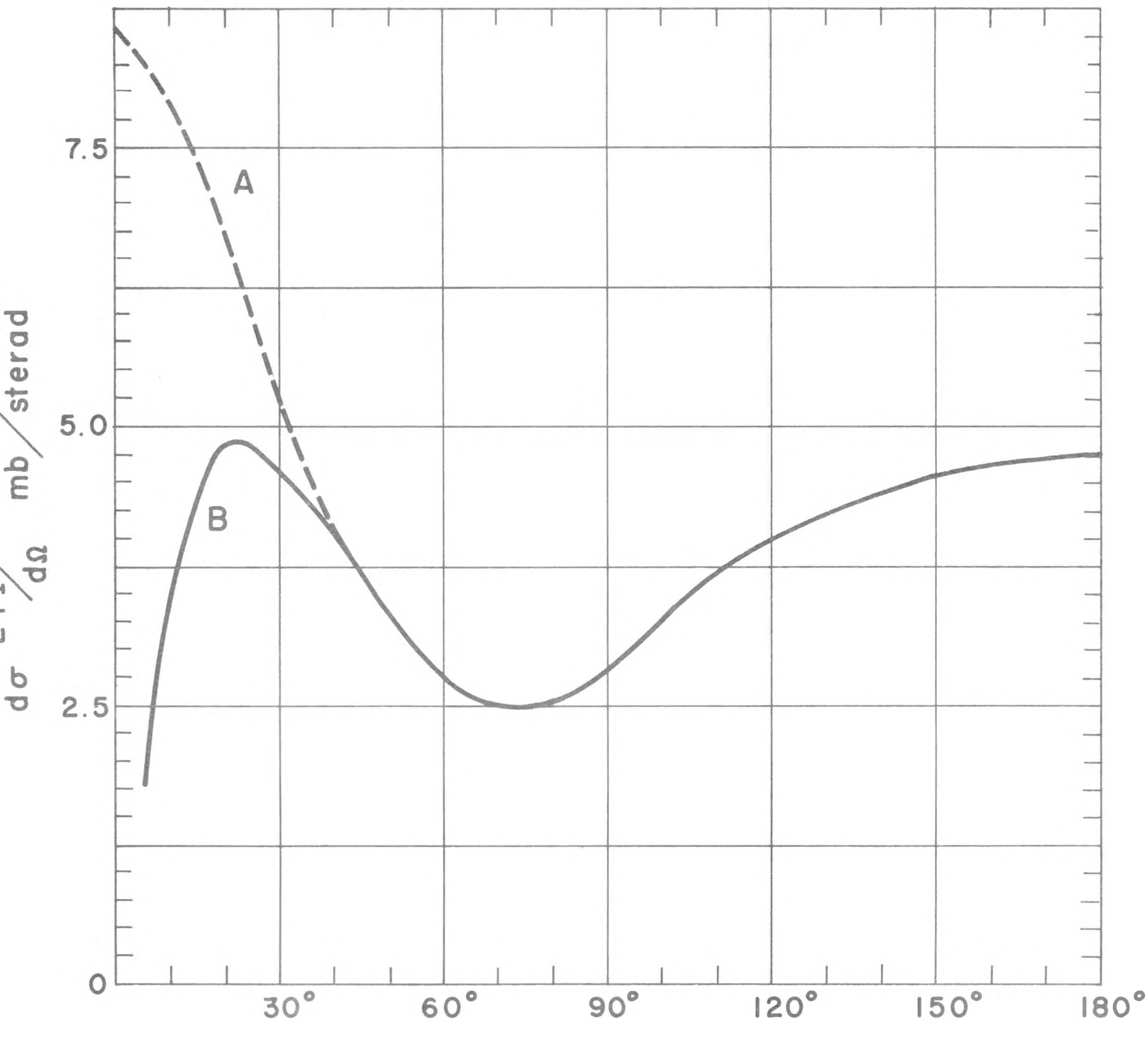
5. Feynman diagram for double scattering. The two particle transition operators, t_i, are represented by circles, the nucleons by heavy lines, the mesons by dashed lines.

6. (a) and (b) are the f^2-potential corrections to single scattering. (c) is an example of a ladder correction to single scattering. (d) and (e) are three-meson potential corrections to single scattering.

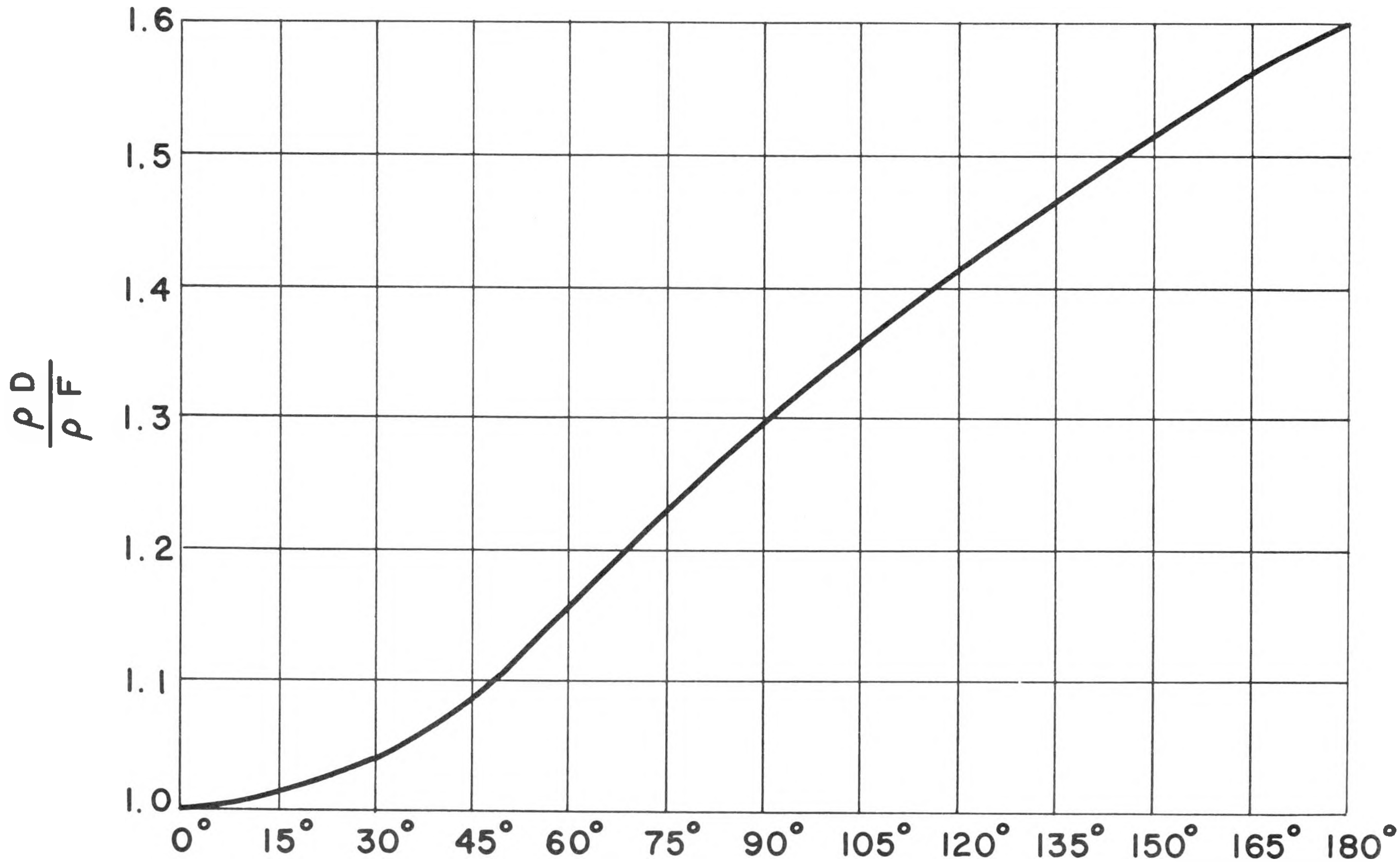
FIGURE CAPTIONS - continued

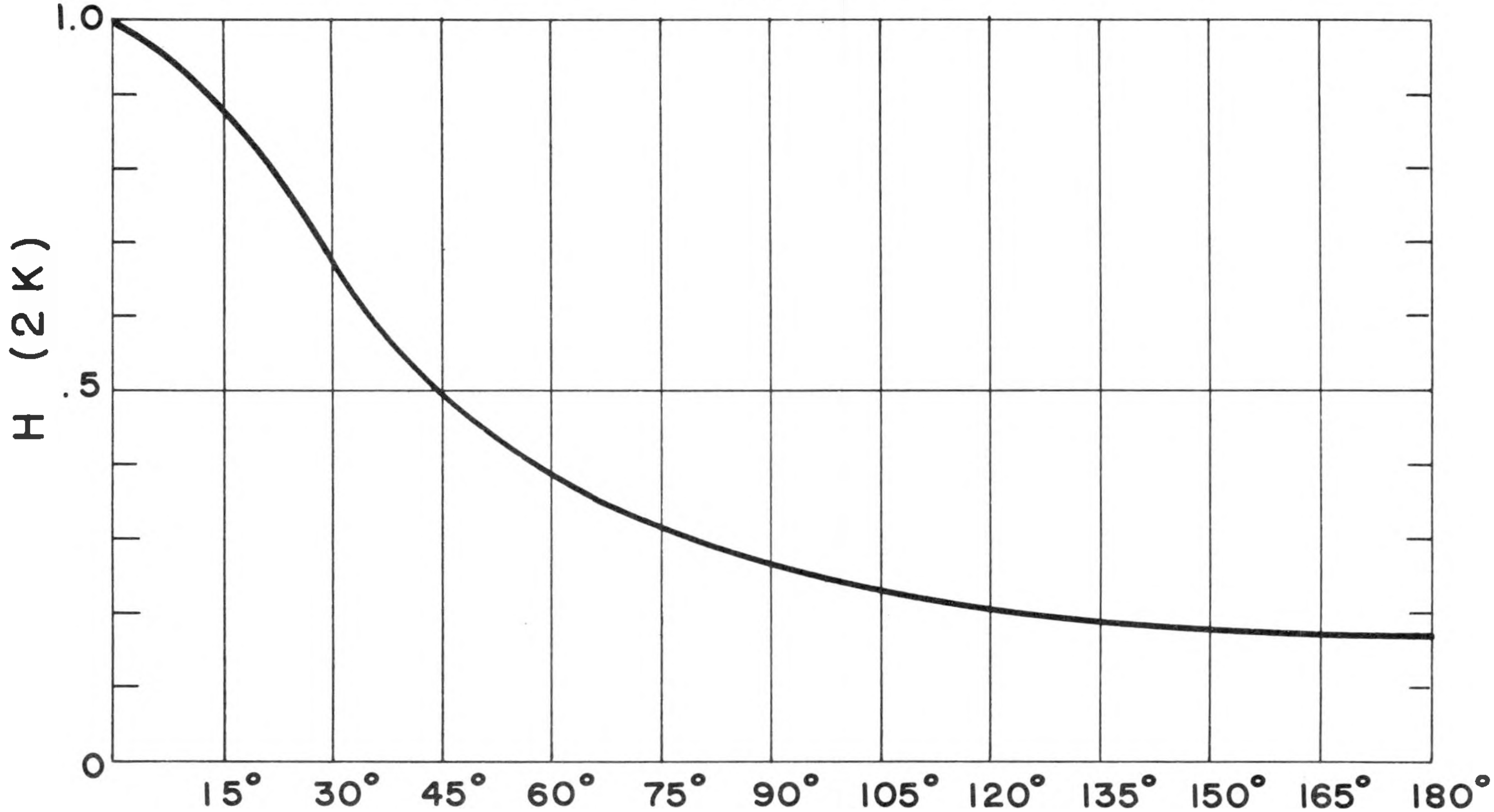
7. The absorption correction to $d\sigma^E/d\Omega$ at 85 Mev ($k'/\mu = 1.03$). Curve A is $d\sigma^E/d\Omega$ uncorrected for absorptive scattering; curve B is $d\sigma^E/d\Omega$ corrected for absorptive scattering, with $\sigma_{abs} = 8$ mb.

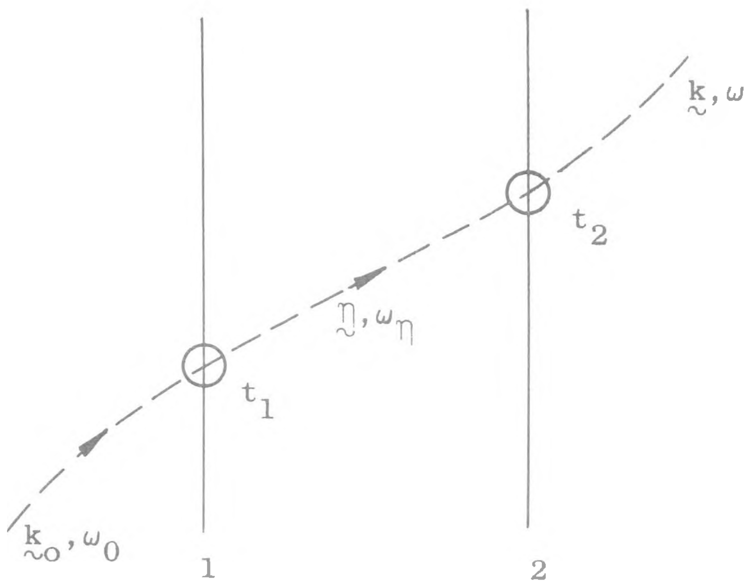

10
5
0
mb/sterad
dσ^E/dΩ
A
B
15°
30°
45°
60°
75°
90°
105°
120°
135°
150°
165°
180°

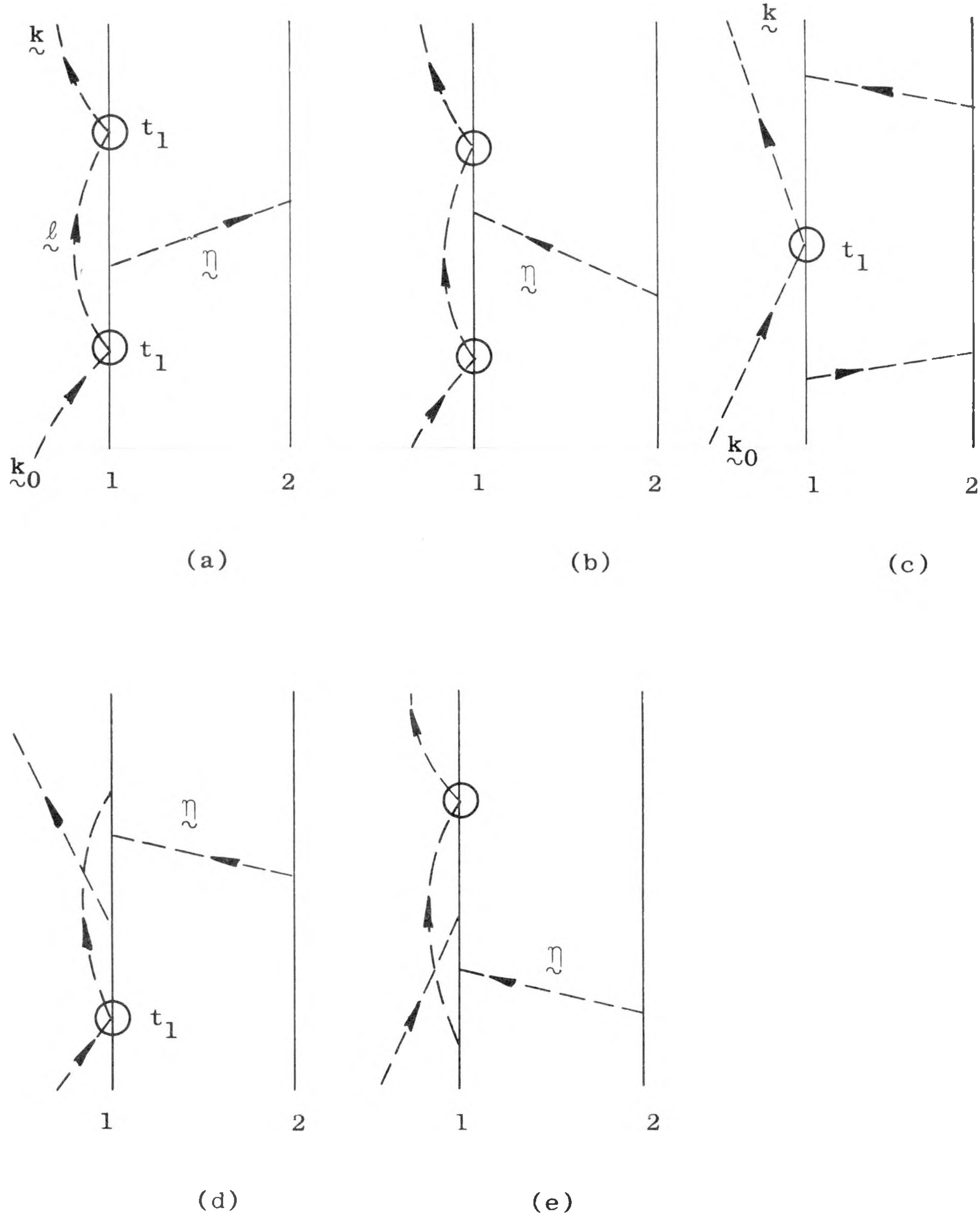


7.5
5.0
2.5
0
A
B
$d\sigma^{E+I}/d\Omega$ mb/sterad
30°
60°
90°
120°
150°
180°


1.6
1.5
1.4
1.3
1.2
1.1
1.0
ρD / ρF
0° 15° 30° 45° 60° 75° 90° 105° 120° 135° 150° 165° 180°


1.0
.5
0
H (2K)
15°
30°
45°
60°
75°
90°
105°
120°
135°
150°
165°
180°


k, ω
t_2
η, ω_η
t_1
k_0, ω_0
1
2


k
t1
ℓ
η
t1
k0
1
2
(a)
η
1
2
(b)
k
t1
k0
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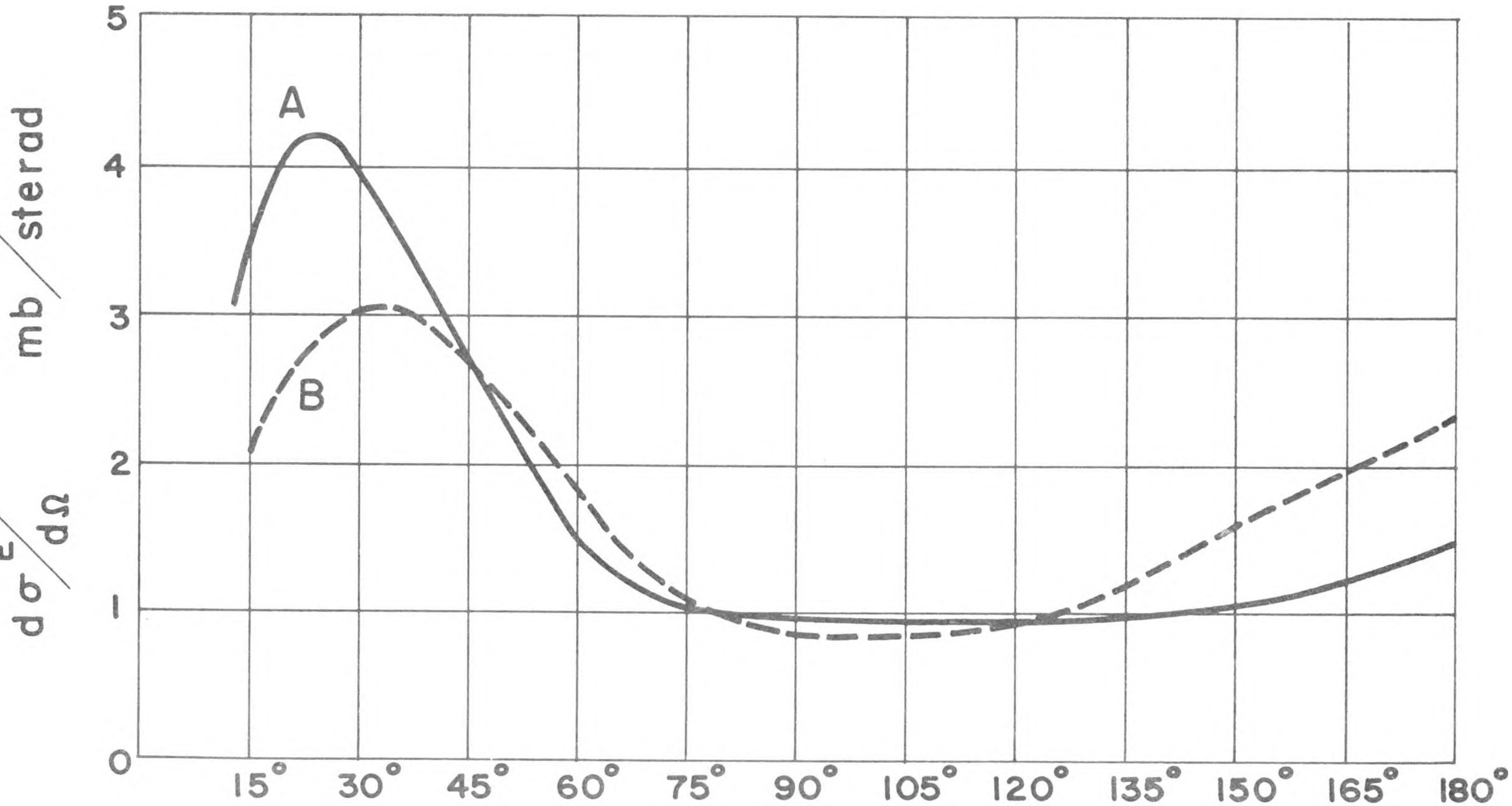
(d)
η
1
2
(e)



5
4
3
2
1
0
mb/sterad
dσ^E/dΩ
A
B
15°
30°
45°
60°
75°
90°
105°
120°
135°
150°
165°
180°