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COO-1469-232

STEREOMATRIX 3-D DISPLAY SYSTEM

August 1973

Department of Computer Science
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Price: Paper Copy $4.00
Microfiche $1.45

Printed in the United States of America
USAEC Technical Information Center, Oak Ridge, Tennessee

UIUCDCS-R-73-590

COO-1469-0232
Distribution Category UC-38

THE STEREOMATRIX 3-D DISPLAY SYSTEM

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This work was supported in part by Contract No. US AEC AT(11-1) 1469 and was submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering, at the University of Illinois.

ACKNOWLEDGMENT

The author would like to thank Professor W. J. Poppelbaum who originally proposed this project. Thanks are also due Professor W. J. Kubitz for continued support and helpful discussions.

Thanks are due my wife, Carole, who made the beautiful drawings in this thesis.

THE STEREOMATRIX 3-D DISPLAY SYSTEM

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STEREOMATRIX is a large screen interactive 3-D laser display system which presents computer-generated wire figures stereoscopically. The presented image can be rotated, translated, and scaled by the system user and the perspective of the image is changed according to the position of the user. A cursor may be positioned in three dimensions to identify points and allows communication with the computer.

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1. INTRODUCTION

Large screen displays have long intrigued the imagination of man. There are many applications where information presented by large displays is vital to daily operations. With the advent of powerful lasers and more importantly a means of deflecting their intense light new possiblities in large screen displays were opened. The STEREOMATRIX system was proposed by Professor Poppelbaum to explore some of these new possiblities.

The fundamental concept of the proposed system was to present stereo views to an observer, but to really test display possibilities many unusual features were combined in STEREOMATRIX. The overall system was designed to be a real time, random access, interactive, large screen display. The system was to correct the presented image for the observer's actual position and the observer could rotate, translate and scale the presented three dimensional image.

The basic method chosen to produce a three dimensional display relies on the stereopair intersection points, P_L and P_R, as illustrated in Figure 1. For each point in the display volume a stereopair of points are generated by the hardware of the system and projected on a screen by two cross-polarized laser beams. By wearing a pair of analyzing glasses the observer's left and right eyes are constrained to see only the respective left and right points. The observer will then perceive a single point which is formed a distance behind the screen at the intersection of the extended left eye and right eye image lines. To further enhance the stereopair three dimensional effect the perspective of the figure is changed as the observer moves. An observer position detector determines the X_o, Z_o coordinates of the observer and the screen images are modified in size, separation and angular position. For a data point P(x,y,z) the system uses the following simple, geometrically-derived equations to generate the stereopair points.

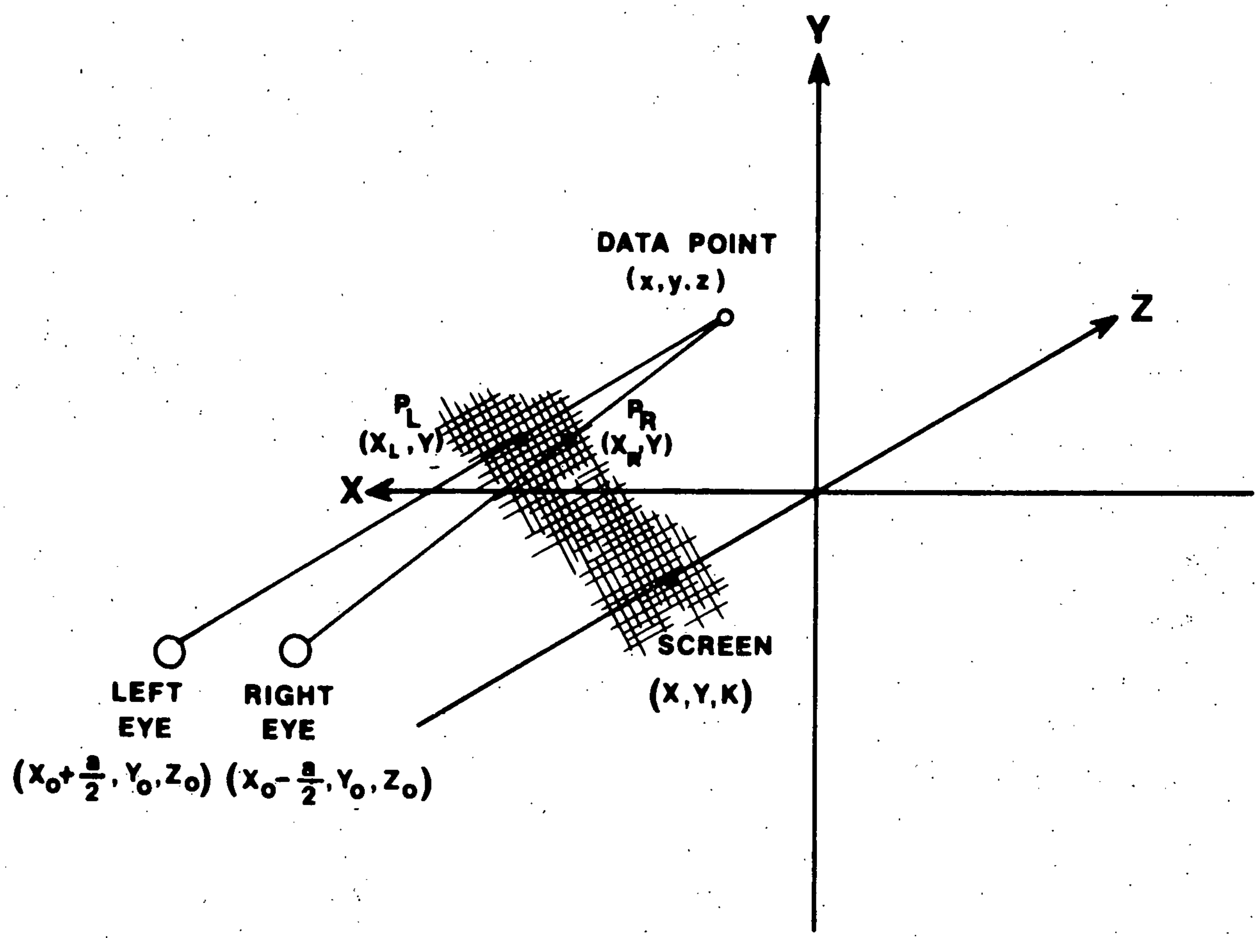

Figure 1. Stereopair Display Points

$$X_L = X_o + \frac{a}{2} + \frac{Z_o - K}{Z_o - z}(x - X_o - \frac{a}{2})$$

$$X_R = X_o - \frac{a}{2} + \frac{Z_o - K}{Z_o - z}(x - X_o + \frac{a}{2})$$

$$Y = \frac{Z_o - K}{Z_o - z}(y)$$

The derivation of these equations is included as Appendix I. These equations differ from previously published equations (Verma, 1972) by the screen position constant K, the sign of z, and the sign of a/2.

To accomplish the desired rotation and translation on command, the STEREOMATRIX system generates a 4 x 4 transformation matrix and continuously modifies the appropriate coefficients. Each incoming data point is premultiplied by this matrix.

Scaling is accomplished by controlling the gain of an operational amplifier and a three dimensional cursor is multiplexed in to allow the observer to identify points to a computer.

1.2 STEREOMATRIX System Components

The six system components are shown in Figure 2. An (x,y,z) triplet for each picture point is accepted in digital or analog form by the perspective transformer. This analog computer generates a stereopair of points from each picture triplet using the perspective equations. Variables in the equations are the observer's position and inputs from the coefficient generator which specify any rotations and translations the observer has requested.

The coefficient generator is basically a single purpose digital computer which responds to commands from a remote control box and computes new elements in a three-by-four matrix which determines rotations and translations.

The position detector is an infrared sensing system which uses digital shaft encoders to measure angles. Analog computation is then performed and X_o,Z_o coordinates are supplied to the transformer.

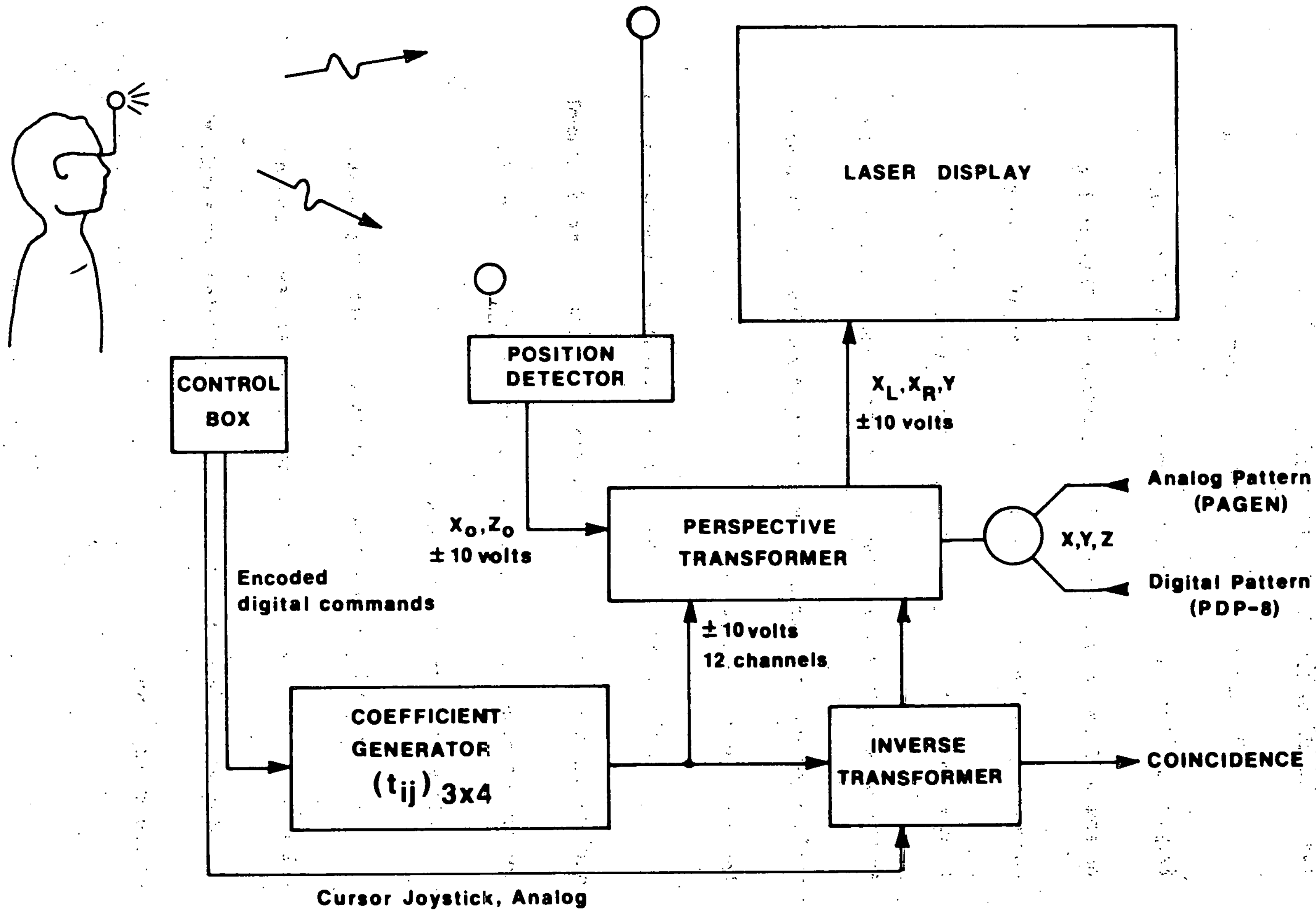

Figure 2. STEREOMATRIX Block Diagram

The inverse transformer accepts the analog X,Y,Z input from the cursor joystick and performs an analog inverse transformation to maintain the cursor stationary in the viewing volume regardless of the rotations and translations performed on the input pattern.

The control box encodes the push button switches which allow the observer to select an axis of rotation or translation. It also contains the cursor joystick and on-off switch as well as slow and normal operation switches.

The laser display is a random access, ultrasonic-deflected system. The observer wears perpendicularly polarized glasses to view right and left eye images on a rear projection screen.

Figure 3 is a pictorial drawing of the complete system.

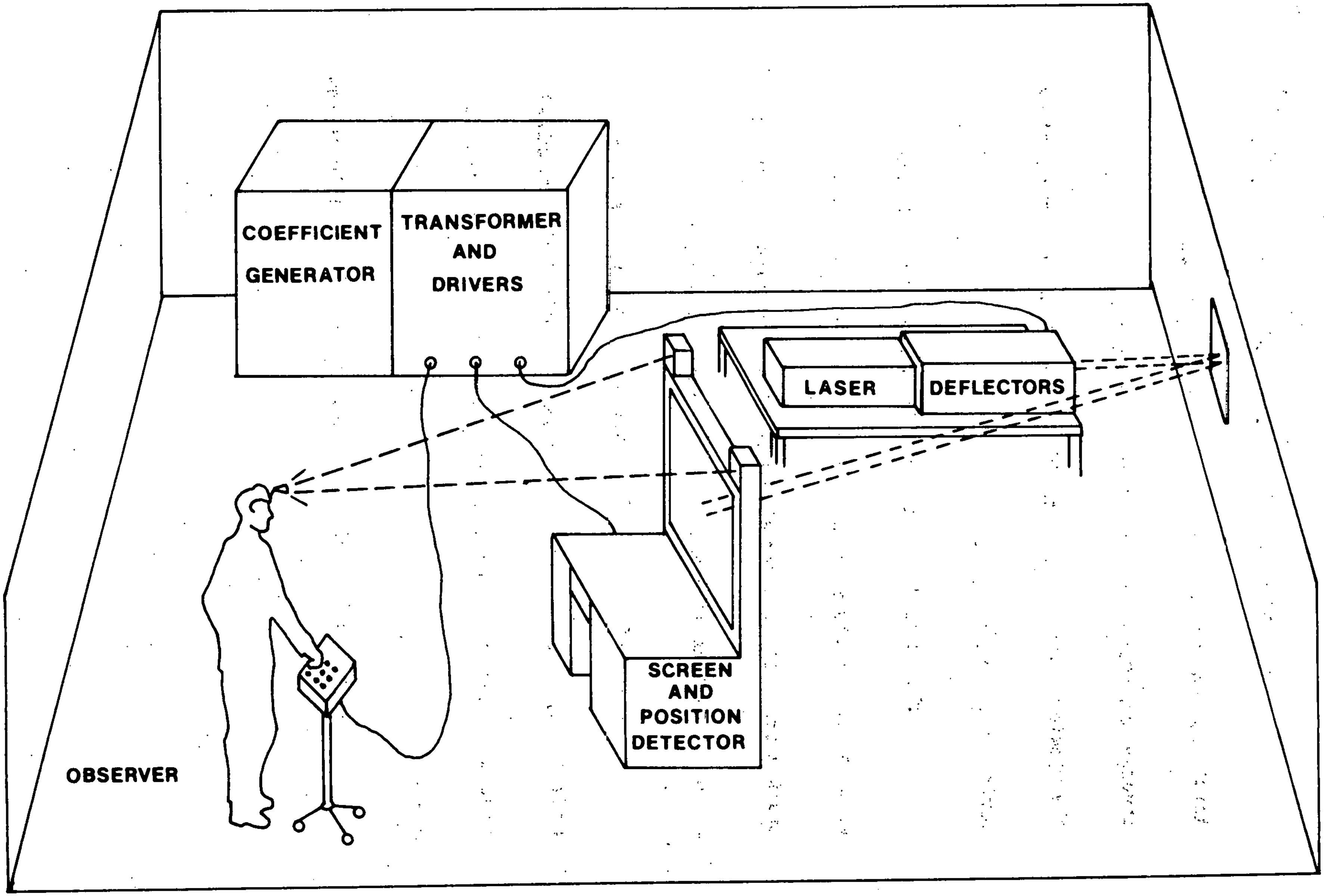

Figure 3. STEREOMATRIX Pictorial

2. OBSERVER POSITION DETECTOR

The observer must wear analyzing glasses to separate the two polarized images so that each eye sees only one image. Mounted on the glasses is a battery-powered infrared source. The angular position of this source is detected by two rotating mirror-phototransistor units which contain optically encoded discs. The geometry of the position detector is shown in Figure 4. The resulting equations for X_o, Z_o are as follows,

$$X_o = \frac{D}{2} \frac{\sin(\theta - \phi)}{\sin(\theta + \phi)}$$

$$Z_o = \frac{-D \sin\theta \sin\phi}{\sin(\theta + \phi)} + K$$

Notice that these equations differ by the sign of Z_o and the screen position constant K from previously published equations (Pirnat, 1972).

The action of the position detector is as follows. At the instant that the rotating mirror reflects the infrared to the phototransistor detector strobe, the optically encoded disc is read to eight bits accuracy of sine and cosine for both

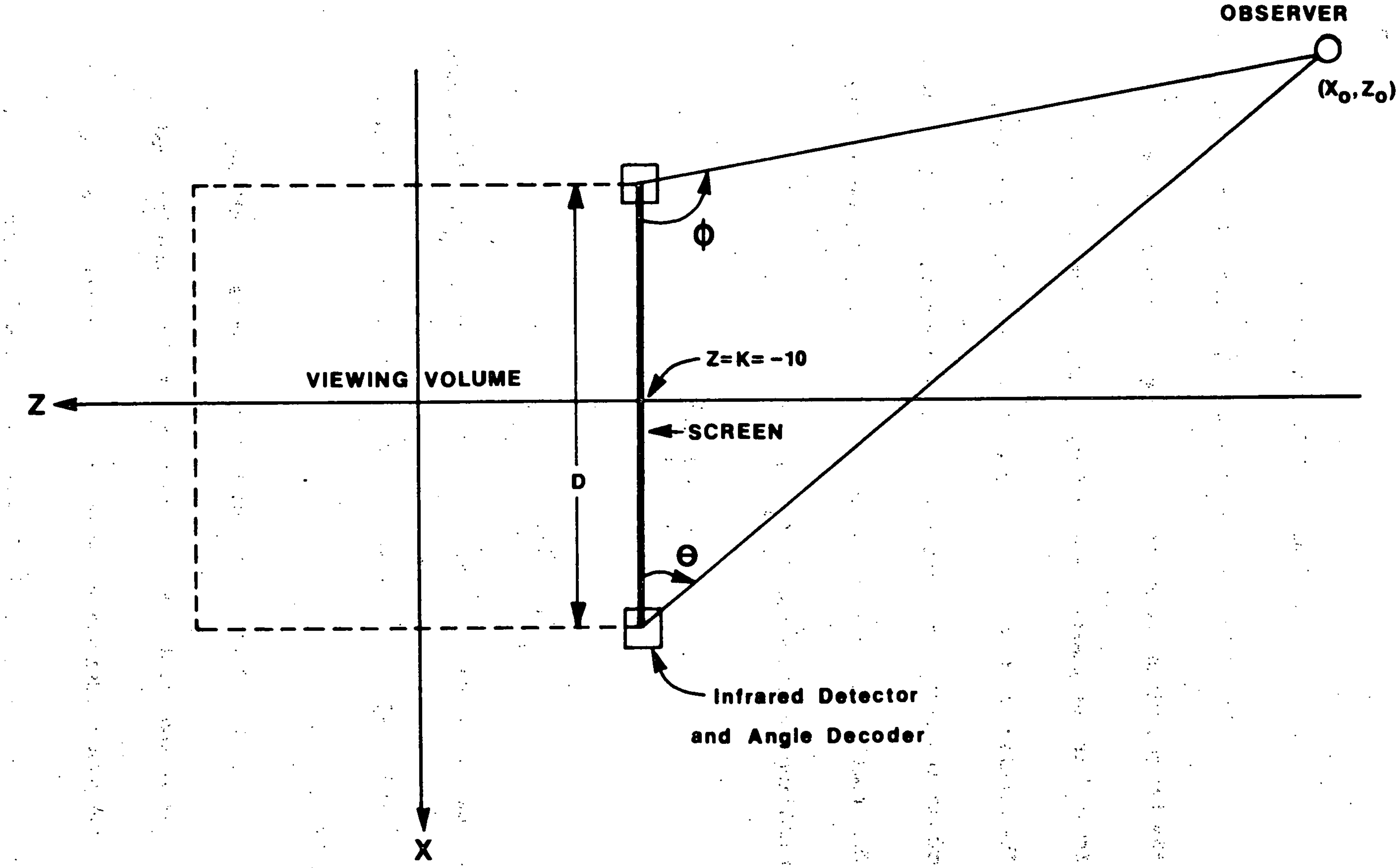

Figure 4. Observer Position Detector Geometry

of the two angles, four values in all. In synchronism with the overall system, these digital values are converted to analog and the computations of X_o and Z_o are performed. The resultant analog voltages are transmitted by cable to the STEREOMATRIX Transformer.

Notice that for a realistic three dimensional system there must be an identical scale factor between the viewing volume and the observer space if we are going to correct the presented image for observer motion.

In this system the observer may exist between Z equal -3 to -10 feet. At our chosen scale factor of 5 volts per foot, this is -15 to -50 volts. For a practical signal range the position detector actually transmits $\frac{Z_o}{10}$, a signal range of -1.5 to -5.0 volts.

3. CONTROL BOX

The control box provides observer interaction with the system. The observer may reset the system from the control box. This initializes the rotation and translation matrix of coefficients to the augmented unit matrix, $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Scaling is controlled only by the Forward and Backward buttons and resets only when the system is switched on.

The observer may select an axis of rotation or a direction of translation and execute the desired amount of movement by pressing Forward or Backward. Speed of movement is selectable as Slow or Normal.

A cursor of three small crossed lines may be multiplexed in by pressing the Cursor On switch. The cursor may be positioned in three dimensions with a joystick. This would be useful for identifying points to a computer.

The schematic of the control box is given in Figure 5. The thirteen digital commands are stored and encoded into eight wires. The analog joystick is not shown as it is simply three

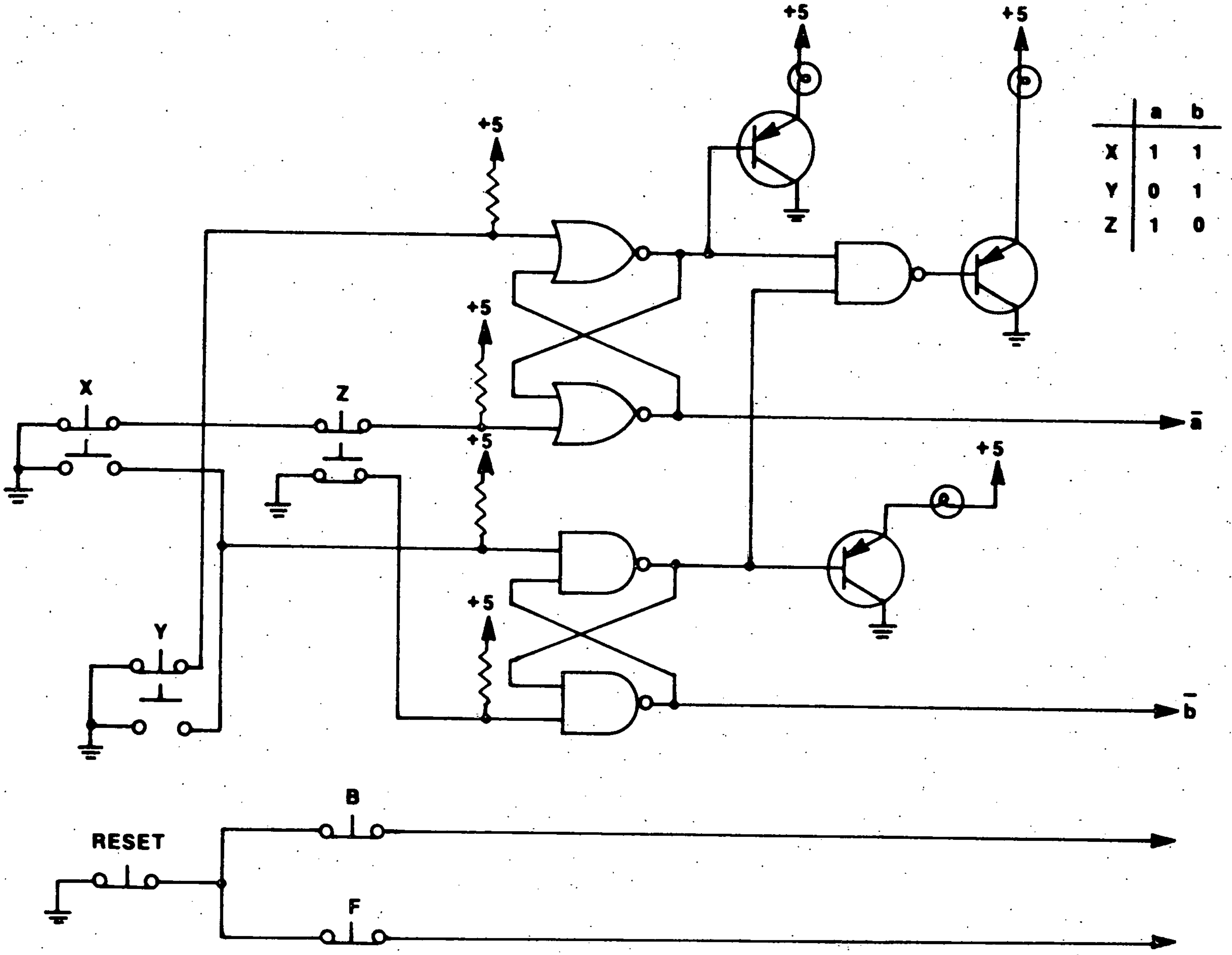

Figure 5. Partial Schematic of Control Box

pots and a commercial mechanical assembly. All selected switches are lighted to indicate to the operator the state of the machine.

4. COEFFICIENT GENERATOR

When the operator commands a rotation or a translation, he is actually commanding a change is the coefficient matrix which describes the transformation from data space to display space. When reset, the transformation is the simple augmented unit matrix given in Chapter 3. All twelve of the possible elements of that matrix are stored in digital registers and converted to analog to drive the STEREOMATRIX Transformer. When a translation is desired along the X-axis, for example, t_{14} is periodically incremented. Similarly, a Y translation involves incrementing only t_{24}, and a Z translation affects only t_{34}.

A rotation is far more complicated as up to eight of the transformation coefficients must be changed each time frame. A rotation about a selected axis may be accomplished by premultiplication of the transformation coefficient matrix with a matrix operator . The matrix operator for an X rotation is,

$$L_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}.$$

A single premultiplication by L_x rotates the image through an angle θ. Observe that a second premultiplication by the same operator produces terms like $(\cos^2\theta - \sin^2\theta) = \cos 2\theta$. Thus we may incrementally rotate the image to any desired multiple of θ by repeated premultiplication. (See Appendix II for an example of a 90° rotation). In STEREOMATRIX $\theta = 2^\circ$ and the new coefficient matrix rotated by 2° is computed for each picture frame. The order of computation and the incremental new values of the coefficients are listed in Table I. All of the new coefficients are stored as they are calculated and at the beginning of each frame a complete new coefficient matrix is gated to the output digital-to-analog converters.

Coefficient Computed	X-Rotation	Y-Rotation	Z-Rotation	X-Trans.	Y-Trans.	Z-Trans.
t_{11}^{n+1}	t_{11}^{n}	$Ct_{11}^{n} + St_{31}^{n}$	$Ct_{11}^{n} + St_{21}^{n}$	t_{11}^{n}	t_{11}^{n}	t_{11}^{n}
t_{21}^{n+1}	$Ct_{21}^{n} + St_{31}^{n}$	t_{21}^{n}	$Ct_{21}^{n} - St_{11}^{n}$	t_{21}^{n}	t_{21}^{n}	t_{21}^{n}
t_{31}^{n+1}	$Ct_{31}^{n} - St_{21}^{n}$	$Ct_{31}^{n} - St_{11}^{n}$	t_{31}^{n}	t_{31}^{n}	t_{31}^{n}	t_{31}^{n}
t_{12}^{n+1}	t_{12}^{n}	$Ct_{12}^{n} + St_{32}^{n}$	$Ct_{12}^{n} + St_{22}^{n}$	t_{12}^{n}	t_{12}^{n}	t_{12}^{n}
t_{22}^{n+1}	$Ct_{22}^{n} + St_{32}^{n}$	t_{22}^{n}	$Ct_{22}^{n} - St_{12}^{n}$	t_{22}^{n}	t_{22}^{n}	t_{22}^{n}
t_{32}^{n+1}	$Ct_{32}^{n} - St_{22}^{n}$	$Ct_{32}^{n} - St_{12}^{n}$	t_{32}^{n}	t_{32}^{n}	t_{32}^{n}	t_{32}^{n}
t_{13}^{n+1}	t_{13}^{n}	$Ct_{13}^{n} + St_{33}^{n}$	$Ct_{13}^{n} + St_{23}^{n}$	t_{13}^{n}	t_{13}^{n}	t_{13}^{n}
t_{23}^{n+1}	$Ct_{23}^{n} + St_{33}^{n}$	t_{23}^{n}	$Ct_{23}^{n} - St_{13}^{n}$	t_{23}^{n}	t_{23}^{n}	t_{23}^{n}
t_{33}^{n+1}	$Ct_{33}^{n} - St_{23}^{n}$	$Ct_{33}^{n} - St_{13}^{n}$	t_{33}^{n}	t_{33}^{n}	t_{33}^{n}	t_{33}^{n}
t_{14}^{n+1}	t_{14}^{n}	$Ct_{14}^{n} + St_{34}^{n}$	$Ct_{14}^{n} + St_{24}^{n}$	$t_{14}^{n} - X_0$	t_{14}^{n}	t_{14}^{n}
t_{24}^{n+1}	$Ct_{24}^{n} + St_{34}^{n}$	t_{24}^{n}	$Ct_{24}^{n} - St_{14}^{n}$	t_{24}^{n}	$t_{24}^{n} - Y_0$	t_{24}^{n}
t_{34}^{n+1}	$Ct_{34}^{n} - St_{24}^{n}$	$Ct_{34}^{n} - St_{14}^{n}$	t_{34}^{n}	t_{34}^{n}	t_{34}^{n}	$t_{34}^{n} - Z_0$

TABLE I.

5. ANALOG TRANSFORMER

The Transformer is a two megaHertz analog computation unit which accepts the coefficient matrix (t_{ij}) and the data points $P(x,y,z)$ and then performs the matrix multiplication to transform the data into display space points $P(x_t,y_t,z_t)$.

$$(t_{ij})\ (X) = (X_t)$$

At this point the image has been rotated or translated according to the commands of the operator. Now to present a 3-D image we must generate the two distinct images for each eye, P_L and P_R. This is accomplished by another analog computation in the Transformer. The equations for this last analog computation are,

$$X_L = X_o + \frac{a}{2} + \frac{Z_o - K}{Z_o - z}\left(x - X_o - \frac{a}{2} \right)$$

$$X_R = X_o - \frac{a}{2} - \frac{Z_o - K}{Z_o - z}\left(x - X_o + \frac{a}{2} \right)$$

$$Y = \frac{Z_o - K}{Z_o - z}\left(y \right)$$

Where K = Screen position = -10 volts,

(x,y,z) = data point,

$(X_o,\ Y_o,\ Z_o)$ = observer position.

See appendix I for the simple, geometrical derivation of these equations.

A block diagram of the Transformer is given in Figure 6. Not shown is a digital interface section which allows cable communication with a PDP-8. Analog data and the cursor coordinates arrive on the left side. The Cursor multiplex generator is turned on and off by command of the observer at the control box. When the cursor is selected the two analog gates pass data to the summer cards at alternate times. Cursor coordinates are passed 1/4 of the time since they form a very small picture element. Longer time spent passing the cursor would make it too bright. The 371 cards perform the analog transformation equations which are as follows,

$$X_t = t_{11}x + t_{12}y + t_{13}z + t_{14}$$

$$Y_t = t_{21}x + t_{22}y + t_{23}z + t_{24}$$

$$Z_t = t_{31}x + t_{32}y + t_{33}z + t_{34}$$

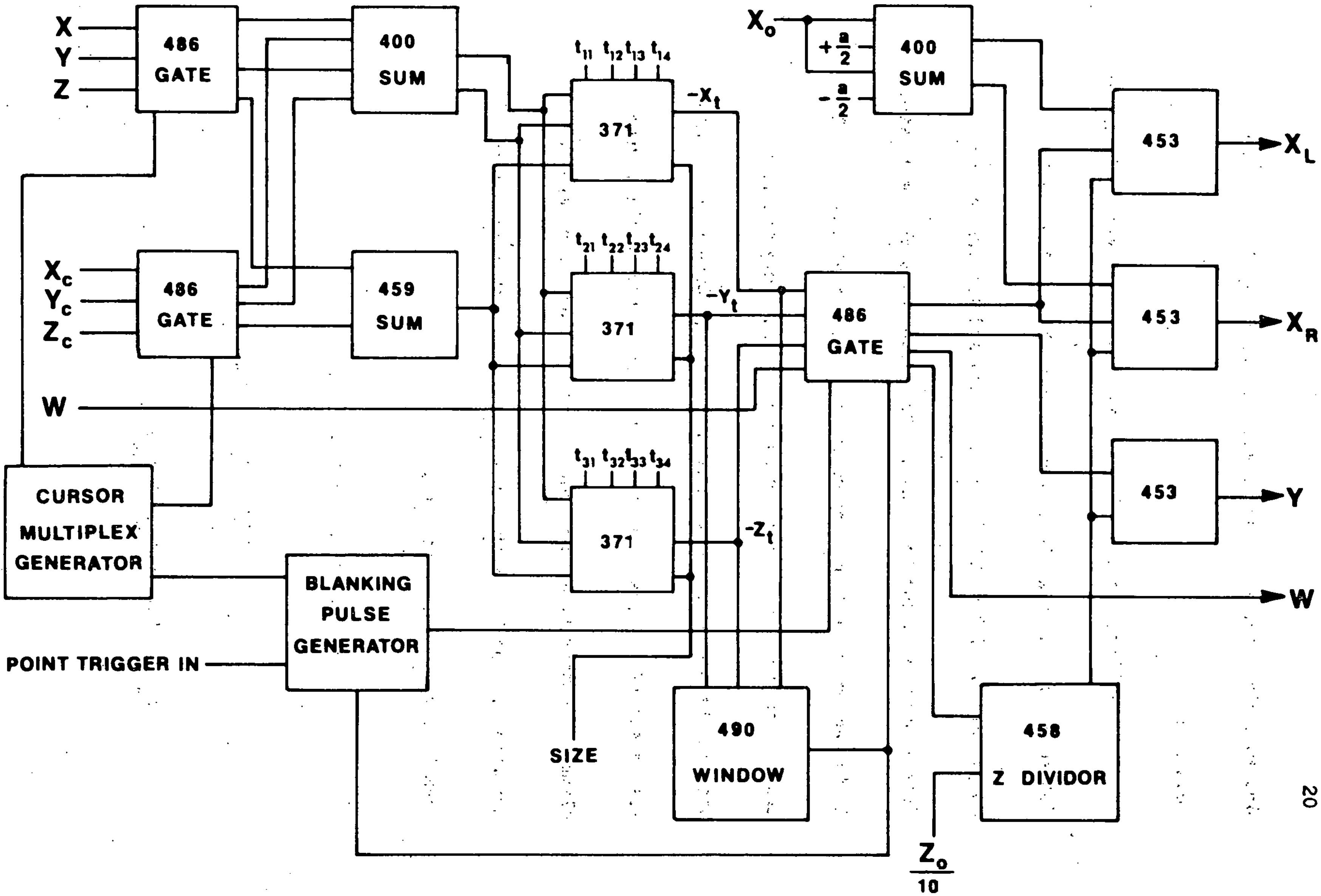

Figure 6. Transformer Block Diagram

Size control is also achieved here with an LED-phototransistor combination in the feedback loop of the output operational amplifier. The overall frequency response of this card is 2 MHz.

The transformed data (X_t, Y_t, Z_t) is then tested for excessive magnitude and if too large the next analog gate is turned off. If amplitudes are less than 10 volts the signals are passed on to the stereopair generator. Here the stereopair equations are executed. The 458 card forms the $(Z_o - K)/(Z_o - z)$ term. Then the 453 card does the final multiplication and summation to complete the stereopair generation.

The frequency response of the stereopair generator is determined by the 458 divider card. This is about 500 KHz, and is a satisfactory figure since the deflection system is limited to approximately 150 KHz.

6. CURSOR SYSTEM

The cursor system is included to allow a user to identify points of interest, to draw pictures, and to designate points to the computer. The cursor is a small 3-D cross which may be positioned anywhere in the 3-D viewing space by moving the joystick on the control box. The cursor is stationary in the display space, so rotations and translations of a figure do not move the cursor about. This is accomplished despite the successive unordered matrix multiplications performed by the coefficient generator simply by calculating the inverse of the latest value of the coefficient matrix. This inverse coefficient matrix is then used to premultiply the (X_c, Y_c, Z_c) cursor coordinates before they are input to the display system. In the display system the coordinates are subsequently premultiplied by the coefficinet matrix. These two premultiplications form the identity matrix and the true cursor coordinates are then displayed. Figure 7 is a block diagram of the cursor system. Notice that the output

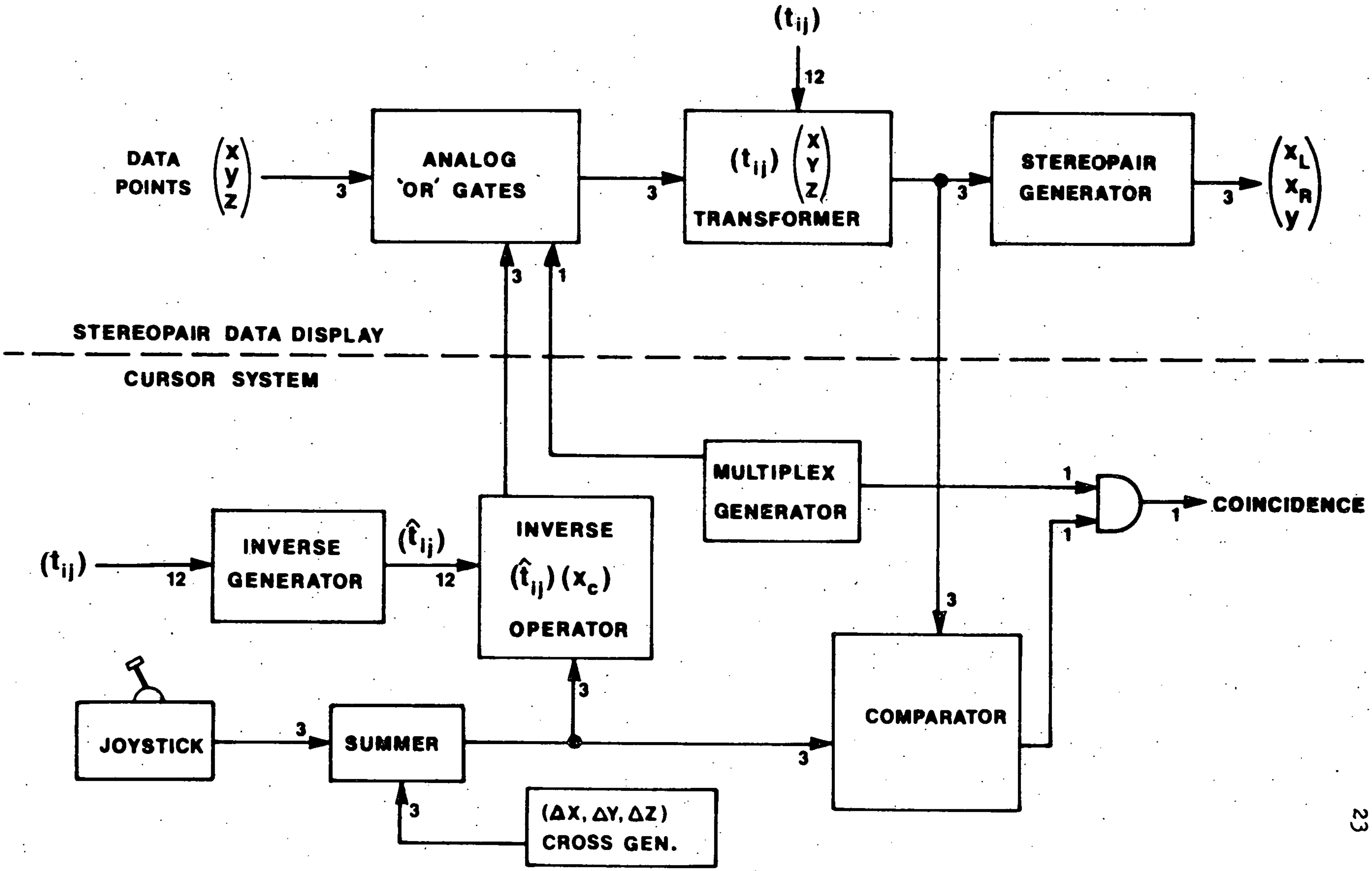

Figure 7. Cursor System Diagram

coincidence signal must be a gated signal as the comparator should see the true cursor coordinates at both inputs when the cursor is being displayed.

It is necessary to go this complicated route to produce the cursor as a simple input of the straight analog cursor signals to the Transformer would result in the cursor rotating and translating with the displayed image. A user would find it quite impossible to position the cursor after any sequence of operations as his simple right-directed movement of the joystick could correspond to any direction of the screen depending on the preceding operations and their order of occurrance.

Figure 8 is a block diagram of the actual Inverse Matrix Generator. (t_{ij}), the coefficient matrix comes in on the left and is used to form all of the elements of the inverse matrix. The problem is simplified in our system because we started with the unit matrix and successively multiply by matrices whose determinants equal one. Now since Det (A)(B) = (Det A)(Det B) we see that the determinant of the coefficient matrix is always

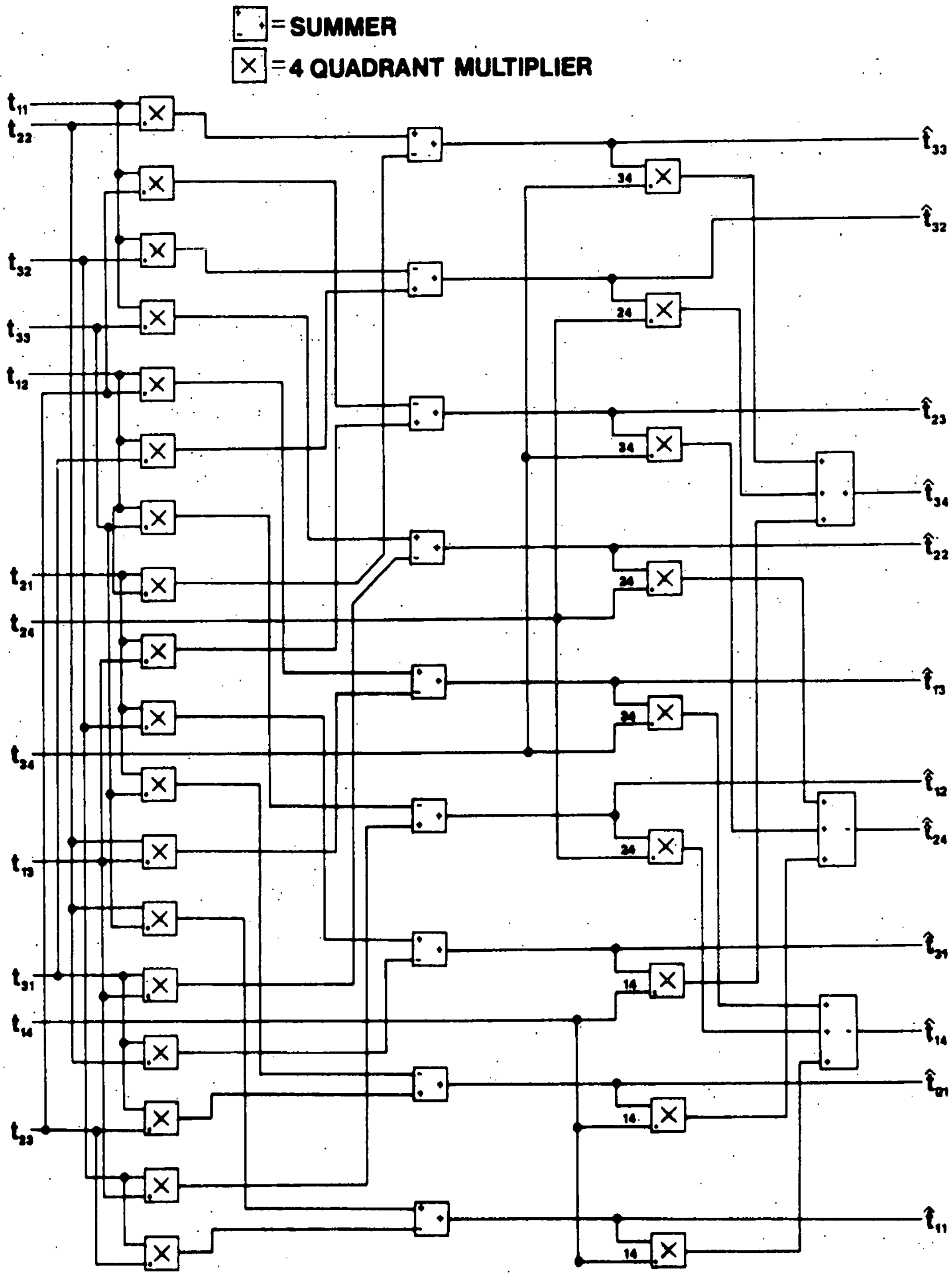

Figure 8, Inverse Matrix Generator

equal to one. Thus, there is no necessity in this case for the usual division by the determinant of the matrix. This reduces the original design (Cheng, 1971) by fifteen multipliers, a summing card, and a polarity correction card.

Figure 9 is a diagram of the inverse operator card. Three of these cards perform the premultiplication of the cursor coordinates by the inverse matrix.

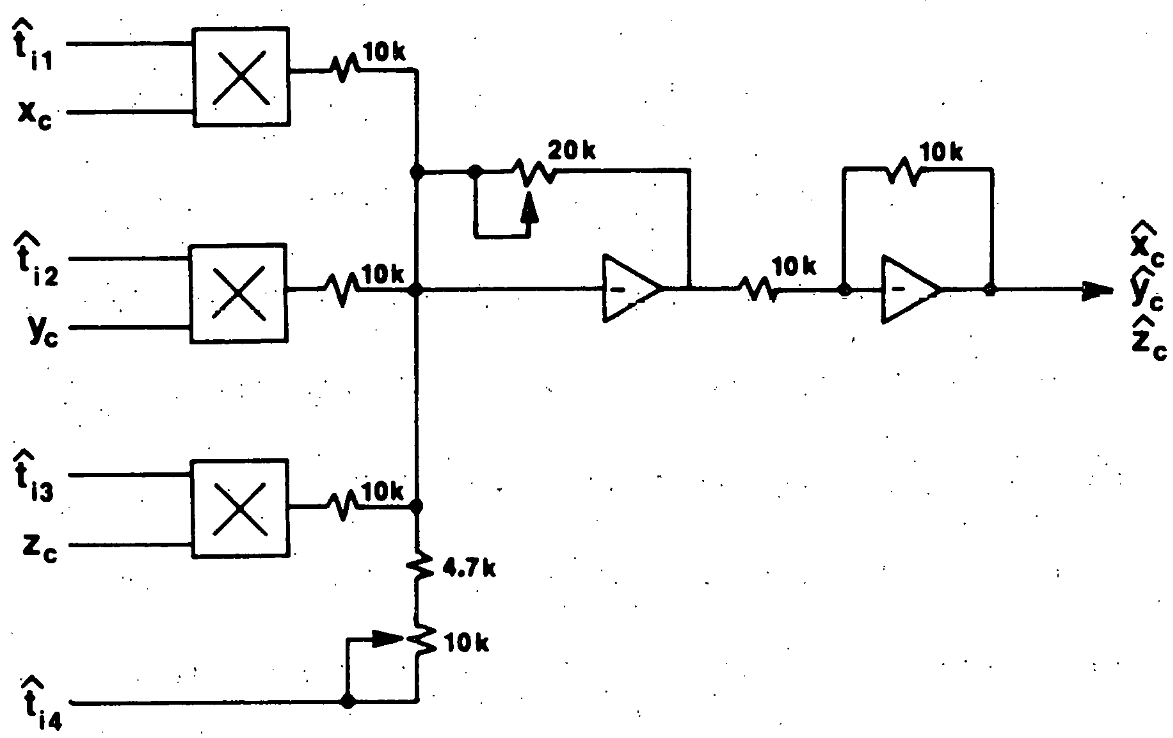

Figure 9. Inverse Operator Card

7. LASER DISPLAY SYSTEM

By far the most difficult part of the overall STEREO-MATRIX project was the Laser Display System. To accomplish a high-speed, random access, large screen display requires solutions to extremely difficult problems. The most difficult problems are the light source and the deflection of that light. The solutions attempted in this project required four state-of-the-art or near state-of-the-art devices. As is often the case with such devices their reliability proved to be extremely poor. Hence, work was often impeded by failures of these devices.

The Argon ion laser light source was obtained early in the project at a time when they were very new. Neither of the two manufacturers at that time could produce a reliable unit.

The three ultrasonic light deflection cells were specially built to our specifications at considerable effort by several manufacturers and required eighteen months to build rather than their estimated 120 days. Even then we had to accept units which fell short of the desired performance as the best that could be produced. These deflectors then proceeded to fail with only a few hours of use. Progress on the complete display was thus discouragingly slow due to these difficulties.

7.1 DISPLAY BRIGHTNESS

Although it was hoped early in the project that a high intensity arc lamp or some similar inexpensive incoherent light source could be used, the requirement to control the light and produce a small bright spot on the screen quickly dictates the selection of coherent laser light. To maximize brightness and produce an appealling display the Argon ion laser with its green 5145 A^{o} light seemed the best choice. This was the most powerful laser available producing light nearest the 5600 A^{o} peak of the eye sensitivity curve. Economics required that we beam split a single powerful laser rather than purchase a laser for each eye view.

A reasonable estimate of the laser power was obtained from other large screen work (Baker & Rugari, 1966). This Rome AFB display was powered by a He-Ne 6328 A^{o} laser driven at about 60-70 milliwatts, and produced a screen brightness of 5-10 foot-lamberts. This is a dim display requiring a darkened room for viewing. Since a brightness similar to normal television (20-40 foot-lamberts) was desired for STEREOMATRIX to allow viewing in a normally-lighted room, it was conservatively estimated (Baker, 1968) that one watt of 5145 A^{o} Argon light would provide this brightness by assuming an optical efficiency of 50% and a screen gain of 0.5.

Using these values we can calculate the expected luminance in foot-lamberts. The one watt beam is split so each eye sees only that half beam which passes the appropriate analyzer. So an effective 0.5 watt beam encounters the optics for an additional reduction to 0.25 watt. Now at 5100 A^{o} 2.8×10^{12} Photons = 1.1 micro-watt = 275 microLumens. Thus,

$$(0.25 \text{ watt}) \times \frac{275 \text{ Lumens}}{1.1 \text{ watt}} = 62.5 \text{ Lumens.}$$

If the screen is 3 by 4 feet and our image covers 10% of the screen area, we have

$$\frac{62.5 \text{ Lumens}}{(0.1)(12 \text{ Ft}^2)} = 52.0 \text{ foot-candles.}$$

With the estimated gain for our rear projection screen of 0.5, the brightness is then,

$$B = g\ I$$

$$B = 0.5\ (52.0) = 26 \text{ foot-lamberts.}$$

This is a quite reasonable brightness. Unfortunately some important factors have been overlooked. First, the optical efficiency estimate was based on a mirror deflection system and we finally decided to use the much more lossy ultrasonic cells. These cells deflect at best 50% of the input. Two cells in series to do X and Y deflection means (50%)(50%) = 25%. Furthermore to use the ultrasonic cells required many more optical operations on the beam than was expected. This introduced more loss. Finally, the one watt laser proved capable of producing only about 500 milliwatts output.

Expensive coatings improve this situation to the point that the twenty-two surfaces used in the optical design yield an overall transmittance of about 80%. However to obtain the optimum spot size at the screen it is necessary to limit the expanded beam.

This reduces the power available by a factor of about one-fifth. The deflection loss, the coatings, and the beam limiter can be combined to arrive at a more accurate optical efficiency of 4% instead of the earlier figure of 50%. The 26 foot-lamberts brightness figure then becomes 2.1 foot-lamberts. This figure agrees quite well with the visual experience.

7.2 LASER

The Argon ion laser uses a hemispherical resonator with the flat mirror replaced by a Littrow prism single wavelength selector. The resonant cavity is thirty-six inches long. The power output is approximately 500 milliwatts in the 5145 A° green line.

208-volt three-phase power is full-wave rectified to supply 175 volts and 35 amperes DC to the laser tube. Fifty-one transistors form a powerful current regulator which dissipates several kilowatts at maximum power conditions. Cooling water is circulated through the transistor heat sink at all times when the laser is running. The laser tube is also water cooled. Figure 10 is the circuit diagram of the laser power supply. Not shown is the three-phase full-wave rectifier bridge of 1N1188 diodes which supply DC to the plus and minus terminals on the left of the diagram.

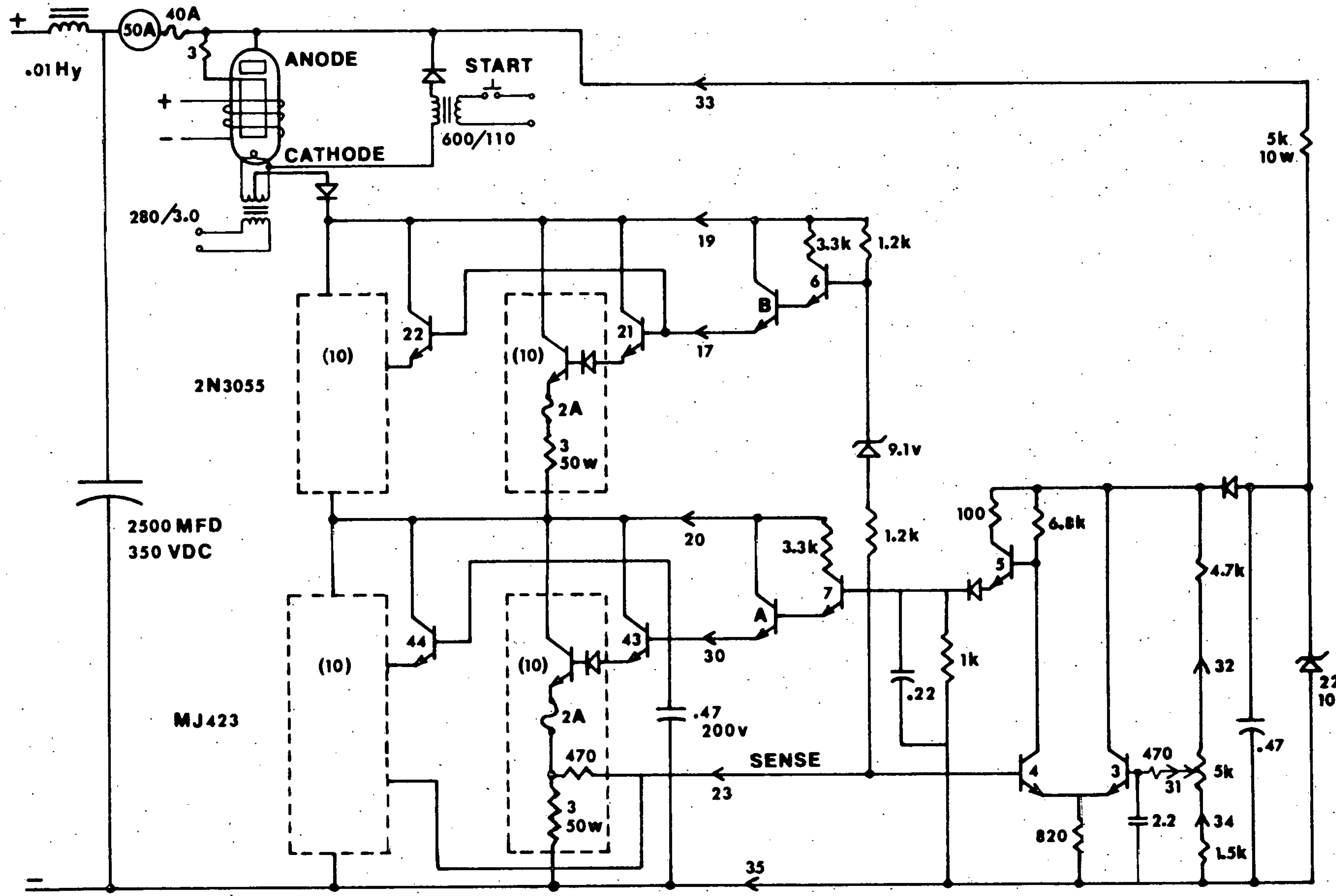

Figure 10. Laser Power Supply

7.3 ULTRASONIC DEFLECTION

A random access display requires a high speed deflection system. Ultrasonic deflectors have the smallest access times of the available light deflection systems. A one-inch aperture lead molybdate crystal is driven by a lithium niobate transducer over a frequency range of 80 to 160 MegaHertz to create a variable diffraction grating and consequently a variable deflection angle. The limitations on such a system are the very small angular deflection obtained (10 milliradians), the large light loss (only 40-50% is deflected), and the time required to establish a frequency in the crystal (the length of the crystal aperture divided by the velocity of sound in the crystal is 6.7 microseconds in this system).

The electronic driver for these deflectors must supply 2 to 3 watts to the transducer over the wide frequency range. A variable capacitance diode is used to vary the frequency and a nonlinear amplifier compensates to provide an overall linear deflection.

7.4 OPTICS

Figure 11 illustrates the optics required in this system. The incoming laser beam is expanded from approximately two millimeters to thirty millimeters by the first two lenses. Then the beam is restricted and reduced about one axis only by the cylinder lens. The vertical line of light at this point passes through the common Y deflector. The second cylinder lens then reforms the beam to a circular pattern before splitting. At the beam splitter half the light continues straight forward to be deflected according to the right-eye signal and the other half of the beam goes through a double reflection before passing through the left-eye deflector, X_L. The two double boxes immediately following the beam splitter indicate the quarter wave plate polarization rotator and analyzer necessary to obtain different polaizations for each eye view.

The final outputs pass through the 500 mm. and 25 mm. inverting telescope which magnifies the deflection angles by a factor of 20. The beams then reflect from a mirror which is not shown and travel a distance of 28 feet to the rear projection screen.

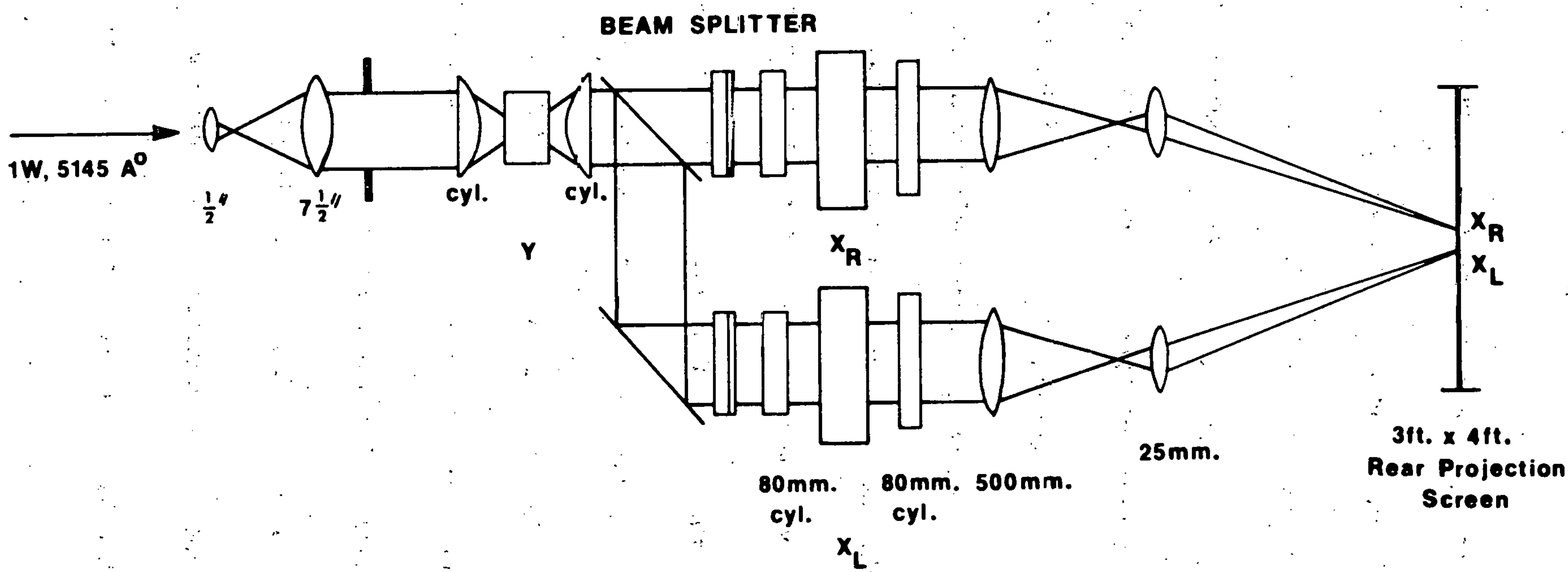

Figure 11. Optical Diagram

8. CONCLUSIONS AND RECOMMENDATIONS

The overall electronics of the system work reasonably well. The 3-D display lacks resolution and brightness and could be significantly improved by use of a more powerful laser. The resolution would be much better with a high quality beam expander. The present expander must be limited to one-fourth the maximum available aperture to produce a good spot on the screen.

The reliability of the deflector drivers needs to be much improved and their output power level should be better controlled to give a more uniform screen illumination.

The observer position detector needs some work to stabilize its output.

The multipliers in the stereopair transformer exhibit considerable drift and should be improved or replaced.

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APPENDIX I

STEREOPAIR EQUATIONS

Referring to Figure A.1 for an observer at (X_o,Z_o) and a data point at (X,Z), the image point P is the screen intersection. By similar triangles,

$$b = \frac{Z_o - K}{Z_o - Z}(X_o - X).$$

Then $$P = X_o - b = X_o + \frac{Z_o - K}{Z_o - Z}(X_o - X)$$

For an eye separation of a, the observer's left eye would be at $X_o + a/2$ and his right eye at $X_o - a/2$. Hence the respective equations are:

$$P_L = X_o + \frac{a}{2} + \frac{Z_o - K}{Z_o - Z}(X - X_o - \frac{a}{2})$$

$$P_R = X_o - \frac{a}{2} + \frac{Z_o - K}{Z_o - Z}(X - X_o + \frac{a}{2}).$$

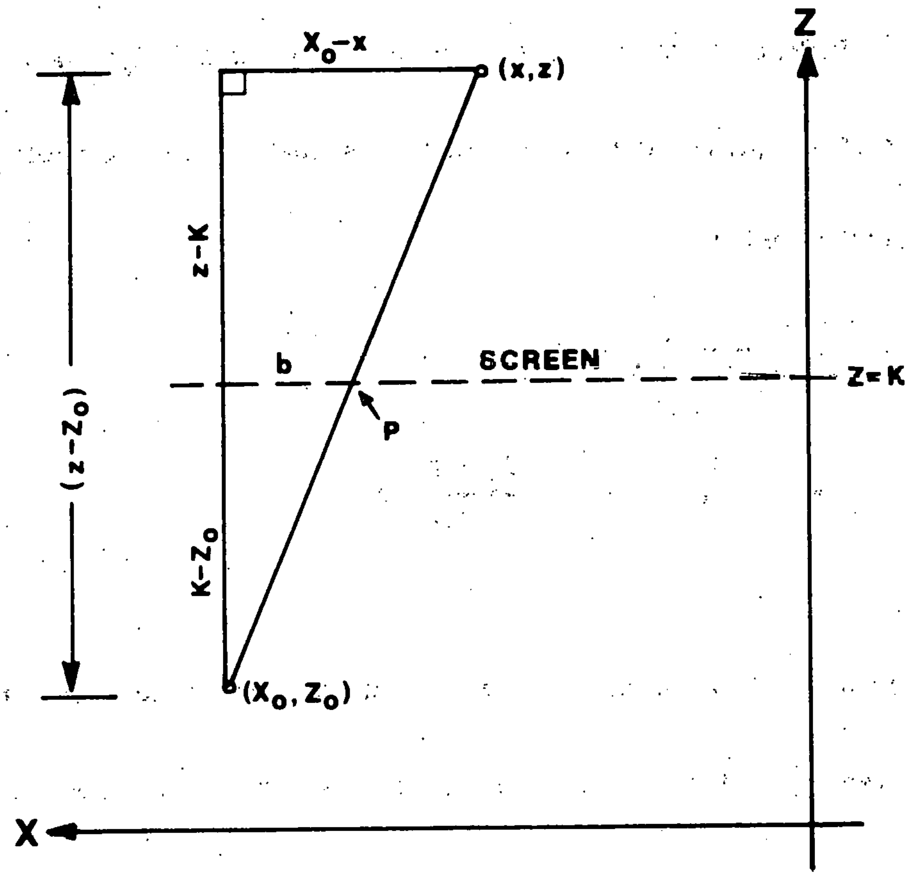

Figure A.1. Stereopair Geometry

APPENDIX II

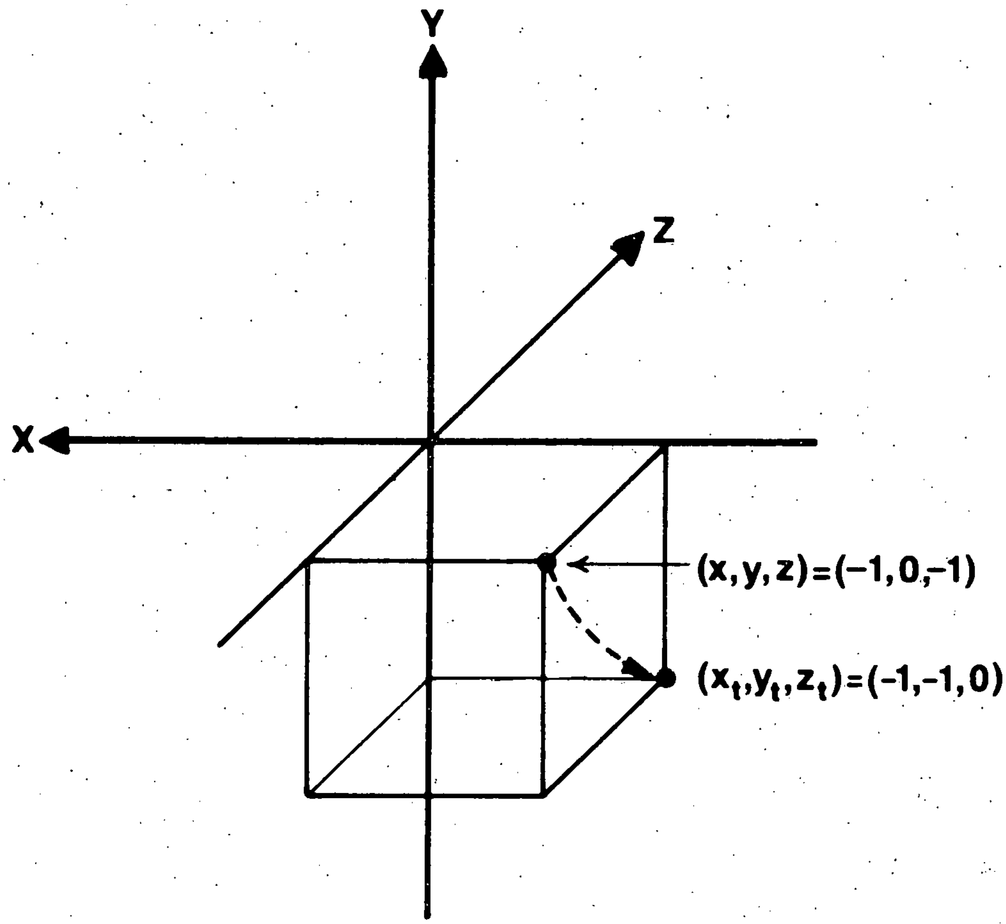

Figure A.2. 90-Degree Forward X Rotation

VITA

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