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RESONANCE EFFECTS IN ONE-REGION THORIUM BREEDER REACTORS

INTRODUCTION

The value of η_R (an average value of $\nu \frac{\sigma_f}{\sigma_a}$ in the resonance and epi-thermal energy regions) for U(233) has not been firmly established as yet. To indicate what effect η_R has on critical mass and breeding ratio of a U(233) - Th(232) power breeder, a homogeneous slurry system of Th(232) O_2 and U(233) O_2 in D_2O at 300°C was investigated with η_R as a parameter; spherical reactors having various radii and thorium concentrations were considered.

SUMMARY

The critical mass decreased almost linearly with the increasing η_R/η_T whereas the resonance effect and initial breeding ratio increased approximately linearly with increasing η_R/η_T.

The resonance effect was found to be more significant than usually estimated ($\epsilon_{23} P_{23} \simeq 1.15$).

The critical mass would be 7% greater if the value of $\eta_R/\eta_T \simeq 0.9$ rather than 1.0, in a 7-ft. one-region core containing 300 gm/ℓ of Th(232) and enough U(233) to be critical; in this comparison, the resonance effect would be 2% less; the initial breeding ratio would be 5% less; and the initial breeding gain would be 50% less.

These calculations show a need for more reliable information than presently available concerning the resonance integrals of U(233) and its variation with U(233) concentration.

RESULTS

The critical mass increased almost linearly with a decrease in η_R/η_T but was fairly insensitive to core size for radii greater than 5 feet and Th(232) concentration greater than 100 gm/ℓ (Fig. 1, 2).

At reasonable concentrations of Th ($200 \leq Th \leq 500$) for a one-region power breeder and for practical core sizes ($5' \leq R \leq 7'$), the value of $\epsilon_{23} P_{23}$ varies from 1.21 to 1.05 depending on the thorium concentration and the value for η_R/η_T. Thus, assuming $\epsilon_{23} P_{23} = 1$ (neglect of resonance effects), the critical mass could be overestimated by as much as 37%. The value of $\epsilon_{23} P_{23}$ is quite insensitive to changes in core size for this range of radii.

The value of $\epsilon_{23} P_{23}$ is approximately given by the following equation for the ranges specified in the previous paragraph.

$$\epsilon_{23} P_{23} \simeq \left[\frac{N_{02}}{1300} - .013\right] \frac{\eta_R}{\eta_T} - \frac{N_{02}}{2600} + 1.01 \qquad \text{(Fig. 3,4)}$$

If one were to consider resonance absorption, but <u>no</u> resonance fissions, the maximum permissible Th(232) concentration would be 300 gm/ℓ from criticality considerations.

A one-region reactor will not breed (IBR $<$ 1.0) if the core radius were 4 feet or less even with $\eta_R/\eta_T = 1$ for Th(232) concentrations less than 700 gm/ℓ. If $\eta_R/\eta_T \simeq 1$, a 5' core containing not less than 350 gm/ℓ of Th(232) would breed. If η_R/η_T were decreased, the core radius and/or Th concentration must be increased to maintain the same breeding ratio. If $\frac{\eta_R}{\eta_T} \simeq .925$ it would take at least 700 gm/ℓ of Th(232) to breed in a 5' core. For $\eta_R/\eta_T \simeq .90$, a larger core size must be used; for a 6' core, BR $\geq$ 1 if the Th(232) concentration were greater than 250 gm/ℓ. To obtain a BR $\geq$ 1 with $\frac{\eta_R}{\eta_T}$ = .8 a 7' core and a Th(232) concentration between 175 and 550 gm/ℓ are required.

Approximate equations for IBR as a function of η_R/η_T and R have been fitted to the curves obtained.

For N_{02} = 500 gm/ℓ.

$$IBR \simeq 0.625 - \frac{0.27}{R} - \frac{3.7}{R^2} + 0.625 \frac{\eta_R}{\eta_T}$$

For N_{02} = 300 gm/ℓ.

$$IBR \simeq 0.87 - \frac{0.30}{R} - \frac{4.9}{R^2} + 0.38 \frac{\eta_R}{\eta_T}$$

Both of these equations apply over the ranges

$$5' \leq R \leq 7'$$

$$.8 \leq \frac{\eta_R}{\eta_T} \leq 1.0$$

If Th(232) concentration lies between 200 and 500 gm/ℓ, it appears that there exists a "critical" value of η_R/η_T for which the breeding ratio is independent of Th(232) concentration. These values are shown in the following table.

Core radius	"critical" $\frac{\eta_R}{\eta_T}$
7'	0.89
6'	0.85
5'	0.81
4'	0.78

For values of η_R/η_T greater than the "critical" value, B.R. increases with increasing Th concentration while for η_R/η_T less than the "critical" value, the breeding ratio decreases with increasing Th(232) concentration. (Fig. 5).

For all values of $\frac{\eta_R}{\eta_T} \angle 1.0$, there is an optimum value of Th(232) concentration for which the B.R. has a maximum value. This optimum thorium concentration decreases as the reactor radius increases and as $\frac{\eta_R}{\eta_T}$ decreases. (Fig. 6,7)

PROCEDURE

Calculation of Resonance Escape Probability

The resonance escape probability can be defined in the following manner:

$$P \equiv 1 \; - \text{probability of absorption.}$$

Using the asymptotic solution for $\phi(E)$ [Fermi Age theory[1]] in an infinite medium, the flux will be given by[(1)]

$$\phi \simeq \frac{1}{\xi \, \Sigma_s E}$$

Also,

$$\text{probability of absorption} = \frac{\int \Sigma_a^i \;\; \phi dE}{1} \simeq \int \Sigma_a^i \, \frac{dE}{\xi \Sigma_s E}$$

$$= \frac{N_i}{\xi \Sigma_s} \; R_a^i$$

$$\text{where} \quad R_a^i \equiv \int \sigma_a^i \; \frac{dE}{E} \equiv \text{resonance absorption integral.}$$

$$P \simeq 1 - \frac{N_i}{\xi \, \Sigma_s} \quad R_a^i \simeq \exp \left[- \frac{N_i}{\Sigma_s \xi} \;\; R_a^i \right]$$

If the infinite dilution value of R_a^i were used, the value of P would be considerably smaller than the observed values. From experiments with Th(232) and U(238), it has been found that R_a^i is a function of N_i. For Th(232), this function is given as (2)

$$R_a^{02} = 8.33 \left(\frac{\Sigma_s}{N_{02}}\right)^{.253}$$

For U(233), there is no data available for the resonance absorption integral and so an estimate was made. It was assumed that the effect of N_i on R_a^i would be the same for U(233) as for U(235). The resonance escape probability for U(235) was obtained from a 53-group criticality study by Safonov.[3] These values for P_{25} were then used to calculate R_a^{25} as a function of N_{25}. Using infinite dilution resonance absorption integrals of 1215 b. and 1060 b. for U(233) and (235), respectively, (obtained from cross section curves)[4] the value of R_a^{23} was found to be

$$R_a^{23} \simeq 130 \, \ell n \left[.926 \frac{\xi \Sigma_s}{N_{23}} \right] \text{ (b)}$$

in the region

$$10^{-3} \leq \frac{N}{\xi \Sigma_s} \leq 10^{-1} \text{ (b.}^{-1}\text{)}$$

Leakage Calculations

Three probabilities of leakage were considered:

1. Leakage while at thermal energies $\left(\frac{L^2B^2}{1 + L^2B^2}\right)$

2. Leakage while slowing down from fission energies to thermal $\left(\frac{B^2 \tau_T}{1 + B^2 \tau_T}\right)$

3. Leakage while slowing down from fission energies to resonance $\left(\frac{B^2 \tau}{1 + B^2 \tau}\right)$

For a spherical core B^2 was taken as (1)

$$B^2 = \left(\frac{\pi}{R + \epsilon}\right)^2$$

where ϵ = extrapolation length = .71 λ_{Tr}.

The thermal diffusion length was calculated as a function of U(233) and Th(232) concentration.

The age to thermal and resonance energies was obtained from published values(5) for D_2O and evaluated at 300°C.

The following values were used throughout this report:

$$\tau = 168 \text{ cm}^2$$

$$\tau_T = 212 \text{ cm}^2$$

Derivation of Equations for Criticality

Starting with one fast neutron

$$1 = \frac{P_{02} \; P_{23}}{(1 + B^2 \tau_T)(1 + L^2 B^2)} f \, \eta_T + \int \eta_R \, \Sigma_a^{23} \, \phi \, dE$$

$$\simeq \frac{P_{02} \, P_{23} \; f \; \eta_T}{(1 + B^2 \tau_T)(1 + L^2 B^2)} + \int \eta_R \frac{\Sigma_a^{23}}{\xi \, \Sigma_s} \; \frac{P_{02} \; P_{23}}{1 + B^2 \tau} \; \frac{dE}{E}$$

Now

$$P \equiv 1 - \int \Sigma_a \, \phi \, dE = 1 - \int \Sigma_a \frac{P}{\xi \, \Sigma_s} \frac{dE}{E}$$

$$1 - P = \int \Sigma_a \frac{P}{\xi \, \Sigma_s} \frac{dE}{E}$$

By definition let $\bar{P} \equiv \frac{1 + P}{2}$

substitute these definitions above

$$1 = \frac{P_{02}\, P_{23}\, f \quad \eta_T}{(1+B^2\tau_T)(1+L^2B^2)} + \frac{\eta_R\, \overline{P_{02}}}{1 + B^2\tau}\, (1 - P_{23}) \left(\Sigma_{aT}^{23} + \Sigma_{aT}^{02}\right)$$

Resonance Effects

If we compare the critical condition with the usual one for a thermal reactor; i.e.,

$$1 = \frac{\eta\, f\, P \epsilon}{(1+L^2B^2)(1+B^2\tau_T)}$$

where $P \equiv P_{02}\, P_{23} \quad \eta \equiv \eta_T$

we see that

$$\epsilon \equiv 1 + \frac{(1 + B^2\tau_T)\,(1 + L^2\, B^2)}{1 + B^2\tau} \quad \frac{\eta_R}{\eta_T} \quad \frac{\overline{P_{02}}\,(1 - P_{23})}{P_{02}\, P_{23}\, f}$$

For Initial Breeding Ratio

By definition

NPR $\equiv$ neutron production rate $= \eta_T \quad \Sigma_{aT}^{23} \quad \phi_T + \eta_R \int \Sigma_a^{23} \quad \phi dE$

However,

$$\phi \simeq \frac{NPR}{\xi \Sigma_s} \quad \frac{P_{02}\, P_{23}}{1 + B^2 T} \; \frac{1}{E}$$

we then get

$$NPR' \equiv \frac{NPR}{\phi_T} = \frac{\eta_T \quad \Sigma_{aT}^{23} \;(1 + B^2\tau)}{1 + B^2\tau - \eta_R\; \bar{P}_{02}(1 - P_{23})}$$

From the definition of initial breeding ratio

$$\mathrm{IBR}\ \frac{\Sigma_{aT}^{02}\ \phi_T + \int \Sigma_a^{02}\ \phi\ dE}{\Sigma_{aT}^{23}\ \phi_T + \int \Sigma_a^{23}\ \phi\ dE} \simeq \frac{\Sigma_{aT}^{02}\ \phi_T + \int \Sigma_a^{02}\ \frac{NPR}{\xi \Sigma_s}\ \frac{P_{02}\ P_{23}}{1 + B^2 \tau}\ \frac{dE}{E}}{\Sigma_{aT}^{23}\ \phi_T + \int \Sigma_a^{23}\ \frac{NPR}{\xi \Sigma_s}\ \frac{P_{02}\ P_{23}}{1 + B^2 \tau}\ \frac{dE}{E}}$$

using the value of NPR' from above and the critical condition, we find

$$\mathrm{IBR} = \frac{P_{02}\ P_{23}\ \eta_T (1+B^2\tau) + (1+L^2B^2)(1+B^2\tau_T)\ \left[\eta_R\ \bar{P}_{02}(1-p_{23}) + \eta_T\ \bar{P}_{23}(1 - P_{02}) - (1 + B\tau)\right]}{(1 + L^2\ B^2)(1 + B^2\tau_T)\ \left[(1 + B^2\tau) + (\eta_T - \eta_R)\ \bar{P}_{02}\ (1 - P_{23})\right]}$$

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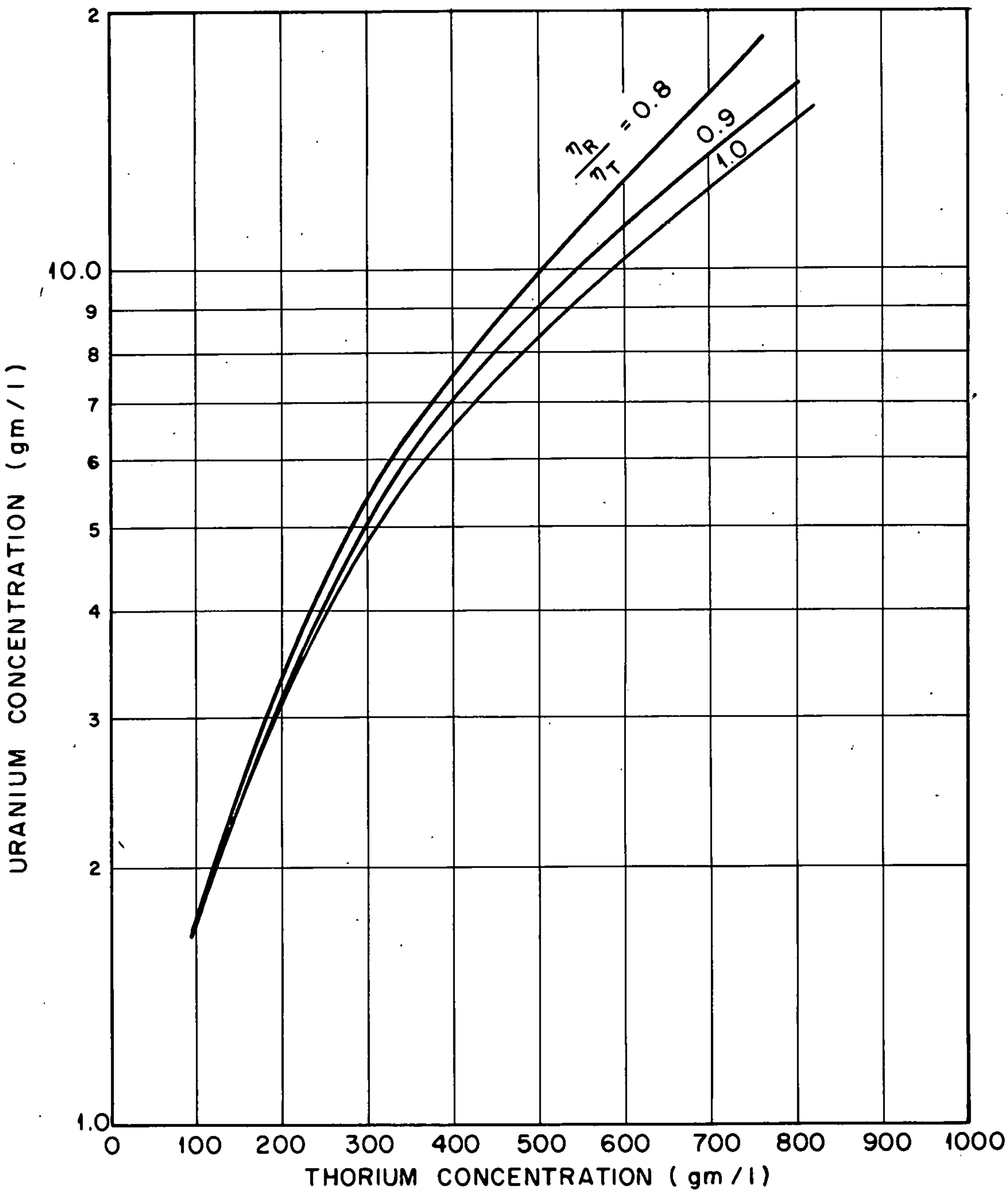

Fig. 1. Critical Concentration (η = 2.25; R = 5′; T = 300°C)

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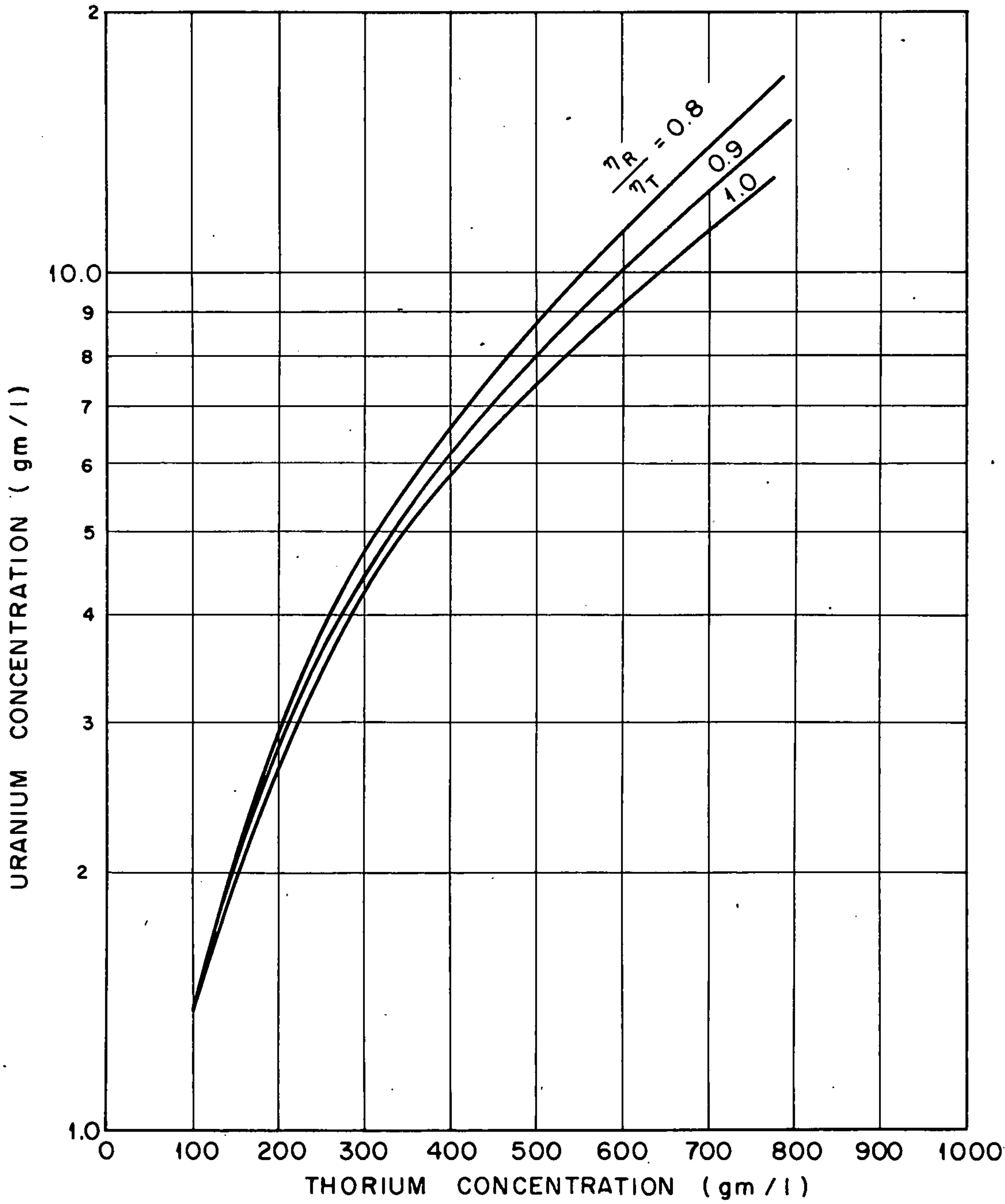

Fig. 2. Critical Concentration (η = 25; R = 7'; T = 300°C)

12

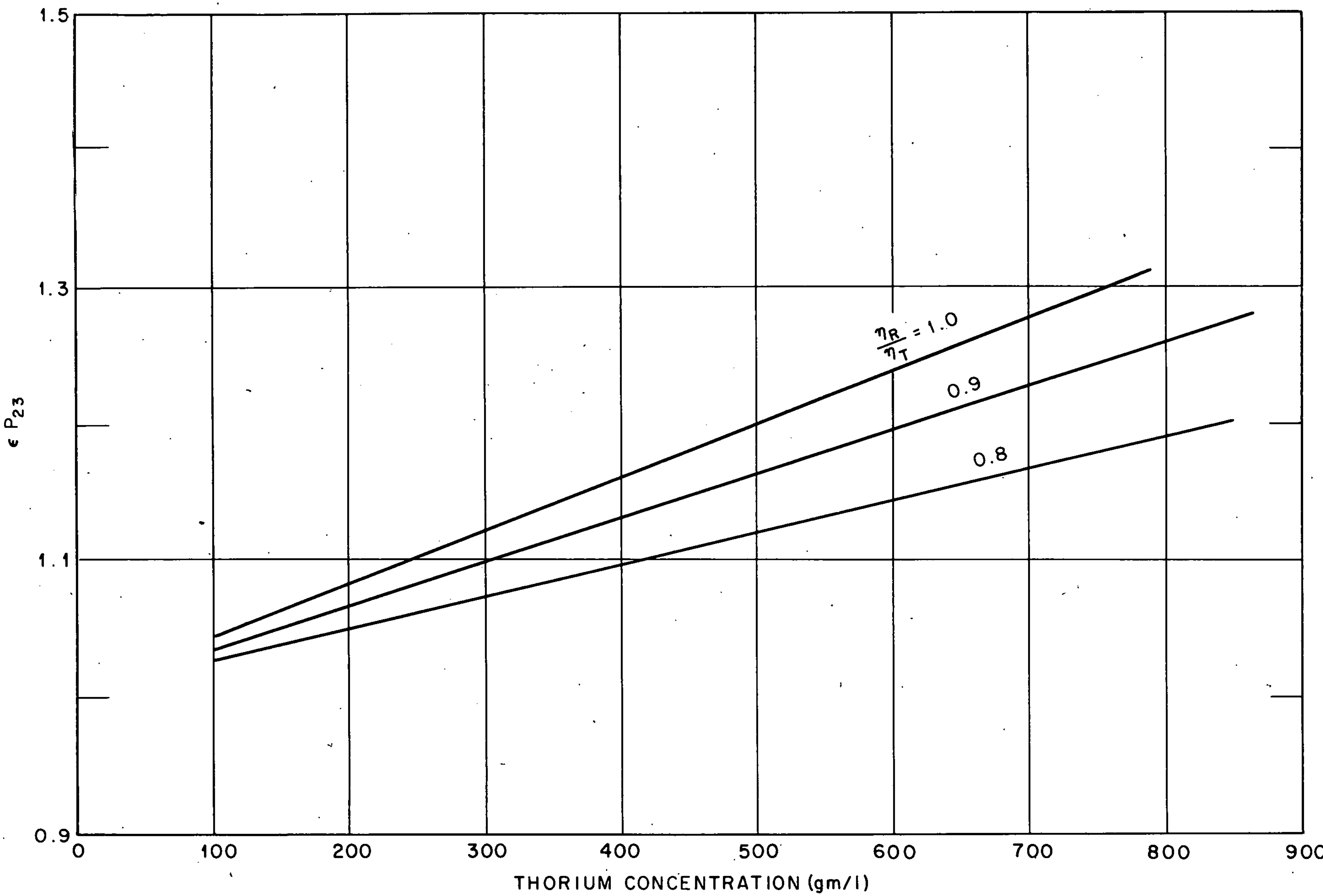

Fig. 3. Resonance Effects (R = 5', η_T = 2.25, T = 300°C, k = 1.0)

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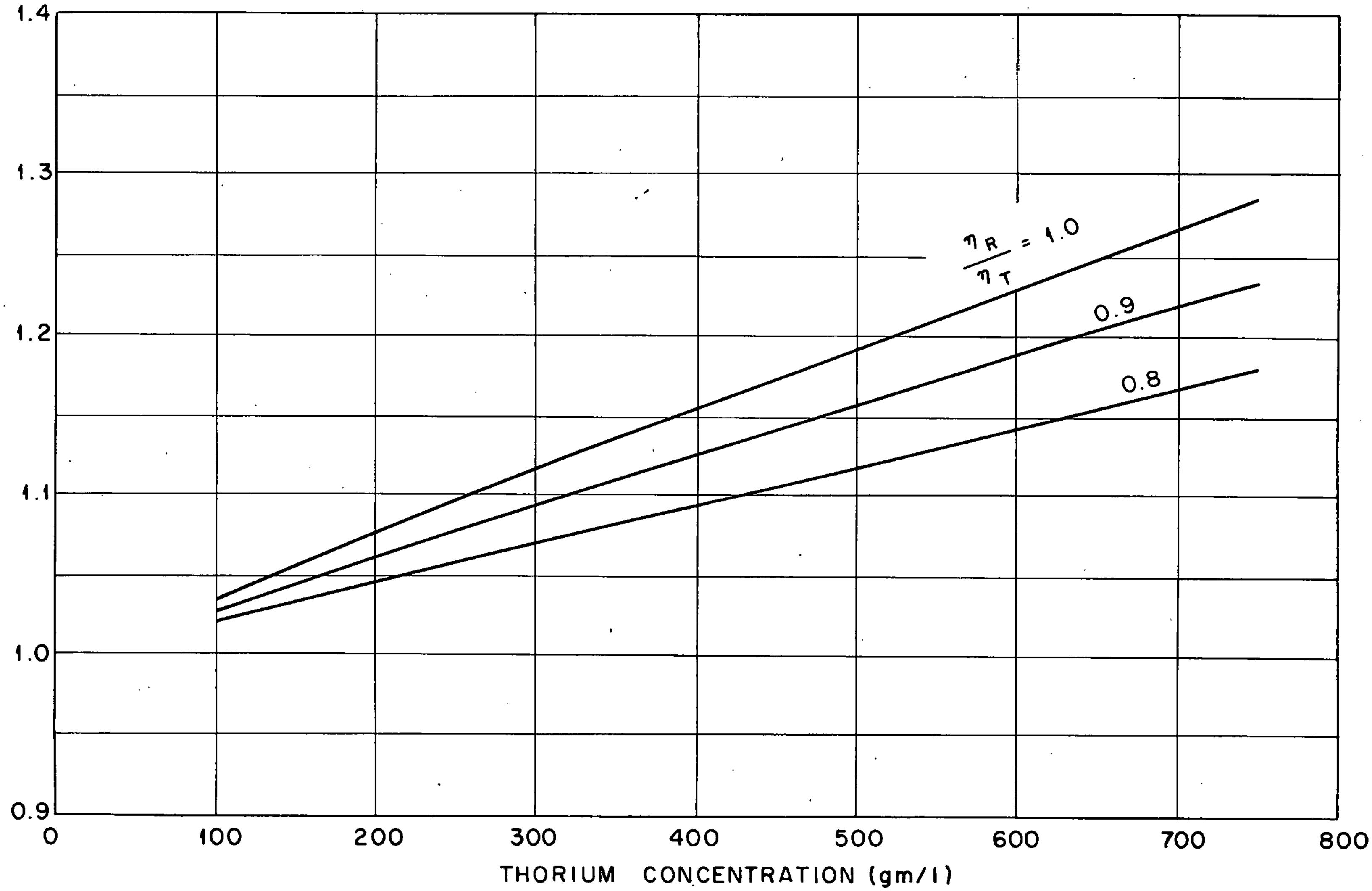

Fig. 4. Resonance Effects ($R = 7'$; $\eta_T = 2.25$; $T = 300°C$; $k \approx 1.0$)

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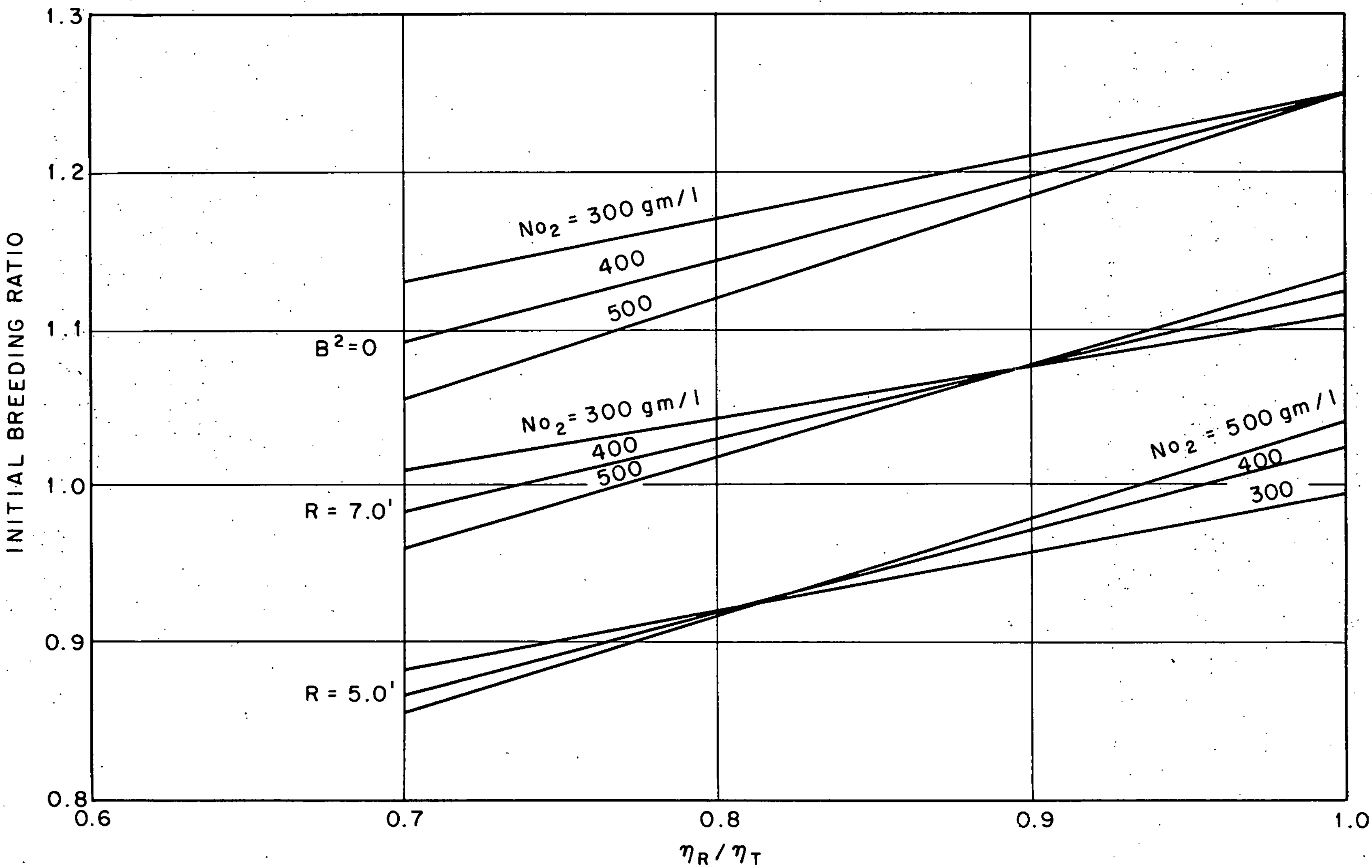

Fig. 5. Summary of Initial Breeding Ratio Variation (η_T= 2.25, T = 300°C, k = 1.0)

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15

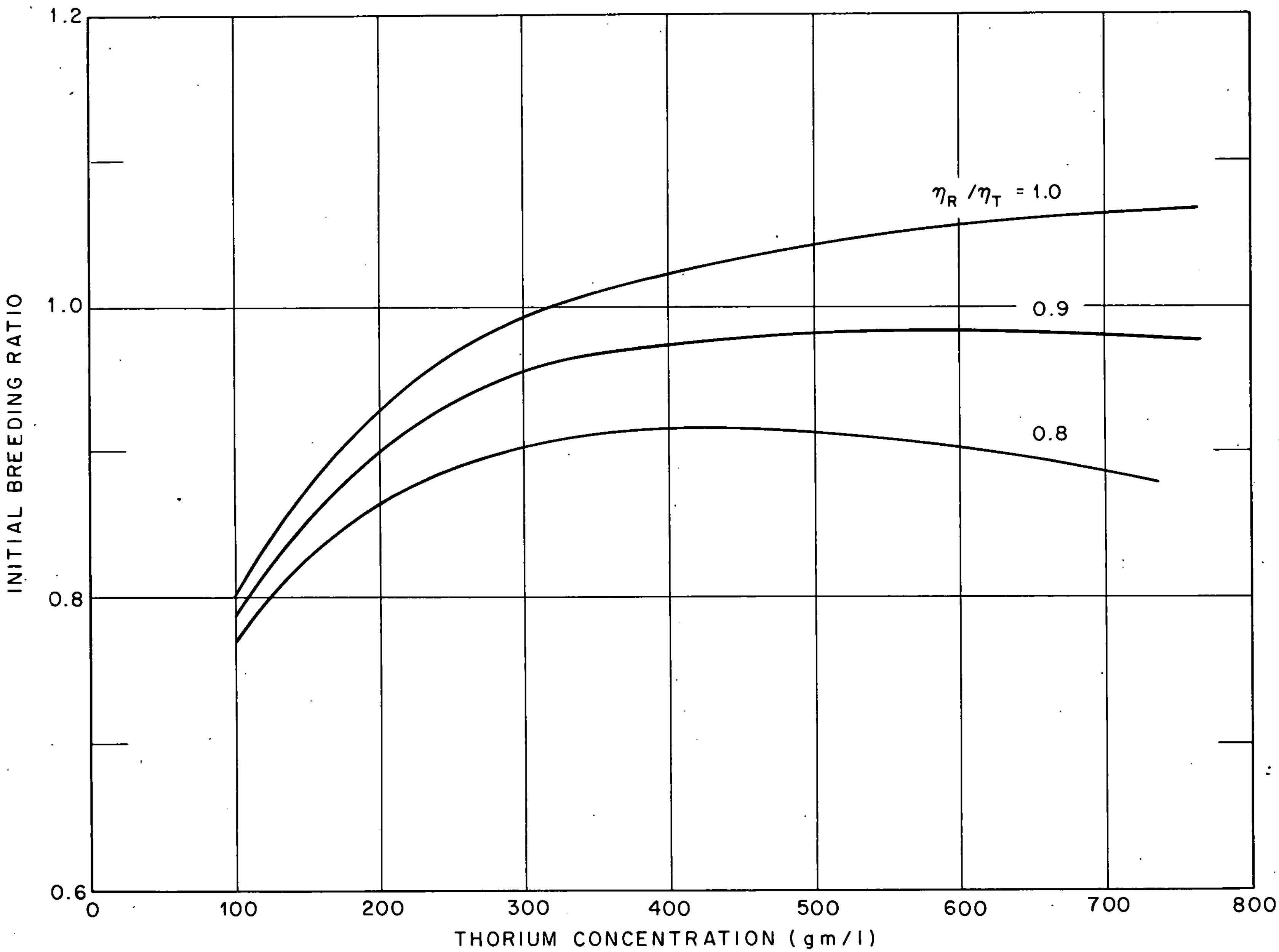

Fig. 6. Initial Breeding Ratio Variation (R = 5', η_R = 2.25, T = 300°C, k = 1.0)

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16

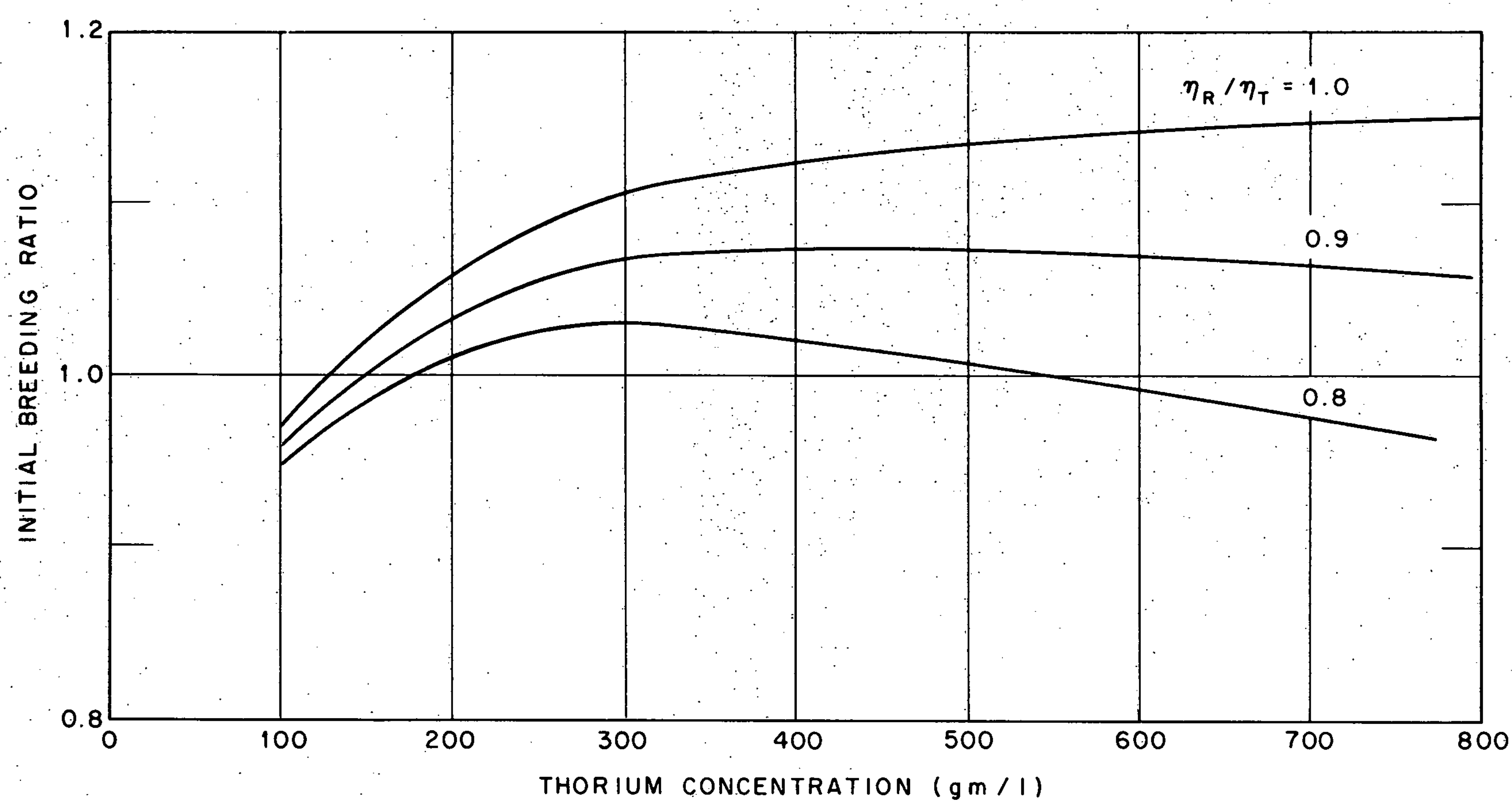

Fig. 7. Initial Breeding Ratio Variation (R = 7', η_T = 2.25, T = 300°C, k = 1.0)

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NOMENCLATURE

Symbol	Meaning	Units
B	Geometric Buckling	cm^{-1}
E	Energy	ev
L	Thermal Diffusion Length	cm
N_i	Number Density	$(b.\text{-}cm)^{-1}$
P_i	Resonance Escape Probability	---
R	Core Radius	cm
R_a	Resonance Absorption Integral	b.
IBR	Initial Breeding Ratio	---
NPR	Neutron Production Rate	$cm^{-3}\ s.^{-1}$
ϵ_i	Ratio of Total Fission to Thermal Fission Rate	---
ξ	Mean Letheragy Loss per Collision	---
ϕ	Neutron Flux	$cm^{-2}\ s.^{-1}$
ν	Neutrons Produced per Fission	---
η	Neutrons per Absorption in Fissionable Material	---
σ_f^i	Microscopic Fission Cross-Section	b.
σ_a^i	Microscopic Absorption Cross-Section	b.
Σ_s	Macroscopic Scattering Cross-Section	cm^{-1}
Σ_a^i	Macroscopic Absorption Cross-Section	cm^{-1}
τ	Fermi Age to Resonance	cm^{-2}

Subscript T refers to thermal

Subscript R refers to resonance

NUCLEAR CONSTANTS AT 300°C

ξ	0.508
$\sigma_s^{D_2O}$	10.5 b.
$\lambda_{Tr}^{D_2O}$	3.711 cm
τ	168 cm^2
τ_T	212 cm^2
ν_{23}	2.50
η_T^{23}	2.25
σ_{fT}^{23}	346 b.
σ_{aT}^{23}	381 b.
σ_{aT}^{02}	4.5 b

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