

DISTRIBUTIONS OF STRAIGHT SECTIONS IN MARK V

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Using the method and example of a machine described in MURA-TBE/DWK-1, we can test the variation in σ for an arbitrary arrangement of sectors. We plot trace ΔM as a function of the position of a straight section in the accelerator and then neglecting terms in the total trace with coefficients θ^2 and higher powers of θ , we use

$$\text{trace } M = \text{trace } M_0 + \sum \theta_k \text{ trace } \Delta M_k$$

where M_0 is the matrix for no straight sections and θ_k is the length of the straight section in the k^{th} position in the sector and trace ΔM_k is read off of the graph. This computation of the trace gives different answers depending upon where the array of straight sections lies in the sector. By trial we must find the maximum and the minimum total trace to determine the difference in σ or in ν for particles of different radii in the same Mark V; because at different radii, the array of straight

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sections occurs at different positions relative to the position of the positive or negative field gradient.

As shown in MURA-TBE/DWK 1 a straight section occurring in a focussing sector produces

$$\text{Trace } \Delta M_{14} = e C_{1-F} a_F + f C_{1-F} C_F + g d_{1-F} a_F + h_{1-F} C_F$$

which is from

$$M_{14} = \begin{pmatrix} 1 & \theta \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a_{1-F} & b_{1-F} \\ C_{1-F} & d_{1-F} \end{pmatrix} \begin{pmatrix} ef \\ gh \end{pmatrix} \begin{pmatrix} a_F & b_F \\ C_F & d_F \end{pmatrix}$$

and F is the fraction of a focussing segment between the straight section and the end of the focussing segment.

For radial motion:

$$e = \cosh \psi_- \quad \text{with } \psi = \frac{\sqrt{n} \pi}{N}$$

$$f = \frac{1}{\sqrt{n_-}} \sinh \psi_-$$

$$g = \sqrt{n_-} \sinh \psi_-$$

$$h = \cosh \psi_-$$

and

$$a_F = \cos F \psi_+$$

$$C_F = -\sqrt{n_+} \sin F \psi_+$$

$$C_{1-F} = -\sqrt{n_+} \sin (1-F) \psi_+$$

$$d_{1-F} = \cos (1-F) \psi_+$$

For the axial, or Z, motion interchange the numbers used for ψ_+ and ψ_- in the formulae and interchange the numbers used for n_- and n_+ .

Straight sections occurring in a defocussing sector produce

$$\text{Trace } \Delta M_3 = a g_{1-F} e_F + b g_{1-F} g_f + C h_{1-F} e_F + d h_{1-F} g_F$$

which is from

$$M_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e_{1-F} & f_{1-F} \\ g_{1-F} & h_{1-F} \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_F & f_F \\ g_F & h_F \end{pmatrix}$$

and for radial motion

$$\begin{aligned} a &= \cos \psi_+ \\ b &= \frac{1}{\sqrt{n_+}} \sin \psi_+ \\ c &= -\sqrt{n_+} \sin \psi_+ \\ d &= \cos \psi_+ \end{aligned}$$

and

$$\begin{aligned} g_{1-F} &= \sqrt{n_-} \sinh (1-F) \psi_- \\ h_{1-F} &= \cosh (1-F) \psi_- \\ e_F &= \cosh F \psi_- \\ g_F &= \sqrt{n_-} \sinh F \psi_- \end{aligned}$$

with the previous interchanges for calculating axial motion with these formulae.

The graphs show the variation of these increments of traces for different locations of a straight section in the sector. It is merely necessary to add up the trace ΔM 's for the straight sections. For example:

If we want to know how much the number of betatron oscillations around the accelerator varies with the position of the straight section, we note that since trace $M = 2 \cos \sigma$,

$\Delta \nu_{(F)} = - \frac{N \theta}{4 \pi \sin \sigma}$ trace $\Delta M_{(F)}$; and since $\Delta \nu_{(F)}$ depends upon F , ν will vary by the maximum variation in $\Delta \nu_{(F)}$.

For one straight section, the radial trace $\Delta M = -48$ for the largest value and for the smallest value trace $\Delta M = -3.5$ thus $\delta \nu_{\max} = \frac{N \theta}{4 \pi \sin \sigma} (44.5)$ is the variation in ν_r .

For our example of $37 = N$ sectors and $\sigma = 119^\circ$, and A 20 cm straight section

$$\delta \nu_{\max} = .0067 (44.5) = .30 \text{ waves.}$$

If we space three straight sections uniformly in a sector,

$\sum_k \text{tr } \Delta M_k = -48 - 27 - 7.5 = -82.5$ for one of them between focussing and defocussing magnets and

$\sum_k \text{tr } \Delta M_k = -3.5 - 2(35) = -73.5$ for two of the three symmetrically placed in the defocussing sector.

Thus

$\delta \nu_{\max} = .0067 (9.0) = .06$ waves which is a great improvement over one straight section.

Testing four straight sections and also two straight sections a quarter of a sector apart both give

$\int \nu_r \sim .33$ which is much worse than three straight sections.

The Z motion for three straight sections has

$\int z = .077$ waves. One might safely go to 50 cm straight sections three per sector with $\int \nu$ no worse than .2. For such long straight sections, the terms in θ^2 might become important.

σ 's or ν 's could be kept much more constant if the separation of straight sections is slightly less than the length of a sector, in this case another periodicity would be introduced into the machine and the question of how bad the stop bands would become would have to be examined.