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Verification of Multiplicative Renormalization of a Model with Spontaneously Broken Symmetry*

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Abstract

We verify explicitly at the one loop level the multiplicative renormalization program, performed in an Abelian gauge model with spontaneously broken symmetry. By utilizing the two parameter gauge freedom which was introduced in a previous note, the off-, as well as on-, mass-shell Green's functions can be made finite after renormalization.

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I. Introduction

In this note, we report on one loop verification of the renormalization program[1], performed in the Abelian gauge model with spontaneously broken symmetry which was considered in an earlier publication[2].

While the renormalizability of models of this nature has been established through general consideration[3], essentially by heavy use of Ward-Takahashi identities[4], we deem it nonetheless a meaningful exercise to perform some low order calculations to ascertain that "things go as expected".

We shall carry out the calculation in the class of renormalization gauges reported earlier[2,5]. These gauges are associated with two continuous parameters: (i) κ^2, which can be identified as the "mass" of the gauge excitation field φ, and (ii) ξ, a numerical parameter which in a loose sense plays the role of the ξ-parameter of Lee and Yang[6]. To be more specific, we confine our consideration to the choice $\xi=1$ and $\kappa^2=m_r^2$ [7] <u>in the tree approximation</u>, where m_r is the physical mass of the vector particle A_μ in the model.

We emphasize that the above choice is made for the tree approximation only. In fact the major contribution of this note is to point out that because of the two parameter gauge freedom, it is possible to write power series in coupling constants for ξ and κ^2, with coefficients so adjusted that the φ propagator has its pole at the vector meson mass (m_r) even after renormalization. The off-mass-shell Green's functions (such as the mixed

A_μ - φ propagator and the longitudinal part of the A_μ propagator) can be made finite simultaneously.

One may question the necessity to have well defined off-mass-shell Green's functions, so long as the S-matrix elements are finite. We concur with this view in part. However, in practice when radiative corrections, such as self energy insertions, are taken into account in a problem, well behaved off-mass-shell Green's functions should facilitate divergence isolation and thus the calculation.

It remains to be pointed out that we are going to deal with only the infinite parts of the renormalization constants. The finite parts of the Ward-Takahashi identities are therefore not verified in this work[8].

II. Model and Renormalization Program

The model[2,9] consists of a vector meson A_μ, a scalar meson χ, a gauge excitation field φ, and a fermion field ψ. We denote the left-handed portion of ψ as

$$\psi_L = \frac{1}{2}\left(1 + i\gamma_5\right)\psi \ , \tag{1a}$$

and the right-handed portion as

$$\psi_R = \frac{1}{2}\left(1 - i\gamma_5\right)\psi \ . \tag{1b}$$

After the Higgs-Kibble[10] mechanism has been invoked, the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2} g^2 v^2 A_\mu A^\mu$$
$$+ gvA_\mu \partial^\mu \varphi - \frac{1}{2} \partial_\mu \varphi \, \partial^\mu \varphi - \frac{1}{2}\left(\bar{\mu}^2 + hv^2\right)\varphi^2$$

$$- \frac{1}{2} \partial_\mu \chi \, \partial^\mu \chi - \frac{1}{2}\left(\bar{\mu}^2 + 3hv^2\right)\chi^2$$

$$-\left(\bar{\mu}^2 + hv^2\right) v \chi + g \, A_\mu\left(\chi \, \partial^\mu \varphi - \varphi \partial^\mu \chi\right)$$

$$- g^2 v \chi \, A_\mu \, A^\mu - \frac{1}{2} g^2 \, A_\mu \, A^\mu\left(\varphi^2 + \chi^2\right)$$

$$- hv\chi\left(\varphi^2 + \chi^2\right) - \frac{h}{2}\left(\varphi^2 + \chi^2\right)^2$$

$$- \bar{\psi}\left(\gamma_\mu \frac{1}{i} \partial^\mu - g \, \gamma_\mu \, A^\mu \frac{1}{2} \, (1 + i\gamma_5)\right)\psi$$

$$- \frac{f}{\sqrt{2}} \, v \, \bar{\psi} \, \psi - \frac{f}{\sqrt{2}} \bar{\psi} \, \psi \, \chi - \frac{f}{\sqrt{2}} \bar{\psi} \, \gamma_5 \, \psi \, \varphi + \mathcal{L}' \quad , \quad (2)$$

here $\bar{\mu}$ is the bare mass of the complex scalar field $\bar{\varphi} = \frac{1}{\sqrt{2}}\ (v + \chi + i\varphi)$ in the limit when the symmetry is not broken and $v/\sqrt{2}$ is the vacuum expectation value of $\bar{\varphi}$. $\mathcal{L}'$ depends on the gauge used[2]. We choose to impose on the physical states the subsidiary condition

$$\left(\partial_\mu A^\mu - \alpha \, \beta^{-1} \, m_r \, \varphi\right) \, |\Psi^{\text{physical}}\rangle = 0 \qquad (3)$$

where m_r is the physical mass of A_μ. In this case

$$\int d^4x \, \mathcal{L}' = \int d^4x \, \left\{- \frac{1}{2} \left(\beta \, \partial_\mu \, A^\mu - \alpha \, m_r \, \varphi\right)^2\right\}$$

$$-i \, \mathrm{Tr} \ln\left(\frac{-\partial^2 + \alpha\beta^{-1} \, m_r \, gv + \alpha\beta^{-1} \, m_r \, g\chi}{- \partial^2 + m_r^{\,2}}\right) \qquad (4)$$

where the second term on the right generates compensating ghost loops[11].

Renormalization is accomplished by rescaling both the fields and the mass and coupling parameters according to

$$A_\mu = Z_A^{\,1/2}\left(A_\mu\right)_r \, , \qquad \varphi = Z_\varphi^{\,1/2}\varphi_r \, , \qquad \chi = Z_\chi^{\,1/2}\chi_r \, ,$$

$$\psi_L = Z_L^{\,1/2}\left(\psi_L\right)_r \, , \qquad \psi_R = Z_R^{\,1/2}\left(\psi_R\right)_r \, ,$$

$$\bar{\mu}^2 = Z_{\bar{\mu}}\, \bar{\mu}_r^{\,2}\,, \qquad v = Z_v v_r\,, \qquad g = Z_g g_r\,,$$

$$h = Z_h h_r\,, \qquad f = Z_f f_r\,, \qquad \alpha = Z_\alpha\,, \text{ and } \quad \beta = Z_\beta\,. \tag{5}$$

Quantities with the subscript r are said to be renormalized.

This multiplicative renormalization program is regarded as successful, if by adjusting the Z's, all the S-matrix elements are made finite, while the Ward-Takahashi identities remain true. We claim that, at least at the one loop level, even the off-mass-shell Green's functions can be rendered finite.

It can be shown that for any arbitrary but finite α and β,

$$Z_g = Z_A^{-1/2} + \text{finite part.} \tag{6}$$

This differs from the situation in the unitarity gauge[1], which corresponds to the limit $\alpha \to \infty$, where the relation in Eq. (6) is not true.

We introduce the further notations

$$Z\text{'s} = 1 + L\text{'s}\,, \tag{7}$$

where the L's are at least of the order of square of g_r, $\sqrt{h_r}$, and f_r.

The physical masses of $A_\mu(\varphi)$, χ, and ψ are, respectively,

$$m_r = g_r v_r\,, \tag{8a}$$

$$\mu^2 = 2h_r v_r^{\,2}\,, \tag{8b}$$

and

$$m_e = f_r v_r/\sqrt{2}\,. \tag{8c}$$

The perturbation prescription is summarized in the appendix.

III. Results —

We will not spell out the details of the calculation, since they are straightforward, tedious, and mostly unilluminating.

What we have done is to extract, to second order in g_r, $\sqrt{h_r}$, and f_r, the divergent parts of all the radiative correction to $\langle \chi \rangle$, the two point, and the three point functions by using $\mathcal{L}_{int} + \mathcal{L}_{gauge}$ of the appendix. Renormalizability then dictates that these divergent terms should be cancelled out by the corresponding counter terms in $\mathcal{L}_{counter}$. Note that these are non-trivial requirements, since there are altogether twelve conditions due to $\langle \chi \rangle$ and the propagators[12], plus seven due to various three point vertices[13]. On the other hand, there are twelve renormalization constants only.

In the vacuum expectation $\langle \chi \rangle$ and self-energy calculation, there are occasions when we encounter quadratically divergent integrals. Although not necessary, we find it most convenient to define them by the dimension-interpolation technique of 't Hooft and Veltman[14].

All in all, we find that the following set of renormalization constants will lead to the vanishing $\langle \chi \rangle$ and finite on and off-mass-shell propagators and vertices[15]

$$L_A = -2L_g = -x$$

$$L_\varphi = L_\chi = 2x - 2y$$

$$L_L = -x - y$$

$$L_R = -y$$

$$L_v = 2x - y$$

$$L_h = -6x + 4y + 6\left(m_r/\mu\right)^2 x - 8\left(m_e/\mu\right)^2 \left(m_e/m_r\right)^2 x + 10\left(h/16\pi^2\right)\Gamma(2 - \tfrac{1}{2}n)$$

$$L_f = -3/2\ x + 2y$$

$$L_\alpha = -x + y$$

$$L_\beta = 1/2\ x$$

and

$$L_{\mu}\bar{\mu}_r^2 = -\left(L_h + 2L_v\right)\frac{\mu^2}{2} + \lim_{n\to 4}\left\{-\frac{g^2}{16\pi^2}\frac{n-1}{\left(m_r^{\ 2}\right)^{1-\frac{1}{2}n}}\right.$$

$$-\frac{h}{16\pi^2}\frac{1}{\left(m_r^{\ 2}\right)^{1-\frac{1}{2}n}} - \frac{3h}{16\pi^2}\frac{1}{(\mu^2)^{1-\frac{1}{2}n}}$$

$$\left.+\left(\frac{m_e}{m_r}\right)^2\frac{g^2}{16\pi^2}\frac{n}{\left(m_e^{\ 2}\right)^{1-\frac{1}{2}n}}\right\}\Gamma\left(1-\frac{1}{2}n\right)$$

with
$$x = \frac{g^2}{16\pi^2}\lim_{n\to 4}\Gamma(2-\tfrac{1}{2}n)$$

and
$$y = \left(\frac{m_e}{m_r}\right)^2 \lim_{n\to 4}\frac{g^2}{16\pi^2}\Gamma(2-\tfrac{1}{2}n)\ ,$$

where Γ's are the gamma functions.

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Appendix.[16]

We want to write down the ingredients for developing a perturbation series. In the following, all operators are in the interaction representation.

The propagators in the Feynman-'t Hooft-Stueckelberg gauge are

$$\langle\left(A^{\mu}(x)\ A^{\nu}(y)\right)_{+}\rangle = \frac{1}{i}\ \frac{g^{\mu\nu}}{-\partial^2 + m_r^{\ 2} - i\epsilon}\ \delta(x-y)\ ,$$

$$\langle\left(A^{\mu}(x)\ \varphi(y)\right)_{+}\rangle = \langle\left(\varphi(x)\ A^{\mu}(y)\right)_{+}\rangle = 0\ ,$$

$$\langle\left(\varphi(x)\ \varphi(y)\right)_{+}\rangle = \frac{1}{i}\ \frac{1}{-\partial^2 + m_r^{\ 2} - i\epsilon}\ \delta(x-y)\ ;$$

$$\langle\left(\chi(x)\ \chi(y)\right)_{+}\rangle = \frac{1}{i}\ \frac{1}{-\partial^2 + \mu^2 - i\epsilon}\ \delta(x-y)\ ,$$

and

$$\langle\left(\psi(x)\ \overline{\psi}(y)\right)_{+}\rangle = \frac{1}{i}\ \frac{1}{m + \gamma : \frac{1}{i}\partial - i\epsilon}\ \delta(x-y)\ .$$

The interaction Lagrangian is decomposed into three pieces

$$\mathcal{L}_I = \mathcal{L}_{counter} + \mathcal{L}_{int} + \mathcal{L}_{gauge}$$

where

$$\begin{aligned}
\mathcal{L}_{counter} = & - \left(Z_\mu\, \overline{\mu}_r^{\ 2} + Z_h\, Z_v^{\ 2}\, h_r\, v_r^{\ 2}\right) Z_v\, Z_\chi^{\ 1/2}\, v_r\, \chi \\
& + \left(Z_A - 1\right)\left[-\tfrac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \tfrac{1}{2}\left(\partial_\mu A^\mu\right)^2 - \tfrac{1}{2} m_r^{\ 2} A_\mu^{\ 2}\right. \\
& - \tfrac{1}{2}\left(Z_g^{\ 2}\, Z_v^{\ 2} - 1\right) m_r^{\ 2}\, Z_A\, A_\mu A^\mu \\
& - \tfrac{1}{2}\left(Z_\beta^{\ 2} - 1\right) Z_A \left(\partial_\mu A^\mu\right)^2 \\
& + \left(Z_\varphi - 1\right)\left(-\tfrac{1}{2}\,\partial_\mu \varphi\, \partial^\mu \varphi - \tfrac{1}{2}\, m_r^{\ 2}\, \varphi^2\right)
\end{aligned}$$

$$- \frac{1}{2}\left[\left(Z_\mu - 1\right)\bar{\mu}_r^{\,2} + \left(Z_h Z_v^{\,2} - 1\right)h_r v_r^{\,2} + \left(Z_\alpha^{\,2} - 1\right)m_r^{\,2}\right]Z_\varphi \varphi^2$$

$$+ \left(Z_g Z_v - Z_\alpha Z_\beta\right)m_r Z_A^{\,1/2} Z_\varphi^{\,1/2} A_\mu \partial^\mu \varphi$$

$$+ \left(Z_\chi - 1\right)\left(- \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} \mu^2 \chi^2\right)$$

$$- \frac{1}{2}\left[\left(Z_\mu - 1\right)\bar{\mu}_r^{\,2} + 3\left(Z_h Z_v^{\,2} - 1\right)h_r v_r^{\,2}\right]Z_\chi \chi^2$$

$$+ \left(Z_R - 1\right)\left(- \bar{\psi}_R \gamma_\mu \frac{1}{i} \partial^\mu \psi_R\right) + \left(Z_L - 1\right)\left(- \bar{\psi}_L \gamma_\mu \frac{1}{i} \partial^\mu \psi_L\right)$$

$$- \left(Z_f Z_v Z_L^{\,1/2} Z_R^{\,1/2} - 1\right) m_e \bar{\psi} \psi$$

$$+ \left(Z_g Z_A^{\,1/2} Z_\varphi^{\,1/2} Z_\chi^{\,1/2} - 1\right)g_r A^\mu \left(\chi \partial_\mu \varphi - \varphi \partial_\mu \chi\right)$$

$$- \left(Z_g^{\,2} Z_v Z_A Z_\chi^{\,1/2} - 1\right)g_r m_r A_\mu A^\mu \chi$$

$$- \frac{1}{2}\left(Z_g^{\,2} Z_A Z_\chi - 1\right)g_r^{\,2} A_\mu A^\mu \chi^2$$

$$- \frac{1}{2}\left(Z_g^{\,2} Z_A Z_\varphi - 1\right)g_r^{\,2} A_\mu A^\mu \varphi^2$$

$$- \left(Z_h Z_v Z_\chi^{\,3/2} - 1\right) \mu \left(\frac{1}{2} h_r\right)^{1/2} \chi^3$$

$$- \left(Z_h Z_v Z_\chi^{\,1/2} Z_\varphi - 1\right) \mu \left(\frac{1}{2} h_r\right)^{1/2} \chi \varphi^2$$

$$- \frac{1}{4}\left(Z_h Z_\chi^{\,2} - 1\right)h_r \chi^4$$

$$- \frac{1}{2}\left(Z_h Z_\chi Z_\varphi - 1\right)h_r \chi^2 \varphi^2$$

$$- \frac{1}{4}\left(Z_h Z_\varphi^{\,2} - 1\right)h_r \varphi^4$$

$$+ \left(Z_g\, Z_A^{1/2}\, Z_L - 1\right) g_r\, \overline{\psi}\, \gamma_\mu\, \frac{1}{2}\left(1 + i\gamma_5\right) \psi\, A^\mu$$

$$- \left(Z_f\, Z_L^{1/2}\, Z_R^{1/2}\, Z_\chi^{1/2} - 1\right)\left(m_e/m_r\right) g_r\, \overline{\psi}\, \psi\, \chi$$

$$- \left(Z_f\, Z_L^{1/2}\, Z_R^{1/2}\, Z_\varphi^{1/2} - 1\right)\left(m_e/m_r\right) g_r\, \overline{\psi}\, \gamma_5\, \psi\, \varphi\, ,$$

$$\begin{aligned}\mathcal{L}_{int} = {} & g_r\, A^\mu\left(\chi\, \partial_\mu \varphi - \varphi\, \partial_\mu \chi\right) - g_r\, m_r\, A^\mu\, A_\mu\, \chi \\ & - \frac{1}{2}\, g_r^{\,2}\, A_\mu\, A^\mu\left(\chi^2 + \varphi^2\right) - \mu\left(\frac{1}{2}\, h_r\right)^{1/2} \chi\left(\chi^2 + \varphi^2\right) \\ & - \frac{1}{4}\, h_r\left(\chi^2 + \varphi^2\right)^2 + g_r\, \overline{\psi}\, \gamma_\mu\, \frac{1}{2}\left(1 + i\gamma_5\right) \psi\, A^\mu \\ & - \left(\frac{m_e}{m_r}\right) g_r\, \overline{\psi}\, \psi\, \chi - \left(\frac{m_e}{m_r}\right) g_r\, \overline{\psi}\, \gamma_5\, \psi\, \varphi\, ,\end{aligned}$$

and

$$\int d^4x\, \mathcal{L}_{gauge} = -i\mathrm{Tr}\ell n \left(1 + \frac{\left(Z_\alpha Z_\beta^{-1} Z_g Z_v - 1\right) m_r^{\,2} + Z_\alpha Z_\beta^{-1} Z_g Z_\chi^{1/2} m_r g_r \chi}{-\partial^2 + m_r^{\,2}}\right)$$

Note that the relation

$$h_r\, v_r^{\,2} + \overline{\mu}_r^{\,2} = 0$$

is true, which results in vanishing $\langle\chi\rangle$ in the tree approximation.

The perturbation series is now obtained by the standard procedure.

Footnotes and References.

(1) The execution of this program in the unitarity guage has been carried out by T. Appelquist and H. Quinn (private communication).

(2) Y.-P. Yao, "Quantization and Gauge Freedom in a Theory with Spontaneously Broken Symmetry" (to be published in Phys. Rev.). Notations and metric used in this note are the same as those in this reference.

(3) G. 't Hooft, Nucl. Phys. B33, 173 (1971); B35, 167 (1971); B.W. Lee, Phys. Rev. D5, 823 (1972); B.W. Lee and J. Zinn-Justin, Phys. Rev. D5, 3121 (1972), D5, 3137 (1972), D5, 3155 (1972); E.S. Fradkin and I.V. Tyntin, "Theory of Neutral Gauge Fields with Spontaneous Symmetry Breaking," P.N. Lebedev Physical Institute preprint, N55 (1972).

(4) For example, A.A. Slanov, "Generalized Ward Identities and Gauge Fields," Kiev preprint (1971).

(5) A somewhat more restricted class of gauges with $\kappa^2 = \xi m^2$ has been considered by K. Fujikawa, B.W. Lee and A.I. Sander (to be published in Phys. Rev.).

(6) T.D. Lee and C.N. Yang, Phys. Rev. 128, 885 (1962).

(7) E.C.G. Stueckelberg, Helv. Phys. Acta 11, 299 (1938); R.P. Feynman, Acta Phys. Polon. 24, 697 (1963); G. 't Hooft (see reference 3).

(8) In fact, we do not expect them to be satisfied because of the Adler-Bell-Jackiw type anomaly. See D. Gross and R. Jackiw, M.I.T. preprint (1972) and C. Bouchiat, J. Iliopoulos, and Ph. Meyer, Orsay preprint (1972).

(9) T. Appelquist and H. Quinn, Phys. Letters 39B, 229 (1972).

(10) P.W. Higgs, Phys. Letters 12, 132 (1964), Phys. Rev. Letters 13, 508 (1964), Phys. Rev. 145, 1156 (1965); T.W.B. Kibble, Phys. Rev. 155, 1554 (1966); G. Guralnik, C.R. Hagen, and T.W.B. Kibble, Phys. Rev. Letters 13, 585 (1964); F. Englert and R. Brout, Phys. Rev. Letters 13, 321 (1964).

(11) L.D. Fadde'ev and V.N. Popov, Phys. Letters 25B, 29 (1967).

(12) There are three conditions on $\langle (A^{\mu} A^{\nu})_{+} \rangle$, which lead to finite renormalized mass and coefficients multiplied to $g^{\mu\nu}$ and $\partial^{\mu}\partial^{\nu}$. One condition is required to give finite $A_{\mu} - \varphi$ mixing. Finite renormalized wave function and mass for φ and χ yield two conditions each. Finally, there are three conditions on ψ.

(13) We have not checked the four point functions. However, if anything, their divergence behavior should be milder and we are hopeful of consistency.

(14) G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972). For further references and other contribution, see G.M. Cicuta, SLAC preprint, submitted to XVI International Conference in High Energy Physics, Batavia, Illinois (September 6-13, 1972).

(15) The induced couplings are also finite.

(16) Readers interacting weakly with this subject may perform appendectomy at this point.