OAK RIDGE NATIONAL LABORATORY

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Subject: DESCRIPTION AND OPERATING MANUAL FOR THE THREE GROUP, THREE REGION REACTOR CODE FOR ORACLE

To: Distribution

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THE THREE GROUP, THREE REGION REACTOR WITH SHELLS

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I. Introduction

The first part of this report is intended to be an outline of the theory of the three group, three region reactor with shells. Derivational detail is avoided and may be found in another report.[1] It is hoped, however, that sufficient information is presented to satisfy the average user of the three group, three region reactor code.

The second section is written as an operation manual for the main code and the two auxiliary codes. The form of input data and the mechanics of code running are treated in considerable detail.

II. Theory

A. Differential Equations and Boundary Conditions

1. Differential equation. Nine equations apply to the regions:

$$D_{ij} \left\{\Phi''_{ij}(r) + \frac{\gamma}{r} \Phi'_{ij}(r) + B_i^2\right\} - \left\{\Sigma_{a_{ij}} + \Sigma_{x_{ij}}\right\} \Phi_{ij}(r) + T_{ij}(r) = 0 \qquad (1)$$

$$T_{1j}(r) = \left\{f_{1j}\Phi_{1j}(r) + f_{2j}\Phi_{2j}(r) + f_{3j}\Phi_{3j}(r)\right\} (1 - \delta_j) \qquad (2)$$

$$T_{2j}(r) = \left\{f_{1j}\Phi_{1j}(r) + f_{2j}\Phi_{2j}(r) + f_{3j}\Phi_{3j}(r)\right\} \delta_j + \Sigma_{x_{1j}}\Phi_{1j}(r) \qquad (3)$$

$$T_{3j}(r) = \Sigma_{x_{2j}}\Phi_{2j}(r) \qquad (4)$$

(1) Soon to be published.

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where

i denotes group,

j denotes region,

D = diffusion coefficient,

Φ = flux,

Φ', Φ'' = first, second derivatives with respect to r,

γ = 0, 1, 2 for slab, cylinder, or sphere,

r = distance from origin,

B_i^2 = unreflected buckling for i^{th} group,

Σ_a = absorption cross section,

Σ_x = transfer cross section; i.e., probability per unit path length a neutron be transferred from one group to the one below it,

T = source term,

$f_{ij} = \nu\Sigma_{f_{ij}}$; Σ_f = fission cross section,

δ = fraction of neutrons born into the second group.

2. Boundary conditions for the differential equation. The boundary conditions are:

$$I_j^\gamma \left\{R_{ij}\Phi_{ij}(I_j) + 2D_{ij}S_{ij}\Phi'_{ij}(I_j)\right\} = J_j^\gamma \left\{P_{ij}\Phi_{i,j+1}(J_j) + 2D_{i,j+1}Q_{ij}\Phi'_{i,j+1}(J_j)\right\} \quad (5)$$

$$I_j^\gamma \left\{P_{ij}\Phi_{ij}(I_j) - 2D_{ij}Q_{ij}\Phi_{ij}(I_j)\right\} = J_j^\gamma \left\{R_{ij}\Phi_{i,j+1}(J_j) - 2D_{i,j+1}S_{ij}\Phi_{i,j+1}(J_j)\right\} \quad (6)$$

$$I_3^\gamma \left\{R_{i3}\Phi_{i3}(I_3) + 2D_{i3}S_{i3}\Phi'_{i3}(I_3)\right\} = 0 \quad (7)$$

$$\Phi'_{i1}(0) = 0 \quad (8)$$

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where I_j denotes the inner radius of the j^{th} shell, J_j denotes the outer radius of the j^{th} shell, and P, Q, R, and S are defined in the notes on anisotropic scattering in thin shells by R. R. Coveyou and R. R. Bate.(1) Briefly, they may be described as follows:

If one considers, at most, two collisions of a neutron in a thin shell and seeks expressions for the currents, casting these expressions into familiar diffusion theory form, there results:

$$\text{for the forward exit current,} \quad \frac{P\Phi(I)}{4} - \frac{QD\Phi'(I)}{2} ; \qquad (9)$$

$$\text{for the backward exit current,} \quad \frac{\overline{R}\Phi(I)}{4} - \frac{\overline{S}D\Phi'(I)}{2} . \qquad (10)$$

The P, Q, $\overline{R}$, and $\overline{S}$ which are coefficients of the usual diffusion theory terms are somewhat complicated functions of the shell constants. It is to be noted that in (5), (6), and (7) $R = (1 - \overline{R})$, $S = (1 + \overline{S})$.

Now, defining

$$\alpha_{ij} \equiv \frac{\Sigma a_{ij} + \Sigma x_{ij}}{D_{ij}} - B^2$$

$$\rho_{ij} \equiv \frac{\Sigma x_{ij}}{D_{i+1,j}} \qquad (11)$$

$$T^+_{ij} \equiv \frac{T_{ij}}{D_{ij}}$$

$$f^+_{ij} \equiv \frac{f_{ij}}{D_{1j}} ,$$

(1) Appendix.

(1) may be written as

$$\Phi''_{ij}(r) + \frac{\gamma}{r}\,\Phi'_{ij}(r) - \alpha_{ij}\Phi_{ij}(r) + T^{+}_{ij}(r) \;=\; 0 \quad , \tag{12}$$

while (2), (3), and (4) become

$$T^{+}_{1j}(r) \;=\; \left\{f^{+}_{1j}\Phi_{1j}(r) + f^{+}_{2j}\Phi_{2j}(r) + f^{+}_{3j}\Phi_{3j}(r)\right\}(1 - \delta_j) \quad , \tag{13}$$

$$T^{+}_{2j}(r) \;=\; \left\{f^{+}_{1j}\Phi_{1j}(r) + f^{+}_{2j}\Phi_{2j}(r) + f^{+}_{3j}\Phi_{3j}(r)\right\}\delta_j + \rho_{1j}\Phi_{1j}(r) \quad , \tag{14}$$

$$T^{+}_{3j}(r) \;=\; \rho_{2j}\Phi_{2j}(r) \quad . \tag{15}$$

B. The Difference Equation and Boundary Conditions

1. Difference equation. The difference equation which approximates (12) with an error of order h^4 where h is the lattice spacing,

$$\begin{aligned} &\left\{1 + \frac{h^2\gamma^2}{12r^2} - \frac{h^2\gamma}{6r^2} - \frac{\alpha h^2}{12}\right\} \frac{\Phi(r + h) - 2\Phi(r) + \Phi(r - h)}{h^2} \\ &+ \left\{\frac{\gamma}{r} - \frac{h^2\gamma^2}{12r^2} - \frac{\alpha\gamma h^2}{12r} + \frac{\gamma h^2}{6r^3}\right\} \frac{\Phi(r + h) - \Phi(r - h)}{2h} - \alpha\Phi(r) + T(r) \\ &- \frac{\gamma}{r}\,\frac{T(r + h) - T(r - h)}{2h} \quad \frac{T(r + h) - 2T(r) + T(r - h)}{h^2} \;=\; 0 \quad . \end{aligned} \tag{16}$$

Multiplying through by $\frac{h^2}{2}$, (16) is put into the form

$$P_k\Omega_{k+1} - Q_k\Omega_k + R_k\Omega_{k-1} + S_k \;=\; 0 \tag{17}$$

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where

$$\Omega_k = \text{flux at space point } k,$$

$$P_k = \frac{1}{2} + \frac{\gamma h}{4r} + \frac{\gamma(\gamma - 2)\, h^2}{24\, r^2} - \frac{\gamma(\gamma - 2)\, h^3}{48\, r^3} - \frac{\alpha^*}{24} - \frac{\alpha^* \gamma h}{48 r} \,, \tag{18}$$

$$Q_k = 1 + \frac{\gamma(\gamma - 2)\, h^2}{12\, r^2} + \frac{5\alpha^*}{12} \,, \tag{19}$$

$$R_k = \frac{1}{2} - \frac{\gamma h}{4r} + \frac{\gamma(\gamma - 2)\, h^2}{24\, r^2} + \frac{\gamma(\gamma - 2)\, h^3}{48\, r^3} - \frac{\alpha^*}{24} + \frac{\alpha^* \gamma h}{48 r} \,, \tag{20}$$

$$S_k = A_k T^*_{k+1} + \frac{5}{12} T^*_k + B_k T^*_{k-1} \tag{21}$$

$$A_k = \frac{\gamma h}{48r} \quad \frac{1}{24}$$

$$B_k = \frac{1}{24} \quad \frac{\gamma h}{48r} \,,$$

and now

$$\alpha^*_{ij} \equiv \alpha_{ij} h_j^2 = \left\{ \frac{\Sigma_{a_{ij}} + \Sigma_{x_{ij}}}{D_{ij}} - B^2 \right\} h_j^2 \tag{22}$$

$$\rho^*_{ij} \equiv \rho_{ij} h_j^2 = \frac{\Sigma_{x_{ij}} h_j^2}{D_{i+1,j}}$$

$$f^*_{ij} \equiv f^+_{ij} h_j^2 = \frac{f_{ij} h_j^2}{D_{1j}}$$

$$D^*_{ij} \equiv \frac{D_{ij}}{h_j}$$

$$T^*_k \equiv h^2 T^+_k \,.$$

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Note subscript k refers to the space point. The P_k, Q_k, R_k, and S_k above should not be confused with the P, Q, R, and S of the shells. They are not the same nor are they related.

2. Inner shell boundary conditions for the difference equation.

a. Form II of original boundary conditions. Defining quantities which will hereafter be used to describe the shells,

$$
\begin{aligned}
2\beta_1 &= PS + RQ \\
2\beta_2 &= RS + PQ \\
2\beta_3 &= S^2 - Q^2 \\
2\beta_4 &= R^2 - P^2 ,
\end{aligned}
\tag{23}
$$

saying

$$\Phi'(r) = \frac{M_k}{h}\,\Omega_{k+1} + \frac{N_k}{h}\,\Omega_k + \frac{L_k}{h}\,\Omega_{k-1} + \frac{h}{12}\left\{T^+(r + h) - T^+(r - h)\right\} \tag{24}$$

where

$$M_k = \frac{1}{2} + \frac{\gamma h}{6r} - \frac{\gamma h^2}{12r^2} - \frac{\alpha^*}{12} \tag{25}$$

$$N_k = \frac{\gamma h}{3r} \tag{26}$$

$$L_k = -\frac{1}{2} + \frac{\gamma h}{6r} + \frac{\gamma h^2}{12r^2} + \frac{\alpha^*}{12} \tag{27}$$

and finally defining

$$Y \equiv \left(\frac{J}{I}\right)^{\gamma} \sim 1 + \gamma\,\frac{t}{I}, \quad t = \text{shell thickness} , \tag{28}$$

(5) and (6) may be written as

$$2D_1^*\beta_2M_1\Omega_{I+1} + (\beta_4 + 2D_1^*\beta_2N_1)\,\Omega_I + 2D_1^*\beta_2L_1\Omega_{I-1} + \frac{2D_1^*\beta_2}{12}\,(T^*_{I+1} - T^*_{I-1}) \;=$$

$$= \; Y2D_2^*\beta_1M_2\Omega_{J+1} + Y2D_2^*\beta_1N_2\Omega_J + Y2D_2^*\beta_1L_2\Omega_{J-1} + \frac{2YD_2^*\beta_1}{12}\,(T^*_{J+1} - T^*_{J-1}) \;; \qquad (29)$$

$$2D_1^*\beta_3M_1\Omega_{I+1} + (2D_1^*\beta_3N_1 + \beta_2)\,\Omega_I + 2D_1^*\beta_3L_1\Omega_{I-1} + \frac{2D_1^*\beta_3}{12}\,(T^*_{I+1} - T^*_{I-1}) \;=\; Y\beta_1\Omega_J \qquad (30)$$

This is a new form of the boundary conditions at the inner shells.

b. Space points in the shells. The subscripts on D^*, M, N, and L indicate in this instance, for 1, the region interior to the shell and, for 2, the region exterior to the shell. Two ficticious points have been introduced above; namely, I+1 and J-1 points. The sequence of points at a shell is shown in Figure 1.

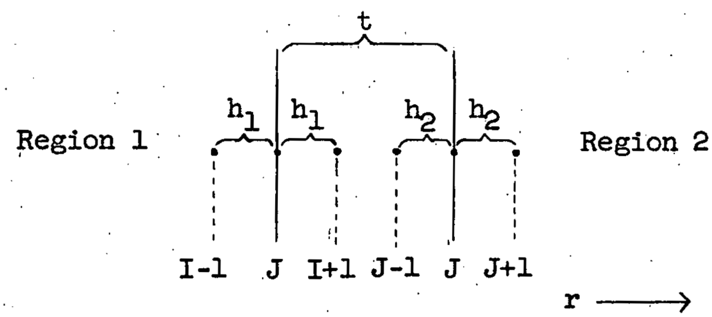

Figure 1. Illustration of sequence of space points at a shell

It is as though the shell has been removed. J-1, J, and J+1 points have properties of Region 2 while I-1, I, and I+1 points have properties of Region 1. Shell properties appear only in the β's and in Y. Note that the lattice spacings need not be the same in Regions 1 and 2. Even though it should happen $h_1 > t - h_2$ such that I+1 lies to the right of J-1, the order of Figure 1 is always considered to hold. In the case of no shell, t = 0 and I+1 coincide with J+1, provided $h_1 = h_2$.

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c. Reduction of form II to region interior difference equation form. Now, by eliminating Ω_{J+1}, Ω_J, and Ω_{I-1} from (29), (30), (17) at I, and (17) at J, one gets the form

$$P_{I+1}\Omega_{J-1} - Q_{I+1}\Omega_{I+1} + R_{I+1}\Omega_I + S_{I+1} = 0 \quad . \tag{31}$$

Elimination of Ω_{J+1}, Ω_I, and Ω_{I-1} gives the form

$$P_{J-1}\Omega_J - Q_{J-1}\Omega_{J-1} + R_{J-1}\Omega_{I+1} + S_{J-1} = 0 \quad . \tag{32}$$

(31) and (32) have the form of the difference equation within a region. Upon computation of the coefficients the shell indeed has been removed. The coefficients are:

$$P_{I+1} = 2\left(1 + \frac{\gamma t}{I}\right) R_I(R_J M_J - P_J L_J)\, \beta_1 \tag{33}$$

$$Q_{I+1} = -2D_1^*(R_I M_I - P_I L_I)\left\{\frac{\beta_2 P_J}{D_2^*} - 2\beta_3(Q_J M_J + P_J N_J)\right\}$$

$$R_{I+1} = \frac{R_I P_J \beta_4}{D_2^*} - 2\beta_2\left\{R_I(M_J Q_J + P_J N_J) - P_J \frac{D_1^*}{D_2^*}(Q_I L_I + R_I N_I)\right\}$$

$$- 4D_1^*\beta_3(Q_I L_I + R_I N_I)(M_J Q_J + P_J N_J)$$

$$S_{I+1} = R_I P_J\left\{2\beta_2 \frac{D_1^*}{D_2^*} W_I - 2\beta_1 W_J\left(1 + \frac{\gamma t}{I}\right)\right\} - 4D_1^* R_I \beta_3 W_I(P_J N_J + M_J Q_J)$$

and

$$P_{J-1} = -\frac{R_I P_J \beta_4}{D_1^*} - 2\beta_2\left\{P_J(R_I N_I + Q_I L_I) - \frac{D_2^*}{D_1^*} R_I(P_J N_J + Q_J M_J)\right\} \tag{34}$$

$$+ 4D_2^*\beta_3(R_I N_I + Q_I L_I)(P_J N_J + Q_J M_J)$$

$$Q_{J-1} = 2D_2^*(R_J M_J - L_J P_J)\left\{\frac{R_I\beta_2}{D_1^*} + 2\beta_3(R_I N_I + Q_I L_I)\right\}$$

$$R_{J-1} = -2\left(1 - \frac{\gamma t}{I}\right) P_J \beta_1 (R_I M_I - P_I L_I)$$

$$S_{J-1} = 2P_J R_I\left\{\beta_1 W_I\left(\frac{\gamma t}{I} - 1\right) + \frac{D_2^*}{D_1^*}\beta_2 W_J\right\} + 4D_2^* P_J \beta_3 W_J (R_I N_I + Q_I L_I)$$

where

$$W_I = \frac{1}{12}(T^*_{I+1} - T^*_{I-1}) - \frac{L_I S_I}{R_I} \tag{35}$$

$$W_J = \frac{1}{12}(T^*_{J+1} - T^*_{J-1}) - \frac{M_J S_J}{P_J} . \tag{36}$$

3. Other boundary conditions of the difference equation.

a. Introduction of Γ, Δ method. Now assuming the relation

$$\Omega_k = \Gamma_k \Omega_{k+1} + \Delta_k , \tag{37}$$

it can be seen

$$\Gamma_k = \frac{P_k}{Q_k - R_k \Gamma_{k-1}} \tag{38}$$

$$\Delta_k = \frac{\Gamma_k(R_k\Delta_{k-1} + S_k)}{P_k} \tag{39}$$

b. Boundary condition at the origin. Using (8), the boundary condition at the origin,

$$\Gamma_0 = \frac{2(1+\gamma)}{2(1+\gamma)+\alpha^*} \tag{40}$$

$$\Delta_0 = \frac{2S_0^*}{2(1+\gamma)+\alpha^*} = \frac{2\left(\frac{5}{12}\,T_0^* + \frac{1}{12}\,T_1^*\right)}{2(1+\gamma)+\alpha^*} \; . \tag{41}$$

c. Boundary condition at the outer boundary. Defining

$$\beta_5 = \frac{S_N}{R_N} \tag{42}$$

$$X_N = \frac{1}{12}\left\{T_{N+1}^* - T_{N-1}^*\right\} \tag{43}$$

where N subscript indicates the outside boundary, the outer boundary condition becomes

$$\Omega_{N+1} = -\frac{2\beta_5 D_1^* X_N + \Delta_N\left\{1 + 2\beta_5 D_1^*\left(\frac{L_1\Delta_{N-1}}{\Delta_N} + N_1 + L_1\Gamma_{N-1}\right)\right\}}{2\beta_5 D_1^* M_1 + \Gamma_N\left\{1 + 2\beta_5 D_1^*(N_1 + L_1\Gamma_{N-1})\right\}} \tag{44}$$

Note that subscripts on D, L, M, and N are unnecessary since there is one medium only with which to deal. They have been retained for consistency.

At this point the difference equation has been revealed and the boundary conditions put into suitable form. The balance of the paper will be concerned with the codes.

III. Codes

A. Main Code, Three Group, Three Region

1. Code logic. The calculation runs as follows:

1) A flat source is introduced in fissionable regions; zero source elsewhere.

2) First group Γ_0 and Δ_0 are calculated; then, by means of (40) and (41), the Γ's and Δ's are calculated to N_1, the space point of the inner surface of the first shell.

3) Γ's and Δ's are calculated through the first shell by using (31) - (36), (40), and (41).

4) Γ's and Δ's are calculated through the second region, the second shell, and the third region to the outside boundary.

5) (44) is satisfied at the outside boundary.

6) (37) allows computation of the Ω_k. After each Ω_k is computed, the source at k to the next group is found. The calculation works in to r = 0 from the outside boundary.

7) With the computed sources, the calculation is repeated for the second group. Sources for the third group are computed.

8) The third group calculation is done.

9) The third group fluxes are added over the reactor and normalized corresponding to the differential operation

$$\frac{\int_R \Phi_3(\underline{r})\, d\underline{r}}{\int_R d\underline{r}} = \Sigma^i \text{ ; } i \sim \text{iteration .}$$

10) k is computed by the quotient $\frac{\Sigma^i}{\Sigma^0}$, Σ^0 being a fixed number, the largest number the Oracle can accommodate.

11) The third group fluxes and the sources for the next iteration are divided by k to keep them within range of the machine numbers. Each third group flux is then compared with the corresponding flux from the previous iteration. When the scaled third-group fluxes from two succeeding iterations differ by no more than ϵ, the convergence criterion which is initially set to one-half, ϵ is divided by two and the calculation repeats starting with the first group and the new sources.

12) When k is computed, it is scaled by 10^{-2} and punched out in decimal convention. ϵ is then punched. The programmer follows ϵ until he is satisfied with the convergence. He then has the option of asking for all fluxes to be punched out. The initial conditions will be punched following the fluxes. If he chooses, the programmer may dispense with punching the fluxes and have only the initial conditions punched.

Since the input for the three group, three region code uses the constants (22), and since it has been recognized that these are not the most convenient quantities to speak of or to compute, a constant preparation program has been coded which uses more familiar quantities and prepares input to 3G3R.

2. Code operation. Instructions on running 3G3R and the form of the output are as follows:

Instructions for running 3G3R, Program Number 6.

Operation	Result
1. Place program 6 under reader.	
2. 90 7FA.	Reads program into memory starting at memory cell 7FA.
3. Place initial conditions under reader.	
4. 43 25A.	Sums all words in the memory to see if program was read in correctly. If the sum is correct, goes to beginning of code, reads in initial conditions, and starts computing. The sum check may be used only once. If it is desired to check the sum again, the sum check code must be read in. For this give 90 25A then 43 25A.
5. When satisfied with convergence, set break point toggle.	Computation stops on the right of 1CE.
6. Remove break point and give 41 ICC.	Fluxes punched if desired. Initial conditions punched always.
7. Place next case initial conditions under reader.	
8. 43 001.	
9. Hereafter give 43 001 for each new case.	

Running time per case is 6 - 10 minutes.

3. Form of output. The form of 3G3R output is

Symbol	Quantity	Numbers System in which Punched	Scaling
k^1	first iteration k	decimal	10^{-2}
ϵ^1	first iteration convergence criterion	hex	1
k^2	.	.	
ϵ^2	.	.	
.	.	.	
.	.	.	
k^n	n^{th} iteration k	decimal	10^{-2}
ϵ^n	n^{th} iteration convergence criterion	hex	1
FFFFFFFFFF	word of F's to mark start of first group fluxes	-	-
$\Omega_{1k}^{(1)}$	first group fluxes at 51 space points	decimal	-
FFFFFFFFFF	word of F's to mark start of second group fluxes	-	-
$\Omega_{2k}^{(1)}$	second group fluxes	decimal	-
FFFFFFFFFF	work of F's to mark start of third group fluxes	-	-
$\Omega_{3k}^{(1)}$	third group fluxes	decimal	-
FFFFFFFFFF	word of F's to mark start of problem parameters	-	-
$\bar{X}$	case number	hex	-
N_1	region 1 space points - produced as $0000N_1 0000N_1$	hex	-
N_2	region 2 space points	hex	-

(1) Fluxes are punched from the outside boundary to r = 0. There are 44 region fluxes and 7 shell and boundary fluxes; i.e., 51 space points.

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Symbol	Quantity	Numbers System in which Punched	Scaling
N_3	region 3 space points	hex	-
ϵ	final convergence criterion	hex	-
γ	geometry factor	hex	2^{-2}
t_1	shell 1 thickness	decimal	2^{-10}
h_1	region 1 lattice spacing	decimal	2^{-10}
t_2	shell 2 thickness	decimal	2^{-10}
h_2	region 2 lattice spacing	decimal	2^{-10}
h_3	region 3 lattice spacing	decimal	2^{-10}
α_{11}^*	group 1, region 1 α^*	decimal	2^{-7}
α_{12}^*	group 1, region 2 α^*	decimal	2^{-7}
α_{13}^*	group 1, region 3 α^*	decimal	2^{-7}
D_{11}^*	group 1, region 1 D^*	decimal	2^{-7}
D_{12}^*	group 1, region 2 D^*	decimal	2^{-7}
D_{13}^*	group 1, region 3 D^*	decimal	2^{-7}
f_{11}^*	group 1, region 1 f^*	decimal	2^{-7}
f_{12}^*	group 1, region 2 f^*	decimal	2^{-7}
f_{13}^*	group 1, region 3 f^*	decimal	2^{-7}
ρ_{11}^*	group 1, region 1 ρ^*	decimal	2^{-7}
ρ_{12}^*	group 1, region 2 ρ^*	decimal	2^{-7}
ρ_{13}^*	group 1, region 3 ρ^*	decimal	2^{-7}
β_{111}	β_1 for group 1, shell 1	decimal	2^{-1}
β_{112}	β_1 for group 1, shell 2	decimal	2^{-1}
β_{211}	β_2 for group 1, shell 1	decimal	2^{-1}
β_{212}	β_2 for group 1, shell 2	decimal	2^{-1}
β_{311}	β_3 for group 1, shell 1	decimal	2^{-1}

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Symbol	Quantity	Numbers System in which Punched	Scaling
β_{312}	β_3 for group 1, shell 2	decimal	2^{-1}
β_{411}	β_4 for group 1, shell 1	decimal	2^{-1}
β_{412}	β_4 for group 1, shell 2	decimal	2^{-1}
β_{511}	β_5 for group 1, shell 1	decimal	2^{-1}
β_{512}	β_5 for group 1, shell 2	decimal	2^{-1}
α^*_{21}	group 2, region 1 α^*	decimal	2^{-7}
α^*_{22}	group 2, region 2 α^*	decimal	2^{-7}
⋮	⋮	⋮	⋮
ρ^*_{23}	group 2, region 3 ρ^*	decimal	2^{-7}
β_{121}	β_1 for group 2, shell 1	decimal	2^{-1}
⋮	⋮	⋮	⋮
β_{522}	β_5 for group 2, shell 2	decimal	2^{-1}
α^*_{31}	group 3, region 1 α^*	decimal	2^{-7}
⋮	⋮	⋮	⋮
β_{532}	β_5 for group 3, shell 2	decimal	2^{-1}

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B. Constant Preparation Routine

1. Code logic. The constant preparation routine accepts more familiar constants than (22) and arranges and punches them in suitable form for input to 3G3R. It also optimizes the N's in the following sense.

(18) and (20) show, for $\gamma = 0$, if $\alpha^* \geqq 12$ the difference equation breaks down since P_k and R_k become less than 0. It has been arbitrarily said, then, that α^* be no larger than 4. As is seen from (22), α^* depends on the lattice spacing. CPR tests α^*'s for all combinations of N_1, N_2, and N_3 subject to the conditions $N_1 + N_2 + N_3 = 44$, $N_1 \geq 4$, $N_2 \geq 4$, $N_3 \geq 4$. The set of N's which minimizes the worst α^* is chosen. There follows a test to see if the $\alpha^* < 4$. Should an α^* be larger than 4, it is punched out in decimal followed by the group and region it characterizes. The machine stops after all the bad α^*'s are punched.

2. Oracle decimal convention. Before giving the form of the input to CPR, a word on decimal fixed point and floating point notation is in order. In decimal fixed point convention, the first digit is the sign digit. A 0 indicates a positive number; and F indicates a negative number. The decimal point lies after the sign digit. For example, 0.0032 would be written as 0003200000 while -0.01 appears as F010000000. This convention applies to fractions. To represent numbers outside the range of machine numbers, floating point decimal convention may be used. In this convention the magnitude of the number is scaled by powers of ten until it is the largest fraction possible. It is then written as a decimal fixed point number. Immediately following the magnitude, the exponent of ten is written so as to bring the fraction to its proper value. The last several digits in the word are reserved for the value of the exponent. A 0 for the first digit indicates a positive exponent; and F indicates a negative

exponent. For example, 320 would be written as 0320000000, 0000000003 while -0.01 takes the form F100000000, F000000001. A 0 must be written as 0000000000, 8000000000. Note that these conventions may not be used by the programmer arbitrarily. He must have a code equipped to accept them.

3. Form of input. The form of input to CPR is as follows:

Memory Position	Symbol	Quantity	Notation Convention	Scaling
28C	g	Group number. If negative, 3G3R does 2 groups; if positive, 3G3R does 3 groups. See below.	binary fixed point	-
28D	X	Flux and case numbers. See below.	binary fixed point	-
28E	γ(1)	Geometry factor.	binary fixed point	2^{-2}
28F	R_1, R_1 exponent	Region 1 thickness.	decimal floating point	-
291	R_2, R_2 exponent	Region 2 thickness.	decimal floating point	-
293	R_3, R_3 exponent	Region 3 thickness.	decimal floating point	-
295	t_1, t_1 exponent	Shell 1 thickness.	decimal floating point	-
297	t_2, t_2 exponent	Shell 2 thickness.	decimal floating point	-

(1) For slab 0000000000; cylinder 2000000000; sphere 4000000000.

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Memory Position	Symbol	Quantity	Notation Convention	Scaling
299	h_1 exponent	Region 1 lattice spacing to be filled in by CPR. Put in 0's.	-	-
29B	$\Sigma_{a_{11}}$ exponent	Group 1, region 1 absorption cross section.	decimal floating point	-
29D	$\Sigma_{a_{21}}$ exponent	Group 2, region 1 absorption cross section.	decimal floating point	-
29F	$\Sigma_{a_{31}}$ exponent	Group 3, region 1 absorption cross section.	decimal floating point	-
2A1	$\Sigma_{x_{11}}$ exponent	Group 1, region 1 transfer cross section.	decimal floating point	-
2A3	$\Sigma_{x_{21}}$ exponent	Group 2, region 1 transfer cross section.	decimal floating point	-
2A5	$\Sigma_{x_{31}}$ (1) exponent	Group 3, region 1 transfer cross section.	decimal floating point	-
2A7	D_{11} exponent	Group 1, region 1 diffusion coefficient.	decimal floating point	-
2A9	D_{21} exponent	Group 2, region 1 diffusion coefficient.	decimal floating point	-
2AB	D_{31} exponent	Group 3, region 1 diffusion coefficient.	decimal floating point	-

(1) Σ_{x_3}'s are obviously zero. Put in as 0000000000, 8000000000.

Memory Position	Symbol	Quantity	Notation Convention	Scaling
2AD	$\nu\Sigma_{f_{11}}$ exponent	ν x group 1, region 1 fission cross section.	decimal floating point	-
2AF	$\nu\Sigma_{f_{21}}$ exponent	ν x group 2, region 1 fission cross section.	decimal floating point	-
2B1	$\nu\Sigma_{f_{31}}$ exponent	ν x group 3, region 1 fission cross section.	decimal floating point	-
2B3	δ_1 exponent	Fraction of neutron born into group 2 in region 1.	decimal floating point	-
2B5	h_2 exponent	Region 2 lattice spacing. Filled in by CPR. Put in O's.	-	-
2B7	$\Sigma_{a_{12}}$ exponent	Region 2 lattice spacing. Filled in by CPR. Put in O's.	decimal floating point	-
2B9	$\Sigma_{a_{22}}$ exponent	"	"	-
2BB	$\Sigma_{a_{32}}$ exponent	"	"	-
2BD	$\Sigma_{x_{12}}$ exponent	"	"	-
2BF	$\Sigma_{x_{22}}$ exponent	"	"	-

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Memory Position	Symbol	Quantity	Notation Convention	Scaling
2C1	$\Sigma_{x_{32}}$[1] exponent	Region 2 lattice spacing. Filled in by CPR. Put in 0's.	decimal floating point	-
2C3	D_{12} exponent	"	"	-
2C5	D_{22} exponent	"	"	-
2C7	D_{32} exponent	"	"	-
2C9	$\nu\Sigma_{f_{12}}$ exponent	"	"	-
2CB	$\nu\Sigma_{f_{22}}$ exponent	"	"	-
2CD	$\nu\Sigma_{f_{32}}$ exponent	"	"	-
2CF	δ_2 exponent	Fraction of neutrons born into group 2 in region 2.	"	-
2D1	h_3 exponent	Region 3 lattice spacing. Filled in by CPR. Put in 0's.	-	-

(1) Σ_{x_3}'s are obviously zero. Put in as 0000000000, 8000000000.

Memory Position	Symbol	Quantity	Notation Convention	Scaling
2D3	$\Sigma_{a_{13}}$ exponent	Region 3 lattice spacing. Filled in by CPR. Put in O's.	decimal floating point	-
2D5	$\Sigma_{a_{23}}$ exponent	"	"	-
2D7	$\Sigma_{a_{33}}$ exponent	"	"	-
2D9	$\Sigma_{x_{13}}$ exponent	"	"	-
2DB	$\Sigma_{x_{23}}$ exponent	"	"	-
2DD	$\Sigma_{x_{33}}$[(1)] exponent	"	"	-
2DF	D_{13} exponent	"	"	-
2E1	D_{23} exponent	"	"	-
2E3	D_{33} exponent	"	"	-

(1) Σ_{x_3}'s are obviously zero. Put in as 0000000000, 8000000000.

Memory Position	Symbol	Quantity	Notation Convention	Scaling
2E5	$\nu\Sigma_{f_{13}}$ exponent	Region 3 lattice spacing. Filled in by CPR. Put in 0's.	decimal floating point	-
2E7	$\nu\Sigma_{f_{23}}$ exponent	"	"	-
2E9	$\nu\Sigma_{f_{33}}$ exponent	"	"	-
2EB	δ_3 exponent	Fraction of neutrons born into group 2 in region 3.	"	-
2ED	B_1^2 exponent	Unreflected group 1 buckling.	"	-
2EF	B_2^2 exponent	Unreflected group 2 buckling.	"	-
2F1	B_3^2 exponent	Unreflected group 3 buckling.	"	-
2F3	β_{111}	β_1 for group 1, shell 1.	decimal fixed point	2^{-1}
2F4	β_{112}	β_1 for group 1, shell 2.	"	"
2F5	β_{211}	β_2 for group 1, shell 1.	"	"
2F6	β_{212}	β_2 for group 1, shell 2.	"	"
2F7	β_{311}	β_3 for group 1, shell 1.	"	"
2F8	β_{312}	β_3 for group 1, shell 2.	"	"
2F9	β_{411}	β_4 for group 1, shell 1.	"	"

Memory Position	Symbol	Quantity	Notation Convention	Scaling
2FA	β_{412}	β_4 for group 1, shell 1.	decimal fixed point	2^{-1}
2FB	β_{51}	β_5 for group 1.	"	"
2FC	β_{121}	β_1 for group 2, shell 1.	"	"
2FD	β_{122}	β_1 for group 2, shell 2.	"	"
2FE	β_{221}	β_2 for group 2, shell 1.	"	"
2FF	β_{222}	β_2 for group 2, shell 2.	"	"
300	β_{321}	β_3 for group 2, shell 1.	"	"
301	β_{322}	β_3 for group 2, shell 2.	"	"
302	β_{421}	β_4 for group 2, shell 1.	"	"
303	β_{422}	β_4 for group 2, shell 2.	"	"
304	β_{52}	β_5 for group 2.	"	"
305	β_{131}	β_1 for group 3, shell 1.	"	"
306	β_{132}	β_1 for group 3, shell 2.	"	"
307	β_{231}	β_2 for group 3, shell 1.	"	"
308	β_{232}	β_2 for group 3, shell 2.	"	"
309	β_{331}	β_3 for group 3, shell 1.	"	"
30A	β_{332}	β_3 for group 3, shell 2.	"	"
30B	β_{431}	β_4 for group 3, shell 1.	"	"
30C	β_{432}	β_4 for group 3, shell 2.	"	"
30D	β_{53}	β_5 for group 3.	"	"

4. Variations of the 3G3R code. The possibility of doing a two-group calculation exists. If g is negative, the second group calculation in 3G3R is skipped. However, something must be put in the second group positions. The correct procedure is to make the second and third group constants the same. All δ_j should be zero.

$\underline{X}$ gives the programmer the choice of punching fluxes in 3G3R, the first three digits controlling. If a group flux is desired, a 2 should be placed in the corresponding place among the first three digits. For example, if first and third group fluxes are to be punched, $\underline{X}$ appears as 2020000000; if all three fluxes are wanted, $\underline{X}$ is written as 2220000000. The remaining seven digits may be used to identify the reactor.

Note that if there are no shells, the situation may be expressed by setting $t_1 = t_2 = 0$; $\beta_1 = \beta_2 = 1$; $\beta_3 = \beta_4 = 0$. $\beta_5 = 0$ requires the flux be 0 at the outside boundary while $\beta_5 = 1$ demands the return current be 0.

To treat a two region reactor, collapse one of the shells, place an arbitrary boundary, and make the region constants on either side of the collapsed shell the same.

When a shell is collapsed, note that three fluxes are repeated in the punch out of 3G3R; namely, Ω_{J+1}, Ω_J, Ω_{J-1}.

5. Code operation. Running instructions for CPR, Program V are:

Operation	Result
1. Place CPR, Program V under reader.	
2. 90 394.	Load CPR into memory starting with memory cell 394.

Operation	Result
3. Place initial conditions under reader.	Sum check is done and if correct, goes to main part of code. If bad α^*'s punch, review initial conditions.
4. 43 394	
5. At end of punching, advance tape a short distance.	Puts a double space at the end of each case. This divides cases for 3G3R.
6. Use correction subroutines to alter basic case for new cases. See below.	
7. After all corrections give 41 OOA.	
8. Continue correcting and using 41 OOA for rest of cases.	

Running time per case is approximately one minute.

6. Initial conditions correction tapes. Three types of corrections may be made:

1) Straight correction.

2) Decimal fixed point to binary fixed point.

3) Decimal floating point to binary floating point.

For 1, make a correction tape of the form (/ indicates single space)

address of correction / correction / next address / next correction / etc.

... / FFFFFFFFFF /

Place under reader and give 43 152.

For 2, the tape has the form

address of correction / correction in decimal fixed point / etc.

... / FFFFFFFFFF /

Load under reader and give 43 159.

For 3, the tape should be

address of correction / magnitude / exponent / etc.

... / FFFFFFFFFF /

Place under reader and give 43 163.

Examples of correction tape form:

1) Change case number and geometry,

28D / 2220000002 / 28E / 4000000000 / FFFFFFFFFF / ;

2) Change β_5's,

2FB / 0500000000 / 304 / 0500000000 / 30D / 0500000000 / FFFFFFFFFF / ;

3) Change R_2 and R_3,

291 / 0100000000 / 0000000002 / 293 / 0150000000 / 0000000002 /

FFFFFFFFFF / .

C. Group Constant Preparation Routine.

For a limited class of reactors a code is available which computes group constants and prepares the input for CPR. The code may be used for light water moderated and reflected reactors only. Two elements in addition to H, O, and U may be used in the core. Only water is allowed in the reflector. The theory is that described by Perry.(1)

1. Form of input. The input to GCPR is as follows:

Memory Position	Symbol	Quantity	Convention	Scaling
2E3	m	Cross section indication. See below.	hex	-
2E4	g	Group number. See CPR section.	hex	-

(1) Perry, "A Conceptual Design of a Pressurized Water Package Power Reactor", ORNL 1613, p. 124 ff.

Memory Position	Symbol	Quantity	Notation Convention	Scaling
2E5	$\underline{\bar{X}}$	Flux and case number. See CPR section.	hex	-
2E6	δ	Geometry factor. See CPR section.	hex	2^{-2}
2E7	R_1 exponent	Region 1 thickness.	decimal floating point	-
2E9	R_2 exponent	Region 2 thickness.	"	-
2EB	R_3 exponent	Region 3 thickness.	"	-
2ED	t_1 exponent	Shell 1 thickness.	"	-
2EF	t_2 exponent	Shell 2 thickness.	"	-
2F1	δ_1	Fraction of neutrons born into group 2 in region 1. See below.	decimal fixed point	1
2F2	δ_2	Fraction of neutrons born into group 2 in region 2. See below.	"	1
2F3	δ_3	Fraction of neutrons born into group 2 in region 3. See below.	"	1

Memory Position	Symbol	Quantity	Notation Convention	Scaling
2F4 ↓ 30E	β's	See CPR.	decimal fixed point	2^{-1}
30E	"	"	"	"
30F	$N_c(H)$	Nuclear density of H in core.	"	10^{-24}
310	$N_R(H)$	Nuclear density of H in reflector.	"	"
311	$N_c(O)$	Nuclear density of O in core.	"	"
312	$N_R(O)$	Nuclear density of O in reflector.	"	"
313	N(O)	Nuclear density of U in core.	"	"
314	N(4)[(1)]	Nuclear density of element 4 in core.	"	"
315	$\xi(4)$	Average lethargy loss per collision for element 4.	"	1
316	$\frac{1}{3\{1-\bar{\mu}(4)\}}$	$\bar{\mu}$ = average cosine of scattering angle.	"	"
317	N(5)[(1)]	Nuclear density of element 5 in core.	"	10^{-24}
318	$\xi(5)$	Average lethargy loss per collision for element 5.	"	1

(1) Use volume fractions for alloys.

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Memory Position	Symbol	Quantity	Notation Convention	Scaling
319	$\frac{1}{3\{1-\bar{\mu}(5)\}}$	$\bar{\mu}$ = average cosine of scattering angle.	decimal fixed point	1
31A	$\sigma_{in}\bar{\xi}$	Inelastic scattering factor. See below.	"	10^{21}
31B	w	Unreflected half width of prism.	"	10^{-3}
31C	ℓ,h	Unreflected half width of prism <u>or</u> half height of cylinder.	"	"
31D	R	Radius of GCPR model.	"	"

2. Variations and special cases of GCPR. m is a positive or negative number (for convenience, all 0's or all F's) which, when positive, allows the programmer to insert his own third region constants and, when negative, asks the program to insert its second region constants into the third region position.

The δ's are considered to be normally 0. When they are other than 0, change memory position 029 from 607FE847FD to 607FD847FD. It was realized too late the inconsistency of limiting fissionable material to region 1 and yet having δ_2 and δ_3 specified in the initial conditions. They must be zero unless the regions are to be permuted.

σ in $\bar{\xi}$ allows for inelastic scattering in the manner described by Perry. The inelastic scatterer must be element 4.

Note that 31B and 31C need not always be filled. For finite systems, slab geometry fills both, cylindrical geometry fills 31C and spherical geometry fills neither.

Regions may be permuted by setting the break point. When the machine stops on the left of 13F, the region constants may be punched out and read into the proper places. Returning to the left of 13F punches the constants in the new order. The table below gives the positions of the constants.

Region	Memory Positions of Constants
1	3B9 - 3C3
2	3C4 - 3CE
3	3CF - 3D9

3. Code operation.

	Operation	Result
1.	Select from the library of cross section elements 4 and 5.	
2.	Place program under reader; 90 35C.	Loads GCPR routine into memory starting with memory cell 35C.
3.	Place element 4 cross section tape under reader; 90 27B.	Loads 31 values of $3\sigma_h$ and 31 values of σ_a into 4 position.
4.	Place element 5 cross section tape under reader; 90 31E.	Loads 31 values of $3\sigma_h$ and 31 values of σ_a into 5 position.
5.	Place initial conditions under reader; 43 3A0.	Checks the sum. If correct, goes into routine.
6.	At end of punch out, give a short tape advance.	Puts double space between cases.

Operation	Result
7. To change initial conditions for new cases, see section on corrections below.	
8. To use CPR, program V_2, insert correction tape number 1.	Alters CPR to accept output of GCPR.

4. Corrections to initial conditions. If no element 5 is used, correction routine number 1 may be read into element 5 positions with no other reading-in required. Give a 90 31E. With element 5, the correction routine is placed in positions which are used during the main routine. In this case, correction routine number 2 must be read in prior to each set of corrections. Give 90.

Two corrections are possible: straight corrections and decimal binary corrections. The correction routines are so arranged that the straight corrections are given first in the same manner as described in CPR section followed by a word of F's. Then the decimal to binary corrections are given, again followed by a word of F's. The routines complete the straight corrections and go automatically to the decimal-to-binary corrections. When these are finished, the main routine is entered at the proper place.

To summarize:

Correction routine number 1 (no element 5, read in once)

90 31E

With corrections under reader, 43 323.

Correction routine number 2 (with element 5, read in each time)

90 3E8

With corrections under reader, 43 3ED.

APPENDIX

Treatment of Anisotropic Scattering in Thin Shells

Roger R. Bate and Robert R. Coveyou

We assume that the angular distribution of neutrons entering the shell is

$$\left\{\frac{\Phi(I)\,\mu}{2} - \frac{3D\Phi'(I)\,\mu^2}{2}\right\} d\mu\ , \quad 0 \le \mu \le 1\ . \tag{1}$$

Then the angular distribution of neutrons penetrating to depth z without collisions is

$$\left\{\frac{\Phi(I)\,\mu}{2} - \frac{3D\Phi'(I)\,\mu^2}{2}\right\} e^{-\frac{\tau z}{\mu}} d\mu\ , \quad 0 \le \mu \le 1\ ; \tag{2}$$

the angular distribution before scattering of neutrons in slice (z, z + dz) which will be scattered in slice (z, z + dz) is

$$\left\{\frac{\Phi(I)\,\mu}{2} - \frac{\mu^2 3D\Phi'(I)}{2}\right\} e^{-\frac{\tau z}{\mu}} \frac{\sigma dz}{\mu} d\mu =$$

$$= \sum_{n=0}^{\infty}\left(\frac{2n+1}{2}\right)\left[\int_0^1 \sigma\left\{\frac{\Phi(I)}{2} - \frac{3D\Phi'(I)\,\mu'}{2}\right\} e^{-\frac{\tau z}{\mu'}} P_n(\mu')\, d\mu'\right] P_n(\mu)\, d\mu\, dz =$$

$$= \frac{1}{2}\left[\frac{\sigma\Phi(I)E_2(\tau z)}{2} - \frac{3\sigma D\Phi'(I)E_3(\tau z)}{2}\right] d\mu\, dz + \frac{3}{2}\left[\frac{\sigma\Phi(I)E_3(\tau z)}{2} - \frac{3\sigma D\Phi'(I)E_4(\tau z)}{2}\right] d\mu\, dz\, \mu + \tag{3}$$

$$+ \frac{5}{2}\left[\frac{\sigma\Phi(I)\left\{3E_4(\tau z) - E_2(\tau z)\right\}}{4} - \frac{3\sigma D\Phi'(I)\left\{3E_5(\tau z) - E_3(\tau z)\right\}}{4}\right] d\mu\left(\frac{3\mu^2 - 1}{2}\right) dz$$

$$+ \ldots$$

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The scattering kernel expanded in spherical harmonics is

$$K(\mu_0) = \sum_{n=0}^{\infty}\left(\frac{2n+1}{2}\right) K_n P_n(\mu_0) = \frac{1}{2}K_0 + \frac{3}{2}K_1\mu_0 + \frac{5}{2}K_2\left(\frac{3\mu_0^2-1}{2}\right) . \quad (4)$$

We now state without proof a theorem: If the angular distribution of neutrons before scattering is given by

$$D(\mu)\, d\mu = \sum_{n=0}^{\infty} \frac{2n+1}{2} D_n P_n(\mu)\, d\mu , \quad (5)$$

and the scattering kernel is

$$K(\mu_0) = \sum_{n=0}^{\infty}\left(\frac{2n+1}{2}\right) K_n P_n(\mu_0) , \quad (6)$$

then the resulting distribution after one scattering is

$$R(w)\, dw = \sum_{n=0}^{\infty} \frac{2n+1}{2} K_n D_n P_n(w)\, dw . \quad (7)$$

Applying this theorem to distribution in equation (3) the angular distribution of neutrons after <u>first</u> scattering in slice (z, z + dz) becomes

$$\frac{K_0}{2}\left[\frac{\sigma\Phi(I)E_2(\tau z)}{2} - \frac{3\sigma D\Phi'(I)E_3(\tau z)}{2}\right] d\mu\, dz \; +$$

$$+ \frac{3K_1}{2}\left[\frac{\sigma\Phi(I)E_3(\tau z)}{2} - \frac{3\sigma D\Phi'(I)E_4(\tau z)}{2}\right] \mu\, d\mu\, dz \; + \quad (8)$$

$$+ \frac{5K_2}{2}\left[\frac{\sigma\Phi(I)\left\{3E_4(\tau z) - E_2(\tau z)\right\}}{4} - \frac{3\sigma D\Phi'(I)\left\{3E_5(\tau z) - E_3(\tau z)\right\}}{4}\right] \frac{3\mu^2-1}{2}\, d\mu\, dz$$

$$+ \ldots$$

The angular distribution of neutrons before the second scattering in slice (w, w + dw) which have been first scattered in slice (z, z + dz) and will be scattered in slice (w, w + dw) is

$$\sigma\left[\frac{K_0}{2}\left\{\frac{\sigma\Phi(I)E_2(\tau z)}{2} - \frac{3\sigma D\Phi'(I)E_3(\tau z)}{2}\right\} + \frac{3K_1}{2}\left\{\frac{\sigma\Phi(I)E_3(\tau z)}{2} - \frac{3\sigma D\Phi'(I)E_4(\tau z)}{2}\right\}\mu + \right.$$

$$\left. + \frac{5K_2}{2}\left\{\frac{\sigma\Phi(I)\left\{3E_4(\tau z) - E_2(\tau z)\right\}}{4} - \frac{3\sigma D\Phi'(I)\left\{3E_5(\tau z) - E_3(\tau z)\right\}}{4}\right\}\left(\frac{3\mu^2 - 1}{2}\right)\right] \tag{9}$$

$$e^{-\frac{\tau(w - z)}{\mu}}\frac{dw}{\mu}\,d\mu\,dz = \qquad w > z$$

Expanding in spherical harmonics

$$= \left[\frac{\sigma^2 K_0}{4}\left\{\frac{\Phi(I)E_2(\tau z)}{2} - \frac{3D\Phi'(I)E_3(\tau z)}{2}\right\}E_1(\tau w - \tau z) + \right.$$

$$+ \frac{3\sigma\, K_1}{4}\left\{\frac{\Phi(I)E_3(\tau z)}{2} - \frac{3D\Phi'(I)E_4(\tau z)}{2}\right\}E_2(\tau w - \tau z) +$$

$$+ \frac{5\sigma^2 K_2}{4}\left\{\frac{\Phi(I)\left\{3E_3(\tau z) - E_2(\tau z)\right\}}{4} - \frac{3D\Phi'(I)\left\{3E_5(\tau z) - E_3(\tau z)\right\}}{4}\right\}\cdot$$

$$\left.\cdot\left\{\frac{3}{2}E_3(\tau w - \tau z) - \frac{1}{2}E_1(\tau w - \tau z)\right\}\right]du\,dz\,dw +$$

$$+ \left[\frac{3\sigma^2 K_0}{4}\left\{\frac{\Phi(I)E_2(\tau z)}{2} - \frac{3D\Phi'(I)E_3(\tau z)}{2}\right\}E_2(\tau w - \tau z) + \right.$$

$$+ \frac{9\sigma^2 K_1}{4}\left\{\frac{\Phi(I)E_3(\tau z)}{2} - \frac{3D\Phi'(I)E_4(\tau z)}{2}\right\}E_3(\tau w - \tau z) +$$

$$+ \frac{15\sigma^2 K_2}{4} \left\{ \frac{\Phi(I) \left\{3E_4(\tau z) - E_2(\tau z)\right\}}{4} - \frac{3D\Phi'(I) \left\{3E_5(\tau z) - E_3(\tau z)\right\}}{4} \right\} \cdot$$

$$\cdot \left\{ \frac{3}{2} E_4(\tau w - \tau z) - \frac{1}{2} E_2(\tau w - \tau z) \right\} \Bigg] \mu \, d\mu \, dz \, dw +$$

$$+ \left[\frac{5\sigma^2 K_0}{4} \left\{ \frac{\Phi(I) E_2(\tau z)}{2} - \frac{3D\Phi'(I)}{2} \right\} \left\{ \frac{3}{2} E_3(\tau w - \tau z) - \frac{1}{2} E_1(\tau w - \tau z) \right\} + \right.$$

$$+ \frac{15\sigma^2 K_1}{4} \left\{ \frac{\Phi(I) E_3(\tau z)}{2} - \frac{3D\Phi'(I)}{2} \right\} \left\{ \frac{3}{2} E_4(\tau w - \tau z) - \frac{1}{2} E_2(\tau w - \tau z) \right\} +$$

$$+ \frac{25\sigma^2 K_2}{4} \left\{ \frac{\Phi(I) \left\{3E_4(\tau z) - E_2(\tau z)\right\}}{4} - \frac{3D\Phi'(I) \left\{3E_5(\tau z) - E_3(\tau z)\right\}}{4} \right. \cdot$$

$$\cdot \left\{ \frac{9}{4} E_5(\tau w - \tau z) - \frac{3}{2} E_3(\tau w - \tau z) + \frac{1}{4} E_1(\tau w - \tau z) \right\} \Bigg] \frac{3\mu^2 - 1}{2} \, d\mu \, dz \, dw \quad .$$

Applying the theorem in equation (7), the angular distribution of neutrons which suffered the first collision in slice $(z, z + dz)$ and the second collision in slice $(w, w + dw)$ is

$$\sigma^2 \left[\Phi(I) E_2(\tau z) - 3D\Phi'(I) E_3(\tau z) \right] \left[\frac{K_0^2}{8} E_1(\tau w - \tau z) + \frac{3K_0 K_1 \mu}{8} E_2(\tau w - \tau z) + \right.$$

$$\left. + \frac{5K_0 K_2}{16} \left(\frac{3\mu^2 - 1}{2} \right) \left\{ 3E_3(\tau w - \tau z) - E_1(\tau w - \tau z) \right\} \right] d\mu \, dz \, dw + \qquad (10)$$

$$+ \sigma^2 \left[\Phi(I) E_3(\tau z) - 3D\Phi'(I) E_4(\tau z) \right] \left[\frac{3K_0 K_1}{8} E_2(\tau w - \tau z) + \frac{9K_1^2 \mu}{8} E_3(\tau w - \tau z) + \right.$$

$$\left. + \frac{15K_1 K_2}{16} \left(\frac{3\mu^2 - 1}{2} \right) \left\{ 3E_4(\tau w - \tau z) - E_2(\tau w - \tau z) \right\} \right] d\mu \, dz \, dw +$$

$$+\ \sigma^2\left[\Phi(I)\left\{3E_4(\tau z)-E_2(\tau z)\right\}-3D\Phi'(I)\left\{3E_5(\tau z)-E_3(\tau z)\right\}\right]\cdot$$

$$\cdot\left[\frac{5K_0K_2}{32}\left\{3E_3(\tau w-\tau z)-E_1(\tau w-\tau z)\right\}+\frac{15K_1K_2\mu}{32}\left\{3E_4(\tau w-\tau z)-E_2(\tau w-\tau z)\right\}+\right.$$

$$\left.+\ \frac{25K_2^2}{64}\left(\frac{3\mu^2-1}{2}\right)\left\{9E_5(\tau w-\tau z)-6E_3(\tau w-\tau z)+E_1(\tau w-\tau z)\right\}\right]d\mu\ dz\ dw\ .$$

When $w < z$ equation (10) becomes

$$\sigma^2\left[\Phi(I)E_2(\tau z)-3D\Phi'(I)E_3(\tau z)\right]\left[\frac{K_0^2}{8}E_1(\tau z-\tau w)-\frac{3K_0K_1\mu}{8}E_2(\tau z-\tau w)+\right.$$

$$\left.+\ \frac{5K_0K_2}{16}\left(\frac{3\mu^2-1}{2}\right)\left\{3E_3(\tau z-\tau w)-E_1(\tau z-\tau w)\right\}\right]d\mu\ dz\ dw\ +$$

$$+\ \sigma^2\left[\Phi(I)E_3(\tau z)-3D\Phi'(I)E_4(\tau z)\right]\left[-\frac{3K_0K_1}{8}E_2(\tau z-\tau w)+\frac{9K_1^2\mu}{8}E_3(\tau z-\tau w)-\right.$$

$$\left.-\ \frac{15K_1K_2}{16}\left(\frac{3\mu^2-1}{2}\right)\left\{3E_4(\tau z-\tau w)-E_2(\tau z-\tau w)\right\}\right]d\mu\ dz\ dw\ + \tag{11}$$

$$+\ \sigma^2\left[\Phi(I)\left\{3E_4(\tau z)-E_2(\tau z)\right\}-3D\Phi'(I)\left\{3E_5(\tau z)-E_3(\tau z)\right\}\right]\cdot$$

$$\cdot\left[\frac{5K_0K_2}{32}\left\{3E_3(\tau z-\tau w)-E_1(\tau z-\tau w)\right\}-\frac{15K_1K_2\mu}{32}\left\{3E_4(\tau z-\tau w)-E_2(\tau z-\tau w)\right\}+\right.$$

$$\left.+\ \frac{25K_2^2}{64}\left(\frac{3\mu^2-1}{2}\right)\left\{9E_5(\tau z-\tau w)-6E_3(\tau z-\tau w)+E_1(\tau z-\tau w)\right\}\right]d\mu\ dz\ dw\ .$$

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From equation (2), the uncollided forward exit current is

$$\int_0^1 \left\{ \frac{\Phi(I)\mu}{2} - \frac{3D\Phi'(I)\mu^2}{2} \right\} e^{-\frac{\tau t}{\mu}} \, d\mu = \frac{1}{2}\left[\Phi(I)E_3(T) - 3D\Phi' E_4(T) \right] , \tag{12}$$

where $T = \tau t$.

If we multiply the distribution in equation (8) by the probability that a neutron will go from z to t without collision and integrate over μ, we get the single scattered forward exit current

$$\begin{aligned}
&= \frac{\sigma K_0}{4} \int_0^t \left[\Phi(I)E_2(\tau z) - 3D\Phi'(I)E_3(\tau z) \right] \int_0^1 e^{-\frac{\tau(t - z)}{\mu}} \, d\mu \, dt \; + \\
&+ \frac{3\sigma K_1}{4} \int_0^t \left[\Phi(I)E_3(\tau z) - 3D\Phi'(I)E_4(\tau z) \right] \int_0^1 e^{-\frac{\tau(t - z)}{\mu}} \, \mu \, d\mu \, dt \; + \\
&+ \frac{5\sigma K_2}{16} \int_0^t \left[\Phi(I) \left\{ 3E_4(\tau z) - E\ (\tau z) \right\} - 3D\Phi'(I) \left\{ 3E_5(\tau z) - E_3(\tau z) \right\} \right] \int_0^1 e^{-\frac{\tau(t - z)}{\mu}} (3\mu^2 - 1) \, d\mu \, dt \; = \\
&= \frac{\sigma K_0}{4\tau} \int_0^T \left[\Phi(I)E_2(x) - 3D\Phi'(I)E_3(x) \right] E_2(T - x) \, dx \; + \\
&+ \frac{3\sigma K_1}{4\tau} \int_0^T \left[\Phi(I)E_3(x) - 3D\Phi'(I)E_4(x) \right] E_3(T - x) \, dx \; + \\
&+ \frac{5\sigma K_2}{16\tau} \int_0^T \left[\Phi(I) \left\{ 3E_4(x) - E_2(x) \right\} - 3D\Phi'(I) \left\{ 3E_5(x) - E_3(x) \right\} \right] \left[3E_4(T - x) - E_2(T - x) \right] dx \, .
\end{aligned} \tag{13}$$

Similarly, the single scattered backward exit current is

$$\frac{\sigma K_0}{4\tau}\int_0^T\left[\Phi(I)E_2(x) - 3D\Phi'(I)E_3(x)\right] E_2(x)\,dx -$$

$$- \frac{3\sigma K_1}{4\tau}\int_0^T\left[\Phi(I)E_3(x) - 3D\Phi'(I)E_4(x)\right] E_3(x)\,dx + \tag{14}$$

$$+ \frac{5\sigma K_2}{16\tau}\int_0^T\left[\Phi(I)\left\{3E_4(x) - E_2(x)\right\} - 3D\Phi'(I)\left\{3E_5(x) - E_3(x)\right\}\right]\left[3E_4(x) - E_2(x)\right] dx .$$

Similarly, from equations (10) and (11) we get the double scattered forward exit current.

$$\frac{\sigma^2}{\tau^2}\int_0^T\int_0^y\left[\Phi(I)E_2(x) - 3D\Phi'(I)E_3(x)\right]\left[\frac{K_0}{8}E_1(y - x)E_2(T - y) + \frac{3K_0K_1}{8}E_2(y - x)E\ (T - y) +\right.$$

$$\left.+ \frac{5K_0K_2}{16}\left\{\frac{3}{2}E_4(T - y) - \frac{1}{2}E_2(T - y)\right\}\left\{3E_3(y - x) - E_1(y - x)\right\}\right] dx\,dy +$$

$$\frac{\sigma^2}{\tau^2}\int_0^T\int_0^y\left[\Phi(I)E_3(x) - 3D\Phi'(I)E_4(x)\right]\left[\frac{3K_0K_1}{8}E_2(y - x)E_2(T - y) + \frac{9K_1^2}{8}E_3(y - x)E_3(T - y) +\right. \tag{15}$$

$$\left.+ \frac{15K_1K_2}{16}\left\{\frac{3}{2}E_4(T - y) - \frac{1}{2}E_2(T - y)\right\}\left\{3E_4(y - x) - E_2(y - x)\right\}\right] dx\,dy +$$

$$\frac{\sigma^2}{\tau^2}\left[\Phi(I)\left\{3E_4(x) - E_2(x)\right\} - 3D\Phi'(I)\left\{3E_5(x) - E_3(x)\right\}\right]\left[\frac{5K_0K_2}{32}\left\{3E_3(y - x) - E_1(y - x)\right\}\cdot\right.$$

$$\cdot E_2(T - y) + \frac{15K_1K_2}{32}\left\{3E_4(y - x) - E_2(y - x)\right\}E_3(T - y) +$$

$$+ \frac{25K_2^2}{64} \left\{9E_5(y - x) - 6E_3(y - x) + E_1(y - x)\right\}\left\{\frac{3}{2} E_4(T - y) - \frac{1}{2} E_2(T - y)\right\}\Bigg] dx\, dy +$$

$$+ \frac{\sigma^2}{\tau^2} \int_0^T\int_y^T \Big[\Phi(I)\, E_2(x) - 3D\Phi'(I)\, E_3(x)\Big] \Bigg[\frac{K_0^2}{8} E_1(x - y)\, E_2(T - y) - \frac{3K_0K_1}{8} E_2(x - y)\, E_3(T - y) +$$

$$+ \frac{5K_0K_2}{16} \left\{\frac{3}{2} E_4(T - y) - \frac{1}{2} E_2(T - y)\right\} \left\{3E_3(x - y) - E_1(x - y)\right\}\Bigg] dx\, dy +$$

$$+ \frac{\sigma^2}{\tau^2} \int_0^T\int_y^T \Big[\Phi(I)\, E_3(x) - 3D\Phi'(I)\, E_4(x)\Big] \Bigg[- \frac{3K_0K_1}{8} E_2(x - y)\, E_2(T - y) + \frac{9K_1^2}{8} E_3(x - y)\, E_3(T - y) -$$

$$- \frac{15K_1K_2}{16} \left\{\frac{3}{2} E_4(T - y) - \frac{1}{2} E_2(T - y)\right\}\left\{3E_4(x - y) - E_2(x - y)\right\}\Bigg] dx\, dy +$$

$$+ \frac{\sigma^2}{\tau^2} \int_0^T\int_y^T \Big[\Phi(I)\left\{3E_4(x) - E_2(x)\right\} - 3D\Phi'(I)\left\{3E_5(x) - E_3(x)\right\}\Big] \cdot$$

$$\cdot \Bigg[\frac{5K_0K_2}{32} \left\{3E_3(x - y) - E_1(x - y)\right\} E_2(T - y) - \frac{15K_1K_2}{32}\left\{3E_4(x - y) - E_2(x - y)\right\} E_3(T - y) +$$

$$+ \frac{25K_2^2}{64} \left\{\frac{3}{2} E_4(T - y) - \frac{1}{2} E_2(T - y)\right\} \left\{9E_5(x - y) - 6E_3(x - y) + E_1(x - y)\right\}\Bigg] dx\, dy \quad .$$

The formula for the backward double scattered exit current is the same as equation (15) except that the terms $E_n(T - y)$ are replaced by $(- 1)^n E_n(y)$.

If we expand the E functions in "logarithmic" power series and perform the integrations equation (12), the uncollided forward exit current becomes:

$$\frac{\Phi(I)}{4}\left\{1 - 2T + \frac{(3 - 2\gamma)\,T^2}{2} - T^2\ell nT + O(T^3)\right\} - \frac{D\Phi'(I)}{2}\left\{1 - \frac{3T}{2} + \frac{3T^2}{2} + O(T^3)\right\} \quad ; \qquad (12')$$

equation (13), the single scattered forward exit current, becomes:

$$\frac{\sigma\Phi(I)}{\tau 4}\left\{T\left(K_0 + \frac{3K_1}{4}\right) + T^2\left(\frac{2\gamma - 3}{2}K_0 - \frac{3K_1}{2}\right) + K_0T^2\ell nT + O(T^3)\right\} -$$

$$- \frac{3D\Phi'(I)\,\sigma}{4\tau}\left\{T\left(\frac{K_0}{2} + \frac{K_1}{2}\right) + T^2\left(\frac{2\gamma - 7}{8}K_0 - \frac{7K_1}{8} - \frac{5\gamma K_2}{32}\right) + T^2\ell nT\left(\frac{K_0}{4} - \frac{5K_2}{32}\right) + O(T^3)\right\} \quad ; \qquad (13')$$

equation (14), the single scattered backward exit current, becomes:

$$\frac{\Phi(I)\,\sigma}{4\tau}\left\{T\left(K_0 - \frac{3K_1}{4}\right) + T^2\left(\frac{2\gamma - 3}{2}K_0 + \frac{3K_1}{2}\right) + K_0T^2\ell nT + O(T^3)\right\} -$$

$$- \frac{3D\Phi'(I)\,\sigma}{4\tau}\left\{T\left(\frac{K_0}{2} - \frac{K_1}{2}\right) + T^2\left(\frac{2\gamma - 7}{8}K_0 + \frac{7K_1}{8} - \frac{5\gamma k_2}{32}\right) + T^2\ell nT\left(\frac{K_0}{4} - \frac{5K_2}{32}\right) + O(T^3)\right. \quad . \qquad (14')$$

Equation (15) the double-scattered forward exit current, becomes:

$$\frac{\sigma^2\Phi(I)}{16\tau^2}\left[K_0^2(3 - 2\gamma - 2\ell nT)\,T^2 + \frac{9K_1^2T^2}{4} + O(T^3)\right] -$$

$$- \frac{3D\Phi'(I)\,\sigma^2}{32\tau^2}\left[K_0^2(3 - 2\gamma - 2\ell nT)\,T^2 + 3K_1^2T^2 + \frac{5K_0K_2}{4}(\gamma T^2 + T^2\ell nT) + O(T^3)\right] \quad . \qquad (15')$$

The double-scattered backward exit current becomes:

$$\frac{\sigma^2\Phi(I)}{16\tau^2}\left[K_0^2(3 - 2\gamma - 2\ell nT)\,T^2 - \frac{9K_1^2T^2}{4} + O(T^3)\right] -$$

$$- \frac{3D\Phi'(I)\,\sigma^2}{32\tau^2}\left[K_0^2(3 - 2\gamma - 2\ln T)\,T^2 - 3K_1^2T^2 + \frac{5K_0K_2}{4}(\gamma T^2 + T^2\ln T) + O(T^3)\right] . \tag{16}$$

The sum of expressions (12'), (13'), and (15') gives the forward exit current

$$\begin{aligned}
&\frac{\Phi(I)}{4}\Bigg[1 + T\left\{- 2 + \frac{\sigma}{\tau}\left(K_0 + \frac{3K_1}{4}\right)\right\} + T^2\left\{\frac{3 - 2\gamma}{2} - \ln T + \frac{\sigma}{\tau}\left(\frac{2\gamma - 3}{2}K_0 + K_0\ln T - \frac{3K_1}{2}\right) + \right.\\
&\left. + \frac{\sigma^2}{4\tau^2}\left(K_0^2(3 - 2\gamma - 2\ln T) + \frac{9K_1^2}{4}\right)\right\} + O(T^3)\Bigg] - \frac{D\Phi'(I)}{2}\Bigg[1 + T\left\{- \frac{3}{2} + \frac{3\sigma}{4\tau}(K_0 + K_1)\right\} + \\
&+ T^2\left\{\frac{3}{2} + \frac{3\sigma}{8\tau}\left(\frac{2\gamma - 7}{2}K_0 - \frac{7K_1}{2} - \frac{5\gamma K_2}{8} + 3K_0\ln T - \frac{5K_2}{8}\ln T\right) + \frac{3\sigma^2}{16\tau^2}\left(K_0^2(3 - 2\gamma - 2\ln T) + \right.\right.\\
&\left.\left. + 3K_1^2 + \frac{5K_0K_2}{4}(\gamma + \ln T)\right)\right\}\Bigg] .
\end{aligned} \tag{17}$$

The sum of expressions (14') and (16) give the backward exit current

$$\begin{aligned}
&\frac{\Phi(I)}{4}\left[\frac{\sigma T}{\tau}\left\{K_0 - \frac{3K_1}{4}\right\} + T^2\left\{\frac{\sigma}{\tau}\left(\frac{2\gamma - 3}{2}K_0 + K_0\ln T + \frac{3K_1}{2}\right) + \frac{\sigma^2}{4\tau^2}\left(K_0^2(3 - 2\gamma - 2\ln T) - \frac{9K_1^2}{4}\right)\right\}\right] - \\
&- \frac{D\Phi'(I)}{2}\left[\frac{3\sigma T}{4\tau}(K_0 - K_1) + T^2\left\{\frac{3\sigma}{8\tau}\left(\frac{2\gamma - 7}{2}K_0 + \frac{7K_1}{2} - \frac{5\gamma K_2}{8} + K_0\ln T - \frac{5K_2}{8}\ln T\right) + \right.\right.\\
&\left.\left. + \frac{3\sigma^2}{16\tau^2}\left(K_0^2(3 - 2\gamma - 2\ln T) - 3K_1^2 + \frac{5K_0K_2}{4}(\gamma + \ln T)\right)\right\}\right] .
\end{aligned} \tag{18}$$

Hence, if we define

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$$P = 1 + T\left[-2 + \frac{\sigma}{\tau}\left(K_0 + \frac{3K_1}{4}\right)\right] + T^2\left[\frac{3 - 2\gamma}{2} - \ln T + \frac{\sigma}{\tau}\left(\frac{2\gamma - 3}{2} K_0 + K_0 \ln T - \frac{3K_1}{2}\right)\right.$$
$$\left. + \frac{\sigma^2}{4\tau^2}\left\{K_0^2(3 - 2\gamma - 2\ln T) + \frac{9K_1}{4}\right\}\right] . \tag{19}$$

$$Q = 1 + T\left[-\frac{3}{2} + \frac{3\sigma}{4\tau}(K_0 + K_1)\right] + T^2\left[\frac{3}{2} + \frac{3\sigma}{8\tau}\left\{K_0\left(\frac{2\gamma - 7}{2} + 3\ln T\right) -\right.\right.$$
$$\left.\left. - \frac{7K_1}{2} - \frac{5K_2}{8}(\gamma + \ln T)\right\} + \frac{3\sigma^2}{16\tau^2}\left\{K_0^2(3 - 2\gamma - 2\ln T) + 3K_1^2 + \frac{5K_0K_2}{4}(\gamma + \ln T)\right\}\right] . \tag{20}$$

$$R = \frac{\sigma T}{\tau}\left(K_0 - \frac{3K_1}{4}\right) + T^2\left[\frac{\sigma}{\tau}\left\{K_0\left(\frac{2\gamma - 3}{2} + \ln T\right) + \frac{3K_1}{2}\right\} +\right.$$
$$\left. + \frac{\sigma^2}{4\tau^2}\left\{K_0^2(3 - 2\gamma - 2\ln T) - \frac{9K_1^2}{4}\right\}\right] . \tag{21}$$

$$S = \frac{3\sigma T}{4\tau}(K_0 - K_1) + T^2\left[\frac{3\sigma}{8\tau}\left\{K_0\left(\frac{2\gamma - 7}{2} + \ln T\right) + \frac{7K_1}{2} - \frac{5K_2}{8}(\gamma + \ln T)\right\} +\right.$$
$$\left. + \frac{3\sigma^2}{16\tau^2}\left\{K_0^2(3 - 2\gamma - 2\ln T) - 3K_1^2 + \frac{5K_0K_2}{4}(\gamma + \ln T)\right\}\right] . \tag{22}$$

Then the forward exit current is

$$\frac{P\Phi(I)}{4} - \frac{QD\Phi'(I)}{2} , \tag{23}$$

and the backward exit current is

$$\frac{R\Phi(I)}{4} - \frac{SD\Phi'(I)}{2} , \qquad (24)$$

dropping the error terms.

It is clear that the current incident upon the exterior of the shell can be treated in exactly the same way. The above treatment was based on an infinite slab; we can approximate to a large radius spherical shell as follows, remembering that the above expressions are currents per unit area. The boundary conditions are

$$I^2 \left\{ \frac{\Phi_1(I)}{4} + \frac{D_1\Phi_1'(I)}{2} \right\} = J^2 \left\{ \frac{P\Phi_2(J)}{4} + \frac{QD_2\Phi_2'(J)}{2} \right\} + I^2 \left\{ \frac{R\Phi_1(I)}{4} - \frac{SD_1\Phi_1'(I)}{2} \right\} \qquad (25)$$

and

$$J^2 \left\{ \frac{\Phi_2(J)}{4} - \frac{D_2\Phi_2'(J)}{2} \right\} = I^2 \left\{ \frac{P\Phi_1(I)}{4} - \frac{QD_1\Phi_1'(I)}{2} \right\} + J^2 \left\{ \frac{R\Phi_2(J)}{4} + \frac{SD_2\Phi_2'(J)}{2} \right\} , \qquad (26)$$

or

$$I^2 \left[(1 - R)\, \Phi_1(I) + 2D_1(1 + S)\, \Phi_1'(I) \right] = J^2 \left[P\Phi_2(J) + 2D_2Q\Phi_2'(J) \right] \qquad (25')$$

and

$$I^2 \left[P\Phi_1(I) - 2D_1Q\Phi'(I) \right] = J^2 \left[(1 - R)\, \Phi_2(J) - 2D_2(1 + S)\, \Phi_2'(J) \right] , \qquad (26')$$

where $J = I + t$ is the outer radius of the shell.

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