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BUCKLING OF GRAPHITE MODERATED LATTICES CONTAINING SEVEN ROD FUEL CLUSTERS

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ABSTRACT

Buckling and intracell flux measurements have been made on three graphite moderated exponential assemblies each having seven rod enriched uranium fuel clusters. Theoretical values of the buckling are calculated and compared to the experimental values. In evaluating the resonance escape probability, two values are used for the lethargy spread of resonance neutrons. The larger value of 5.6, which appears in most of the literature, compares less favorably with the experimental values than does the smaller value 2.6. The theoretical buckling compares favorably with experimental values for the two larger lattice spacings and is about 10 per cent low for the smallest lattice spacing.

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I INTRODUCTION

Buckling and intracell flux measurements have recently been completed on graphite moderated exponential assemblies. A complete description of the experiment and the results will be given in another report. The present paper will cite a few of the results and outline the method used to calculate the theoretical buckling of lattices composed of 7 rod fuel clusters.

The fuel clusters used in the three lattices to be considered in this paper were made with enriched uranium similar to that which is to be used in the Sodium Reactor Experiment. The formulas used to calculate the lattice constants are the same as those used to evaluate the Sodium Reactor Experiment [1]. Consequently, the accuracy of the buckling calculation of the exponential assemblies gives and indication of the reliability of the SRE calculations.

II LATTICE DESCRIPTION

Figure 1 shows a cross section of the 7 rod fuel cluster. The individual fuel slugs are 0.750 ± .005 inch diameter cylinders of enriched uranium, the enrichment being 2.812 atomic per cent U^{235}. The density is 19.00 grams per cm^3. The fuel cylinders fit snugly into holes in an aluminum container (density = 2.64 gm/cm^3) which in turn fits snugly into holes in the graphite lattice. The dimensions shown on Figure 1 are the same as those of the SRE fuel cluster. Since the thermal neutron absorption and scattering macroscopic cross sections of aluminum are nearly the same as those of sodium, this fuel cluster should be a reasonably good representation of the actual SRE fuel cluster.

The fuel clusters are 5 feet long and are arranged in a square lattice. The nominal lattice spacings are 7, 9.5, and 12 inches. Table I gives the exact average spacing, the mean density of the graphite associated with each lattice, and the number of cells in each assembly. It is assumed that there are no voids in the structure, since the graphite density is obtained by dividing the total mass of graphite by the total volume of the assembly less the volume of the fuel clusters.

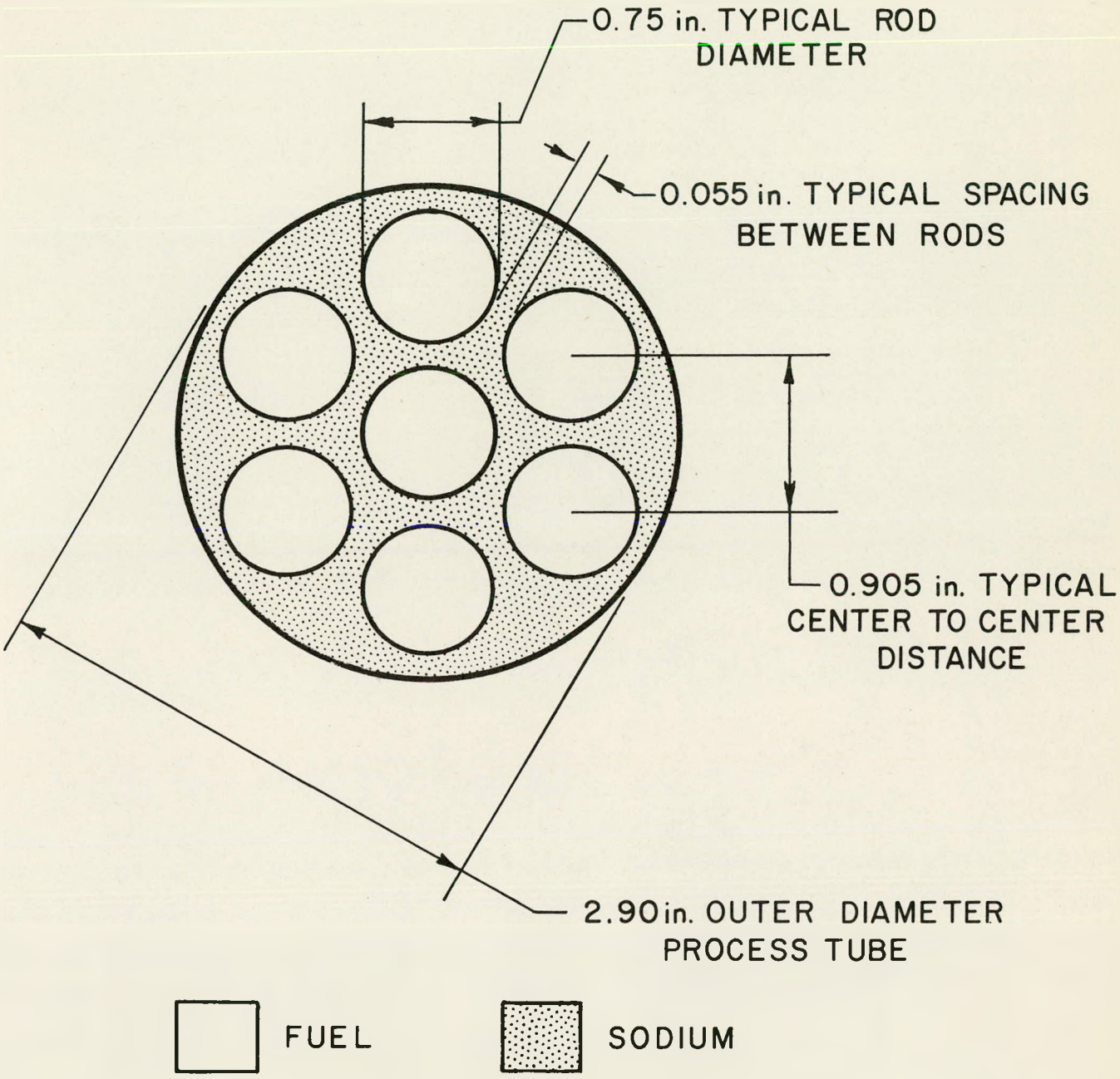

Fig. 1 Seven Rod Fuel Cluster 9693-4629

TABLE I

LATTICE GEOMETRY

Lattice Spacing (inches)	Number of cells	Graphite Density (gms/cm^3)
7.026	36	1.704
9.511	25	1.711
12.006	16	1.706

Volume fractions for each material in a cell were calculated using the dimensions listed in this section. The results are given in Table II.

TABLE II

MATERIAL VOLUME FRACTIONS

Material	7 in. lattice	9.5 in. lattice	12 in. lattice
Uranium	0.06265	0.03419	0.02145
Aluminum	0.06881	0.03755	0.02357
Graphite	0.86854	0.92826	0.95498

III NUCLEAR DATA

The values used for the various nuclear cross sections and other data are given in Table III. They are consistent with the values used for the SRE calculations in Ref. 1.

The thermal diffusion length for the graphite being used was measured and found to be 49 ± 1 cm. This is consistent with the value of 52.0 cm for graphite with a density of 1.60 gm/cm^3. The graphite absorption cross section is derived from this.

TABLE III

THERMAL NEUTRON DATA

	σ_s barns	σ_a (2200) barns
U^{238}	8.3	2.75
U^{235}	10.0	687.
Graphite	4.80	4.75×10^{-4}
2S Aluminum	1.41	0.222

$$\nu = 2.46$$

$$1 + \alpha = 1.184$$

Fission spectrum cross sections for 2.812% enriched uranium

$$\sigma_{tot} = 7.10 \text{ barns}$$

$$\sigma_f = 0.306 \text{ barns}$$

$$\sigma_{in} = 1.84 \text{ barns}$$

$$\sigma_c = 0.093 \text{ barns}$$

$$\sigma_{el} = 4.86 \text{ barns}$$

$$\nu\sigma_f = 0.816 \text{ barns}$$

Resonance Neutron Data

$$\left(\int_{E_2}^{E_1} \sigma_{res} \frac{dE}{E} \right)_{eff} = 7.6 \left[\frac{1}{F} + 3.40 \frac{S}{M} \right] \text{ barns} \qquad \ldots (1)$$

$$\lambda_u = 0.472 \text{ cm}^{-1} \text{ at } 20^\circ \text{ C for } 2.812\% \text{ enrichment}$$

$$\varkappa_m = \begin{cases} 0.1682 \text{ cm}^{-1} & \text{if } \ln u = 2.6 \\ 0.1167 \text{ cm}^{-1} & \text{if } \ln u = 5.6 \end{cases}$$

$$\sigma_s \text{ (graphite)} = 4.70 \text{ barns}$$

IV LATTICE CALCULATIONS

The formulas used in calculating the lattice constants are the same as the ones used for the SRE calculations in Ref. 1. The relevant formulas are repeated below for convenience. The symbols employed have their usual meaning unless otherwise defined.

A. CROSS SECTIONS

The flux-averaged absorption and transport cross sections over the cell are given by

$$\bar{\Sigma} = \frac{\sum_i \bar{\phi}_i V_i \Sigma_i}{\sum_i \bar{\phi}_i V_i} \qquad \ldots (2)$$

where V_i represents the volume fractions and the summation is over all materials in the cell. Thermal absorption cross sections are Maxwell averages. The transport cross section is taken to be $\sigma_{tr} = (1-\bar{\mu})\sigma_s + \sigma_a$.

B. THERMAL UTILIZATION

$$f = \frac{(\bar{\phi}_1 V_1 + \bar{\phi}_3 V_3)\Sigma_a \text{ (fuel)}}{\sum_i \bar{\phi}_i V_i \Sigma_{ai}} \qquad \ldots (3)$$

C. THERMAL DIFFUSION LENGTH

$$L^2 = \frac{1}{3\bar{\Sigma}_a \bar{\Sigma}_{tr}} \qquad \ldots (4)$$

D. RESONANCE ESCAPE

$$p = e^{-1/T} \qquad \ldots (5)$$

where

$$T = \frac{\text{neutrons removed from the resonance energy by slowing down in the moderator}}{\text{resonance neutrons absorbed in the fuel.}}$$

The usual formula for T is written as

$$T = \frac{V_m \xi \Sigma_s(\text{mod})}{V_u N_u \left(\int \sigma_{res} \frac{dE}{E} \right)_{eff}} + (E\text{-}1) \qquad \ldots (6)$$

where the subscript "m" refers to graphite and "u" refers to fuel. (E-1) is the well known excess "absorption" term, where "absorption" means removal of neutrons from the resonance energy region by the process of slowing down, and is calculated only for region 5 of the cell model shown in Fig. 2. The inverse diffusion length for resonance neutrons in the graphite is required for its evaluation and is given by

$$\mathscr{L}_m^2 = 3 \Sigma_{tr} \Sigma_{sl} \left[1 - \frac{4}{5} \frac{\Sigma_{sl}}{\Sigma_s + \Sigma_{sl}} \right] \qquad \ldots (7)$$

where the slowing down cross section Σ_{sl} is defined by

$$\Sigma_{sl} = \frac{\xi \Sigma_s}{\ln u} \qquad \ldots (8)$$

It was estimated that the logarithmic energy band for resonance neutrons is ln u = 2.6 when the 1/v tail is deleted from the U^{238} effective resonance absorption integral. We are aware of the value 5.6 which is given in the reactor handbook. However, we have not been able to justify the use of this value. Reference 2 gives the value 3.08 for this quantity, but the 1/v tail is not subtracted from the epicadminm U^{238} capture. When the 1/v tail was subtracted the value 2.6 was obtained.

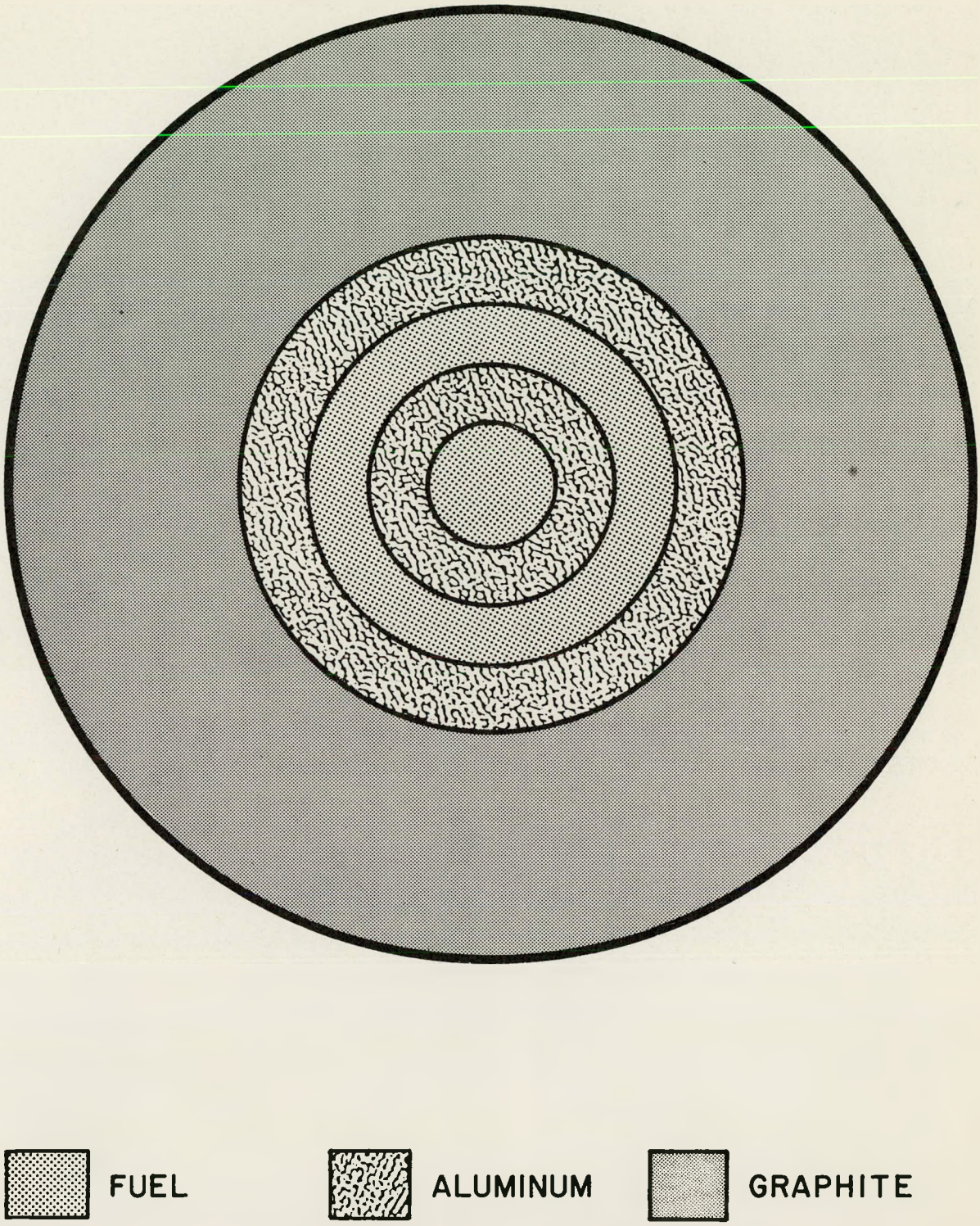

Fig. 2 Five Region Cell Model 9693-4630

The value used for the effective resonance integral in U^{238} with the 1/v absorption removed is

$$\left(\int \sigma_{res} \frac{dE}{E}\right)_{eff} = 7.6\left[\frac{1}{F}+3.40\frac{S}{M}\right] \quad \text{barns} \qquad \ldots(8)$$

The disadvantage factor "F" of the 7 fuel rods for resonance neutrons is taken to be

$$F = \frac{\mathscr{k}_u r_1 \sqrt{7}}{2} \cdot \frac{I_o(\mathscr{k}_u r_1 \sqrt{7})}{I_1(\mathscr{k}_u r_1 \sqrt{7})} \qquad \ldots(9)$$

The value of $\mathscr{k}_u$ was adjusted for changes in temperature and enrichment. The data used are given in Table III.

The "S/M" term in the effective resonance integral was calculated in the manner suggested by Dr. E. R. Cohen, i.e., by taking the surface to be that described by a taunt rubber band placed around the outside of the fuel rods. This is the proper surface to use, provided that a neutron which enters the region inside this surface cannot escape without traversing a fuel rod. This is the case for the 7 rod fuel clusters if scattering in the aluminum is neglected.

E. ETA

$$\eta = \frac{\nu \bar{\sigma}_f(25)}{\bar{\sigma}_a(25)+\frac{1}{\xi}\bar{\sigma}_a(28)} \qquad \ldots(10)$$

F. EPSILON

$$\epsilon = 1 + \frac{\left[(\nu - 1)\,\sigma_f - \sigma_c\right] P}{\sigma_t - \left[\nu\,\sigma_f + \sigma_e\right] P} \qquad \ldots(11)$$

where the cross sections are averaged over the fission spectrum and

$$\sigma_t = \sigma_c + \sigma_f + \sigma_{in} + \sigma_e$$

These cross sections are for the enriched fuel and are given by

$$\sigma_{fuel} = \frac{\mathcal{E}}{1+\mathcal{E}}\ \sigma_{25} + \frac{1}{1+\mathcal{E}}\ \sigma_{28} \qquad \ldots(12)$$

where $\mathcal{E} = N_{25}/N_{28}$

The first collision probability for a fission neutron in the fuel cluster P is obtained from the curves Ref. 3 which gives P for hollow cylinders. The hollow cylinder used here is region 3 of Fig. 2 with the inner fuel rod distributed uniformly on the inside of region 3. The variation of ϵ with temperature is negligible and was not investigated.

G. AGE

$$\tau = \left[\tau_f - (\tau_f - \tau_{in})\frac{\sigma_{in}}{\sigma_{tot}}\ P + \frac{D}{\xi\Sigma_s}\ln\frac{1.44}{5kT}\right]\left(\frac{\rho_o}{\rho}\right)^2 \frac{1}{V_m(1-V_o)} \qquad \ldots(13)$$

where

τ_f = age of fission neutrons to the indium resonance in graphite of density ρ_o.

τ_{in} = age in graphite of inelastically scattered fission neutrons to the indium resonance.

P = collision probability for fission neutrons in the fuel cluster.

ρ = graphite density

V_o = volume fraction of the void

and

$$V_m = \sum_i V_i \frac{(\xi\Sigma_s)_i}{(\xi\Sigma_s)_{graphite}} \qquad \ldots(14)$$

where the summation is over the entire cell.

$\frac{D}{\xi\Sigma_s}$ = 15.01 for epithermal neutrons in graphite of density 1.60 gm/cm^3.

The correction of the age from the indium resonance to thermal energy is made to the energy 5kT. This is in line with recent work by Dr. E. R. Cohen. Dr. Cohen has shown that the correct value, while depending slightly on the absorption of the lattice, is close to this value.

V THERMAL NEUTRON FLUX

The thermal flux distribution was measured in a central cell of the exponential assemblies.[4] The foil activities were adjusted to take into account the finite size of the lattice and then normalized to unity at the center of the fuel cluster. The average flux in each material in the cell is given in Table IV for each of the three lattices. These flux values are used to compute the average thermal absorption and transport cross sections over a cell.

TABLE IV

MATERIAL FLUX VALUES

	7 in.	9.5 in.	12 in.
Central Uranium Rod	1.20	1.22	1.24
Outer Uranium rod	1.98	2.15	2.27
Aluminum	3.20	3.57	3.93
Graphite	7.27	9.63	11.96

VI CALCULATIONS AND RESULTS

Several buckling calculations were made for each lattice spacing using the foregoing procedures. This was done in order to determine the effect of three decisions upon the calculated results. These decisions are:

1. The thermal neutron Maxwellian distribution is characterized by the moderator temperature. To examine the effect of changing the neutron temperature, calculations were made at 20° C and 100° C neutron temperature.

2. The resonance neutrons are characterized by a lethargy spread of 2.6 rather than the usual value of 5.6 which is used in volume I of the reactor handbook, and Glasstone & Edlund. To examine the effect of changing "$\ell n\, u$" calculations were made using the values 2.6 and 5.6.

3. The thermal diffusion length in the graphite used corresponds to 52.0 cm in graphite of density 1.60 gm/cm^3. To examine the effect of changing L_m, calculations were made with L_m = 52, 50, and 47 cm.

The results of the calculations are tabulated in Table V along with the measured values. The L_m shown is the value for graphite of density 1.60.

TABLE V

BUCKLING

Neutron Temperature	ℓn u	$L_{m(cm)}$	$B^2 \times 10^6$ (cm^{-2}) (Lattice Spacing)		
			7 in.	9.5 in.	12 in.
20° C	2.6	52	712	974	857
100° C	2.6	52	695	932	798
20° C	5.6	52	603	918	829
100° C	5.6	52	586	876	770
20° C	2.6	50	708	964	845
20° C	2.6	47	701	951	823
Experimental Values			781 ± 21	967 ± 17	864 ± 9

Curves of the calculated B^2 vs lattice spacing are presented in Fig. 3. The experimental points are also shown. The best fit is obtained with T_n = 20° C, ℓn u = 2.6 and L (graphite) = 52 cm. The measured value of L agrees with the value 52 cm, so this quantity should not be treated as a parameter.

VII DISCUSSION

The purpose of this analysis is to indicate that the lattice formulae which were used lead to a reasonably accurate theoretical value for the buckling of a lattice of the type studied. The assumptions that the neutron temperature is the same as the moderator temperature and that the lethargy spread for resonance neutrons is 2.6 lead to agreement with experimental values to within one per cent

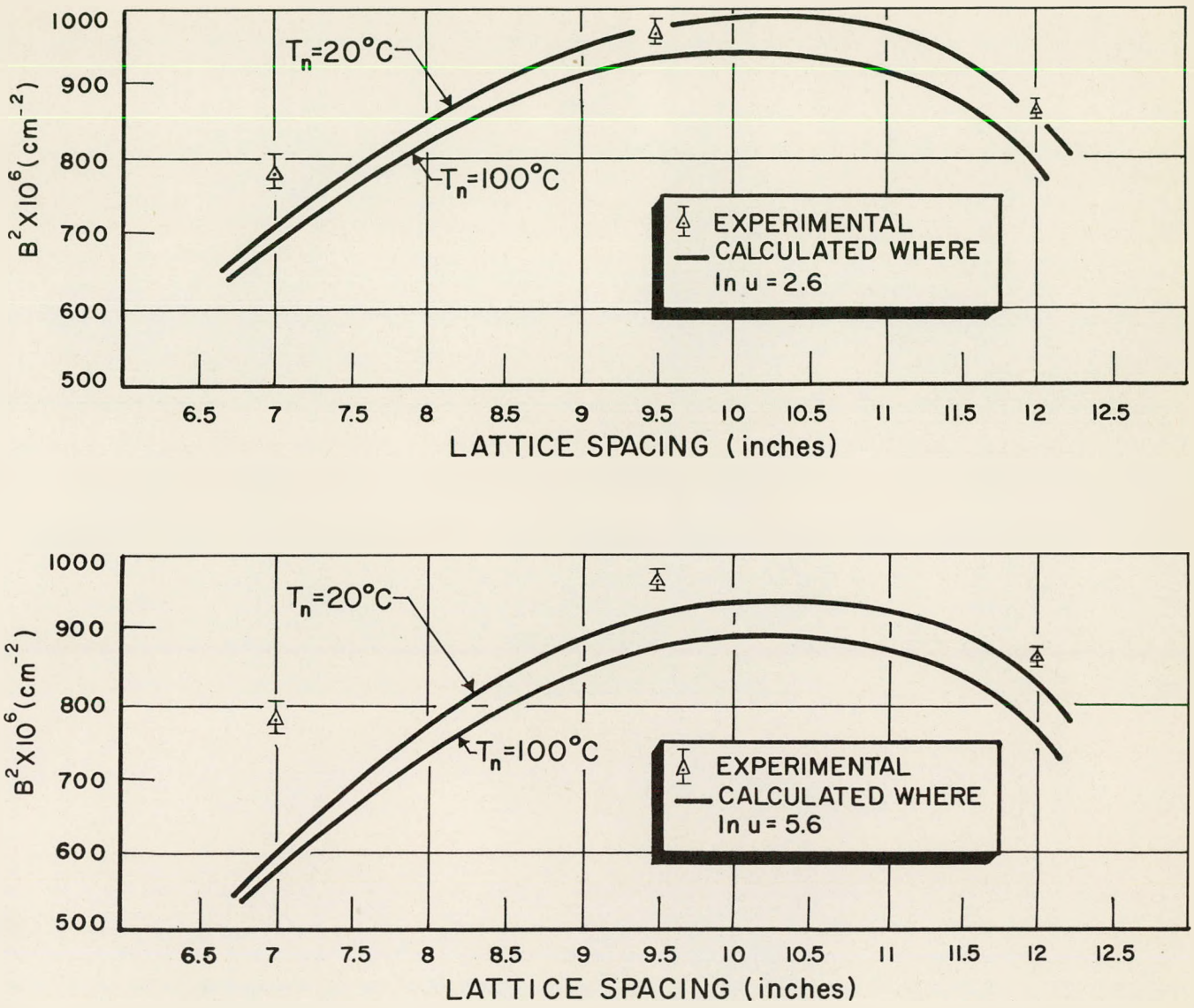

Fig. 3 Lattice Spacing vs Buckling 9693-4613

for the two larger lattice spacings and 8.8 per cent disagreement for the 7 inch lattice.

The reason for repeating the calculations using new values for the neutron temperature and resonance neutron lethargy spread was to determine the effect of reasonable changes in these quantities on the results. These changes lead to less agreement as indicated graphically in Fig. 3. The better fit obtained by using the value 2.6 for the resonance neutron lethargy spread inclines the author to favor the use of this value in calculating the resonance escape probability.

It is not claimed that all uncertainty in the evaluation of the other parameters has been eliminated. However, it is believed that all the important matters, such as deleting the 1/v tail from the effective resonance integral, taking the age to 5kT and the evaluation of the thermal diffusion length have been dealt with satisfactorily. Even though additional experiments and analysis might show that the data cannot be fitted satisfactorily by a suitable choice of neutron temperature and resonance neutron lethargy spread, we are able to calculate the buckling of a lattice to within a few per cent. This gives us confidence in the values found in Ref. 1 for the critical size of the Sodium Reactor Experiment.

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