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Scaling Laws for the Linear Theta Pinch, II: Circulating Power in a High-Field Reactor

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CIRCULATING POWER IN A HIGH-FIELD REACTOR

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ABSTRACT

The dc joule losses per unit length in a theta-pinch compression coil are calculated and compared to the thermonuclear energy production for two different methods of plasma heating. In the first method, conventional staged theta-pinch heating is assumed. In the second method, laser heating by long wavelength irradiation from the ends is assumed. Reactor parameters are calculated, and it is shown that for circulating power fractions to be 20% or less, the plasma radius must be at least a few cm in size.

I. INTRODUCTION

The high-beta CTR program at Los Alamos continues an interest in the straight, open-ended theta-pinch geometry as a possible back-up fusion reactor candidate to toroidal Scyllac. A question of considerable interest is how the linear system will scale up to a reactor. The answer turns out to be sensitive to the choice of magnetic field strength used, with higher field strengths being favored. Such a design concept will probably necessitate the use of a magnetic coil located inside the lithium breeding blanket, from strength of materials and energy storage considerations.

In a recent report,[1] Ellis and Sawyer examined this concept for two different assumptions about the method of plasma heating. In the first method, conventional (magnetic) theta-pinch heating was assumed (i.e., shock heating followed by adiabatic compression). In the second method, long wavelength laser heating was assumed (i.e., classical inverse bremsstrahlung absorption of 10.6 μm CO_2 laser radiation, incident from the ends). Using either method, the reactor length, and hence the plant cost and power output, were shown to be minimized by operating at the highest possible plasma density, and hence, for a given temperature, at the highest possible value of magnetic field.

Scaling laws were derived in Ref. 1 for reactor length, confinement time, ion density, cycle time, etc, as functions of the magnetic field strength, B_o, based upon the assumption that particle end loss is the dominant mechanism limiting plasma confinement. With the further assumption that the plasma was highly compressed (i.e., plasma radius equal to a few ion gyroradii), the thermonuclear energy produced per unit length and the laser energy required for heating were derived. These assumptions lead to values of the plasma radius of 1-2 mm, laser energies of 5-10 MJ, and power plants in the few hundred MWe range.

Such "skinny" plasmas, while theoretically satisfying the plasma physics requirements, lead to power plants which have unrealistically small thermonuclear power outputs. When joule losses in the magnet coil are taken into consideration, such a reactor will become uneconomic to operate.

In this report we extend the previous analysis of Ellis and Sawyer for the high-field, inside-coil concept to include a rudimentary energy balance study for the linear theta-pinch reactor. We calculate the important energy loss mechanism of resistive heating (joule losses) in the compression coil, and by analogy with the RTPR[2,3] (the toroidal Reference Theta Pinch Reactor design), we estimate the overall circulating power fraction for the reactor. Our calculations show that the thermonuclear energy production can be made to dominate over the losses, resulting in an acceptably low value of the circulating power fraction, only if the plasma radius is increased substantially from its theoretical minimum size. The smallest acceptable value of the plasma radius turns out to be a few cm instead of a few mm, with concomitant increases in the size of the reactor power output and the magnetic and laser energy storage requirements.

II. REVIEW OF LINEAR THETA-PINCH SCALING LAWS

In this section we will review some of the scaling results derived in Ref. 1 . These equations are needed to calculate linear theta-pinch reactor parameters. Our assumptions will be stated explicitly as we go along.

The basic coil geometry is shown in Fig. 1. We assume a sharp boundary model for the plasma column, with radius = a, $\beta = 1$, and a common electron and ion temperature of 10 keV during the thermonuclear burn, $kT_e = kT_i = kT = 10$ keV. We further make the assumption that $n\tau = 10^{15}$ cm^{-3} sec for the reactor, based upon RTPR experience.[2,3]

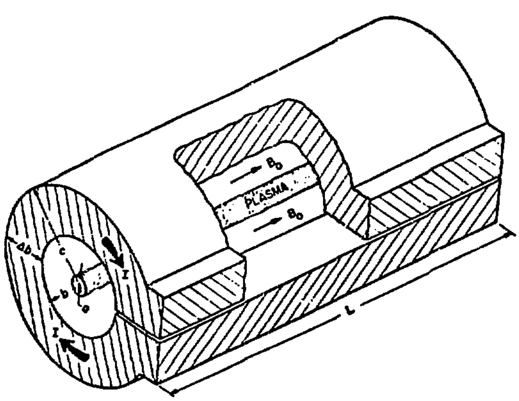

Fig. 1. Linear theta-pinch geometry.

A. Ion Density

The ion density follows directly from pressure balance, $n\,k(T_e + T_i) = \beta\, B_o^2/2\mu_o$. In convenient units,

$$n = 1.24 \times 10^{12}\ B_o^2\ (\mathrm{kG})\ \ \mathrm{cm}^{-3}. \qquad (1)$$

B. Confinement Time

The confinement time (or thermonuclear burn time) follows directly from the Lawson condition, $n\tau = 10^{15}$ cm^{-3} sec, and Eq. (1).

$$\tau = \frac{806}{B_o^2(\mathrm{kG})}\ \mathrm{sec}. \qquad (2)$$

C. Reactor Length

The reactor length can be estimated by assuming that plasma flow from the open ends at essentially the ion thermal velocity is the dominant mechanism limiting plasma confinement. Radial diffusion and axial thermal conduction are assumed small compared to particle end loss. Equating the Freidberg[4] version of the end loss time,

$$\tau_{EL} = \left(\frac{\pi}{2}\right)^{1/2}\left(\frac{m_i}{kT}\right)^{1/2}\ \frac{L}{\eta}\ ,$$

where m_i is the mass of an "average" D-T ion (4.2×10^{-27} kg), L is the total reactor length, and η is a "mirror" parameter [$\eta = \frac{1}{2R}\,(1 + \sqrt{1-\beta})$, with R = 1 for the case of no applied mirrors] to the Lawson confinement time, Eq. (2), yields

$$L = \frac{1.97 \times 10^5}{B_o^2\ (\mathrm{kG})}\ \mathrm{km}\ . \qquad (3)$$

D. Thermonuclear Energy Per Pulse

The thermonuclear energy produced per unit length of plasma in one burning pulse is given by

$$\frac{E_n}{L} = \frac{1}{4}\ \pi\ a^2\ n^2\ Q_n\ \overline{\sigma v}\ \tau\ ,$$

where Q_n is the energy release per reaction, $\overline{\sigma v}$ is the Maxwell-averaged D-T fusion cross section (equal to 1.1×10^{16} cm^3 sec^{-1} at 10 keV), n is the ion density, and τ is the burn time. If we take Q_n = 18.9 MeV/reaction* (which is somewhat pessimistic,

*From 14.1 MeV birth energy per neutron plus 4.8 MeV from the $Li^6(n,\alpha)$ T breeding reaction in the blanket.

since it ignores the 3.52 MeV energy of the trapped alpha particle), $n\tau = 10^{15}$ cm^{-3} sec, and n from Eq. (1) (the pressure balance condition), we obtain

$$\frac{E_n}{L} = 3.22 \times 10^{-5}\, a^2 B_o^2 \, , \qquad (4)$$

where E_n/L is in MJ/m for a in cm and B in kG. For a given magnetic field, E_n/L is thus directly proportional to the plasma cross-sectional area.

E. Pulse-Cycle Time

In a pulsed fusion reactor, the cycle time τ_c (the number of seconds between successive burning pulses) is an adjustable parameter which can be chosen to limit the neutron loading at the first wall, due to primary (uncollided) 14.06 MeV neutrons, to some specified value. In RTPR studies[2,3,5] this value has varied between 2 and 8 MW/m^2. Other factors being equal, τ_c should be chosen as short as possible in order to maximize the duty cycle and hence the utilization of the plant.

The time-averaged wall loading $\overline{P}_w/A$ is defined by

$$\frac{\overline{P}_w}{A} = \frac{E_n}{L} \; \frac{1}{2\pi b \; \tau_c} \; \frac{14.06 \text{ MeV}}{Q_n(\text{MeV})} \, .$$

Substituting Q_n = 18.9 MeV and E_n/L from Eq. (4) yields

$$\tau_c = 3.80 \times 10^4 \; \frac{a^2 B_o^2}{b} \left(\frac{\overline{P}_w}{A}\right)^{-1} , \qquad (5)$$

where τ_c is in seconds for a and b in cm, B_o in kG, and $\overline{P}_w/A$ in MW/m^2.

F. Plant Thermal Power Output

The thermal output power of the plant is defined in terms of the thermonuclear energy production per pulse and the cycle time τ_c:

$$P_{Th} = \frac{E_n}{L} \times L \times \frac{1}{\tau_c} \, .$$

Substituting Eqs. (3), (4), and (5) yields

$$P_{Th} = 1.67 \times 10^7 \; \frac{b}{B_o^2} \left(\frac{\overline{P}_w}{A}\right) , \qquad (6)$$

where P_{Th} is in MW for b in cm, B_o in kG, and $\overline{P}_w/A$ in MW/m^2. Note that P_{Th} is independent of any plasma properties, and for a given wall loading and magnetic field is determined solely by the coil radius b.

G. Plant Electrical Power Output

The electrical power output is related to the thermal power output by

$$P_{elec} = \varepsilon P_{Th} \, , \qquad (7)$$

where ε is an overall thermal conversion efficiency for the power plant. In the RTPR studies[2,3,5] ε turns out to be about 40%.

It can be seen from these equations that unless B_o is large, the reactor size becomes unwieldy. For example if B = 100 kG, b = 10 cm and $\overline{P}_w/A$ = 3.5 MW/m^2 (a commonly quoted value)[2] then the reactor length is 19.7 km (12.2 miles) and the thermal power is 58.5 GW_{Th}. It follows that the way to reduce the plant size, and hence cost, is to make B_o as large as possible.

The plasma ion density, length, and confinement time [Eqs. (1) - (3)] are functions of the magnetic field only. These relationships are plotted in Fig. 2.

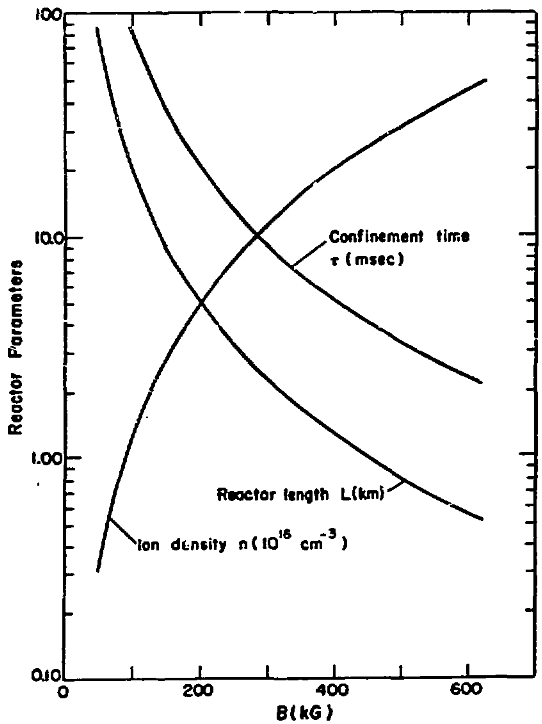

Fig. 2. Reactor length, confinement time, and ion density vs magnetic field strength ($n\tau$ = 10^{15} cm^{-3} sec, kT = 10 keV, β = 1).

III. MAXIMUM PRACTICAL MAGNETIC FIELD

The maximum magnetic field that can be used will be governed by strength of materials since the coil winding must be capable of supporting the magnetic pressure produced by the confinement field, whether dc or pulsed. A survey[1] of the literature on high magnetic field technology indicates that the largest magnetic fields have been obtained in single-turn solenoids.[6] The limits for coils that last many shots are about 600 kG for coils of 1-cm bore and 300 kG for coils of 10-cm bore. One MG is definitely out of reach with present technology.

In this report we will be considering magnetic fields in the several hundred kG range. The yield points of some possible coil materials are plotted against magnetic pressure in Fig. 3, taken from Ref. 1 . Since the coil material should also be a good electrical conductor to minimize joule losses, the best choice appears to be Be-CU, with a yield strength of approximately 150,000 psi. From the magnetic pressure relation,

$$P(\text{psi}) = 0.588\ B^2(\text{kG}) \ ,$$

this corresponds to a maximum magnetic field of slightly over 500 kG. We will limit consideration to magnetic fields of 400 kG or less in the present study. At 400 kG the magnetic pressure exerted on the coil is 94,000 psi.

IV. CONCEPTUAL COIL DESIGN

We will assume that the coil is located inside the lithium breeding blanket for strength reasons, and also to minimize the magnetic energy storage requirements. We further assume that the coil will operate hot. The resistivity of Be-Cu at 500°C is $\eta = 5 \times 10^{-6}\ \Omega$-cm.

The coil thickness Δb (see Fig. 1) will be taken as 10 cm, a compromise between neutron damage effects and strength requirements. Laminar construction (not shown in Fig. 1) will be necessary to avoid skin effects.

The coil radius b depends in a complicated way on the choice of B_o, plasma heating, etc. In Ref. 1 the coil radius b was assumed to have a value of 2 cm, independent of B_o. The "constant-b" assumption simplifies the mathematics but is inconsistent with minimization calculations. The derivation of the relationship $b = b(B_o)$, and the related

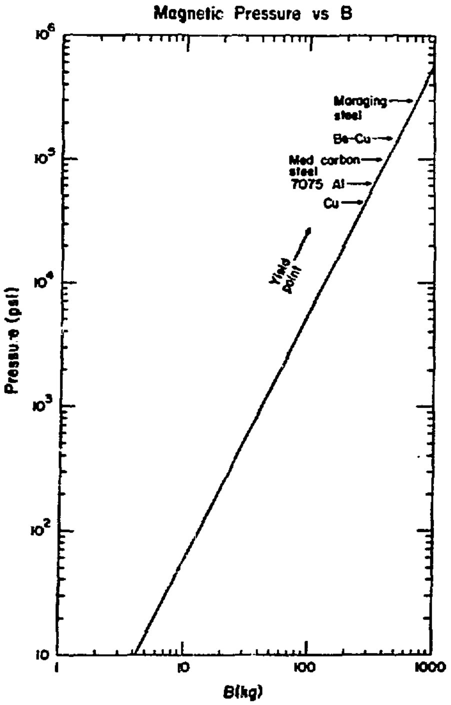

Fig. 3. Magnetic pressure vs magnetic field strength.

relationships $b = b(a)$ and $a = a(B_o)$, constitute one of the primary objectives of this report. These dependences will be derived in Sections VIII and IX.

V. JOULE LOSSES IN THE COMPRESSION COIL

Joule losses in the compression coil of a pulsed reactor are of two types. The eddy current losses arise from time variations of the magnetic field, and can be minimized by laminar construction. The dc, or transport current, losses are associated with the solenoidal field B_o, and are subject only to minor control. In the calculations which follow we will assume that eddy current losses are negligibly small compared to the transport current losses, which may be optimistic.

During one burning pulse the transport current losses per unit length of the reactor are

$$\frac{E_j}{L} = \tau \int_b^{b+\Delta b} \eta\, j^2(r)\ 2\pi r\ dr \ , \qquad (8)$$

where $j(r)$ is the current density distribution in the coil. Equation (8) is minimized when $B(r)$ reaches its steady-state solution, i.e., satisfies the time independent diffusion equation. This solution, which is derived in Appendix A, is

$$B(r) = \begin{cases} B_o & r \leq b \\ B_o \dfrac{\ell n(c/r)}{\ell n(c/b)} & b \leq r \leq c \end{cases} \quad (9)$$

where $c = b + \Delta b$ is the outer coil radius as shown in Fig. 1. Although the compression coil is shown in Fig. 1 as a one-turn solenoid for simplicity, in practice laminations will definitely be required if $B(r)$ attains the steady-state distribution given by Eq. (9) on the millisecond time scales of interest. The skin depth associated with a rise time of 1 msec and a coil resistivity of 5×10^{-6} Ω-cm is 0.7 cm.

The required distribution $j(r)$ is calculated from $\nabla \times B = \mu_o j$ in Appendix B. In mks units the result is

$$j(r) = \frac{1}{r} \frac{B_o}{\mu_o \ell n (c/b)} \quad . \quad (10)$$

By direct integration we then find

$$\frac{E_j}{L} = \frac{2\pi \eta B_o^2 \tau}{\mu_o^2 \ell n (c/b)} \quad . \quad (11)$$

Substituting from Eq. (2) for τ eliminates B_o^2.

$$\frac{E_j}{L} = \frac{3.2 \times 10^5 \eta}{\ell n (1 + \frac{\Delta b}{b})} , \quad (12)$$

where E_j/L is in MJ/m for η in Ω-cm.

Thus joule losses are only weakly geometry dependent, and in particular are independent of the number of turns. This result is true whether the multiple turns (if any) are cylindrically or helically wound, and whether or not they are laminated.

We conclude that once the coil resistivity, η, has been chosen, the joule losses per unit length of the coil are essentially fixed. Since the coil length L is inversely proportional to B_o^2, however, the total joule losses in the reactor are magnetic-field dependent.

VI. MAGNETIC ENERGY STORED IN THE COIL

The magnetic energy stored in the coil includes contributions from the uniform field in the coil bore and the radially dependent field in the coil wall.

$$E_M = L \int_0^{b + \Delta b = c} \frac{B^2(r)}{2\mu_o} 2\pi r \, dr \quad . \quad (13)$$

Substituting Eq. (9) for $B(r)$ and integrating yields

$$E_M = \frac{\pi b^2 L B_o^2}{2\mu_o} (1 + f) \quad , \quad (14)$$

where f represents magnetic energy stored in the coil wall. The function f is derived in Appendix C.

$$f(c/b) = \frac{(c/b)^2}{(\ell n\, c/b)^2} \left\{ \frac{1}{2} \left(1 - \frac{b^2}{c^2}\right) - \frac{b^2}{c^2} (\ell n \frac{c}{b}) (1 + \ell n \frac{c}{b}) \right\} . \quad (15)$$

Finally, substituting from Eq. (3) for L yields

$$E_M = 246\, b^2 (1 + f) \quad , \quad (16)$$

where E_M is in MJ for b in cm.

VII. PLANT CIRCULATING POWER FRACTION

We form the dimensionless ratio of joule losses to thermonuclear energy production from Eqs. (12) and (4):

$$\frac{E_j}{E_n} = \frac{9.94 \times 10^9 \eta(\Omega\text{-cm})}{a^2(\text{cm})\, B_o^2(\text{kG})\, \ell n \left(1 + \frac{\Delta b}{b}\right)} \quad . \quad (17)$$

The ratio E_j/E_n must be less than unity for any viable reactor. For example, in the RTPR toroidal reactor design,[2] E_j/E_n as defined here is 6.2%, compared to the total circulating power fraction of 12.8%. Circulating power fractions greater than about 20% are usually considered unacceptable in a reactor. Therefore we will assume

$E_j/E_n = 1/2\ (0.20) = 0.10$ in the following calculations, which is probably at the upper limit of acceptable values.

In order to obtain closed-form analytic solutions for a and b, we replace the logarithm term in Eq. (17) by its expansion

$$\ln(1 + \frac{\Delta b}{b}) \approx 2 \left[\frac{2b}{\Delta b} + 1\right]^{-1} \quad . \tag{18}$$

For $2b/\Delta b \geq 1$, i.e., for $b \geq 5$ cm, Eq. (18) is accurate to better than 10%. For the smallest value of b discussed in this report, b = 2.93 cm, Eq. (18) is still accurate to better than 20%. With this substitution, Eq. (17) becomes

$$\frac{E_j}{E_n} = 4.97 \times 10^9 \; \frac{\eta\left(1 + \frac{2b}{\Delta b}\right)}{a^2\; B_o^2} , \tag{19}$$

(cm, Ω-cm, kG). Note that E_j/E_n is minimized by operating at the largest possible value of B_o, again emphasizing the desirability of high-field operation.

III. REACTOR PARAMETERS IN THE CASE OF CONVENTIONAL THETA-PINCH HEATING

In conventional theta pinches, plasma heating is accomplished in stages[7] by a combination of shock (or implosion) heating followed by adiabatic compression to the ignition temperature in a rising magnetic field. Using the so called "free-expansion" implosion model (which predicts a first-stage equilibrium position at a/b = 0.76), Ribe[7] has calculated the final temperature after compression, kT_o, as a function of B_o, E_θ, and compression ratio a_o/b:

$$E_\theta\left(\frac{kV}{cm}\right) = 0.244\left(\frac{a_o}{b}\right)^{7/3} [kT_o(keV)]^{1/2}\; B_o(kG). \tag{20}$$

We denote the ignition-state quantities (a_o, n_o, kT_o, B_o) by a subscript "o" to distinguish them from the average quantities during the burn (a, n, kT, B). If we assume kT_o = 5 keV and kT = 10 keV, as predicted by computer burn codes[8] ($kT > kT_o$ because of alpha particle heating), then the average and ignition values are related approximately by $a = \sqrt{2}\, a_o$, $n = 1/2\ n_o$, $B = B_o$, and $kT = 2kT_o$. Thus Eq. (20) becomes

$$b = 0.55\ a \left[\frac{B_o(kG)}{E_\theta\left(\frac{kV}{cm}\right)}\right]^{3/7} , \tag{21}$$

(b in cm for a in cm). Substituting b from Eq. (21) into Eq. (19) yields

$$\frac{E_j}{E_n} = \frac{4.97 \times 10^9\ \eta}{a^2\ B_o^2} \left[1 + \frac{1.10a}{\Delta b}\left(\frac{B_o}{E_\theta}\right)^{3/7}\right] . \tag{22}$$

Equation (22) is quadratic in a^2, with solution

$$a = \frac{-\beta' + \sqrt{\beta'^2 - 4\alpha'\gamma'}}{2\alpha'}\ \text{cm} \tag{23}$$

where $\alpha' = B_o^2(kG)\left[\frac{E_j}{E_n}\right]$

$$\beta' = -\,5.47 \times 10^9\ \frac{\eta\ \Omega\text{-cm})}{\Delta b(cm)}\left[\frac{B_o(kG)}{E_\theta(kV/cm)}\right]^{3/7}$$

$$\gamma' = -\,4.97 \times 10^9\ \eta(\Omega\text{-cm}) \quad .$$

By inspection, a will be minimized by choosing both E_θ and B_o as large as possible. In the following examples we will fix $E_j/E_n = 0.1$, $\eta = 5 \times 10^{-6}$ Ω-cm, Δb = 10 cm, and $P_w/A = 3.5\ W/m^2$, as discussed previously. The reactor parameters are then calculated as follows. By choosing (arbitrary) values for B_o and E_θ, we calculate the plasma radius from Eq.(23). Having B_o, E_θ, and a, we can calculate the coil radius b from Eq. (21). It is then a straightforward matter to calculate the reactor length, burn time, power output, ion density, and duty cycle from Eqs. (1)-(7) of Sec. II. Finally, the magnetic energy storage requirement can be calculated from Eq. (16) of Sec. VI .

<u>Example 1:</u> B_o = 200 kG, E_θ = 2 kV/cm.

These might be reasonable choices for a linear theta-pinch reactor (E_θ has the same value as in RTPR).[5] We calculate a = 5.97 cm and b = 23.6 cm from Eqs. (23) and (21). From Eqs. (1)-(6) and (16) it follows that $n = 5 \times 10^{16}\ cm^{-3}$, τ = 20 msec, L = 4.9 km, τ_c = 6.6 sec, P_{Th} = 34.5 GW(Th), and E_M = 178 GJ. For ε = 0.4, P_{elec} = 13.8 GW(e).

<u>Example 2:</u> B_o = 400 kG, E_θ = 4 kV/cm.

This is a more extreme case. We calculate: $a = 2.00$ cm, $b = 7.92$ cm, $n = 2 \times 10^{17}$ cm^{-3}, $\tau = 5$ msec, $L = 1.2$ km, $\tau_c = 8.8$ sec, $P_{Th} = 2.9$ GW(Th), and $E_M = 29$ GJ. For $\varepsilon = 0.40$, $P_{elec} = 1.2$ GW(e).

In Example 1 the requirements on B_o and E_θ are relatively modest, and yield a large, but conceivable power plant. In Example 2, the values of B_o and E_θ are more difficult to achieve, but the plant size is much smaller. The plant would be smaller still if one could push the field strength up to 500 kG. The reactor length in this case would be only 790 meters, or about 2600 feet. Somewhere in the B_o-E_θ parameter space there will exist optimum values of the electric and magnetic fields which correspond to a "best" linear reactor design.

The plasma radius in Eq. (23) is much more sensitive to B_o than to E_θ, as the final example shows.

Example 3: $B_o = 400$ kG, $E_\theta = 2$ kV/cm.

We calculate: $a = 2.32$ cm, $b = 12.36$ cm, $n = 2 \times 10^{17}$ cm^{-3}, $\tau = 5$ msec, $L = 1.2$ km, $\tau_c = 7.56$ sec, $P_{Th} = 4.5$ GW(Th), and $E_M = 57.5$ GJ. For $\varepsilon = 0.40$, $P_{elec} = 1.8$ GW(e).

Thus reducing E_θ from 4 kV/cm to 2 kV/cm increases a by only 16%, and the power and energy values remain relatively small. The effect of reducing B_o by a factor of two (from 400 kG to 200 kG) is much more dramatic (see examples 1 and 3).

IX. REACTOR PARAMETERS IN THE CASE OF DIRECT LASER HEATING

One proposal for a fusion reactor is based upon a magnetically confined plasma column which is heated to ignition via long-wavelength (e.g., 10.6 μm) laser irradiation from the ends.[9-12] Several potential problem areas have been identified in this scheme which will require further investigation before laser heating of a long plasma column can be considered feasible. These problem areas include the mechanism of energy absorption, the beam-channeling problem , anomalous backscatter effects, plasma instabilities, etc. For the purposes of this study we will assume that these problems have been overcome, and that laser heating is successful. Then, using simple energy arguments for the reactor and the laser similar to those employed above for the conventional linear theta-pinch reactor, we will proceed to calculate the reactor parameters and energy storage requirements, including the energy requirements for the laser.

We will limit consideration here to "direct" laser heating, in which case there is no magnetic compression. The initially "cold" (of order 10 eV) plasma column is assumed to be heated to D-T ignition (∿ 5 KeV) by means of classical inverse bremsstrahlung absorption of the laser beam.

The minimum laser energy required per heating pulse, E_L, may be thought of as having three components: (1) the energy required to heat the plasma, (2) the energy which goes into work against the magnetic field during the accompanying expansion, and (3) energy which is lost due to the open ends, local overheating in the plasma, etc. These terms are derived in Appendix D, where it is shown that

$$E_L = 493\ a^2\ (\text{cm})\ \ \text{MJ}. \tag{24}$$

The laser energy is thus independent of all plasma properties except the cross-sectional area, and minimizing the plasma radius will therefore minimize the laser energy required. It will be shown below, however, that the minimum plasma radius compatible with a given circulating power fraction for the reactor is a function of B_o. Hence E_L is implicitly a function of the magnetic field also.

The coil radius b in a laser-heated reactor can be much smaller than in a conventional theta-pinch reactor because no compressional heating is involved. To avoid alpha particle collisions with the wall we require that b satisfy

$$b = a + 2r_B(\alpha)\quad , \tag{25}$$

where $r_B(\alpha) = 272/B$ (kG) i. the alpha particle gyroradius in cm at 3.52 MeV. (This condition is automatically satisfied in a conventional theta-pinch reactor). Substituting b from Eq. (25) into Eq. (19) again yields a quadratic equation in a, with solution

$$a = \frac{-\beta'' + \sqrt{\beta''^2 - 4\alpha''\gamma''}}{2\alpha''}\quad \text{cm} \tag{26}$$

where $\alpha'' = B_o^2(\text{kG}) \left[\frac{E_i}{E_n}\right]$

$$\beta'' = -\ 9.94 \times 10^9\ \eta(\Omega\text{-cm})/\Delta b(\text{cm})$$

$$\gamma'' = -\left[4.97 \times 10^9 + \frac{5.43 \times 10^{12}}{B(kG)\ \Delta b(cm)}\right]\eta(\Omega\text{-cm}).$$

By inspection a will be minimized, and hence E_L will be minimized, by choosing B_o as large as possible. As in the theta-pinch case, Sec. VIII, we will assume $E_j/E_n = 0.1$, $\eta = 5 \times 10^{-6}$ Ω-cm, Δb = 10 cm, and $\overline{P}_w/A = 3.5$ MW/m^2 as reasonable choices for a high-field reactor. We then consider operation at two values of the magnetic field, B_o = 200 and B_o = 400 kG.

Example 1: B = 200 kG.

In this case we calculate a = 3.78 cm, b = 5.50 cm, n = 5 x 10^{16} cm^{-3}, τ = 20 msec, L = 4.9 km, τ_c = 9.6 sec, P_{Th} = 9.5 GW(Th), E_M = 21.4 GJ, and E_L = 7.0 GJ. For ε = 0.4, P_{elec} = 3.8 GW(e).

From Appendix D, we find L/ℓ_{ab} = 1.9, where ℓ_{ab} is the laser absorption length for 10.6 μ radiation. In this case, where the reactor length is less than two absorption lengths, some laser energy may be lost from the open ends of the pinch. This point is discussed in Appendix D.

Example 2: B = 400 kG.

As in the case of the conventional theta pinch, 400 kG represents a rather extreme value of magnetic field. This case probably yields the minimum practical plasma radius, and hence the minimum laser energy requirements. We calculate a = 1.57 cm, b = 2.93 cm, n = 2 x 10^{17} cm^{-3}, τ = 5 msec, L = 1.2 km, τ_c = 14.6 sec, P_{Th} = 1.07 GW(Th), E_M = 7.4 GJ, and E_L = 1.2 GJ. For ε = 0.4, P_{elec} = 0.43 GW(e).

For B = 400 kG, we calculate ℓ_{ab} = 162 meters and hence L/ℓ_{ab} = 7.7. This value is large enough to imply essentially total absorption of the laser energy by the plasma.

In this problem, since E_θ is not involved, B_o is conveniently the only free parameter. The various reactor parameters can thus be plotted as functions of the magnetic field strength alone. The results are shown in Figs. 4 and 5.

X. DISCUSSION AND CONCLUSIONS

In order to keep the length and power output of a straight reactor to reasonable proportions, it appears that operation at high magnetic fields, in the range of 200 to 400 kG, is desirable. At such high field strengths, it is probable that the compression coil will be located inside the lithium

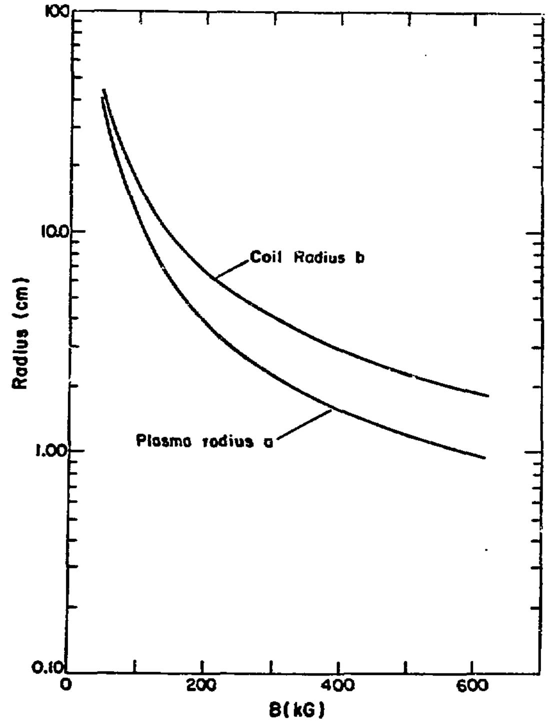

Fig. 4. Plasma radius and coil radius vs magnetic field strength for laser-heated reactor.

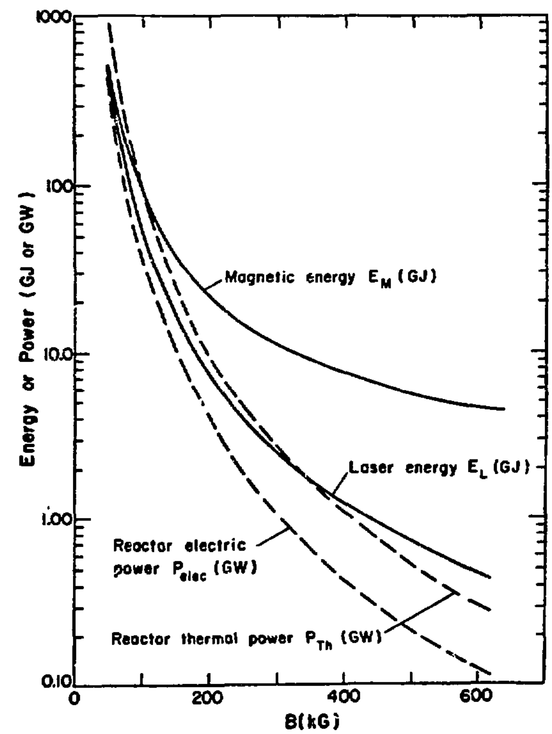

Fig. 5. Energy requirements and lower output vs magnetic field strength for laser-heated reactor.

blanket, as assumed in the present paper. If an outside coil were assumed instead, the minimum value of the coil radius b would increase by perhaps half a meter from the values calculated in this report. This would be a difficult proposition from the standpoint of both strength of materials and magnetic energy storage requirements. An inside coil, on the other hand, is subject to heating and structural damage from the intense bremsstrahlung, neutron, and gamma-ray fluxes it encounters, and a careful study will be required to see if it can survive in such an environment.

In this report we have estimated the joule losses in a theta-pinch coil by assuming that certain of the coil parameters - namely the resistivity and wall thickness - are essentially fixed parameters. Certainly the values used here ($\eta = 5 \times 10^{-6}$ Ω-cm and $\Delta b = 10$ cm) cannot be lowered appreciably without affecting credibility of the reactor. The resistivity of pure copper, for example, is $\sim 1.7 \times 10^{-6}$ Ω-cm at room temperature, a very unlikely operating point for an inside coil.

It is possible that a superconducting or cryogenic magnet coil could overcome the joule-loss problem, but such a magnet would introduce new problems of its own. It would have to be located outside of the lithium blanket because of the neutron flux, and this leads to difficulties involving both the large magnetic field strength and the large magnetic energy storage requirements, if the magnet is operated pulsed. If steady-state operation is assumed instead, the lithium pumping losses and other losses associated with the scheme will lead to a minimum-size requirement on the plasma radius, just as do the conventional joule losses treated here.

Many of the energy losses associated with a fusion reactor have been neglected in the above analysis, such as eddy current losses, pumping losses, etc. On the other hand some sources of recoverable energy have also been neglected; direct conversion from the alpha particle heating and expansion, the recoverable fraction of joule losses in the hot coil, etc. These items have all been lumped together in our basic assumption of a 20% circulating power fraction, based upon energy flow calculations for the RTPR.[2,3,5]

Looking beyond the coil problem, a linear high-field reactor is not without attractions. It is basically a simple design, lends itself to modular construction, and provides ready access from the ends. The reactor thermal output power and magnetic energy storage requirements are modest when compared to some other fusion reactor designs, at least in the 400 kG cases.

Of the two plasma heating methods compared in this study, magnetic heating seems by far the more desirable. It involves generally familiar technology and, on the laboratory scale at least, has been very successful in producing thermonuclear plasmas.

The direct-laser-heated reactor concept appears, by comparison, to be very difficult to implement. Even if all physics problems associated with the method are successfully solved, the laser energy requirements - in the several GJ/pulse range - are enormous by present day standards. Since heating has always been one of the strong points of the linear theta pinch, the role of laser heating in a linear reactor program should probably be one of supplementing magnetic heating. One could imagine, for example, that shock heating might prove more difficult to implement in a reactor than in an experimental environment, due to insulator or other problems. In this case it would be very useful to have a backup candidate for first-stage heating which could eliminate or reduce the need for large applied electric fields. If the laser were used to heat the plasma to 100 eV instead of ~ 5 KeV, the energy savings for the laser would amount to a factor of 20, thereby making the laser problem much more tractable. (We note that laser-staging is possible only in a linear geometry.)

We conclude that plasma radii in the several cm range, which are necessary for economical reactor operation, are compatible with theta-pinch physics and technology, and result in reasonable reactor parameters (length, power output, cycle time, etc). Laser heating is a possible backup candidate for the shock-heating stage. The primary difficulty anticipated is with the compression coil design, and a high-field coil development program is scheduled to be undertaken at LASL in the near future to help define the problem areas.

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APPENDIX A

SOLUTION OF THE TIME-DEPENDENT DIFFUSION EQUATION FOR THE SOLENOIDAL FIELD B(r)

In mks units Maxwell's equations are

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} \qquad \text{(A-1)}$$

$$\nabla \times \overline{B} = \mu_o\left(\overline{j} + \frac{\partial \overline{D}}{\partial t}\right) \qquad \text{(A-2)}$$

$$\nabla \cdot \overline{B} = 0 \qquad \text{(A-3)}$$

$$\nabla \cdot \overline{E} = \frac{\rho}{\varepsilon_o} \qquad \text{(A-4)}$$

where all symbols have their usual meanings. Substituting $\overline{E} = \eta\overline{j}$ into Eq. (A-1), where η is the coil resistivity, and setting $\frac{\partial \overline{B}}{\partial t} = 0$ yields

$$\nabla \times (\eta\overline{j}) = 0 \quad . \qquad \text{(A-5)}$$

Setting $\frac{\partial D}{\partial t} = 0$ in Eq. (A-2), and substituting

$$\overline{j} = \frac{1}{\mu_o} \nabla \times \overline{B} \qquad \text{(A-6)}$$

into Eq. (A-5) yields

$$\left(\frac{\eta}{\mu_o}\right) \nabla \times (\nabla \times \overline{B}) = 0 \quad , \qquad \text{(A-7)}$$

which is the time-independent diffusion equation for $\overline{B}$. Using the vector identity

$$\nabla \times (\nabla \times \overline{B}) = \nabla(\nabla \cdot \overline{B}) - (\nabla \cdot \nabla)\,\overline{B} \quad , \qquad \text{(A-8)}$$

and $\nabla \cdot B = 0$ from Eq. (A-3), gives

$$(\nabla \cdot \nabla)\,\overline{B} = 0 \quad . \qquad \text{(A-9)}$$

The only component of $\overline{B}$ in the present problem is axial; hence $\overline{B} = \hat{z}B$. In cylindrical coordinates Eq. (A-9) becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial B}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 B}{\partial \theta^2} + \frac{\partial^2 B}{\partial z^2} = 0. \qquad \text{(A-10)}$$

Allowing only radial variations in B, Eq. (A-10) can be written as an ordinary differential equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dB}{dr}\right) = 0 \qquad \text{(A-11)}$$

with a first integral:

$$r\frac{dB}{dr} = k_1 \quad . \qquad \text{(A-12)}$$

Here k_1 is an arbitrary constant to be determined by the boundary conditions on B. Integrating again,

$$B(r) = k_1 \ln r + k_2 \quad . \qquad \text{(A-13)}$$

The boundary conditions are that $B(r) = B_o$ at $r = b$ (inside coil surface), and $B(r) = 0$ at $r = c$ (outside coil surface) (refer to Fig. 1). Thus

$$k_1 = \frac{B_o}{\ln(b/c)} \qquad \text{(A-14)}$$

and

$$k_2 = -B_o\frac{\ln c}{\ln(b/c)} \quad . \qquad \text{(A-15)}$$

Substituting Eqs. (A-14) and (A-15) into Eq. (A-13) gives the required solution for B(r).

$$B(r) = B_o\frac{\ln(c/r)}{\ln(c/b)}\,, \quad b \le r \le c. \qquad \text{(A-16)}$$

APPENDIX B

STEADY-STATE CURRENT DENSITY DISTRIBUTION IN A STRAIGHT THETA-PINCH COIL

From the steady-state form of Ampere's Law [Eq. (A-6), Appendix A],

$$\bar{j} = \frac{1}{\mu_o} \nabla \times \bar{B} \quad . \tag{B-1}$$

We find the current density [in the azimuthal (θ) direction] corresponding to the axial (z) magnetic field B(r):

$$j(r) = \frac{1}{\mu_o} \frac{dB(r)}{dr} \quad . \tag{B-2}$$

Substituting Eq. (A-16) for B(r) yields

$$j(r) = \frac{1}{\mu_o} \frac{d}{dr} \left[B_o \frac{\ln(c/r)}{\ln(c/b)} \right] \tag{B-3}$$

$$= -\frac{1}{\mu_o r} \frac{B_o}{\ln(c/b)} \quad , \tag{B-4}$$

which is the desired result.

APPENDIX C

MAGNETIC ENERGY STORED IN COMPRESSION COIL

The stored magnetic energy includes a contribution from the coil wall as well as the bore.

$$E_M = L \int_0^{b + \Delta b = c} \frac{B^2(r)}{2\mu_o} 2\pi r \; dr. \tag{C-1}$$

Breaking E_M into two integrals yields

$$\frac{E_M}{L} = \int_0^b \frac{B_o^2}{2\mu_o} 2\pi r \; dr + \int_b^c \frac{B^2(r)}{2\mu_o} 2\pi r \; dr \; , \tag{C-2}$$

where

$$B(r) = B_o \frac{\ln(c/r)}{\ln(c/b)} \tag{C-3}$$

is our previous result from Appendix A[Eq. (A-16)].

The first integral yields

$$\left(\frac{E_M}{L}\right)_1 = \frac{\pi b^2}{2\mu_o} B_o^2 \quad . \tag{C-4}$$

The second integral can be written in the form

$$\left(\frac{E_M}{L}\right)_2 = \frac{\pi b^2}{2\mu_o} B_o^2 \left\{ \frac{2\left(\frac{c}{b}\right)^2}{\left(\ln \frac{c}{b}\right)^2} \int_b^c \left(\ln \frac{r}{c}\right)^2 \frac{r}{c} \frac{dr}{c} \right\} . \tag{C-5}$$

Making a change of variable to X = r/c in the integral leads to a form given in Dwight's <u>Tables of Integrals</u> (4th Ed., example 612.1):

$$\int_{b/c}^1 (\ln X)^2 \, X \, dX = \left[\frac{X^2}{2} (\ln X)^2 - \frac{X^2}{2} \ln X + \frac{X^2}{4} \right]_{b/c}^1 . \tag{C-6}$$

Evaluating Eq. (C-6) at the indicated integration limits and substituting with Eq. (C-4) into Eq. (C-2) yields

$$\frac{E_M}{L} = \frac{\pi b^2 B_o^2}{2\mu_o} \left\{ 1 + \frac{(c/b)^2}{(\ln c/b)^2} \left[\frac{1}{2} \left(1 - \frac{b^2}{c^2}\right) + \frac{b^2}{c^2} \left(\ln \frac{b}{c}\right) \left(1 - \ln \frac{b}{c}\right) \right] \right\} . \tag{C-7}$$

The first term in Eq. (C-7) is associated with the uniform field region in the coil bore, while the second is associated with magnetic field distributed logarithmically with radius in the coil wall. If we identify the nonuniform B contribution to the energy integral by f, then

$$E_M = \frac{\pi b^2 L B_o^2}{2\mu_o} (1 + f) \quad , \tag{C-8}$$

which is in the desired form.

APPENDIX D

LASER ENERGY REQUIREMENTS

The calculation of the minimum laser energy, E_L, required per pulse can conveniently be divided into three parts: (1) the energy E_{L1} required to raise the plasma temperature from its starting value to the ignition temperature of 5 keV; (2) the energy E_{L2} expended in doing work against the magnetic field; and (3) the energy E_{L3} wasted in unavoidable losses such as excess local heating above 5 keV, losses from the open ends of the device, etc. The total laser energy required is

$$E_L = E_{L1} + E_{L2} + E_{L3} . \quad \text{(D-1)}$$

We will now derive these terms.

1. Thermal Energy E_{L1}. The thermal energy density associated with a Maxwellian plasma is $\frac{3}{2}$ $nk(T_e + T_i)$. From the pressure balance condition,

$$nk(T_e + T_i) = \beta \frac{B^2}{2\mu_o} , \quad \text{(D-2)}$$

the thermal energy content of the plasma column is thus

$$E_p = \left(\frac{3}{2}\right) \times \left(\frac{\beta B^2}{2\mu_o}\right) \times \left(\pi a^2 L\right) , \quad \text{(D-3)}$$

where a is the column radius and L is the length. Ellis and Sawyer[1] have previously shown that β is essentially unity at the ignition point after laser heating, irrespective of the assumed starting conditions. Substituting L from Eq. (3) (in Sec. II) thus yields

$$E_p = 370\ a^2(\text{cm}) \quad \text{MJ}, \quad \text{(D-4)}$$

i.e., E_p depends only on the plasma radius.

The calculation of L in Eq. (3) is for kT = 10 keV. Hence the laser energy E_{L1} required to raise the plasma to 5 keV is 1/2 E_p, or

$$E_{L1} = 185\ a^2(\text{cm}) \quad \text{MJ}. \quad \text{(D-5)}$$

[If desired, we could express E_{L1} in terms of the ignition radius a_o instead of the average radius a. Since $a = \sqrt{2}\ a_o$ (see Sec. VIII), the result is $E_{L1} = 370\ a_o^2(\text{cm})$ MJ].

2. Expansion Energy E_{L2}. When the plasma is heated it expands, doing work E_{L2} against the constant magnetic field B_o. The calculation of E_{L2} is simplified if we assume β = constant = unity during the expansion. In this case the plasma behaves like an ideal gas. From the first law of thermodynamics, $\Delta Q = \Delta E + \Delta W$, we have

$$\frac{\Delta E}{\Delta Q} + \frac{\Delta W}{\Delta Q} = 1 , \quad \text{(D-6)}$$

where ΔE is the energy which goes into internal energy (i.e., heating):

$$\Delta E = C_v\ \Delta T \equiv E_{L1} , \quad \text{(D-7)}$$

and ΔW is the energy which goes into work .

$$\Delta W = E_{L2} . \quad \text{(D-8)}$$

The quantity ΔQ is the total energy supplied by the laser for heating and expansion; for our case of expansion at constant pressure,

$$\Delta Q = E_{L1} + E_{L2} = C_p\ \Delta T . \quad \text{(D-9)}$$

(Here C_v and C_p are the specific heats at constant volume and constant pressure, respectively).

Substituting Eqs. (D-7) and (D-9) into $\Delta E/\Delta Q$ yields

$$\frac{\Delta E}{\Delta Q} = \frac{C_v}{C_p} = \frac{1}{\gamma} = \frac{3}{5} , \quad \text{(D-10)}$$

for a monatomic gas. Thus 60% of the laser input energy goes into heating and 40% into work. Combining Eqs. (D-6)-(D-10) yields $E_{L2} = 2/3\ E_{L1}$.

In practice beta will be an increasing function of temperature, becoming approximately unity at temperatures of a few kilovolts (Ellis and Sawyer, Ref. 1). The effect of variable β during the heating process is to reduce the amount of laser energy which goes into work. This reduction is estimated to be about 50%, in which case we obtain

$$E_{L2} = \frac{1}{3} E_{L1} \quad . \tag{D-11}$$

3. Energy Losses E_{L3}. In addition to the energy requirements E_{L1} and E_{L2}, the laser must furnish any additional energy lost from the open ends of the pinch or absorbed in overheating at the ends. In order that the central regions of the long column reach 5 keV, some overheating at the input ends will occur.

The absorption length for 10.6 μm CO_2 laser radiation has been derived in Ref. 1 , based on recent calculations by Johnston and Dawson.[13]

$$\ell_{ab} = \frac{2.12 \times 10^8 \,(kT)^{7/2}}{\ell n[1.54 \times 10^5 \,(kT)^{3/2}]} \frac{1}{B^4} \quad , \tag{D-12}$$

where ℓ_{ab} is in km for kT in keV and B in kG. Using Eq. (3) for the reactor length L and evaluating Eq. (D-12) at kT = 5 keV yields

$$\frac{\ell_{ab}}{L} = \frac{2.09 \times 10^4}{B_o^2 \text{ (kG)}} \quad . \tag{D-13}$$

Eq. (D-13) shows that $L \geq \ell_{ab}$ for $B_o \geq 145$ kG, which essentially defines the operating regime for a laser-heated reactor. Otherwise the reactor would have to be placed in a resonant cavity of several km length to avoid excessive light losses.

If $L \gg \ell_{ab}$, overheating losses will be a maximum but radiation lost out the ends will be a minimum, and vice versa for $L \ll \ell_{ab}$. These effects thus tend to be mutually exclusive, and their cumulative effect tends to be constant. Ellis and Sawyer[1] have previously estimated the overheating effect in the 400 kG case ($L = 7.4\, \ell_{ab}$) as roughly a factor of two in energy penalty. If we take this as a general result and neglect any side-scattering losses from the beam we obtain

$$E_{L3} = E_{L1} + E_{L2} \quad . \tag{D-14}$$

Using Eq. (D-11) this becomes

$$E_{L3} = \frac{4}{3} E_{L1} \quad . \tag{D-15}$$

We now combine terms to obtain the overall energy E_L.

$$E_L = E_{L1} + E_{L2} + E_{L3} \tag{D-16}$$

$$= \frac{8}{3} E_{L1} \quad .$$

Substituting E_{L1} from Eq. (D-5) yields

$$E_L = 493\, a^2 \text{(cm)} \quad \text{MJ}, \tag{D-17}$$

which is the desired result.

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