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CURRENT QUARKS, CONSTITUENT QUARKS, AND SYMMETRIES OF RESONANCE DECAYS

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A B S T R A C T

The transformation between "current" quarks and "constituent" quarks recently suggested by Melosh is examined with respect to its predictions for pionic decays of resonances. It implies the use of $SU(6)_W$ for classifying particle states but not for describing decay processes. Instead, pion emission proceeds via $\Delta L_z = 0, \pm 1$, where L is the internal ("quark") orbital angular momentum. This decay symmetry is called $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$. It is proven equivalent for any decay $A \rightarrow B + \pi$ (where A, B are arbitrary $q\bar{q}$ or qqq hadrons) to the $^3P_0$ quark-pair creation model for such decays, as formulated by Micu, Colglazier, Petersen and Rosner. The roles of final orbital angular momenta ℓ and of $SU(3) \times SU(3)$ subgroups of $SU(6)_W$ are also discussed, and some new predictions are made for decays of meson resonances below 1700 MeV.

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DISCLAIMER

INTRODUCTION

The quark model has met with remarkable success in hadronic physics, both as a "constituent" model for hadrons [1] and as a model for their weak and electromagnetic currents [2),3)].

The two quark pictures are associated with two different SU(6) algebras of operators. The algebra $SU(6)_{W,strong}$ introduced by Lipkin and Meshkov [4] seems appropriate for the classification of the lowest lying hadrons: they appear to fall into simple irreducible representations of this symmetry. Thus, this "constituent" picture places the observed mesons in $\underset{\sim}{1}$ and $\underset{\sim}{35}$ dimensional representations and the observed baryons in $\underset{\sim}{56}$ and $\underset{\sim}{70}$ dimensional representations of $SU(6)_{W,strong}$.

A different SU(6) algebra introduced by Dashen and Gell-Mann [5], which one may call $SU(6)_{W,currents}$, is obeyed by the so-called "good" charges in the infinite momentum limit [6]. These charges are the integrals over local densities which are directly or indirectly measurable in weak and electromagnetic processes. Moreover, since the isovector axial charge is included in this algebra, and pionic decays of resonances $A \rightarrow B + \pi$ are related by PCAC to the matrix element of this axial charge, one expects that $SU(6)_{W,currents}$ may provide a description of some purely hadronic processes as well.

Conceptually, constituent quarks and current quarks, and hence their respective $SU(6)_W$ algebras, are not necessarily identical. The success of CVC means that the SU(3) subalgebras are identical, but an attempt to identify particle states with pure representations of $SU(6)_{W,currents}$ leads to a number of difficulties. Notable among these are the prediction that Adler-Weisberger relations [7] should be well-saturated with the lowest $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryon resonances (which they are not [7),8)]), and that anomalous magnetic moments of baryons should vanish (which they do not) [5]. Moreover, although a proton is well-classified as a state of three "constituent" quarks, it looks considerably more complicated when probed with currents. In such a description (based on deep inelastic lepton scattering [9]) the proton seems to contain additional quark-antiquark pairs and perhaps some neutral "glue" as well [3]. Hence, although physically observable transitions are best described in the language of $SU(6)_{W,currents}$, the fact that the states themselves are complicated mixtures of representations of this group [10] has been a substantial obstacle up to now to concrete applications of this symmetry.

Recently Melosh [11] has constructed the transformation between the two $SU(6)_W$ algebras in the free quark model. He finds that this transformation V, acting on the axial charge F^{i5}, produces a transformed axial charge $\tilde{F}^{i5} \equiv V^{-1}F^{i5}V$ with relatively simple properties in the constituent quark picture.

Abstracting from this "free quark" property, several authors [12),13)] have assumed that a similarly simple $\tilde{F}^{i5}$ exists in the interacting theory as well. The matrix elements of $\tilde{F}^{i5}$ between pure states of $SU(6)_{W,strong}$ (corresponding to our idealization of the physical particles) are then relatively straightforward to compute, and imply a number of relations among pionic decays of resonances.

In this paper we examine the general structure of such predictions. We find that the $SU(6)_{W,strong}$ introduced in Ref. 4), while perhaps appropriate for classifying particle states, does not follow for collinear processes such as resonance decays. This "wrong" symmetry will be referred to as "naïve" $SU(6)_W$. Instead, in the present scheme, some relatively simple selection rules govern pionic transitions between states classified according to $SU(6)_{W,strong}$. In terms of an abstractly defined internal orbital angular momentum L_z, these transitions turn out to involve only $\Delta L_z = 0, \pm 1$. This "possibly right" decay symmetry will thus be referred to as $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$.

We find that for any hadronic decay $A \rightarrow B + \pi$, the symmetry $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$, motivated by the work of Ref. 11), is equivalent to the successful phenomenological "$^3P_0$" model of Micu [14)], Colglazier and Rosner [15)] and Petersen and Rosner [16)]. The phenomenological prescription [14)-19)] known as "ℓ broken $SU(6)_W$" [16),17)] is found to be equivalent to $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ for many cases of physical interest, but a special case of this symmetry (and hence essentially unmotivated on theoretical grounds) in general. We have also derived the predictions of the chiral $SU(3) \times SU(3)_{strong}$ subalgebra of $SU(6)_{W,strong}$, and discuss their relation to other symmetries in some specific mesonic cases. Various phenomenological applications to decays of the mesons in the "R" region (mass around 1700 MeV) are mentioned.

The paper is organized as follows. Sections 1 and 2 are devoted, respectively, to the constituent and current quark approaches to $SU(6)_W$ as applied to hadronic vertices. These sections, as well as the beginning of Section 3 which deals with the explicit expressions for decay amplitudes, are expository in nature and follow closely the treatments of Refs. 11)-13). Then, in the remainder of Section 3, we discuss in some detail all possible pionic decays of meson resonances up to $L=2$. This Section includes predictions of interest to experimentalists. Section 4 treats the equivalences and differences among the various above-mentioned pictures of hadronic decays. Finally, in Section 5, we summarize our results and indicate where further work is needed.

1. CONSTITUENT QUARKS

Particle states at rest appear to be well described by the quark model with orbital excitations. By taking all mesons as quark-antiquark pairs and all baryons as three quarks one can then understand a number of features of the hadron spectrum.

The symmetry associated with the above classification is $U(6)\times U(6)\times O(3)$. The first $U(6)$ refers to quarks and the second to antiquarks, while the $O(3)$ describes the orbital angular momentum L. A number of multiplets of this group are either entirely filled or well on their way to being so. There is strong evidence for mesonic states belonging to $(\underset{\sim}{6},\bar{\underset{\sim}{6}})$, $L^P=0^-,1^+$ and 2^- and for the baryonic multiplets $[(\underset{\sim}{56},\underset{\sim}{1}),\ L^P=0^+]$, $[(\underset{\sim}{70},\underset{\sim}{1}),\ L^P=1^-]$ and $[(\underset{\sim}{56},\underset{\sim}{1}),\ L^P=2^+]$. Other mesonic and baryonic multiplets may exist roughly degenerate with the $L=2$ levels just mentioned.

Clearly this $U(6)\times U(6)\times O(3)$ symmetry, which we shall call the rest symmetry, has to be badly violated in resonance decays since it forbids, e.g., $\Delta\to N\pi$ or $\rho\to\pi\pi$. This is, of course, not surprising since decay amplitudes involve moving particles for which the rest symmetry should not hold. Lipkin and Meshkov [20)] pointed out that the reflection and rotation invariance properties of collinear processes, such as two-body decays, lead to invariance under "W spin" or $SU(2)_W$. For a particle moving along the $\hat{z}$ axis, the generators of W spin may be written

$$W_x = P_{int} S_x \quad ; \quad W_y = P_{int} S_y \quad ; \quad W_z = S_z \tag{1.1}$$

where P_{int} is the intrinsic parity of the system. For a single free quark moving in the $\hat{z}$ direction one would thus write the $SU(2)_W$ generators as follows

$$W_x = \beta \frac{\sigma_x}{2} \quad ; \quad W_y = \beta \frac{\sigma_y}{2} \quad ; \quad W_z = \frac{\sigma_z}{2} \tag{1.2}$$

Combining $SU(2)_W$ with $SU(3)$ one is thus led to consider $SU(6)_{W,strong}$ as the natural candidate for this "relativistic spin symmetry". In a constituent quark model, the generators of this algebra would then be written

$$\begin{aligned} W^i &= \sum_{q,\bar{q}} \frac{\lambda^i}{2} + \frac{\bar{\lambda}^i}{2} \\ W^i_x &= \sum_{q,\bar{q}} \frac{\lambda^i}{2} \beta \frac{\sigma_x}{2} + \frac{\bar{\lambda}^i}{2} \beta \frac{\sigma_x}{2} \\ W^i_y &= \sum_{q,\bar{q}} \frac{\lambda^i}{2} \beta \frac{\sigma_y}{2} + \frac{\bar{\lambda}^i}{2} \beta \frac{\sigma_y}{2} \\ W^i_z &= \sum_{q,\bar{q}} \frac{\lambda^i}{2} \frac{\sigma_z}{2} + \frac{\bar{\lambda}^i}{2} \frac{\sigma_z}{2} \end{aligned} \tag{1.3}$$

These expressions for the generators clearly commute with $\sum_{q,\bar{q}} \alpha_z$, which, for free quarks, is the generator of Lorentz boosts in the $\hat{z}$ direction. This last property of $SU(6)_{W,strong}$ for free quarks suggests that perhaps the physical $SU(6)_{W,strong}$ might be an appropriate algebra for particles moving along the $\hat{z}$ axis, and, in particular, for decay amplitudes or for collinear processes. It should be pointed out, however, that this is an assumption which is not really well motivated. It allows us to classify moving states knowing their rest frame classification. In the rest frame both quark spin and W spin are well defined and one finds that for

$$\begin{aligned} \text{quarks} \qquad & \vec{W} = \vec{S} \\ \text{antiquarks} \qquad & W_z = S_z \\ & W_{x,y} = -S_{x,y} \end{aligned} \tag{1.4}$$

where $\vec{S}$ stands for ordinary quark spin. These relations follow trivially from $\beta q = +q$ and $\beta \bar{q} = -\bar{q}$. As is well known, one finds from Eqs. (1.4) the so-called W-S flip [21], e.g.,

$$|\rho(J_z = 1)\rangle \sim |W=1, W_z = 1\rangle$$

but
$$|\rho(J_z = 0)\rangle \sim -|W=0, W_z = 0\rangle \qquad (1.5)$$

while
$$|\pi\rangle \sim -|W=1, W_z = 0\rangle$$

Under Lorentz boosts in the $\hat{z}$ direction, L does not remain a good quantum number but L_z will continue to be conserved (as well as W_z, of course).

Even though $SU(6)_W$ appears to be an approximate symmetry for the classification of one particle (moving) states it need not be a symmetry of the whole Hilbert space. An illustrative example of a similar situation is found in the case of the hydrogen atom. The energy levels of the H atom possess an extra degeneracy beyond that of rotational invariance: states of different angular momentum but the same principal quantum number have the same energy. It may be shown that the generators of an $O(4)$ symmetry commute with the Hamiltonian of the H atom [22]: however, the scattering of bound states does not possess this symmetry. Thus $O(4)$ is only useful for analyzing the spectrum of isolated states and not for the dynamics of particle interactions.

If, however, one does assume $SU(6)_{W,\text{strong}}$ for collinear processes such as three-point functions ["naïve" $SU(6)_W$], one has separate conservation of L_z and W_z. It is precisely this restriction which appears to be violated in nature [15),16),18)].

Hence, we can only conclude that the generators of $SU(6)_{W,\text{strong}}$ approximately commute with the strong interaction Hamiltonian when sandwiched between one-particle states, and that $SU(6)_{W,\text{strong}}$ is not, even approximately, a vertex symmetry. For this reason phenomenological models were proposed several years ago with the intention of relaxing $SU(6)_{W,\text{strong}}$ symmetry somewhat.

It was shown by Carlitz and Kislinger [23] that one can construct $SU(6)_W$ invariant vertices by working with states of quark spin but allowing an additional quark-antiquark pair to be produced with $L_z = 0$, $P = C = +$, and internal quantum numbers of an SU(3) singlet. This pair shares its quarks among the two outgoing particles in the decay. On the basis of this picture, it was guessed that a useful modification of naïve $SU(6)_W$ would result from allowing the pair to have $L_z = \pm 1$ as well as $L_z = 0$ [15]. This model has come to be called the "$^3P_0$ picture", [14),15] since the $q\bar{q}$ pair is assumed to have all the quantum numbers of the vacuum. Actually the name is somewhat misleading, since $^3P_0$ implies a definite relation between $L_z = \pm 1$ and $L_z = 0$. In contrast, the model assumes the relative admixtures of these two states to be arbitrary.

In various practical applications [14)-19] it has been shown that the $^3P_0$ picture is equivalent to breaking only those naïve $SU(6)_W$ relations which link different final relative orbital momenta ℓ between the two resonance decay products. Those naïve $SU(6)_W$ predictions referring to a given ℓ are preserved. This scheme is the "ℓ broken $SU(6)_W$" referred to in the Introduction. As we shall show, the simplest place where ℓ broken $SU(6)_W$ is only a special case of (and not equivalent to) the $^3P_0$ picture is in $L = 2 \rightarrow L = 1 + \text{pion}$ decays.

Two subgroups of $SU(6)_{W,\text{strong}}$ have also been applied to hadron decays. These are the coplanar [15),24] and chiral [12),13] versions of $U(3) \times U(3)$.

In the picturesque language of constituent quarks, the possibility of transverse motion of these quarks inside a hadron suggests that one studies the subset of the generators (1.3) commuting both with a Lorentz boost along the $\hat{z}$ axis and a transverse boost (say α_x in the free quark model). This gives rise to the coplanar $[\tilde{U}(3) \times U(3)]$ $\beta\sigma_y$ first discussed by Dashen and Gell-Mann [25]. This symmetry was explicitly applied to decays in Ref. 24) and was found to be weaker than the $^3P_0$ picture but consistent with it.

The chiral version of $U(3) \times U(3)_{\text{strong}}$ generated by W^i and W_z^i is somewhat better motivated theoretically since in any interacting quark model these generators should always commute with boosts along the $\hat{z}$ axis. On the other hand, from the current quark point of view, as will be seen in the next Section, the chiral $U(3) \times U(3)_{\text{currents}}$ is well established while the existence of tensor currents, which can be constructed in a current quark model and used to extend the chiral algebra to an $SU(6)_{W,\text{currents}}$ algebra, requires an additional assumption. Hence it is of some interest to study the implications and the breaking pattern of the chiral $SU(3)\times SU(3)_{\text{strong}}$ subalgebra of $SU(6)_{W,\text{strong}}$.

2. CURRENT QUARKS

To discuss current-quark algebraic structures let us start from the well-known chiral $SU(3)\times SU(3)$ algebra [2] of vector and axial vector charges

$$F^i(t) = \int d^3x\, \mathfrak{F}_0^i(x) \tag{2.1}$$

$$F^{i5}(t) = \int d^3x\, \mathfrak{F}_0^{i5}(x) \tag{2.2}$$

where the current densities $\mathfrak{F}_\mu^i(x)$ and $\mathfrak{F}_\mu^{i5}(x)$ are well defined measurable physical operators in weak and electromagnetic transitions. In a "current-quark" model, one would write these vector and axial vector current densities as

$$\mathfrak{F}_\mu^i(x) = \bar{q}(x)\gamma_\mu \frac{\lambda^i}{2} q(x) \tag{2.3}$$

$$\mathfrak{F}_\mu^{i5}(x) = \bar{q}(x)\gamma_\mu\gamma_5 \frac{\lambda^i}{2} q(x) \tag{2.4}$$

From these expressions and the canonical anticommutation relations of the quark fields one derives the equal time commutators:

$$[F^i(t), F^j(t)] = i f_{ijk} F^k(t)\ ; \ [F^i(t), F^{j5}(t)] = i f_{ijk} F^{k5}(t)$$

$$[F^{i5}(t), F^{j5}(t)] = i f_{ijk} F^k(t) \tag{2.5}$$

This $SU(3)\times SU(3)$ algebraic structure is then assumed for physical vector and axial vector charges independently of any underlying quark structure. As is well known, this algebra leads, with the help of PCAC, to Adler-Weisberger sum rules. Insofar as they have been tested these are remarkably successful. In view of this, one can attempt, following Dashen and Gell-Mann, to enlarge the algebra. With the equal time commutation relations of the quark model one can clearly define an U(12) algebra generated by 144 charges defined as integrals over the densities

$$\bar{q}(x)\,\Gamma\,\frac{\lambda^i}{2}\,q(x)$$

where Γ is one of the 16 Dirac covariants. All these current densities are in principle directly or indirectly measurable: for example, although nothing in nature seems to be coupled directly to a tensor current $\mathcal{F}^{i}_{\mu\nu}(x)$, represented in the quark model by $\bar{q}(x)\,\sigma_{\mu\nu}\,(\lambda^{i}/2)q(x)$, this current nevertheless appears on the right-hand side of commutators of more directly observable operators, e.g., in $[\mathcal{F}^{i}_{\mu}(x),\ \partial^{\nu}\ \mathcal{F}_{\nu}^{i5}(y)]_{x_0=y_0}$. In this sense all 144 current densities could be meaningful physical operators provided, of course, that the algebraic structure of the current quark model can be abstracted that far.

It is important to realize that contrary to the situation in the constituent quark approach one has here an algebraic structure which is generated by well defined operators: they are integrals over local current densities and their Lorentz transformation properties are given "by definition". Let us go in particular to the infinite momentum frame. As usual we define as "good" operators those charges whose matrix elements do not vanish when taken between finite mass states with infinite momentum along, say, the $\hat{z}$ direction [6]. There are 35 independent good operators *) whose quark model expressions are given by

$$\begin{aligned}
F^i(t) &= \int d^3x\, \mathcal{F}^i_0(x) &&= \int d^3x\, q^{+}(x)\, \frac{\lambda^i}{2}\, q(x)\\
F^i_x(t) &= \int d^3x\, \mathcal{F}^i_{23}(x) &&= \int d^3x\, q^{+}(x)\,\beta\sigma_x \frac{\lambda^i}{2}\, q(x)\\
F^i_y(t) &= \int d^3x\, \mathcal{F}^i_{31}(x) &&= \int d^3x\, q^{+}(x)\,\beta\sigma_y \frac{\lambda^i}{2}\, q(x)\\
F^i_z(t) &= \int d^3x\, \mathcal{F}^{i5}_3(x) &&= \int d^3x\, q^{+}(x)\,\sigma_z \frac{\lambda^i}{2}\, q(x)
\end{aligned} \qquad (2.6)$$

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*) Properties of good currents and indeed the Melosh transformation itself may be reformulated on a light-like surface. This is perhaps a more exact expression of the infinite momentum limit.

These charges generate at infinite momentum an SU(6) algebra which we will call $SU(6)_{W,\text{currents}}$. The algebraic similarity between these F_{α}^{i} or "current-quark generators" and the W_{α}^{i} or "constituent quark generators" defined in the previous section is now obvious. It is, however, extremely important not to confuse or to identify too quickly the two sets of generators of these $SU(6)_{W}$ algebras. The F_{α}^{i} 's are explicitly integrals over local expressions with well defined Lorentz transformation properties, but they have not been assumed to commute with the strong interaction Hamiltonian. The W_{α}^{i} 's, on the other hand, are supposed to commute approximately with the strong interaction Hamiltonian at least between one-particle states but there is no a priori reason why they should be integrals over local measurable densities. (In fact, as we shall see below, one finds that even in the free quark model [11] the F_{α}^{i} 's do not commute with the "strong interaction" Hamiltonian and that the W_{α}^{i} 's are not local charges.) Hence one has two sets of possibly distinct operators which generate the same $SU(6)_{W}$ algebraic structure. A fundamental question is of course the relation, if any, between the two sets of generators. From the successes of the conserved vector current hypothesis (CVC) we are forced to identify the W^{i} and the F^{i}, i.e., the generators of ordinary SU(3) of strong interactions and the integrals of the time components of the vector currents. To identify the full sets of generators of $SU(6)_{W,\text{strong}}$ and $SU(6)_{W,\text{currents}}$ would, however, be incorrect. A strong objection to such an identification comes from the Adler-Weisberger relation which, when saturated with resonances necessarily implies representation mixing, i.e., particles which belong to different $SU(6)_{W,\text{strong}}$ representations are connected by the axial charge and thus at least a piece of their wave function must lie in the same representation of $SU(6)_{W,\text{currents}}$. One could argue, perhaps, that if the Adler-Weisberger sum rule is saturated with resonances (N,Δ) belonging to the same $SU(6)_{W,\text{strong}}$ multiplet, the result obtained for $-G_A/G_V$, 5/3, is not too far removed from the correct value. Dashen and Gell-Mann [5] have shown, however, that in the same approximation, i.e., classifying the nucleon octet and $\Delta(1238)$ decimet in a $\underset{\sim}{56}$ of $SU(6)_{W,\text{currents}}$ [as well as in a $\underset{\sim}{56}$ of $SU(6)_{W,\text{strong}}$] the anomalous magnetic moments of all $\frac{1}{2}^{+}$ baryons and the magnetic transitions between the $\frac{1}{2}^{+}$ octet and $\frac{3}{2}^{+}$ decimet are predicted to vanish. This is clearly unacceptable. One arrives at the conclusion that there must be sizeable representation mixing and thus that the generators of the two $SU(6)_{W}$ algebras cannot be identified. This being the case one may try to relate the two algebras by a unitary transformation. In fact there is no compelling argument in favour of such an assumption: it is motivated by the feeling that the similarity in these algebraic structures is a fundamental property of the hadronic world. If to the quantum numbers corresponding to particle classifications under either $SU(6)_W$ one adds all necessary quantum numbers to obtain two bases of the Hilbert space, a unitary transformation relating one of these bases to the other must exist. This does not imply that the generators of the two $SU(6)_W$ algebras are, by themselves, related through a unitary transformation but it makes such an assumption somewhat more plausible. Furthermore Melosh [11] showed recently that in the free quark model such a unitary transformation does in fact exist. We will postulate that such a transformation exists in general and write

$$W_\alpha^i = V F_\alpha^i V^{-1} \tag{2.7}$$

where V is unitary and will be called the Melosh transformation. This transformation V thus expresses the general idea that hadrons can be simultaneously described as simple constituent quark states where strong interactions are concerned and as rather complicated structures when currents are involved. It systematizes in a compact way all $SU(6)_{W,\text{currents}}$ representation mixing for all hadrons. To make this last point clear let us recall that W_z^0 and F_z^0 act like quark spin operators for constituent and current quarks respectively. Both operators commute with J_z and thus make it possible to define "orbital" angular momenta [26] by

$$L_z(W) = J_z - W_z^0$$

and

$$L_z(F) = J_z - F_z^0 \tag{2.8}$$

We may then classify states according to their $SU(6)_W$ quantum numbers and angular momentum. For example the $SU(6)_{W,\text{strong}}$ classification of the lowest lying mesons would be $\underset{\sim}{35} \oplus \underset{\sim}{1}$, $L_z(W) = 0$ while the first excited states would be $\underset{\sim}{35} \oplus \underset{\sim}{1}$, $L_z(W) = -1,0,+1$.

The Melosh transformation V must obviously be invariant under rotations around the $\hat{z}$ axis i.e., $[V,J_z] = 0$. Let us suppose that $[V,F_z^0] \neq 0$ so that F_z^0 and W_z^0 are different operators. A state with a simple constituent quark spin structure will then in general be expected to correspond to a mixture of different current quark spin states. Similarly, while the constituent orbital angular momentum $L_z(W)$ will be simple, one will have a mixture of current orbital angular momenta $L_z(F)$ since $J_z = W_z^0 + L_z(W) = F_z^0 + L_z(F)$, and J_z commutes with the Melosh transformation.

To conclude this section on current quarks we recall the explicit construction of the Melosh transformation in the free quark model. Such a model is defined by the Hamiltonian

$$H_{free} = \int d^3x \, q^+(x) \left\{ -i\vec{\alpha}\cdot\vec{\partial} + \beta m \right\} q(x) \tag{2.9}$$

The generators of $SU(6)_{W,currents}$ are given, in this model, by the expressions appearing in Eqs. (2.6). They do not commute with H_{free} and hence cannot be identified with the generators of $SU(6)_{W,strong}$. In this sense, the free quark model, although trivial, exhibits some of the complexity of the full problem. As shown by Melosh with

$$V_{free} = \exp(i \, Y_{free})$$

and

$$Y_{free} = \frac{1}{2}\int d^3x \, q^+(x) \arctan\left(\frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{m}\right) q(x) \tag{2.10}$$

one finds, through Eq. (2.7), the following set of operators

$$\begin{aligned}
W^i_{,free} &= F^i_{,free} \\
W^i_{x,free} &= F^i_{x,free} + \int d^3x \, q^+(x) \frac{1}{\kappa}\left\{\frac{1}{1+\kappa}\frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{m} - i\right\}\gamma_5 \frac{\partial_x}{m}\frac{\lambda^i}{2} q(x) \\
W^i_{y,free} &= F^i_{y,free} + \int d^3x \, q^+(x) \frac{1}{\kappa}\left\{\frac{1}{1+\kappa}\frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{m} - i\right\}\gamma_5 \frac{\partial_y}{m}\frac{\lambda^i}{2} q(x) \\
W^i_{z,free} &= F^i_{z,free} + \int d^3x \, q^+(x) \frac{1}{\kappa}\left\{\frac{1}{1+\kappa}\frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{m} - i\right\}\sigma_z \frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{m}\frac{\lambda^i}{2} q(x)
\end{aligned} \tag{2.11}$$

where

$$\kappa = \left[1 + \left(\frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{m}\right)^2\right]^{1/2}$$

These transformed operators can be shown to commute with H_{free} and are thus appropriate as $SU(6)_{W,strong}$ generators. Some of the properties of the Melosh transformation are worth pointing out. The first remarkable property of V_{free} is that it is non-local in the transverse directions and hence, as shown explicitly by Eqs. (2.11), the symmetries of the Hamiltonian are generated by non-local operators! Introducing the (current) quark spin and orbital angular momentum operators

$$\Sigma_{i,\mathrm{free}} = \int d^3x \, q^+(x) \frac{\sigma_i}{2} q(x)$$
$$L_{i,\mathrm{free}} = i\epsilon_{ijk} \int d^3x \; x_k \, q^+(x) \partial_j q(x) \qquad (2.12)$$

we can describe the properties of Y_{free} under $SU(6)_{W,\mathrm{currents}} \times O(2)$ as follows: Y_{free} is the SU(3) singlet member of a $\underset{\sim}{35}$ with $\Delta J_z = 0$, $\Delta\Sigma_{z,\mathrm{free}} = \pm 1$. Although Y_{free} only changes $L_{z,\mathrm{free}}$ by ± 1 the presence of the power series in $(\vec{\gamma}_\perp \cdot \vec{\partial}_\perp)/m$ implies that Y_{free} can mix essentially all total angular momenta. Let us note also that the W^i_α have simple algebraic properties under $SU(6)_{W,\mathrm{currents}} \times O(2)$: they transform as the sum of a $\underset{\sim}{35}$ with $\Delta S_z = \Delta L_z = 0$ and of a $\underset{\sim}{35}$ with $\Delta S_z = -\Delta L_z = \pm 1$.

3. THE MODEL AND ITS APPLICATIONS TO MESON RESONANCES

a) The $SU(6)_W$ $(\Delta L_z = 0, \pm 1)$ model

Given the existence of a unitary transformation between constituent quarks and current quarks we may now proceed, with the help of PCAC, to a "theory" of pionic transitions. As hadrons do seem to fall into irreducible representations of $SU(6)_{W,\mathrm{strong}}$, this algebra is an approximate symmetry of the strong interaction hamiltonian (at least for one-particle states). This does not imply that $SU(6)_{W,\mathrm{strong}}$ can be considered as an approximate symmetry in the whole Hilbert space. Indeed, as discussed in Section 1 one finds that $SU(6)_{W,\mathrm{strong}}$ is totally unacceptable as a vertex symmetry. On the other hand, the PCAC relation

$$\partial^\mu F^{i5}_\mu = c\,\phi^i \qquad (i = 1, \dots 8) \qquad (3.1)$$

allows us to write pionic three-point functions as follows:

$$\langle B\pi^i | A \rangle \propto \left(\frac{m_B^2 - m_A^2}{c} \right) \langle B | F^{i5} | A \rangle \qquad (3.2)$$

Thus, PCAC reduces those hadronic three-point functions involving at least one pseudoscalar meson to the matrix elements of the axial charge between one-particle states for which $SU(6)_{W,\mathrm{strong}}$ is expected to be a good approximate symmetry. But the axial charges F^{i5} are of course generators of $SU(6)_{W,\text{currents}}$ and they are assumed to be related by the (unitary) Melosh transformation to the $SU(6)_{W,\text{strong}}$ generators. Hence any assumption about the Melosh transformation V will reflect itself in pionic decays of hadrons. Let us formalize this connection by defining the Melosh transformation as the unitary transformation of the Hilbert space which relates the constituent quark representation [i.e., in $SU(3)\times SU(3)_{\text{strong}}$ or in $SU(6)_{W,\text{strong}}$] of a hadron state h to its current quark representation [i.e., in $SU(3)\times SU(3)_{\text{currents}}$ or in $SU(6)_{W,\text{currents}}$]:

$$|h;\text{current quark}\rangle = V\,|h;\text{constituent quark}\rangle \tag{3.3}$$

The matrix elements of the axial charges F^{i5}, given by

$$\langle h';\text{current quark}|\,F^{i5}\,|h;\text{current quark}\rangle$$

can then be rewritten as

$$\langle h';\text{constituent quark}|\,V^{-1}F^{i5}V\,|h;\text{constituent quark}\rangle \tag{3.4}$$

As discussed in the previous section, in the free quark model, where V_{free} was explicitly constructed by Melosh, one finds that the transformed axial charge has a remarkably simple algebraic structure: it transforms, under $SU(3)\times SU(3)_{\text{strong}}$ as the sum of the $\{(\underset{\sim}{8},\underset{\sim}{1})_0,0\}$ - $\{(\underset{\sim}{1},\underset{\sim}{8})_0,0\}$ and $\{(\underset{\sim}{3},\bar{\underset{\sim}{3}})_1,-1\}$ - $\{(\bar{\underset{\sim}{3}},\underset{\sim}{3})_{-1},1\}$ representations *). Equivalently, the transformed axial charge transforms, under $SU(6)_{W,\text{strong}}$, as the sum of the $\{\underset{\sim}{35},L_z=0\}$ and $\{\underset{\sim}{35},L_z=\pm 1\}$ representations.

We will assume in this paper that these simple algebraic properties of the transformed free quark axial charge can be extended to the physical axial charge. This is of course an extremely strong assumption: it can be somewhat motivated by the following argument.

As long as the Melosh transformation is bilinear in quark fields these algebraic properties of the axial charge are in fact the most general ones. Indeed, for the axial charges belonging to $SU(3)$ octets with $\Delta J_z=0$, a Melosh transformation bilinear in quarks which is an $SU(3)$ singlet and

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*) We have used the standard notation $\{(\underset{\sim}{A},\underset{\sim}{B})_{S_z},L_z\}$ to label irreducible representations of $SU(3)\times SU(3)$ at infinite momentum along the z axis, corresponding to helicity S_z+L_z and to total quark spin S_z.

commutes with J_z, can only transform them to octets with $\Delta S_z = 0$ and $\Delta S_z = \pm 1$. This is equivalent to the $SU(3) \times SU(3)_{strong}$ or $SU(6)_{W,strong}$ transformation properties of the free transformed axial charge. The question then is of course why V should be bilinear in quarks. We do not have any compelling argument in favour of this assumption except that it is the simplest way to connect two remarkable properties of hadronic physics: both currents and hadronic states seem to behave in some sense like free quark currents and free quark states respectively. A bilinear Melosh transformation allows us to connect these two "free quark" approximations.

Before using Eq. (3.4) to compute pionic decays of hadrons we still have to connect real hadronic states to free constituent quark states. Although we know of no satisfactory prescription to make this connection, especially since we do not know the strong interaction Hamiltonian, it seems reasonable to assume that physical one-particle states can be constructed by acting with some operator U on free quark states [11]. The success of $SU(6)_{W,strong}$ or its subgroup $SU(3) \times SU(3)_{strong}$ in classifying hadronic states can then be understood if U is assumed to transform as an $SU(6)_{W,strong}$ or $SU(3) \times SU(3)_{strong}$ singlet.

To summarize, our model for pionic decays is based on the following assumptions

1) PCAC;
2) "free constituent quark" classification of hadrons in $SU(6)_{W,strong}$ or $SU(3) \times SU(3)_{strong}$ (U transforms as a singlet)
3) $V^{-1}F^{i5}V$ transforms as the sum of the $\{(\underset{\sim}{8},\underset{\sim}{1})_0,0\} - \{(\underset{\sim}{1},\underset{\sim}{8})_0,0\}$ and $\{(\underset{\sim}{3},\underset{\sim}{\bar{3}})_1,-1\} - \{(\underset{\sim}{\bar{3}},\underset{\sim}{3})_{-1},1\}$ representations of $SU(3) \times SU(3)_{strong}$ or equivalently as the sum of the $\{\underset{\sim}{35},\ L_z = 0\}$ and $\{\underset{\sim}{35},\ L_z = \pm 1\}$ representations of $SU(6)_{W,strong}$.

In the remainder of this section we will refer to our model as the $SU(6)_W[\Delta L_z = 0, \pm 1]$ model.

b) Classification according to $U(3) \times U(3)$ and $SU(6)_W$

We shall need the reduction of the $(\underset{\sim}{6},\underset{\sim}{\bar{6}}),L$ states of $U(6) \times U(6) \times O(3)$ to combinations of $U(3) \times U(3) \times O(2)_{L_z}$ or $SU(6)_W \times O(2)_{L_z}$ states. Once we reduce the states of given quark spin S and z component S_z, the extension to states of arbitrary L will be trivial.

The relation between quark-spin and W-spin states was already pointed out in Section 1. In the discussion of $[U(3)\times U(3)]_{S_z}$ we need only recall that $\sigma_z q = S_z q$ while $\sigma_z \bar{q} = -S_z \bar{q}$. Then Table Ia) shows the necessary relations between the rest symmetry, chiral $U(3)\times U(3)$, and $SU(6)_W$ classifications. For simplicity we consider only octets, avoiding the discussion of nonet couplings except in the context of phenomenological interpretations.

States of arbitrary L with given helicity are then expressed in terms of those in Table Ia) via Clebsch-Gordan coefficients

$$|J S L \lambda\rangle = \sum_{S_z + L_z = \lambda} (S\, S_z\, L\, L_z | J \lambda)\, |S\, S_z\rangle \qquad (3.5)$$

The J^{PC} values of the specific states considered here are shown in Table 1b *).

c) Predictions for specific processes

The reduced matrix elements within a given set of decays $L \to L' + \pi$ in chiral $U(3)\times U(3)$ may be labelled by the initial L_z^i, the final L_z^f, and the total helicity λ. In addition, for reduced matrix elements of three octets $(\underset{\sim}{8},\underset{\sim}{1}) \to (\underset{\sim}{8},\underset{\sim}{1}) \otimes (\underset{\sim}{8},\underset{\sim}{1})$ and $(\underset{\sim}{1},\underset{\sim}{8}) \to (\underset{\sim}{1},\underset{\sim}{8}) \otimes (\underset{\sim}{1},\underset{\sim}{8})$, the relative phase of the two depends on the product of the charge parities of A(L), B(L') and π. In practice for the cases we consider, this leads to an extra degree of freedom in the $L_z^i = L_z^f = \lambda = 1$ amplitudes, which will be expressed via a parameter μ.

Since chiral $SU(3)\times SU(3)$ is the weakest of the symmetries we consider, our Tables will give its predictions, with a simple "dictionary" at the end of each table for converting to the stronger symmetries.

The reduced matrix elements in chiral $SU(3)\times SU(3)$, called $a^{L_z^f L_z^i}$, are normalized so that in the $SU(6)_W$ ($\Delta L_z = 0, \pm 1$) limit

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*) It is necessary to take some care in choosing a consistent set of conventions for the behaviour of $\{[U(3)\times U(3)]_{S_z}, L_z\}$ representations under charge conjugation and parity. For example, one $\{(\underset{\sim}{8},\underset{\sim}{1})_0 - (\underset{\sim}{1},\underset{\sim}{8})_0, 0\}$ representation represents a pion with $C = +1$ and another, different $\{(\underset{\sim}{8},\underset{\sim}{1})_0 - (\underset{\sim}{1},\underset{\sim}{8})_0, 0\}$ representation, an helicity zero B with $C = -1$.

$$a_{\lambda}^{L_z^f L_z^i} = a^{L_z^f L_z^i} \tag{3.6}$$

except for reduced chiral $SU(3) \times SU(3)$ matrix elements which vanish in this limit ($\mu = 0$). The helicity amplitudes g_{λ} for $A(L) \to B(L') + \pi$ in $SU(6)_W$ [$\Delta L_z = 0, \pm 1$] are

$$\begin{aligned} g_{\lambda} = \sum_{L_z^f, L_z^i} & \xi^A(\lambda - L_z^i)\,(S^A\, \lambda - L_z^i\; L\; L_z^i \,|\, J^A \lambda) \\ & \times \xi^B(\lambda - L_z^f)\,(S^B\, \lambda - L_z^f\; L'\; L_z^f \,|\, J^B \lambda) \\ & \times \left\{ \begin{matrix} \underset{\sim}{35} & \underset{\sim}{35} \\ (\underset{\sim}{8}, 2W^B+1) & (\underset{\sim}{8}, 3) \end{matrix} \middle| \begin{matrix} \underset{\sim}{35} \\ (\underset{\sim}{8}, 2W^A+1) \end{matrix} \right\}_{\alpha_1} \left\{ \begin{matrix} \underset{\sim}{8} & \underset{\sim}{8} \\ B & \pi \end{matrix} \middle| \begin{matrix} \underset{\sim}{8} \\ A \end{matrix} \right\}_{\alpha_2} \\ & \times (W^B\, \lambda - L_z^f\; 1\; L_z^f - L_z^i \,|\, W^A\, \lambda - L_z^i)\; a^{L_z^f L_z^i} \end{aligned} \tag{3.7}$$

The first two Clebsch-Gordan coefficients in Eq. (3.7) are those coming from Eq. (3.5). The next term is an $SU(3)$ scalar factor [a Clebsch-Gordan coefficient of $SU(6)$ [27]]. The index α_1 denotes symmetric or antisymmetric coupling of $\underset{\sim}{35} \to \underset{\sim}{35} \otimes \underset{\sim}{35}$; it is chosen so that the $SU(2)$ scalar factor [Clebsch-Gordan coefficient of $SU(3)$ [28]] denotes the proper d type or f type coupling as dictated by the product of charge parities of A, B and π:

$$\begin{aligned} \mathcal{C}(A)\,\mathcal{C}(B)\,\mathcal{C}(\pi) &= + \quad : \quad \alpha_2 = \text{"d-type"} \\ \mathcal{C}(A)\,\mathcal{C}(B)\,\mathcal{C}(\pi) &= - \quad : \quad \alpha_2 = \text{"f-type"} \end{aligned} \tag{3.8}$$

In the isoscalar factors, we shall take the specific isomultiplets $A = B = K$ (the $I = \frac{1}{2}$, $Y = +1$ states) for convenience. The phases ξ^A and ξ^B arise from the relation between quark spin and W spin states [see, e.g., Eq. (1.5)], and are

$$\begin{aligned} \xi^{A,B}(S_z) &= +1 \quad : \quad S_z = +1 \\ &= -1 \quad : \quad \text{otherwise} \end{aligned} \tag{3.9}$$

Finally, the last Clebsch-Gordan coefficient in Eq. (3.7) expresses W spin conservation. Note that the value of W_z associated with the pion $(W = 1)$

need not be zero. It is zero only in the case of "naïve" $SU(6)_W$, where $\Delta L_z = L_z^f - L_z^i = 0$.

The Tables are:

Table II : $L=1 \to L=0$
Table III : $L=2 \to L=0$
Table IV : $L=1 \to L=1$
Table V : $L=2 \to L=1$.

We shall discuss the relations among various symmetries implied by these results in the next section. Meanwhile it is helpful to point out where comparisons with data may be made.

d) Applications to experiment

The case of $L=1 \to L=0$ decays has already been studied in a formalism equivalent to $SU(6)_W$ ($\Delta L_z = 0, \pm 1$) [15]. The $\Delta L_z = \pm 1$ transitions were found to be dominant, as measured in the $B \to \omega\pi$ decay. Recently these fits are being repeated with kinematic factors appropriate to the use of PCAC [29]; qualitative features are expected to remain the same.

Present information about $L=2 \to L=0$ decays allows one to check some predictions of Table III. It is useful to rewrite these predictions in terms of the $\ell=1$ and $\ell=3$ amplitudes a_P and a_F (see the "dictionary" at the bottom of the Table). One then obtains the results shown in Table VI. We quote here only partial width predictions. No decays $(2^{-\pm} \to 1^{--})$ have been observed as yet which would allow for comparison with helicity amplitude ratio predictions (in contrast to the case $B \to \omega\pi$.) In constructing Table VI we have used the fact that Table III refers to "K" $\to$ "K"π. These values are then converted to specific processes using $SU(3)$ and (where necessary) a nonet ansatz for couplings. A convenient table for performing such conversions in general is given in Ref. 24).

The experimental predictions have been obtained by using the decay width $\Gamma(g(1700) \to 2\pi) = 40$ MeV, an approximate value suggested by a recent analysis [30] of the CERN-Munich data. The masses of decaying states are all taken at the idealized values of 1700 MeV for the purpose of barrier factor calculations. We use zero-radius forms for these factors, as suggested by the successful application of SU(3) to the known $L=2 \rightarrow L=0$ decays [31]. In fact, the interested reader is referred to that reference for a quotation of many SU(3) related results which will not be mentioned here, and is urged to recalculate any partial widths on the basis of exact masses and various barrier factors. The results quoted are somewhat sensitive to these choices. For example, the kinematic factor arising from the use of PCAC is such that

$$\Gamma_\ell(A \rightarrow B\pi) = \frac{p\,(m_A^2 - m_B^2)^2}{m_A^2}\, a_\ell^2\, c_\ell^2 \qquad (3.10)$$

where p is the final c.m. three-momentum, a_ℓ are the reduced matrix elements (a_P and a_F for Table VI, as defined in Table III), and the squares of the c_ℓ are shown in Table VI. Use of Eq. (3.10) increases the prediction for $\Gamma(g \rightarrow \omega\pi)$ to 26 MeV and that for $\Gamma(\omega_3 \rightarrow \rho\pi)$ to 80 MeV. These values are closer to the fit of Graham and Yoon, Ref. 31), but in our opinion the data do not yet allow one to decide conclusively in favour of such a kinematic factor in all cases. Note, for example, that the simple zero-radius f wave barrier factor agrees with SU(3) for the ratio $g \rightarrow K\bar{K}/g \rightarrow \pi\pi$, as it seems to do in a number of other cases.

The prediction for $A_3 \rightarrow \rho\pi$ ($\ell = 3$) is notable. No evidence for this decay has been found so far either in $\ell = 1$ or $\ell = 3$.

For $L=1 \rightarrow L=1$ decays (Table IV) one expects one f wave amplitude and two independent p wave amplitudes in ℓ broken $SU(6)_W$. A partial wave decomposition of the helicity amplitudes for the various processes, shows however, that there are no purely f wave decays. Hence simple tests analogous to those implied by Table III are not possible. Various processes which are purely p wave depend on different combinations of p wave amplitudes. However, one does have [in $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ or ℓ broken $SU(6)_W$] the following relations

$$\frac{\tilde{\Gamma}[B \rightarrow \pi_N(980)\pi]}{\tilde{\Gamma}[D_{(\eta\text{-like mixing})} \rightarrow A_1\pi]} = \frac{4}{3} \qquad (3.11)$$

and

$$\frac{\tilde{\Gamma}[B \to A_1 \pi]}{\tilde{\Gamma}[D_{(\eta\text{-like mixing})} \to \pi_N(980)\pi]} = 3 \qquad (3.12)$$

In both equations $\tilde{\Gamma}$ stands for the partial width with barrier factors removed. Since the B is so close to $A_1\pi$ threshold, this leads to a rather small physical width for $B \to A_1\pi$ (a few MeV, based on taking ~ 20 MeV for $D \to \pi_N \pi$). However, the strong intrinsic B-A_1-π coupling should be observable in B exchange processes, such as $\pi^- p \to A_1^0 n$. To date we know of no firm evidence for this process, however.

Table V contains a number of predictions for the S wave partial widths in decays that involve higher partial waves as well. The most useful of these can be written as

$$\tilde{\Gamma}_{\ell=0}[\rho(2^{--}) \to A_2\pi] = a_S^2/200 \qquad (3.13)$$

$$\tilde{\Gamma}_{\ell=0}[A_3(2^{-+}) \to f_0\pi] = 3a_S^2/800 \qquad (3.14)$$

$$\tilde{\Gamma}_{\ell=0}[\rho'(1^{--}, L=2) \to A_1\pi] = a_S^2/240 \qquad (3.15)$$

$$\tilde{\Gamma}_{\ell=0}[\omega'(1^{--}, L=2) \to B\pi] = a_S^2/160 \qquad (3.16)$$

If the A_3 is a resonance, $f_0\pi$ is its dominant decay mode. Then

$$\frac{\Gamma_{\ell=0}[\rho(2^{--}) \to A_2\pi]}{\Gamma_{\ell=0}[A_3(2^{-+}) \to f_0\pi]} = \frac{4}{3} \qquad (3.17)$$

implies a considerable value for the numerator. We hesitate to quote a predicted partial width since it is not clear whether the whole $\ell = 0$ $f_0\pi$ bump usually called A_3 is indeed resonant. It is interesting that there does seem to be evidence for a πA_2 enhancement around 1700 MeV [28]. A partial wave analysis of this effect, for example in the channel

$$\pi^- + p \longrightarrow \rho^-(2^{--}) + p \qquad (3.18)$$
$$\hookrightarrow A_2^- \pi^0$$
$$\hookrightarrow K^- K^0$$
$$\hookrightarrow \pi^- \pi^+$$

could be performed using multi-particle spectrometers such as Omega at CERN.

There are two d wave amplitudes in ℓ broken $SU(6)_W$ and three in $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$. In ℓ broken $SU(6)_W$ [but not in $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$!] the following relations hold:

$$\tilde{\Gamma}[\omega(2^{--}) \to B\pi] = \frac{3}{160}(a_D^1)^2 \qquad (3.19)$$

$$\tilde{\Gamma}[A_3(2^{-+}) \to D_{(8)}\pi] = \frac{1}{160}(a_D^1)^2 \qquad (3.20)$$

$$\tilde{\Gamma}[\rho(2^{--}) \to \pi_N(980)\pi] = \frac{1}{40}(a_D^1)^2 \qquad (3.21)$$

$$\tilde{\Gamma}[\rho'(1^-, L=2) \to A_2\pi] = \frac{1}{160}(a_D^0)^2 \qquad (3.22)$$

$$\tilde{\Gamma}[\rho(2^{--}) \to A_1\pi] = \frac{3}{160}(a_D^0)^2 \qquad (3.23)$$

$$\tilde{\Gamma}[\eta(2^{-+}) \to \pi_N(980)\pi] = \frac{1}{160}(a_D^0)^2 \qquad (3.24)$$

A selection rule, which holds in $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ but not in chiral $U(3) \times U(3)$ says

$$A_3 \not\to B\pi \qquad (3.25)$$

Normally this decay would be allowed in a d wave ($\lambda = 1$ only).

4. EQUIVALENCES

a) Rôle of $^3P_0$ model

As mentioned in Section 3, the $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ model is equivalent to the $^3P_0$ model. What is meant by this equivalence is that there is a one-to-one correspondence between independent parameters of both models. Strictly speaking one could argue that the two models cannot be identical since in the $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ model each helicity amplitude for a specific decay channel $A \to B + \pi$ is proportional to $m_A^2 - m_B^2$, because of PCAC, while this factor is not present in the usual formulation of the $^3P_0$ model. However, since the $^3P_0$ model is essentially a phenomenological prescription to break $SU(6)_{W,\mathrm{strong}}$ symmetry for vertices one can always include any function of the external masses in its definition.

The proof of the equivalence between the two models in the above sense is straightforward and not very illuminating. Let us consider the decay amplitudes

$$A(L) \longrightarrow B(L') + \pi$$

$$\text{"}K\text{"} \longrightarrow \text{"}K\text{"} + \pi$$

Helicity amplitudes for these decays in the $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ model can be rewritten as

$$\begin{aligned} g_\lambda = \sum_{L_z} & \begin{Bmatrix} \underset{\approx}{35} & \underset{\approx}{35} & \Big| & \underset{\approx}{35} \\ (\underline{8}, 2W^B+1) & (\underline{8}, 3) & \Big| & (\underline{8}, 2W^A+1) \end{Bmatrix}_{\alpha_1} \begin{Bmatrix} \underset{\sim}{8} & \underset{\sim}{8} & \Big| & \underset{\sim}{8} \\ B & \pi & \Big| & A \end{Bmatrix}_{\alpha_2} \\ & \Big\{ a^{L_z L_z} (W^B \lambda - L_z\ 1\ 0 | W^A \lambda - L_z)(S^A \lambda - L_z\ L\ L_z | J^A \lambda)(S^B \lambda - L_z\ L'\ L_z | J^B \lambda) \\ & + a^{L_z+1\ L_z} (W^B \lambda - L_z - 1\ 1\ 1 | W^A \lambda - L_z)(S^A \lambda - L_z\ L\ L_z | J^A \lambda)(S^B \lambda - L_z - 1\ L'\ L_z+1 | J^B \lambda) \\ & + a^{L_z-1\ L_z} (W^B \lambda - L_z+1\ 1\ -1 | W^A \lambda - L_z)(S^A \lambda - L_z\ L\ L_z | J^A \lambda)(S^B \lambda - L_z+1\ L'\ L_z-1 | J^B \lambda) \Big\} \end{aligned} \tag{4.1}$$

where we have absorbed the factor $m_A^2 - m_B^2$ into the reduced matrix elements $a^{L_z^f L_z^i}$.

In the $^3P_0$ model, on the other hand, the same helicity amplitudes are parametrized as follows

$$\tilde{g}_\lambda = \sum_{L_z} \left\{ \begin{matrix} \underset{\sim}{35} & \underset{\sim}{35} \\ (\underset{\sim}{8}, 2S^B+1) & (\underset{\sim}{8},3) \end{matrix} \middle| \begin{matrix} \underset{\sim}{35} \\ (\underset{\sim}{8}, 2S^A+1)\end{matrix} \right\}_{\beta_1} \left\{ \begin{matrix} \underset{\sim}{8} & \underset{\sim}{8} \\ B & \pi \end{matrix} \middle| \begin{matrix} \underset{\sim}{8} \\ A \end{matrix}\right\}_{\beta_2}$$
$$\Big\{ b^{L_z L_z}(S^B \lambda-L_z\, 1 0|S^A \lambda-L_z)(S^A \lambda-L_z\, L L_z|J^A \lambda)(S^B \lambda-L_z\, L' L_z|J^B \lambda)$$
$$+ b^{L_z+1\, L_z}(S^B \lambda-L_z-1\, 1 1|S^A \lambda-L_z)(S^A \lambda-L_z\, L L_z|J^A \lambda)(S^B \lambda-L_z-1\, L' L_z+1|J^B \lambda)$$
$$+ b^{L_z-1\, L_z}(S^B \lambda-L_z+1\, 1 -1|S^A \lambda-L_z)(S^A \lambda-L_z\, L L_z|J^A \lambda)(S^B \lambda-L_z+1\, L' L_z-1|J^B \lambda)\Big\} \qquad (4.2)$$

It is then simple to show that the coefficients of the $a^{L_z^f L_z^i}$ reduced matrix elements are proportional to the coefficients of the $b^{L_z^f L_z^i}$ reduced matrix elements and that the proportionality factor is independent of L_z. One finds that with the correspondences

$$a^{L_z L_z} \longrightarrow b^{L_z L_z}$$
$$a^{L_z \pm 1\, L_z} \longrightarrow -\frac{4}{3\sqrt{2}}\, b^{L_z \pm 1\, L_z} \qquad (4.3)$$

or

$$a^{L_z L_z} \longrightarrow b^{L_z L_z}$$
$$a^{L_z \pm 1\, L_z} \longrightarrow -\frac{3\sqrt{2}}{4}\, b^{L_z \pm 1\, L_z} \qquad (4.4)$$

the helicity amplitudes g_λ and $\tilde{g}_\lambda$ are identical. The correspondence of Eq. (4.3) is to be used when the coupling

$$A\,(\text{quark spin triplet state}) \longrightarrow B\,(\text{quark spin triplet state}) + \pi$$

is d type in SU(3) (i.e., L+L' even) while Eq. (4.4) is to be used when the same coupling is f type in SU(3) (i.e., L+L' odd).

This thus completes the formal proof that the $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ model and the $^3P_0$ model are identical up to mass factors.

For the case of baryonic decays, the quark spin and W spin of the baryons are the same. The two pictures are manifestly identical for pionic decays in this case.

b) Rôle of ℓ broken $SU(6)_W$

We shall use here the fact that the $^3P_0$ picture and $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ have been proven identical for $A(L) \to B(L') + \pi$. Then let ℓ' be the vector difference of L and L'. One can label amplitudes either by ℓ' and ℓ or by L_z^i and L_z^f. The relation between the two descriptions is

$$b_{\ell',\ell} = \sum_{L_z^i L_z^f} (L L_z^i \, \ell' \, L_z^f - L_z^i | L' L_z^f) \times (\ell' \, L_z^f - L_z^i \; 1 \; L_z^i - L_z^f | \ell \, 0) \, b^{L_z^f L_z^i} \tag{4.5}$$

Note that in the second coefficient the fact that the $^3P_0$ pair has $L = 1$ has been used. This restriction would not be apparent in the $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ language.

In naïve $SU(6)_W$ one has only $L_z^f - L_z^i = \Delta L_z = 0$. The amplitudes $a_{\ell' = \ell}$ all vanish in this case since the second Clebsch-Gordan coefficient is $(\ell 010 | \ell 0) = 0$. Moreover, in naïve $SU(6)_W$, the amplitudes $a_{\ell' = \ell \pm 1, \ell}$ for various ℓ have relations among them. These are discarded in ℓ broken $SU(6)_W$, and the amplitudes $a_{\ell' = \ell \pm 1, \ell}$ treated as free parameters. On the other hand, one never acquires amplitudes of the form $a_{\ell' = \ell, \ell}$. Such amplitudes are expected to be present in the $^3P_0$ picture, however, and thus give rise to additional degrees of freedom.

Exceptions occur in two cases: 1) for $L = L'$, one has $b^{L_z^i L_z^f} = b^{L_z^f L_z^i}$. The sum over L_z^i and L_z^f in Eq. (4.5) then yields no net contribution to $a_{\ell' = \ell, \ell}$ even in $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$; 2) for $L = 0$ $(L' = 0)$ one has only $\ell' = L'$ $(\ell' = L)$ and since $\ell = L' \pm 1$ $(L \pm 1)$ by parity conservation, the amplitude $a_{\ell' = \ell, \ell}$ cannot occur.

Thus, for decays $A(L) \to B(L') + \pi$, ℓ broken $SU(6)_W$ is always a special case of the $^3P_0$ picture [and hence of $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$].

It coincides with the latter only when 1) $L = L'$ or 2) L or L' is zero. The last case applies to all previous phenomenological studies [14)-19)].

c) SU(3) × SU(3) subgroups

The predictions of Tables II-V for chiral $SU(3) \times SU(3)$ relate only amplitudes of a given helicity to one another. If one decomposes helicity states directly into representations of $SU(6)_W \times O(2)_{L_z}$, as we have done, we obtain $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$.

There is another way of obtaining the latter symmetry from the former. In the decays $L = 1 \to L = 0 + \pi$ (for mesons) if one adjoins the predictions of the coplanar $U(3) \times U(3)$ [24)] to those of chiral $U(3) \times U(3)$ [13)] one obtains those of $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ [15)].

To see this explicitly we compare in Table VII the predictions of the two $U(3) \times U(3)$ symmetries, expressed in terms of relations for helicity amplitudes. The coplanar symmetry links together predictions of chiral symmetry for different helicity amplitudes in just such a way as to give $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$.

This relation should be true in general. If we form the algebra containing both the coplanar generators and the chiral generators, it closes upon $SU(6)_W$. The selection rules for pionic transitions are, moreover, the same as in $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$. Since the pion remains bilinear in quarks, it must belong to a $\underset{\sim}{35}$, while the ΔL_z properties have already been defined at the lower symmetry level.

If $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ turns out not to hold, one may thus be able to pin the blame either on coplanar or chiral $SU(3) \times SU(3)$, with the other symmetry continuing to hold. Table VII forms the basis for one such exercise [which is not, however, demanded by present data, since both $SU(3) \times SU(3)$'s seem valid].

5. CONCLUSIONS

In this paper we have explored some consequences of abstracting certain algebraic properties of the Melosh transformation which relates the generators of $SU(6)_{W,\text{strong}}$ and $SU(6)_{W,\text{currents}}$. When PCAC is used to relate pionic decays of resonances to one-particle matrix elements of the axial charge, one obtains a model for the $SU(6)_W$ properties of these pionic transitions. With this model we have understood why a naïve application of $SU(6)_W$ to vertices should fail whenever $\Delta L_z \neq 0$ transitions are possible. Furthermore, this PCAC model has been shown equivalent to the $^3P_0$ model for decays, which has had some phenomenological success. We note, however, that the PCAC model is only formulated to apply to vertices involving a pseudoscalar meson: the $^3P_0$ model has also been applied to transitions involving vector mesons [32]. The application of the Melosh transformation to such decay processes is an interesting open question.

Another phenomenological version of $SU(6)_W$, ℓ broken $SU(6)_W$, has been shown to differ in general from $SU(6)_W$ $[\Delta L_z = 0, \pm 1]$ although in many simple cases the two prescriptions give the same results.

The application of the Melosh transformation thus provides a theoretical basis for understanding the successes of the $^3P_0$ model. The implications of such a transformation for photoproduction processes, vector dominance, and for current induced processes in general, are presently under investigation.

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TABLE I

a) Classification of $|SS_z>$ states of $(\underset{\sim}{6},\underset{\sim}{\bar{6}})$ according to $U(3)\times U(3)$ and $SU(6)_W$ [+)]

	$U(3)\times U(3)$	$SU(6)_W$
$S=1,\quad S_z=1$	$(\underset{\sim}{3},\underset{\sim}{\bar{3}})_1$	$(\underset{\sim}{8},3)_1$
$S_z=0$	$[(\underset{\sim}{8},\underset{\sim}{1})_0+(\underset{\sim}{1},\underset{\sim}{8})_0]/\sqrt{2}$	$-(\underset{\sim}{8},1)_0$
$S_z=-1$	$(\underset{\sim}{\bar{3}},\underset{\sim}{3})_{-1}$	$-(\underset{\sim}{8},3)_{-1}$
$S=0,\quad S_z=0$	$[(\underset{\sim}{8},\underset{\sim}{1})-(\underset{\sim}{1},\underset{\sim}{8})_0]/\sqrt{2}$	$-(\underset{\sim}{8},3)_0$

b) J^{PC} of states considered here

	S = 1	S = 0
L = 0	1^{--}	0^{-+}
L = 1	$2^{++},\ 1^{++},\ 0^{++}$	1^{+-}
L = 2	$3^{--},\ 2^{--},\ 1^{--}$	2^{-+}

+) $SU(6)_W$ representations are written as $[\alpha_{SU(3)},2W+1]_{W_z}$

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TABLE II

$L = 1 \rightarrow L = 0$

$\lambda = 1$	Process	$a^{00}_{\lambda=1}$	$a^{01}_{\lambda=1}$
	$2^{++} \rightarrow 1^{--}$	$+3/16$	$-1/4\sqrt{2}$
	$1^{++} \rightarrow 1^{--}$	$+3/16$	$1/4\sqrt{2}$
	$1^{+-} \rightarrow 1^{--}$	0	$1/4$
$\lambda = 0$		$a^{00}_{\lambda=0}$	$a^{01}_{\lambda=0}$
	$2^{++} \rightarrow 0^{-+}$	$+\sqrt{3}/8$	$-1/2\sqrt{6}$
	$1^{++} \rightarrow 1^{--}$	0	$1/2\sqrt{2}$
	$1^{+-} \rightarrow 1^{--}$	$+3/8\sqrt{2}$	0
	$0^{++} \rightarrow 0^{-+}$	$-(1/8)\sqrt{3/2}$	$-1/2\sqrt{3}$

Dictionary

$SU(6)_W$ $[\Delta L_z = 0, \pm 1]$: $a_{\lambda}^{L_f L_i} = a^{L_f L_i}$ for all λ

ℓ broken $SU(6)_W$: equivalent to

$SU(6)_W$ $[\Delta L_z = 0, \pm 1]$

$a_S \equiv a^{00} + 4\sqrt{2}\, a^{01}/3 \quad (\ell = 0)$

$a_D \equiv a^{00} - 2\sqrt{2}\, a^{01}/3 \quad (\ell = 2)$

"Naïve" $SU(6)_W$: $a^{01} = 0, \quad a_S = a_D$

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TABLE III

$L = 2 \rightarrow L = 0$

$\lambda = 1$	Process	$a^{00}_{\lambda=1}$	$a^{01}_{\lambda=1}$
	$3^{--} \rightarrow 1^{--}$	$-1/2\sqrt{10}$	$+(1/4)\sqrt{3/5}$
	$2^{--} \rightarrow 1^{--}$	$-1/4\sqrt{2}$	$-\sqrt{3}/16$
	$2^{-+} \rightarrow 1^{--}$	0	$-3/8\sqrt{2}$
	$1^{--} \rightarrow 1^{--}$	$-1/4\sqrt{10}$	$-(3/16)\sqrt{3/5}$
$\lambda = 0$		$a^{00}_{\lambda=0}$	$a^{01}_{\lambda=0}$
	$3^{--} \rightarrow 0^{-+}$	$-(1/4)\sqrt{3/5}$	$+3/4\sqrt{10}$
	$2^{--} \rightarrow 1^{--}$	0	$-3/8$
	$2^{-+} \rightarrow 1^{--}$	$-1/4$	0
	$1^{--} \rightarrow 0^{-+}$	$+1/2\sqrt{10}$	$+(3/4)\sqrt{3/20}$

Dictionary

$SU(6)_W$ $[\Delta L_z = 0, \pm 1]$: $a_\lambda^{L_f L_i} = a^{L_f L_i}$ for all λ

ℓ broken $SU(6)_W$: equivalent to
$SU(6)_W$ $[\Delta L_z = 0, \pm 1]$

$a_P \equiv a^{00} + (3/2)\sqrt{3/2}\, a^{01}$ $(\ell = 1)$

$a_F \equiv a^{00} - \sqrt{3/2}\, a^{01}$ $(\ell = 3)$

"Naïve" $SU(6)_W$: $a^{01} = 0$, $a_P = a_F$

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TABLE IV

$L = 1 \rightarrow L = 1$

$\lambda = 2$	Process	$a^{00}_{\lambda=2}$	$a^{11}_{\lambda=2}$	$a^{10}_{\lambda=2}$
	$2^{++} \rightarrow 2^{++}$	0	$-1/4$	0
$\lambda = 1$		$a^{00}_{\lambda=1}$	$a^{11}_{\lambda=1}$	$a^{10}_{\lambda=1}$
	$2^{++} \rightarrow 2^{++}$	$-1/8$	$-\mu/2$	$-3/8\sqrt{2}$
	$2^{++} \rightarrow 1^{++}$	$-1/8$	$+\mu/2$	0
	$2^{++} \rightarrow 1^{+-}$	0	$-1/4\sqrt{2}$	$+3/16$
	$1^{++} \rightarrow 2^{++}$	$-1/8$	$+\mu/2$	0
	$1^{++} \rightarrow 1^{++}$	$-1/8$	$-\mu/2$	$+3/8\sqrt{2}$
	$1^{++} \rightarrow 1^{+-}$	0	$+1/4\sqrt{2}$	$+3/16$
	$1^{+-} \rightarrow 2^{++}$	0	$-1/4\sqrt{2}$	$+3/16$
	$1^{+-} \rightarrow 1^{++}$	0	$+1/4\sqrt{2}$	$+3/16$
	$1^{+-} \rightarrow 1^{+-}$	0	$-\mu$	0
$\lambda = 0$		$a^{00}_{\lambda=0}$	$a^{11}_{\lambda=0}$	$a^{10}_{\lambda=0}$
	$2^{++} \rightarrow 1^{++}$	0	$-1/4\sqrt{3}$	$+(1/4)\sqrt{3/2}$
	$2^{++} \rightarrow 1^{+-}$	$-1/2\sqrt{6}$	0	$-\sqrt{3}/8$
	$1^{++} \rightarrow 2^{++}$	0	$-1/4\sqrt{3}$	$+(1/4)\sqrt{3/2}$
	$1^{++} \rightarrow 0^{++}$	0	$-1/2\sqrt{6}$	$-\sqrt{3}/8$
	$1^{+-} \rightarrow 2^{++}$	$-1/2\sqrt{6}$	0	$-\sqrt{3}/8$
	$1^{+-} \rightarrow 0^{++}$	$+1/4\sqrt{3}$	0	$-(1/4)\sqrt{3/2}$
	$0^{++} \rightarrow 1^{++}$	0	$-1/2\sqrt{6}$	$-\sqrt{3}/8$
	$0^{++} \rightarrow 1^{+-}$	$+1/4\sqrt{3}$	0	$-(1/4)\sqrt{3/2}$

Dictionary, Table IV

$SU(6)_W$ $[L_z = 0, \pm 1]$: $a^{L_f L_i} = a^{L_f L_i}$ for all
$= 0.$

ℓ broken $SU(6)_W$: equivalent to
$SU(6)_W$ $[\Delta L_z = 0, \pm 1]$

$a_P^0 \equiv a^{00} - (3/\sqrt{2})a^{10}$ two independent

$a_P^1 \equiv a^{11} + (3/2\sqrt{2})a^{10}$ $\ell = 1$ combinations

$a_F \equiv a^{00} - a^{11} + (3/\sqrt{2})a^{10}$ $(\ell = 3)$

"Naïve" $SU(6)_W$: $a^{10} = 0$; $a_F = a_P^0 - a_P^1$

TABLE V

$L = 2 \rightarrow L = 1$

$\lambda = 2$	Process	$a^{00}_{\lambda=2}$	$a^{11}_{\lambda=2}$	$a^{10}_{\lambda=2}$	$a^{01}_{\lambda=2}$	$a^{12}_{\lambda=2}$
	$3^{--} \rightarrow 2^{++}$	0	$+\sqrt{3}/8$	0	0	$-1/4\sqrt{3}$
	$2^{--} \rightarrow 2^{++}$	0	$+(1/8)\sqrt{3/2}$	0	0	$+1/2\sqrt{6}$
	$2^{-+} \rightarrow 2^{++}$	0	0	0	0	$+1/4$
$\lambda = 1$		$a^{00}_{\lambda=1}$	$a^{11}_{\lambda=1}$	$a^{10}_{\lambda=1}$	$a^{01}_{\lambda=1}$	$a^{12}_{\lambda=1}$
	$3^{--} \rightarrow 2^{++}$	$+3/8\sqrt{10}$	$-\mu\sqrt{4/15}$	$+1/4\sqrt{5}$	$-1/2\sqrt{15}$	$-1/4\sqrt{30}$
	$3^{--} \rightarrow 1^{++}$	$+3/8\sqrt{10}$	$+\mu\sqrt{4/15}$	$-1/4\sqrt{5}$	$-1/2\sqrt{15}$	$+1/4\sqrt{30}$
	$3^{--} \rightarrow 1^{+-}$	0	$+1/4\sqrt{3/5}$	$-1/2\sqrt{10}$	0	$-1/4\sqrt{15}$
	$2^{--} \rightarrow 2^{++}$	$+3/16\sqrt{2}$	$+\mu/\sqrt{12}$	$+1/8$	$+1/8\sqrt{3}$	$+1/4\sqrt{6}$
	$2^{--} \rightarrow 1^{++}$	$+3/16\sqrt{2}$	$-\mu/\sqrt{12}$	$-1/8$	$+1/8\sqrt{3}$	$-1/4\sqrt{6}$
	$2^{--} \rightarrow 1^{+-}$	0	$-\sqrt{3}/16$	$-1/4\sqrt{2}$	0	$+1/4\sqrt{3}$
	$2^{-+} \rightarrow 2^{++}$	0	$+3/16$	0	$+1/4\sqrt{2}$	0
	$2^{-+} \rightarrow 1^{++}$	0	$-3/16$	0	$+1/4\sqrt{2}$	0
	$2^{-+} \rightarrow 1^{+-}$	0	$-\mu$	0	0	0
	$1^{--} \rightarrow 2^{++}$	$+3/16\sqrt{10}$	$+\mu\sqrt{3/20}$	$+1/8\sqrt{5}$	$+(1/8)\sqrt{3/5}$	$-(1/4)\sqrt{3/10}$
	$1^{--} \rightarrow 1^{++}$	$+3/16\sqrt{10}$	$-\mu\sqrt{3/20}$	$-1/8\sqrt{5}$	$+(1/8)\sqrt{3/5}$	$+(1/4)\sqrt{3/10}$
	$1^{--} \rightarrow 1^{+-}$	0	$-(3/16)\sqrt{3/5}$	$-1/4\sqrt{10}$	0	$-(1/4)\sqrt{3/5}$
$\lambda = 0$		$a^{00}_{\lambda=0}$	$a^{11}_{\lambda=0}$	$a^{10}_{\lambda=0}$	$a^{01}_{\lambda=0}$	$a^{12}_{\lambda=0}$
	$3^{--} \rightarrow 1^{++}$	0	$+3/8\sqrt{5}$	$-(1/2)\sqrt{3/10}$	0	0
	$3^{--} \rightarrow 1^{+-}$	$+(3/8)\sqrt{3/10}$	0	0	$-1/2\sqrt{5}$	0
	$2^{--} \rightarrow 2^{++}$	0	$+(1/8)\sqrt{3/2}$	0	$+1/2\sqrt{3}$	0
	$2^{--} \rightarrow 0^{++}$	0	$+\sqrt{3}/8$	0	$-1/2\sqrt{6}$	0
	$2^{-+} \rightarrow 2^{++}$	$+\sqrt{3/8}$	0	$+1/2\sqrt{6}$	0	0
	$2^{-+} \rightarrow 0^{++}$	$-(1/8)\sqrt{3/2}$	0	$+1/2\sqrt{3}$	0	0
	$1^{--} \rightarrow 1^{++}$	0	$+(3/8)\sqrt{3/10}$	$+1/2\sqrt{5}$	0	0
	$1^{--} \rightarrow 1^{+-}$	$-3/8\sqrt{5}$	0	0	$-(1/2)\sqrt{3/10}$	0

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Dictionary, Table V

$SU(6)_W \quad [\Delta L_z = 0, \pm 1]: \quad a_\lambda^{L_f L_i} = a^{L_f L_i}$ for all λ ; $\mu = 0$

ℓ broken $SU(6)_W$:

$$a^{10}\sqrt{3} + a^{01} = a^{12}\sqrt{2}$$

$$a_S \equiv a^{00} + a^{11}\sqrt{3} + (8/3)\sqrt{2}a^{10} + 4\sqrt{2/3}\, a^{01} \qquad (\ell = 0)$$

$$a_D^0 \equiv a^{00} - (4\sqrt{2}/3)a^{10} \qquad \text{(two independent}$$

$$a_D^1 \equiv a^{11} - (2\sqrt{2}/3)a^{01} \qquad \ell = 2 \quad \text{combinations)}$$

$$a_G \equiv a^{00} - (2/\sqrt{3})a^{11} + \sqrt{2}a^{10} - \sqrt{2/3}a^{01} \qquad (\ell = 4)$$

"Naïve" $SU(6)_W$: $a^{01} = a^{10} = a^{12} = 0$;

$$a_S = a_D^0 + a_D^1\sqrt{3}; \qquad a_G = a_D^0 - 2/\sqrt{3}\; a_D^1$$

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TABLE VI

SPECIFIC PARTIAL WIDTH PREDICTIONS

FOR $L=2 \rightarrow L=0+\pi$

Process type	Specific decay	$\tilde{\Gamma}$ a) a_P^2	a_F^2	Γ, MeV b) (f wave only)
$3^{--} \rightarrow 1^{--} + \pi$	$g \rightarrow \omega\pi$	0	1/105	12 (28)
$3^{--} \rightarrow 1^{--} + \pi$	$\omega_3 \rightarrow \rho\pi$	0	1/35	36
$3^{--} \rightarrow 0^{-+} + \pi$	$g \rightarrow \pi\pi$	0	1/140	40 (input)
$3^{--} \rightarrow 0^{-+} + \pi$	$g \rightarrow K\bar{K}$	0	1/280	5.8 (3)
$2^{--} \rightarrow 1^{--} + \pi$	$\rho(2^{--}) \rightarrow \omega\pi$	1/100	1/150	8
$2^{-+} \rightarrow 1^{--} + \pi$	$A_3 \rightarrow \rho\pi$	1/75	1/50	25
$1^{--} \rightarrow 1^{--} + \pi$	$\rho'(L=2) \rightarrow \omega\pi$ c)	1/180	0	
$1^{--} \rightarrow 0^{-+} + \pi$	$\rho'(L=2) \rightarrow \pi\pi$ c)	1/90	0	

Notes:

a) $\tilde{\Gamma}_\ell = \Gamma_\ell M_R^2 (p/p_0)^{-(2\ell+1)}$ where M_R = mass of the resonance, p is the final c.m. three-momentum, and p_0 is some convenient scale factor whose choice of course determines the magnitude of a_ℓ^2.

b) Predictions based on assuming $\Gamma(g \rightarrow \pi\pi) = 40$ MeV [Ref. 30)]. Experimental numbers, shown in parentheses, are based on Graham and Yoon, [Ref. 31)]. Predictions based on a PCAC kinematic factor are somewhat different and are discussed in the text.

c) Predictions for the 1^{--} states assume no mixing with the $L=0$ multiplet ("radial excitation") expected near the same mass.

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TABLE VII

COMPARISON OF $U(3) \times U(3)$ RELATIONS

FOR $L = 1 \rightarrow L = 0 + \pi$ DECAYS

COPLANAR

$$g_1(2) + g_1(1^{++}) = \sqrt{2}\,g_0(1^{+-})$$
$$g_1(2) = (\sqrt{3}/2)g_0(2)$$
$$\sqrt{2}g_1(1^{+-}) = g_0(1^{++})$$
$$\sqrt{2}g_0(2) - g_0(0) = \sqrt{3}g_0(1^{+-}) \quad \text{a)}$$

CHIRAL

$$g_1(2) + \sqrt{2}g_1(1^{+-}) = g_1(1^{++})$$
$$g_0(2) + \sqrt{2}g_0(0) = -\sqrt{3}g_0(1^{++})$$
$$\sqrt{2}g_0(2) - g_0(0) = \sqrt{3}g_0(1^{+-}) \quad \text{a)}$$

$SU(6)_W$ $[\Delta L_z = \pm 1]$ OR $^3P_0$ PICTURE OR ℓ BROKEN $SU(6)_W$ b)

$g_1(2) = (3/\sqrt{2})D$	$g_0(1^{++}) = \sqrt{2}(S-D)$
$g_0(2) = \sqrt{6}D$	$g_1(1^{++}) = \sqrt{2}(S+D/2)$
$g_0(0) = -\sqrt{3}S$	$g_0(1^{+-}) = S+2D$
	$g_1(1^{+-}) = S-D$

Notes

a) Relation common to coplanar and chiral $U(3) \times U(3)$

b) D and S are conveniently normalized D wave and S wave amplitudes. An equivalent parametrization in terms of $\Delta L_z = 0, \pm 1$ amplitudes is given in Table II.

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