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PDQ--AN IBM-704 CODE TO SOLVE THE TWO-DIMENSIONAL FEW-GROUP NEUTRON-DIFFUSION EQUATIONS

August 1957

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PDQ

AN IBM-704 CODE TO SOLVE THE TWO-DIMENSIONAL FEW-GROUP NEUTRON-DIFFUSION EQUATIONS

by

G. G. Bilodeau, W. R. Cadwell, J. P. Dorsey, J. G. Fairey, R. S. Varga

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I. INTRODUCTION

A. Group Equations

PDQ is a two-dimensional, reactor-design code for the IBM-704 computer; it finds a discrete numerical approximation to the few-group, time-independent, neutron-diffusion equations for a heterogeneous reactor in a two-dimensional rectangular region R with boundary Γ. The variables are either x-y, in rectangular coordinates, or r-z, in cylindrical coordinates. Denoting either set of coordinates by the vector $\vec{x}$, these neutron-diffusion equations are of the form:

$$(1.1) \quad \left\{ -\nabla\cdot[D_i(\vec{x})\nabla\varphi_i(\vec{x})] + \sigma_i(\vec{x})\varphi_i(\vec{x}) = \frac{\chi_i\psi(\vec{x})}{\lambda} + \Sigma^R_{i-1}(\vec{x})\varphi_{i-1}(\vec{x}) \right\}_{i=1}^{k} ,$$

where

$$(1.2) \quad \sigma_i(\vec{x}) \equiv \Sigma^a_i(\vec{x}) + \Sigma^R_i(\vec{x}) + B_z^2 D_i(\vec{x}) \quad , \quad \text{and}$$

$$(1.3) \quad \psi(x) \equiv \sum_{i=1}^{k} [\nu_i \Sigma^f_i(\vec{x})]\varphi_i(\vec{x}) \quad .$$

In addition,

$$(1.4) \quad \Sigma^R_o(\vec{x}) = \Sigma^R_k(\vec{x}) \equiv 0 \quad , \quad \text{and}$$

$$(1.5) \quad \chi_1 = 1 \text{ for } k = 1; \ \chi_k = 0, \ k>1, \ \sum_{i=1}^{k-1} \chi_i = 1 \text{ for } k > 1.$$

For the PDQ code, $k \leq 4$. Specifically, the code finds a discrete numerical approximation to the few-group, time-independent, neutron-diffusion problem corresponding to the largest (in modulus) eigenvalue λ of (1.1). The physical interpretations of the symbols above are

D = the diffusion constant,

Σ^a = the absorption cross section,

Σ^R = the removal cross section,

B_z^2 = the transverse buckling,

χ_i = the integral of the fission spectrum over the lethargy range represented by group i,

ψ = the fission source,

λ = the eigenvalue,

ν_i = the average number of neutrons produced by a fission in group i,

Σ^f = the fission cross section,

φ = the neutron flux.

B. Restrictions of the PDQ Code

The two-dimensional rectangular region R is composed of subregions R_j and C_j and interfaces γ_j. The first restriction imposed by the code is that all internal interfaces γ_j be composed of horizontal and vertical line segments only.

The regions R_j are called diffusion regions, and it is now assumed that the functions $D_i(\vec{x})$, $\sigma_i(\vec{x})$, $\Sigma_i^R(\vec{x})$ are region-wise constant. Thus, for the region R_j, (1.1) reduces to

$$(1.6) \quad \left\{-D_i(\vec{x})\nabla^2\varphi_i(\vec{x}) + \sigma_i(\vec{x})\varphi_i(\vec{x}) = \frac{\chi_i\,\psi(\vec{x})}{\lambda} + \Sigma_{i-1}^R(\vec{x})\varphi_{i-1}(\vec{x})\right\}_{i=1}^{k} .$$

Across any internal interface γ_j between two diffusion regions, the quantities $D_i(\vec{x})\nabla\varphi_i(\vec{x})$ and $\varphi_i(\vec{x})$ are continuous.

The regions C_j with boundary γ_j are called rod regions, and the neutron flux for one or more groups is not defined interior to such a region, but satisfies

$$(1.7) \quad \frac{D_i}{\varphi_i(\vec{s})}\,\frac{\partial\varphi_i(\vec{s})}{\partial n}\Bigg|_{\vec{s}\in\gamma_j} = -C_i ,$$

where C_i is a positive constant, the derivative being taken perpendicular to γ_j in the direction of C_j. For all other groups, the region is treated as a diffusion region. The only restriction on the placement of rod regions is that no two of them intersect or have a common boundary.

On the external boundary Γ of R, ψ and φ_i, where defined, satisfy the same boundary conditions on each segment of Γ. The boundary conditions which may be used are

$$(1.8) \quad \psi(\vec{s}) = \varphi_i(\vec{s}) = 0 , \quad \vec{s}\in\Gamma, \text{ or}$$

$$(1.8') \quad \frac{\partial\varphi_i}{\partial n} = \frac{\partial\psi}{\partial n} = 0 .$$

The latter condition, corresponding to a symmetry axis, is applied half an interval inward from the external boundary, so that the values of the neutron flux at a boundary point and at the first interior point are equal.

A composition number, which identifies a set of parameters D, Σ^a, Σ^R, and $\nu\Sigma^f$ for each lethargy group, is assigned to each diffusion and rod region. The same number may be assigned to several of these regions if the parameters associated with them are identical. There is no limit to the number of such regions that may be present in the rectangular region of interest, but no more than 35 composition numbers may be assigned to them.

In order to solve the differential equations numerically, a mesh of vertical and horizontal lines is imposed on the rectangular region of the plane. Since the intervals between successive mesh lines need not be constant, this mesh placement is done in such a way that all external boundaries and internal interfaces lie exactly on mesh lines. With the mesh, the unknowns in the discrete-space case are then defined to be the values of φ_i, ψ at the intersections of the horizontal and vertical line segments of the mesh. By replacing the differential conditions of (1.6) through (1.8) by difference equations in the unknowns of the discrete case, the discrete problem is thus defined. The derivation of the difference equations for the discrete case is given in the mathematical description of the next section.

The total number of interior mesh points allowed by the code depends upon the core storage of the 704 being used, as follows:

Words in Core	4096	8192	16,384	32,768
Mesh Points	1250	2500	3750	6500

During the running of a problem, the code fully utilizes the storage available before turning to the drums for auxiliary storage. For this reason, the drum transfer time for each problem is the minimum possible under the condition of core storage available.

C. Additional Features of the Code

1. Computer Equipment Required: The code requires at least one core storage unit, one drum unit consisting of four logical drums, six tape units, an on-line card reader, and an on-line printer. Additional core storage is required for problems exceeding 1250 mesh points as already indicated, but

additional drum storage cannot be utilized without changes in the code. Flux and source values are edited onto tape; and, if an off-line printer is not available, a simulator program must be used to print this tape on-line.

2. One-Group Nonhomogeneous Problems: In the case of one-group nonhomogeneous source problems,* the code solves the differential equation

$$-D_1 \nabla^2 \varphi_1 + (\Sigma_1^a + \Gamma_1 B_z^2)\varphi_1 = \frac{\chi_1 \psi}{\lambda}$$

where $\chi_1 = 1$ and λ is the input approximation λ_o. The value of ψ used at each mesh point is the numerical average of the input flux approximations for the compositions in the four quadrants about the point multiplied by the weighted average of the values of $\nu\Sigma^f$ for these four compositions. The code performs a series of iterations to find the flux corresponding to this source and then averages and edits this flux.

3. Series of Problems: An option is provided in the code whereby the converged flux values on tape at the end of one problem may be used as the input flux approximation for another problem. The two problems must be similar in that they must have the same number of groups and the same number of mesh points in each coordinate direction. If, in addition, the input parameters and material configurations of the two problems are similar, the use of this option should result in decreased running time.

The input cards for all problems must contain a flux approximation, but this is not used in the running of those problems that make use of this tape input option.

The converged flux values of one problem may not be used as the input flux approximation for more than one other problem, unless the tape is copied, because this tape is constantly updated during the running of a problem.

4. 45° Symmetry: The code will perform a special type of mesh sweep in the case of square x-y problems which are symmetric with respect to a diagonal.

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* It is recommended that the input parameter ϵ, used to stop the iterations of the problem, be made considerably smaller than the ϵ used for homogeneous problems with two or more groups. The reason for this is that there is no source or eigenvalue calculation in a one-group problem, and hence the only convergence criterion used is that linking the initial and final residuals (see Iteration Routine).

In using this special sweep, only those mesh points on and above the diagonal are considered. Each improved value obtained at a point is stored both in the location corresponding to this point and in the location corresponding to its image about the diagonal. This results in a considerable reduction in running time, since the entire mesh is updated by sweeping slightly more than half of it.

The presence of this symmetry condition does not relieve the necessity of describing in the input the material configuration of the entire mesh, nor does it increase the total number of mesh points in the rectangular array which is permitted on each size computer.

II. THE MATHEMATICAL THEORY FOR THE PDQ CODE

A. Derivation of Difference Equations and a Statement of the Discrete Space Problem

One of the assumptions of Part I was that the region R is rectangular, and a nonuniform mesh of horizontal and vertical line segments is imposed on R so that all internal and external interfaces lie exactly on mesh lines. The differential conditions of (1.6) through (1.8') are now approximated by difference equations. In two dimensions, the employed approximation to (1.6) is by the usual five-point formula. If the coordinates are the rectangular coordinates x-y, then the derivation of the five-point formulas is carried out precisely as in the report WAPD-159, [1, pp. 8-12].*

In the case of cylindrical coordinates r-z, the derivation of the five-point formulas carried out by Mr. L. Hageman is based on the method used in [1] but results in different expressions for the coefficients of the five-point formula due to the change in the coordinates. All the final five-point formulas, for both x-y and r-x geometry, are explicitly given in Section III-F.

The difference equation analog of (1.6) becomes

$$\left\{ A_i \vec{\varphi}_i = \frac{\chi_i \vec{\psi}}{\lambda} + B_i \vec{\varphi}_{i-1} \right\}_{i=1}^{k} , \quad B_1 \equiv 0 \quad , \quad \text{where} \tag{2.1}$$

$$\vec{\psi} \equiv \sum_{i=1}^{k} V_i \vec{\varphi}_i \quad , \quad \text{and} \tag{2.2}$$

$$\chi_1 = 1 \text{ for } k = 1 \quad , \quad \sum_{i=1}^{k-1} \chi_i = 1 \text{ and } \chi_k = 0 \text{ for } k > 1 \quad , \tag{2.3}$$

in either x-y or r-z geometry. The quantities A_i, B_i, and V_i are NxN matrices, where N is the total number of unknowns in the discrete case. In addition, we have for either type of geometry:

---

* Numbers in brackets correspond to references on Reference page.

(2.4)
1. If $A_\ell = \| a_{i,j}^{(\ell)} \|$, then
   a. $a_{j,j}^{(\ell)} > 0$ for all $j = 1,2,\ldots,N$; $a_{i,j}^{(\ell)} \leq 0$ for all $i \neq j$; $\ell = 1,2,\ldots,k$
   b. A_ℓ is nonsingular
   c. A_ℓ^{-1} has all its elements positive.
2. Each B_ℓ, V_ℓ is a diagonal matrix, where
   a. B_1 is the null matrix, and B_ℓ, $\ell > 1$, is a positive diagonal matrix
   b. V_ℓ is a nonnegative diagonal matrix.

In (2.4), part a. of 1. follows by construction, as do parts a. and b. of 2. Part b. of 1. is described in the following section on inner iterations, and part c. of 1. is related [4] to a lemma of Stieljes on matrices.

From (2.1), we define recursively

(2.5) $$\left\{ L_i \equiv A_i^{-1} \left\{ \chi_i I + B_i L_{i-1} \right\} \quad , \quad i = 1,2,\ldots,k \right\}$$
where L_o is the null matrix.

With the definition of the matrices L_i, the matrix problem (2.1) - (2.3) reduces to

(2.6) $$\lambda \vec{\psi} = \left(\sum_{i=1}^{k} V_i L_i \right) \vec{\psi} \quad , \text{ or } \lambda \vec{\psi} = T \vec{\psi} \quad , \text{ where}$$

(2.7) $$T \equiv \sum_{i=1}^{k} V_i L_i \quad .$$

The matrix problem analogously seeks the eigenvector $\vec{\psi}$ of T corresponding to the largest (in modulus) eigenvalue λ^* of T. This has been shown [5] by using the Perron-Frobenius theory to be well-set, and the largest eigenvalue of T is positive, simple, and its corresponding eigenvector has nonnegative components. This implies the convergence of the following iterative procedure.

(2.8)
$$T \vec{\psi}^{(n)} = \vec{S}^{(n)} = \lambda_n \vec{\psi}^{(n+1)} \quad \text{where}$$
$$\lambda_n = \frac{[T \vec{\psi}^{(n)} , \vec{\psi}^{(n)}]}{[\vec{\psi}^{(n)} , \vec{\psi}^{(n)}]} \quad ,$$

and $\vec{\psi}_o$ is assumed to be a positive vector, and

$$\lim_{n \to \infty} \vec{\psi}^{(n)} = \kappa \vec{\psi} , \quad \lim_{n \to \infty} \lambda_n = \lambda^{\#} , \tag{2.9}$$

where κ is some positive constant.

The matrix product $T\vec{\psi}^{(n)}$ defined by (2.8), is obtained implicitly by performing what are called outer iterations. This will be described in Part C of this section.

B. Inner Iterations

We base the solution of (2.1) on the following iteration scheme for either x-y or r-z geometry; see (2.8).

$$\left\{ \begin{array}{l} \left\{ A_i \vec{\varphi}_i^{(n+1)} = \dfrac{\chi_i \vec{\psi}^{(n)}}{\lambda_n} + B_{i-1} \vec{\varphi}_{i-1}^{(n+1)} \right\}_{i=1}^{k} \\ \vec{\psi}^{(n)} = \sum_{i=1}^{k} V_i \vec{\varphi}_i^{(n)} \end{array} \right. \tag{2.10}$$

The formula to be used in finding λ_n will be developed in the next section. Equations (2.10) actually are only a simplified version of the iteration scheme used in PDQ. Several modifications of (2.10) should be included; however, for the purposes of this section, the basic idea indicated by (2.10) is sufficient.

It is clear that the solution for $\vec{\varphi}_i^{(n+1)}$ involves the solution of the following type of problem

$$A\vec{x} = \vec{k} \tag{2.11}$$

for $\vec{x}$ where $\vec{k}$ and A are known; $\vec{x}$ corresponds to $\vec{\varphi}_i^{(n+1)}$.

The method used in finding $\vec{x}$ is the Young-Frankel Successive Overrelaxation Method as described in [2], [10]. Briefly, we rewrite (2.11) as

$$\vec{x} = M\vec{x} + \vec{g} , \quad DA = I - M , \quad \vec{g} = D\vec{k} \tag{2.12}$$

where D is a diagonal matrix with positive elements along the main diagonal, and M has zeros along its main diagonal. If we let $M = \| b_{ij} \|$, the method of solution of (2.12) by the Young-Frankel method can be written as the following iteration scheme:*

---

* These iterations are known as inner iterations.

$$(2.13)\quad x_i^{(n+1)} = \omega\left[\sum_{j=1}^{i-1} b_{ij}x_j^{(n+1)} + \sum_{j=i+1}^{N} b_{ij}x_j^{(n)} + g_i\right] + (1-\omega)\,x_i^{(n)}$$

where x_i, $i = 1,\ldots,N$ are the components of $\vec{x}$ and ω is the so-called over-relaxation factor of Young. If A is symmetric, positive definite, and satisfies Young's property (A)[2] then (2.13) defines a convergent sequence for $0 < \omega < 2$ and the optimum ω, ω_b, is

$$(2.14)\qquad \omega_b = \frac{2}{1+\sqrt{1-\bar{\mu}^2}}$$

where $\bar{\mu}$ is the spectral norm (i.e., the maximum of the moduli of the eigenvalues of M) of the matrix M defined above. Clearly then, each matrix A_i of (2.10) has associated with it an overrelaxation factor ω_i along with a matrix M_i. The fact that each A_i is symmetric, positive definite and satisfies property (A) has been rigorously shown for x-y geometry in [1]. However, this is not so for r-z geometry, for each A_i, as can be seen from the equations in Section III-F, is nonsymmetric. But a careful reading of Young's paper [2] indicates that for the above theory to hold, it is sufficient that A have property (A) and that M have real eigenvalues* with $\bar{\mu} < 1$. We will now proceed to show that this is indeed true for r-z geometry.

That $A = \|a_{ij}\|$ has property (A) is a consequence of the fact that the corresponding matrix $\bar{A} = \|\bar{a}_{ij}\|$ for x-y geometry satisfies property (A)** and $a_{ij} = 0$ if and only if $\bar{a}_{ij} \equiv 0$.

The proof that M has real eigenvalues will follow from the following series of lemmas. We refer to equations in Section III-F for the construction of the matrix $A = \|a_{ij}\|$. The matrix A can be written as

$$A = \begin{pmatrix} A_1 & B_{12} & 0 & \cdots & 0 \\ B_{21} & A_2 & B_{23} & \ddots & \vdots \\ 0 & B_{32} & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & B_{N,N-1} & A_N \end{pmatrix}$$

---

* The report [3, p. 23] asserts that M has real eigenvalues but does not prove it.

**See [1, p. 18].

where $A_k = (a_{ij}^{(k)})$ $\quad i,j = 1,\ldots,q$ and

$$(2.15) \qquad a_{i,j}^{(k)} = 0 \quad j \neq i,\ i \pm 1, \quad a_{i,i}^{(k)} > 0\ , \quad a_{ij}^{(k)} < 0 \quad j = i \pm 1\ .$$

A_k is the matrix associated with the $k^{\underline{th}}$ row of the mesh. The corresponding qxq matrices $B_{k,k+1}$, $B_{k,k-1}$ are diagonal matrices which contain the coefficients of the points on the $(k+1)^{\underline{st}}$ and $(k-1)^{\underline{st}}$ row, respectively, which are connected to points in the $k^{\underline{th}}$ row by the difference equations mentioned above.

Although A is not symmetric, it is true that $B_{k,k+1} = B_{k+1,k}$.

Let E_k be a qxq diagonal matrix so that

$$E_k = \|\alpha_i^{(k)}\| \ , \quad \alpha_i^{(k)} \neq 0 \quad i = 1,\ldots,q\ .$$

Then

$$E_k A_k E_k^{-1} = \|(\alpha_i^{(k)})(a_{i,j}^{(k)})(\alpha_j^{(k)})^{-1}\| \ .$$

By (2.15), this matrix is symmetric if and only if

$$(\alpha_i^{(k)})(a_{i,i+1}^{(k)})(\alpha_{i+1}^{(k)})^{-1} = (\alpha_{i+1}^{(k)})(a_{i+1,i}^{(k)})(\alpha_i^{(k)})^{-1}$$

for $i = 1,\ldots,q-1$, or, if and only if,

$$(2.16) \qquad \begin{cases} [\alpha_{i+1}^{(k)}]^2 = \dfrac{a_{i,i+1}^{(k)}}{a_{i+1,i}^{(k)}}\,[\alpha_i^{(k)}]^2 \\ \qquad\qquad = \dfrac{a_{i,i+1}^{(k)} \cdots a_{1,2}^{(k)}}{a_{i+1,i}^{(k)} \cdots a_{2,1}^{(k)}}\,[\alpha_1^{(k)}]^2 \end{cases}$$

Clearly then, we can make $E_k A_k E_k^{-1}$ symmetric by choosing E_k so that (2.16) holds with $\alpha_1^{(k)}$ an arbitrary constant. We now show

LEMMA 2.1 For any $k,\ell = 1,\ldots,N$ and any $i = 1,\ldots,q-1$

$$\frac{a_{i,i+1}^{(k)}}{a_{i+1,i}^{(k)}} = \frac{a_{i,i+1}^{(\ell)}}{a_{i+1,i}^{(\ell)}}$$

The proof of this is clear from the difference equations since the ratio $a_{i,i+1}^{(k)}/a_{i+1,i}^{(k)}$ depends only on the mesh size in the r direction, which is the same for each row.

LEMMA 2.2 For any $k,\ell = 1,\ldots,N \quad i = 1,\ldots,q-1$

$$\left[\alpha_{i+1}^{(k)}\right]^2 = \left[\alpha_{i+1}^{(\ell)}\right]^2 \text{ if and only if } \left[\alpha_1^{(k)}\right]^2 = \left[\alpha_1^{(\ell)}\right]^2$$

PROOF: This is a consequence of Lemma 2.1 and (2.16).

Since $\alpha_1^{(k)}$ is arbitrary, we can choose $\alpha_1^{(k)} = \alpha_1$, $k = 1,\ldots,N$. Then, it follows that

$$E_k = E_\ell \qquad k,\ell = 1,\ldots,N. \tag{2.17}$$

(we choose each $\alpha_i^{(k)}$ to be positive)

Moreover, by the construction of $\alpha_i^{(k)}$, $E_k A_k E_k^{-1}$ is symmetric for all $k = 1,\ldots,N$.

If we now define E to be diagonal matrix $E = \|E_k\|$ $k = 1,\ldots,N$, then we can conclude

LEMMA 2.3 The matrix EAE^{-1} is symmetric.

PROOF:

$$EAE^{-1} = \begin{pmatrix} E_1A_1E_1^{-1} & E_1B_{12}E_2^{-1} & 0 \ldots 0 \\ E_2B_{21}E_1^{-1} & E_2A_2E_2^{-1} & \ldots\ldots \\ 0 \ldots\ldots\ldots\ldots & & \\ \vdots & & \\ 0 & & \end{pmatrix}$$

To show this matrix is symmetric, it suffices to prove that

$$E_kB_{k,k+1}E_{k+1}^{-1} = E_{k+1}B_{k+1,k}E_k^{-1}$$

However, this is obvious from (2.17). Q.E.D.

We are now in a position to prove

THEOREM 2.1 Let A be the matrix arising from r-z geometry and M be the matrix defined by (2.12). Then M has real eigenvalues.

PROOF: From (2.12), $DA = I - M$ where D is a diagonal matrix. On the other hand, by Lemma 2.3, $A = E^{-1}SE$ where S is symmetric and E is diagonal. Thus,

$$DE^{-1}SE = I - M \ .$$

However, $DE^{-1} = E^{-1}D$ since both are diagonal matrices and, consequently,

$$E^{-1}(DS)E = I - M \ ,$$

so that I - M is similar to the matrix DS. But, DS is similar to $D^{\frac{1}{2}}SD^{\frac{1}{2}}$ since

$$D^{-\frac{1}{2}}(DS)D^{\frac{1}{2}} = D^{\frac{1}{2}}SD^{\frac{1}{2}} .$$

Thus, $D^{\frac{1}{2}}SD^{\frac{1}{2}}$ is symmetric, and hence DS has real eigenvalues. The same is then true for I - M. This proves the Theorem.

It remains to show that $\bar{\mu}$, the spectral norm of M, is less than one. However, this will follow from the fact that the elements of A satisfy

$$a_{ii} > \sum_{j \neq i} |a_{ij}|$$

and corollary 1 of [1, p. 23].

We can now proceed to the precise way in which the numbers $\bar{\mu}$ are computed which, by (2.14), will give rise to the overrelaxation factor needed in the inner iterations.

For x-y geometry, A is symmetric and M is obtained by dividing each row of A by the corresponding element along the main diagonal--this refers to the operation DA of (2.12). We define an iterative scheme as follows:

$$\text{(2.18)} \qquad \begin{cases} \vec{x}_{i+1} = \vec{x}_i + \Theta\vec{\xi}_i \\ \vec{\xi}_i = M\vec{x}_i - \lambda_i\vec{x}_i \\ \lambda_i = (M\vec{x}_i, \vec{x}_i) \,/\, (\vec{x}_i, \vec{x}_i) \end{cases}$$

This is a scheme proposed by Karush and Hestenes [6] to find $\bar{\mu}$, the largest eigenvalue of M.* The value of Θ for convergence is such that $0 < \Theta < \frac{2}{\beta}$ where β = spread of eigenvalues which can safely be assumed to be < 2. Thus, $0 < \Theta \leq 1$ for this problem. The value $\Theta = 1$ has turned out experimentally to be best, and this is the number used**

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* Actually it finds the largest algebraic eigenvalue. However, in this case this is $\bar{\mu}$ since the eigenvalues of M occur in ± pairs. See [2].

** The experimentation for this result was done on an IBM-650 by L. Hageman and G. Bilodeau.

Actually, this scheme has been shown to work only if M is symmetric. However, we can redefine an inner product $(\vec{x},\vec{y})$ into a weighted inner product

$$[\vec{x},\vec{y}] = \sum_i a_{ii} x_i y_i$$

as mentioned in [1, p. 27]. This has the same basic effect as though M were indeed symmetric.

The main advantage of this scheme lies in the fact that the sequence λ_i is a conservative estimate in that $\lambda_i \leq \bar{\mu}$ and, moreover, λ_i forms an increasing sequence. Thus, λ_i is always a good lower bound for $\bar{\mu}$. An upper bound is obtained as described in [1, p. 23], and the convergence of the sequence of eigenvalues is accelerated by the use of Aitken's δ^2 process defined in [1, p. 28].

For r-z geometry, A is no longer symmetric, and we must proceed in a slightly different fashion. For this case, the scheme defined in [1, pp. 27-30] is used.

More details of the actual techniques in the code will be found in the description of the routines in a later section; however, the basic mathematics mentioned above remains unchanged.

C. Outer Iterations

The outer iteration system has been defined in both the equivalent forms (2.9) and (2.10). However, the convergence rate has been accelerated by using the following method. With the notation of (2.9),

$$T\vec{\psi}^{(n)} = \lambda_n \vec{\psi}^{(n+1)}_{\text{☆}}$$

$$\vec{\psi}^{(n+1)} = (1 + \beta_n)\vec{\psi}^{(n+1)}_{\text{☆}} - \beta_n \vec{\psi}^{(n)} \qquad (2.19)$$

$$\lambda_{n+1} = \left[T\vec{\psi}^{(n)},\ T\vec{\psi}^{(n)} \Big/ T\vec{\psi}^{(n)},\ \vec{\psi}^{(n)} \right]$$

We choose the sequence $\beta_\ell = \beta_\ell(m)$ $\ell = 1,\ldots,m$ which minimizes* the absolute value of the difference of λ_n and the converged value, i.e., the largest eigenvalue of T, in m iterations. The underlying theory, based on Chebyshev polynomials, is described in [7].

---

* In the sense as defined in [7, p. 17].

$$(2.20) \qquad \beta_\ell(m) = \frac{\sigma_{k-1} + \sigma_1 + (\sigma_{k-1} - \sigma_1)\cos\frac{2\ell - 1}{2m}\pi}{2\lambda_\ell - \sigma_{k-1} - \sigma_1 - (\sigma_{k-1} - \sigma_1)\cos\frac{2\ell - 1}{2m}\pi}$$

for $\ell = 1,\ldots,m$ where $\sigma_1 < \sigma_2 < \ldots < \sigma_k$ are the distinct eigenvalues of T.

To justify the formula (2.20), we must assume

(a) eigenvalues of T are real and nonnegative;

(b) eigenvectors of T span the vector space associated with T.

On the basis of (a), we set $\sigma_1 = 0$ and also assume that λ_ℓ is very nearly σ_k (the converged value of $\left\{\lambda_\ell\right\}_{\ell=1}^{\infty}$). Then, letting $\bar{\sigma} = \sigma_{k-1}/\sigma_k$, we get*

$$(2.21) \qquad \beta_\ell(m) = \frac{\bar{\sigma}(1 + \cos\frac{2\ell - 1}{2m}\pi)}{2 - \bar{\sigma}(1 + \cos\frac{2\ell - 1}{2m}\pi)} \quad , \quad \ell = 1,2,\ldots,m$$

which is the formula used in PDQ.

The method of getting $\bar{\sigma}$ was introduced and justified in [8]; it is also described in [9].

The amount of time and the number of outer iterations saved by using the scheme of (2.19) over that of (2.8) are discussed with the use of many typical problems in [9] which also contains a detailed account of the use of these Chebyshev polynomials in the PDQ code of which this report is only a brief outline.

The criterion for stopping the outer iterations is based on the two numbers $\underline{\lambda}_n$ and $\bar{\lambda}_n$ which are, respectively,

$$\min_i \frac{(T\vec{\psi}^{(n)})_i}{(\vec{\psi}^{(n)})_i} \quad , \quad \max_i \frac{(T\vec{\psi}^{(n)})_i}{(\vec{\psi}^{(n)})_i} \quad .$$

If $\frac{\bar{\lambda}_n - \underline{\lambda}_n}{2\lambda_n} \leq \epsilon^2$ where ϵ is an input parameter, then the outer iterations come to an end and the problem is done. It has been shown in [1, p. 22] that the converged eigenvalue σ_k is such that, for all n,

$$\underline{\lambda}_n \leq \sigma_k \leq \bar{\lambda}_n \quad .$$

---

* The quantity $\bar{\sigma}$ is defined as the Homogeneous Spectral Norm in [8].

Although the scheme of (2.19) is still not the same that was used in [1], the result still holds and for the same reasons.

We conclude this section with a comment on the scheme defined by (2.19). It has been found useful after each outer iteration to renormalize the vector $\vec{\psi}$ in the following way. Instead of the second equation in (2.19), we now have

$$\vec{\psi}^{(n+1)} = \frac{\left(\frac{\lambda_{n+1}}{\lambda_n}\right) \| \vec{\psi}^{(n)} \|}{(1+\beta_n) \| \vec{\psi}_A^{(n+1)} \| - \beta_n \| \vec{\psi}^{(n)} \|} \left\{ (1+\beta_n) \vec{\psi}_A^{(n+1)} - \beta_n \vec{\psi}^{(n)} \right\} .$$

This does not affect the generation of the Chebyshev polynomials which arise from the $\beta_n \equiv \beta_n(m)$.

D. Additional Mathematical Features

Two special features of the PDQ code will be described now. The first feature is called the negative flux check in each group. From (2.1), we have in each group i,

$$A_i \vec{\varphi}_i = \vec{S}_i \text{ , where } \vec{S}_i = \frac{\chi_i \vec{\psi}}{\lambda} + B_i \vec{\varphi}_{i-1} \quad . \tag{2.22}$$

In assuming only that the initial estimate $\vec{\psi}_o$ of the fission source is nonnegative, then $\vec{S}_i$, corresponding to the source of neutrons for the i-th lethargy group, is always a nonnegative vector. By Part 1.c. of (2.4), we have that A_1^{-1} is a positive matrix, so that the solution of the above equation

$$\vec{\varphi}_i = A_i^{-1} \vec{S}_i \tag{2.23}$$

is clearly a positive vector. In two dimensions, the matrix A_i is not inverted directly, but rather an iterative procedure for finding the unique solution (2.23) is employed. Since only a finite number of iterations is performed in any lethargy group, it is possible for the final vector approximation to the solution of (2.23) to have some negative components.* An internal check has been placed in the code which automatically forces the inner iteration procedure to produce a final approximate vector solution of (2.23) with positive components.

The second feature of the code concerns problems having 45° symmetry. In these problems, the inner iterations treat only the points on or above the 45°

* If the inner iterations corresponded to iteration by nonnegative matrices, and all initial flux guesses were positive, then no negative components could occur in the vector iterates approximating the solution of (2.23). But the iteration described in part B above in general corresponds to matrices of mixed signs [5, §12].

symmetry axis. Obviously, since the number of points which are necessarily swept in this way is decreased approximately by a factor of two as compared with the total number of points, this option results in faster iteration time. It remains to show that an optimum overrelaxation factor ω can be accurately estimated for this new iterative treatment, as for the general method of Section II-B.

Using the notation of Part B above, let

$$DA = I - M \quad , \tag{2.24}$$

where D is the NxN positive diagonal matrix with diagonal entries $\frac{1}{a_{i,i}}$; $A_\ell = \|a_{i,j}\|$ is the matrix of (2.4); and M, the iteration matrix, is the matrix defined above. Previously, use has been made of the fact that M is nonnegative and transitive,* which guarantees the existence of a positive eigenvector $\vec{v}$ of M such that $M\vec{v} = \bar{\mu}\vec{v}$, and $\bar{\mu}$ is greater than or equal to the moduli of all other eigenvalues of M. From this, by using (2.14), the optimum ω is obtained for the iteration procedure derived from A.

If the components of $\vec{v}$ are denoted by $v_{i,j}$, where (i,j) refer to the mesh coordinate of a point, then, by using the fact that the matrix problem possesses 45° symmetry, it follows that

$$v_{i,j} = v_{j,i} \quad . \tag{2.25}$$

Thus, $\tilde{M}$, the iteration matrix associated with sweeping only those points on or above the 45° symmetry axis, which is also nonnegative and transitive, possesses, by virtue of (2.25), the same dominant eigenvalue as M, namely $\bar{\mu}(M)$, and hence the optimum overrelaxation factor associated with $\tilde{M}$ is precisely the same as the one associated with M. Thus, the ω-routine of Part B applies equally well to the case of 45° symmetry.

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* See [5, §4].

III. CODE ROUTINES

A. Restart Routine

Operator initiation of a restart at any time during the running of a problem causes the contents of the title card to be printed on-line and the problem to be continued from its most recent restart point. There is such a point at the beginning of each routine in the code and, in the iteration routine, at the beginning of the calculation in each group and at the beginning of each source calculation. If a restart is attempted before the first restart point is reached, the routine stops, ready to read in the input routine and begin the problem again.

B. Input Routine

The contents of the first input card (title card) are read, printed on-line, and stored for use in identifying off-line output and restarts. If columns 59-60 and 68-72 are not properly punched (see Title Card), the routine stops after restoring the printer without reading the remaining cards.

The input itself is checked to make sure that it violates none of the restrictions given under Input Card Format. In addition, each number is checked to determine whether it was properly specified for fixed- or floating-point conversion. If an error is detected, the routine prints "Requestor Error - Check Card No. XXXX," restores the printer, and stops. The card number printed is either the number of the first card in the input containing an error or a card number which the routine expected and could not find. (In many cases, the card number printed is one greater than the number of the card containing an error.) When an error is found, the remainder of the input is not checked. Conversion from card image to BCD and from BCD to binary is never checked.

C. Input Print Routine

All the numbers in the 1000, 2000, 3000, and 4000 (if present) input series are edited on-line to form a record of the problem solved. The output of this routine, together with that of the picture routine, gives sufficient information to reconstruct the entire problem input.

D. Expander Routine

The composition description given in the 5000 input series is expanded by using the overlay process explained under Input Card Format. This expansion results in one word per mesh point, giving the four compositions surrounding the

point. After the entire composition description has been expanded, the routine checks the resulting set of words and stops if there is any region of the mesh for which no composition has been specified.

E. Picture Routine

A picture of the specified composition layout of the problem is generated on-line by the picture routine. The routine labels compositions 1,2,...,9, A,B,...,Z, corresponding to input composition numbers 1 through 35. A dotted line is shown along each interface and on the four sides. The mesh spacing is not indicated.

The printed picture is limited by the width of the paper to 58 columns. In the case of a problem description exceeding this width, the first 58 columns are printed with no right-hand boundary line indicated. The routine will print the full 115 rows permitted by the code.

F. Coefficient Routine

The difference equations solved by PDQ are as follows:

$$\alpha_1\varphi(x_o,\ y_o + h_1) + \alpha_2\varphi(x_o - h_2,\ y_o) + \alpha_3\varphi(x_o,\ y_o - h_3) + \alpha_4\varphi(x_o + h_4,\ y_o)$$

$$-\alpha_o\varphi(x_o,\ y_o) + \alpha_5\left(\frac{\chi_i\psi}{\lambda}\right) + \alpha_6\varphi_{i-1}(x_o,\ y_o) = 0$$

where, in the general case, in rectangular coordinates,

$$\alpha_1 = \frac{D_1h_2 + D_4h_4}{2h_1}$$

$$\alpha_2 = \frac{D_2h_3 + D_1h_1}{2h_2}$$

$$\alpha_3 = \frac{D_3h_4 + D_2h_2}{2h_3}$$

$$\alpha_4 = \frac{D_4h_1 + D_3h_3}{2h_4}$$

$$\alpha_o = \frac{D_1h_2 + D_4h_4}{2h_1} + \frac{D_2h_3 + D_1h_1}{2h_2} + \frac{D_3h_4 + D_2h_2}{2h_3} + \frac{D_4h_1 + D_3h_3}{2h_4} + \sigma_1\frac{h_1h_2}{4}$$

$$+ \sigma_2\frac{h_2h_3}{4} + \sigma_3\frac{h_3h_4}{4} + \sigma_4\frac{h_4h_1}{4}, \quad \text{where } \sigma_\ell = \Sigma_\ell^a + \Sigma_\ell^R + D_\ell B_z^2 ,$$

$$\alpha_5 = \frac{h_1 h_2}{4} + \frac{h_2 h_3}{4} + \frac{h_3 h_4}{4} + \frac{h_4 h_1}{4}$$

$$\alpha_6 = \Sigma^R_{1,i-1}\left(\frac{h_1 h_2}{4}\right) + \Sigma^R_{2,i-1}\left(\frac{h_2 h_3}{4}\right) + \Sigma^R_{3,i-1}\left(\frac{h_3 h_4}{4}\right) + \Sigma^R_{4,i-1}\left(\frac{h_4 h_1}{4}\right) \quad .$$

Subscripts apply as indicated in Fig. A. Where i-1 is noted as a subscript, the appropriate value from the preceding group is intended.

This is the general equation for all lethargy groups. It is to be noted, then, that Σ^R is defined as zero for group zero and for the last group; and χ is zero, so that α_5 is set to zero for the thermal group.

Zero flux boundaries are handled by setting the appropriate α (α_1, α_2, α_3, α_4, or any appropriate combination of two of these) to zero. Zero derivative boundary conditions are applied by setting the appropriate α to zero, as above, and neglecting the analogous term in the computation of α_o; this, effectively, sets the flux on the descriptive boundary equal to that at the first interior point.

If the point (x_o, y_o) lies on a boundary along which the logarithmic derivative condition is to be applied, Σ^R, Σ^a, and D in the rod region are set to zero; and C, the logarithmic boundary condition value, is multiplied by half the sum of the mesh increments on the interface, the product being added to α_o.

In the case of cylindrical coordinates, x and y in the above equations are replaced by r and z, respectively; and α_2, α_4, and α_o differ as follows:

$$\alpha_2 = \frac{D_2 h_3 + D_1 h_1}{2h_2} - \frac{D_2 h_3 + D_1 h_1}{4\left(r - \frac{h_2}{2}\right)}$$

$$\alpha_4 = \frac{D_4 h_1 + D_3 h_3}{2h_4} + \frac{D_4 h_1 + D_3 h_3}{4\left(r + \frac{h_4}{2}\right)}$$

$$\alpha_o = \frac{D_1 h_2 + D_4 h_4}{2h_1} + \frac{D_2 h_3 + D_1 h_1}{2h_2} - \frac{D_2 h_3 + D_1 h_1}{4\left(r - \frac{h_2}{2}\right)} + \frac{D_3 h_4 + D_2 h_2}{2h_3} + \frac{D_4 h_1 + D_3 h_3}{2h_4}$$
$$+ \frac{D_4 h_1 + D_3 h_3}{4\left(r + \frac{h_4}{2}\right)} + \sigma_1 \frac{h_1 h_2}{4} + \sigma_2 \frac{h_2 h_3}{4} + \sigma_3 \frac{h_3 h_4}{4} + \sigma_4 \frac{h_4 h_1}{4} \quad .$$

The symmetry boundary condition is always applied at r = 0, so that, at points along column 1,

$$\alpha_2 = 0$$

$$\alpha_4 = \frac{D_4 h_1 + D_3 h_3}{2h_4} + \frac{D_4 h_1 + D_3 h_3}{2(h_2 + h_4)}$$

$$\alpha_o = \frac{D_1 h_2 + D_4 h_4}{2h_1} + \frac{D_3 h_4 + D_2 h_2}{2h_3} + \frac{D_4 h_1 + D_3 h_3}{2h_4} + \frac{D_4 h_1 + D_3 h_3}{2(h_2 + h_4)} + \sigma_1 \frac{h_1 h_2}{4} + \ldots$$

A zero flux boundary condition is always applied on the right-hand boundary, so that, for points along the last interior column, $\alpha_4 = 0$; the other coefficients remain unchanged.

The logarithmic derivative boundary condition is applied in r-z geometry analogously to the manner in which it is applied in the rectangular system.

In all cases, the coefficients used are those defined above normalized by the corresponding α_o.

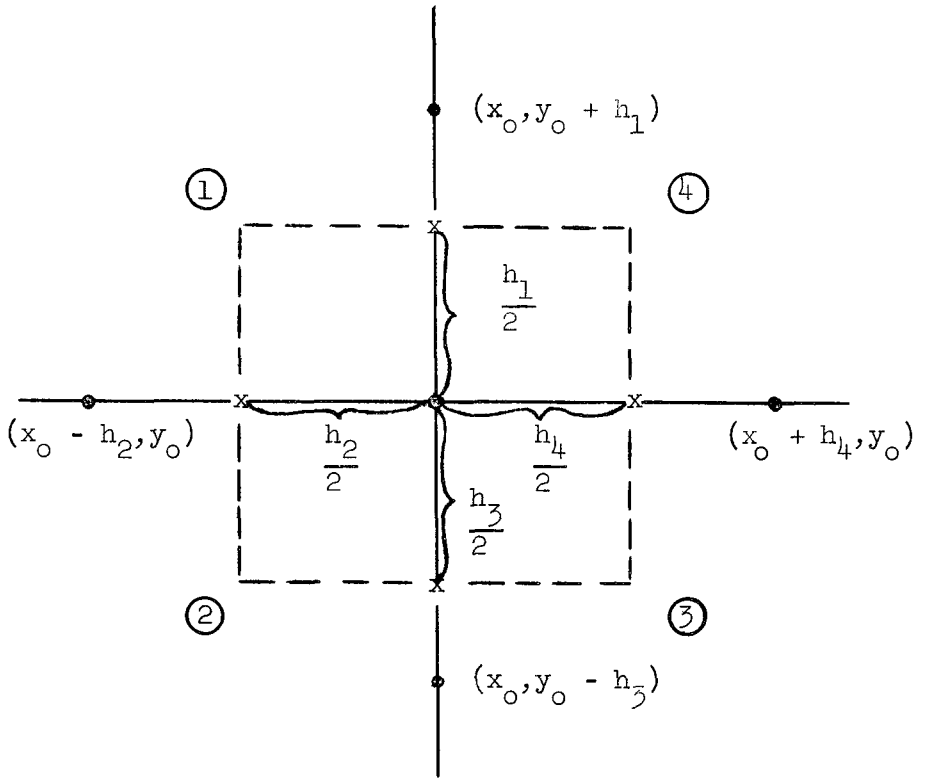


FIGURE A

G. Omega Routine

The purpose of this routine is to estimate the optimum value of omega for each group to be used as the overrelaxation factor in the spatial integration. The optimum omega is a function of $\bar{\mu}(M)$, and is given, as in (2.14), by

$$\omega = \frac{2}{1 + \sqrt{1 - [\bar{\mu}(M)]^2}} \quad . \tag{4.1}$$

The coefficient matrix A resulting from the five-point difference equations is of the form $\|a_{ij}\|$, where $a_{ii} > 0$ and $a_{ij} \leq 0$ if $i \neq j$. The matrix B is related to A as follows:

$$-DA + I + \alpha I = M + \alpha I = B$$

where the diagonal matrix $D = \left\| \frac{1}{a_{ii}} \right\|$. The purpose of the addition of αI is to facilitate the calculation of the eigenvalue of largest absolute value of M in r-z geometry.* The code uses $\alpha = 0.001$.

Given an initial unit vector $\vec{x}^{(o)}$, the general iterative system used is

$$\begin{cases} \lambda^{(i)} + \alpha = \dfrac{[B\vec{x}^{(i)}, \vec{x}^{(i)}]}{[\vec{x}^{(i)}, \vec{x}^{(i)}]} \\ \vec{x}^{(i+1)} = \vec{x}^{(i)} + \theta \vec{\xi}^{(i)} \\ \vec{\xi}^{(i)} = B\vec{x}^{(i)} - (\lambda^{(i)} + \alpha)\vec{x}^{(i)} \end{cases} \tag{4.2}$$

where the inner product above is the weighted inner product of Section II-B. See also (2.18).

X-Y Geometry: The value θ in (4.2) is fixed at unity as mentioned previously. After ten iterations of (4.2), the following are determined for each group:

(a) A lower bound for the estimate of ω, which is (4.1) evaluated with $\bar{\mu}(M) = \lambda^{(10)}$. This is the best lower bound estimate of $\bar{\mu}(M)$, since the numbers $\lambda^{(i)}$ are increasing as mentioned in Section II-B.

(b) An upper bound for the ω estimates, which is (4.1) evaluated with $\bar{\mu}(M) = \bar{\lambda}$ where $\bar{\lambda} = \max_j \dfrac{Bx_j^{(10)}}{x_j^{(10)}} - \alpha$.

---

* Since the eigenvalues of M occur in ± pairs [2], the addition of α to the diagonal of M assures the convergence of the power method to the largest eigenvalue of $M + \alpha I$. Clearly, $\bar{\mu}(M + \alpha I) = \bar{\mu}(M) + \alpha$.

(c) The first estimate of $\bar{\mu}(M)$, which consists of the quotient

$$\frac{[\vec{Bx}^{(10)}, \vec{Bx}^{(10)}]}{[\vec{Bx}^{(10)}, \vec{x}^{(10)}]} - \alpha \equiv \pi^{(10)}.$$

(d) The last four estimates of $\bar{\mu}(M)$ are based on four successive sequences determined by the general formula:

$$\delta_{\ell}^{(i)} = \frac{\left[\delta_{\ell-1}^{(i-1)} - \delta_{\ell-1}^{(i)}\right]^2}{\left[\delta_{\ell-1}^{(i-1)} - \delta_{\ell-1}^{(i)}\right] + \left[\delta_{\ell-1}^{(i-1)} - \delta_{\ell-1}^{(i-2)}\right]} + \delta_{\ell-1}^{(i)}$$

for $\ell = 1$ through $\ell = 4$ where $\delta_{o}^{(i)} = \pi^{(i)}$. The four estimates of $\bar{\mu}(M)$ are $\delta_{\ell}^{(10)}$, which are designated SHK ℓ, $\ell = 1,2,3,4$. The sequence $\delta_{\ell}^{(i)}$ begins when $i = 2\ell + 2$ for all ℓ.

(e) A prediction of a lower and upper bound, $\underline{N}$ and $\bar{N}$, on the number of inner iterations to be done per each outer iteration in the spatial integration is determined by the relationship:

$\underline{N} \equiv$ first positive integer N such that $(\omega - 1)^N \leq \begin{cases} \max(.05,\epsilon) \text{ for a multigroup problem} \\ \epsilon \text{ for a one-group problem} \end{cases}$

and $\bar{N} \equiv$ first positive integer N such that $N(\omega-1)^{N-1} \leq \begin{cases} \max(.05,\epsilon) \text{ for a multigroup problem} \\ \epsilon \text{ for a one-group problem} \end{cases}$

where ϵ is the convergence criterion specified for the spatial integration.

R-Z Geometry: The value of Θ determined by $\frac{1}{\lambda_i + \alpha}$ is used in (4.2) which reduces (4.2) to a power method. After 15 iterations of (4.2), the values (a) through (e) above are determined with the value of (c) being an additional estimate of $\bar{\mu}(M)$.

It can be rigorously shown that $\lim_{i \to \infty} \pi^{(i)} = \bar{\mu}(M)$, and that the sequences $\delta_{\ell}^{(i)}$ will converge at a faster rate as ℓ increases. Since the nature of the convergence of $\delta_{\ell}^{(i)}$ is not precisely known, and since the loss of too many significant figures may occur due to machine limitations, a check is made by the code to insure that the value $\delta_{\ell}^{(f)}$ is acceptable. The value of f is 10 in x-y geometry and 15 in r-z geometry. The condition for acceptance is defined by $\lambda^{(f)} < \delta_{\ell}^{(f)} \leq \bar{\lambda}^{(f)}$. If $\delta_{\ell}^{(f)}$ is not accepted, the code sets the value to zero.

After having made the check, the code chooses that accepted value for which ℓ is greatest as its final estimate of $\bar{\mu}(M)$ to be used in (4.1). If none of the values is acceptable, $\lambda^{(f)}$ is used in (4.1).

The printout during the running of the omega routine includes, for each group, the values of $\lambda^{(f)}$, $\pi^{(f)}$, SHK1, SHK2, SHK3, SHK4, $\underline{\omega}$, ω, $\bar{\omega}$, $\underline{N}$, and $\bar{N}$.

H. Symmetry Setup Routine

If an "8" is punched in column 67 of the title card, this routine deletes from the matrix of coefficients all those coefficients corresponding to points below the main diagonal of the mesh. This permits the iteration routine to sweep only those points on and above this diagonal. Two tape numbers must be manually interchanged when the routine stops after completing this deletion (see Program Stops). The routine is bypassed if column 67 of the title card does not contain an "8."

I. Iteration Routine

The difference equation solved at each mesh point by the iteration routine is

$$\varphi_{i,s,t}^{(n+1)} = \omega_i \left(\alpha_1' \varphi_{i,s,t-1}^{(n+1)} + \alpha_2' \varphi_{i,s-1,t}^{(n+1)} + \alpha_3' \varphi_{i,s,t+1}^{(n)} + \alpha_4' \varphi_{i,s+1,t}^{(n)} + S_{s,t} - \varphi_{i,s,t}^{(n)} \right) + \varphi_{i,s,t}^{(n)}$$

where i is the group number, s and t the column and row numbers, n the inner iteration index, ω_i the overrelaxation factor, $\alpha_1', \alpha_2', \ldots, \alpha_6'$ the normalized coefficients, and

$$S_{s,t} = \alpha_5' \left(\frac{\chi_i \psi_{s,t}^*}{\lambda} \right) + \alpha_6' \varphi_{i-1,s,t} .$$

By using approximate values of φ_{i-1}, φ_i, and ψ^* at each mesh point and an approximate value of λ, the mesh is swept, solving the above equation at each point for an improved value of φ_i. This sweep constitutes one inner iteration. During each such iteration the residual

$$R_i^{(n)} = \sum_{s,t} \left| \varphi_{i,s,t}^{(n+1)} - \varphi_{i,s,t}^{(n)} \right|$$

is formed. Inner iterations are continued in the group, with the source and coefficients unchanged until the flux value at each point is nonnegative and until $R_i^{(n)} \leq \epsilon' R_i^{(0)}$. (Here ϵ' is the larger of 0.05 and the input ϵ except in a one-group problem, where $\epsilon' = \epsilon$.) At this point calculation in group i is terminated, and the inner iteration cycle for group i+1 is begun with the latest values of φ_i and approximate values of φ_{i+1}.

After a set of inner iterations has been performed in each group, the flux values obtained are used to calculate a new source approximation by the formula

$$\psi_{s,t} = \sum_{i=1}^{k} (\nu\Sigma^f)_{i,s,t}\, \varphi_{i,s,t} \quad .$$

The following quantities are then calculated, where m is the outer iteration index and $\varphi^{(m+1)}$ the factor used to extrapolate the source calculation of iteration m+1:

$$\bar{\lambda}^{(m+1)} = \lambda^{(m)} \operatorname*{Max}_{s,t}\left[\frac{\psi_{s,t}^{(m+1)}}{\psi_{s,t}^{*(m)}}\right]$$

$$\underline{\lambda}^{(m+1)} = \lambda^{(m)} \operatorname*{Min}_{s,t}\left[\frac{\psi_{s,t}^{(m+1)}}{\psi_{s,t}^{*(m)}}\right]$$

$$\lambda^{(m+1)} = \lambda^{(m)} \frac{[\vec{\psi}^{(m+1)} \cdot \vec{\psi}^{(m+1)}]}{[\vec{\psi}^{*(m)} \cdot \vec{\psi}^{(m+1)}]}$$

$$K^{(m+1)} = \frac{\dfrac{\lambda^{(m+1)}}{\lambda^{(m)}} \left\| \vec{\psi}^{*(m)} \right\|}{[1 + \Theta^{(m+1)}] \left\| \vec{\psi}^{(m+1)} \right\| - \Theta^{(m+1)} \left\| \vec{\psi}^{*(m)} \right\|}$$

$$\psi_{s,t}^{*(m+1)} = K^{(m+1)} \left\{ [1 + \Theta^{(m+1)}]\, \psi_{s,t}^{(m+1)} - \Theta^{(m+1)}\, \psi_{s,t}^{*(m)} \right\} .$$

This process of calculating improved values of $\varphi_1, \ldots, \varphi_k$, ψ^*, and λ constitutes one outer iteration. These improved values are now used to begin another iteration unless

$$\frac{\bar{\lambda}^{(m+1)} - \underline{\lambda}^{(m+1)}}{2\lambda^{(m+1)}} \leq \epsilon^2 \quad ,$$

in which case the problem is considered converged.

The source calculations of the first two outer iterations are not extrapolated. The third iteration begins an extrapolation cycle which is repeated with variations until the problem is converged.

The source calculation of the first iteration of an extrapolation cycle (iteration m) is not extrapolated. By using the initial residuals in each group of iterations m and m+1, the following quantities are calculated:

$$\bar{\sigma} = \frac{1}{k} \sum_{i=1}^{k} \frac{R_i^{(0),(m+1)}}{R_i^{(0),(m)}}$$

$$\epsilon^{(m+1)} = \frac{\bar{\lambda}^{(m+1)} - \underline{\lambda}^{(m+1)}}{2\lambda^{(m+1)}} \quad .$$

Then, from the recursion formula defining the Chebyshev polynomials

$$T_0(X) = 1$$

$$T_1(X) = X$$

$$T_n(X) = 2X\, T_{n-1}(X) - T_{n-2}(X), \quad n \geq 2$$

the first positive integer ℓ, $\ell \leq 5$, is found for which

$$\frac{\epsilon^{(m+1)}}{T_\ell\left(\frac{2}{\bar{\sigma}} - 1\right)} \leq \epsilon^2 \quad .$$

If the inequality is not satisfied for any such ℓ, it is chosen to be 5. Now ℓ values of $\theta_j(\ell)$ are determined from the formula

$$\theta_j(\ell) = \frac{\bar{\sigma}\left(1 + \cos\frac{2j+1}{2\ell}\pi\right)}{2 - \bar{\sigma}\left(1 + \cos\frac{2j+1}{2\ell}\pi\right)} \qquad j = 0,1,\ldots,\ell-1 \quad .$$

These $\theta_j(\ell)$ form a set of ℓ extrapolation factors which are used in order of decreasing magnitude to extrapolate the source calculations of iterations m+1, m+2, ..., m+ℓ. The source calculation of iteration m+ℓ+1 is not extrapolated, and iteration m+ℓ+2 is the first iteration of a new extrapolation cycle.

Because inner iterations are continued in a group until the flux is nonnegative, it is important that the extrapolated source be nonnegative. To assure this, the largest extrapolation factor, $\theta_{\ell-1}(\ell)$, is used to test the inequality

$$\frac{1 + \Theta_{\ell-1}(\ell)}{\Theta_{\ell-1}(\ell)} \geq \frac{\lambda^{(m+1)}}{\underline{\lambda}^{(m+1)}}$$

before it is used to extrapolate the source of iteration m+1. If the inequality is satisfied, the set of $\Theta_j(\ell)$ is used as indicated above. If the inequality is not satisfied, ℓ is reduced by one, a set of $\Theta_j(\ell - 1)$ is calculated, and the test is made again by using $\Theta_{\ell-2}(\ell-1)$. This process is continued until the inequality is satisfied for some value of $\ell \geq 1$, in which case the extrapolation factors corresponding to this value of ℓ are used, or until $\ell = 0$, in which case no extrapolation is done and an attempt to continue the cycle is made in the following iteration.

At the end of each outer iteration, the routine prints on-line the initial and final residuals and number of inner iterations performed in each group. Also, $\bar{\lambda}$, λ, $\underline{\lambda}$, and the value of $\Theta_j(\ell)$ used to extrapolate the source of this iteration are printed. And for each iteration in which $\bar{\sigma}$ and a set of $\Theta_j(\ell)$ are calculated, the values of $\bar{\sigma}$, $\frac{\epsilon^{(m+1)}}{T_\ell\left(\frac{2}{\bar{\sigma}} - 1\right)}$, and ℓ are printed.

Because a large number of outer iterations is occasionally required to converge a problem, the iteration routine provides a means of terminating the iterations and editing the problem. At the end of each unextrapolated iteration (the first two iterations of the problem and the first and last iterations of each extrapolation cycle), the status of Sense Switch 1 is tested; if it is depressed, the routine stops. The Sense Switch is also tested before beginning the calculation of $\bar{\sigma}$ during the second iteration of a cycle; if it is depressed, the calculation is bypassed, the source is not extrapolated, and the routine stops at the end of the iteration. In both cases, pressing the Start Key causes "Edit Forced" to be printed on-line and the current flux and source to be averaged and edited.

In place of the standard mesh sweeps described, the routine does a faster sweep in the case of square x-y problems which are symmetric with respect to the main diagonal. The difference equation used in this case is identical to that cited, but the solution of this equation is found only at those points on and above the diagonal. Each new value calculated at a point is stored both in the location corresponding to this point and in the location corresponding to its

image about the diagonal. Thus, the flux values at all points are improved without iterating at almost half of them; this effects a considerable savings in iteration time. (This special sweep is performed only if the title card contains an "8" in column 67.)

As may be seen from the difference equation, six quantities are required at each mesh point to complete one sweep of the mesh. For a 2500-point mesh, this means that 15,000 numbers must be available to the computer each sweep. Since several hundred of these sweeps are generally required to converge a problem, the iteration routine must store these numbers in such a way that a minimum amount of time is lost reading them into the computer.

To accomplish this, the routine examines the number of core storage locations in the computer being used and fully utilizes all these locations before turning to the drum for auxiliary storage. The limitations listed in the Introduction on the number of mesh points that may be used with a given size computer assure that all numbers needed for a mesh sweep will fit into core and drum storage. The advantage of this system is seen in the fact that a 2500-point problem on an 8192-word computer requires that 8064 numbers be read from the drum every inner iteration, while the same problem on a 16,384-word computer makes no use of the drum during the inner cycle. Each problem is iterated as fast as is possible under the condition of computer size being used.

J. Average Routine

After a problem has converged, the average routine calculates and prints on-line the composition-integrated areas (volumes), the composition-integrated and averaged flux values for each group, and the composition-integrated and averaged source values

In x-y geometry, the average flux for composition A is given by

$$\bar{\varphi}_A = \frac{\frac{\Sigma h_s h_t}{4} \left(\varphi_{s-1,t-1} + \varphi_{s-1,t} + \varphi_{s,t-1} + \varphi_{s,t}\right)}{\Sigma h_s h_t}$$

where h_s is the interval between columns s-1 and s, and h_t the interval between rows t-1 and t. The sums are taken such that all mesh rectangles within composition A are included. In the case of a mesh rectangle bisected by a symmetry axis, the boundary is taken along this axis and flux values at the pseudo-points thus introduced are taken equal to those at the first interior points.

Similarly, in r-z geometry, the average flux is

$$\bar{\varphi}_A = 2\pi \left[\frac{\Sigma r_{s-1} h_s h_t}{4} \left(\varphi_{s-1,t-1} + \varphi_{s-1,t} + \varphi_{s,t-1} + \varphi_{s,t} \right) \right.$$

$$\left. + \frac{\Sigma h_s^2 h_t}{6} \left(\frac{1}{2}\varphi_{s-1,t-1} + \frac{1}{2}\varphi_{s-1,t} + \varphi_{s,t-1} + \varphi_{s,t} \right) \right]$$

$$\div 2\pi\Sigma \left(r_{s-1} h_s h_t + \frac{1}{2} h_s^2 h_t \right)$$

where r_{s-1} is the interval from the symmetry axis to column s-1.

In the case of both geometries, the numerator and denominator are, respectively, the composition-integrated flux and area (volume).

The composition-integrated source is given by

$$\tilde{S}_A = \Sigma(\nu\Sigma^f)_A \bar{\varphi}_A \quad ,$$

the sum taken over all groups. This divided by the area (volume) of composition A gives the average source.

K. Standard Source Routine

Following the average routine, for two-, three-, and four-group problems, the following quantity, giving the average source for fuel regions at a point, (s,t), is computed for each point of the mesh:

$$S_{s,t} = \sum_{i=1}^{k} \varphi_{i,s,t} \frac{\sum_{\ell=1}^{4} (\nu\Sigma^f)_{i,\ell}}{\sum_{\ell=1}^{4} n_\ell}$$

Here i refers to the group; k is the total number of groups; ℓ refers to the quadrant about the point (s,t) as indicated in Fig. A; $\nu\Sigma^f$ is chosen for the composition corresponding to quadrant ℓ; and n = 1 if the composition in quadrant ℓ is a fuel region (a fuel region here being defined as one for which $\nu\Sigma^f$ in any one or more of the k groups is nonzero); otherwise n is zero. In the case of a point interior to a nonfuel region, the source is set to zero.

L. Special Source Routine

For two-, three-, and four-group problems, a special source, which essentially is a break-down into quadrants of the source defined above, can be computed at each point of the mesh. A "C" or a "P" in column 61 of the title card effects these calculations.

In the terminology used in the preceding section, this source is defined as follows:

$$S_{s,t,\ell} = \sum_{i=1}^{k} \varphi_{i,s,t}(\nu\Sigma^f)_{i,\ell} \qquad \text{for } \ell = 1,2,3,4.$$

Within a nonfuel region, the source is set to zero. Obviously, four values, corresponding to the four quadrants about the point, result at each mesh point.

M. Edit Routine

The only off-line output is that edited onto Tape 4 by the final routine, namely, the pointwise fluxes for each of the k groups and the two sources, where applicable. These edits are optional, the option being specified in the title card. A "P" in column 61 causes the special source to be edited. An "N" in column 62, 63, ..., 66 causes the edit of the corresponding flux or source to be bypassed. (See Title Card.)

The values are edited in such an order that a "picture" is formed, 12 columns being printed across the page (six in the special source edit), and 28 rows down the page (19 in the special source edit). Each page is labeled with the problem title, the eigenvalue at the time of the edit, page number, what is being printed, i.e., flux, with group number, or source, and the proper row and column identification.

All tape editing is subjected to an RTT test.

APPENDIX I. INPUT PREPARATION

A. Title Card

A title card must precede the input deck of each problem. This card is used to identify on-line and off-line output and to specify certain control information.

Columns 1-50 of the card are for problem identification. Any combination of alphabetic and numeric punching may be used and any of these columns may be left blank.

Columns 51-58 must be blank.

Columns 59-60 must contain the problem geometry, either "XY" or "RZ."

Column 61 is used to control the special source routine which calculates four source values at each mesh point. A "C" in this column causes the routine to compute these values and place them on tape for later on-line punching. A "P" causes the routine to compute the values, place them on tape, and edit them onto the output tape for off-line printing. If neither a "C" nor a "P" is present, the entire calculation is bypassed.

Columns 62 66 contain control information for the flux and source edit. The columns refer respectively to φ_1, φ_2, φ_3, φ_4, and S, where S is the source calculated in the standard source routine. An "N" in a given column causes the edit of the corresponding flux or source to be bypassed. If a column does not contain an "N," the corresponding flux or source is edited onto the output tape for off-line printing. The content of a column is not tested if there is no corresponding flux; for example, column 65 in a three-group problem is irrelevant.

Column 67 is normally blank. An "8" should be punched in this column in the case of square x-y problems which are symmetric with respect to the diagonal through (0,0). This causes the iteration routine to sweep only those points on and above the diagonal. Symmetry with respect to the diagonal implies that ss = tt, a = b, and c = d on card 1001, that the 4000 input series need not be specified, and that the geometry specification is symmetric.

Columns 68-72 must contain "PDQ02."

B. Decimal Input

All the input to this code is in fixed-point decimal form with each input card punched according to a definite pattern. Columns 1-7 and 11 are blank, columns 8-10 contain "DEC," and columns 12-15 contain the card number, followed by a comma in column 16. The input itself begins in column 17 and may extend through column 72. Successive numbers are separated by commas, but no comma is allowed following the last number on a card. The first blank column indicates the end of information on a card, and any number punched beyond this blank column is not used. Numbers may be preceded by signs, but only minus signs are necessary.

With the exception of the title card, all input data cards are numbered, and the deck must be arranged in order of increasing card number. A different series of numbers is used for each different type of input. The numbering system is as follows:

1000 series: Miscellaneous parameters and control information.
2000 series: Material- and group-dependent parameters.
3000 series: Mesh intervals in the x or r coordinate direction.
4000 series: Mesh intervals in the y or z coordinate direction.
5000 series: Composition description.

Since the input is in fixed-point form, it is necessary to indicate which of the input numbers are to be converted to floating-point form for use in numerical calculations. This is accomplished by including a decimal point in those numbers which are to be converted. (It should be noted that the number zero need never be accompanied by a decimal point.)

In particular, the numbers on card 1002, all numbers in the 2000 series, and the mesh intervals in the 3000 and 4000 series must contain decimal points. The numbers on card 1001, the row and column numbers in the 3000 and 4000 series, and all numbers in the 5000 series must contain no decimal points. To make this distinction more evident, all numbers which must have decimal points are designated in the following Input Card Format by upper case or Greek letters; all numbers which must not have decimal points are designated by lower case letters.

The only real limitation on the range of input numbers is that contained in the 704 itself. Problems of overflow and underflow may develop, however,

if values are assigned to input constants which are well outside their normal range. The flux editing routine prints four digits to the left of the decimal point and three to the right, and no problem which has a solution in this range should encounter difficulty. The number of digits that may be specified in input numbers is completely arbitrary, but no more than seven or eight significant digits are used by the computer. The only limitation on the total amount of input which may be used is that the total of all numbers in the input, including card numbers, must not exceed 2400.

C. Input Card Format

For purposes of the following description, the point (0,0) is considered to lie in the upper left-hand corner of the mesh. Column numbers increase to the right, and row numbers increase downward.

Card Number	Description
1001	k, n, ss, tt, a, b, c, d
	k: The number of groups ($1 \leq k \leq 4$).
	n: The number of compositions for which input is specified in the 2000 series ($1 \leq n \leq 35$). Input must always be specified for all compositions m, m = 1,2,...,n.
	ss: The last column in the mesh. This is either a zero flux boundary or just outside a symmetry axis ($3 \leq ss \leq 81$).
	tt: The last row in the mesh This is either a zero flux boundary or just outside a symmetry axis ($3 \leq tt \leq 115$). The number of interior mesh points is given by the product (ss-1) (tt-1) and must not exceed the limits given in the Introduction for the size computer being used.
	a: If a = 0, row 0 is a zero flux boundary. If a = 1, there is a symmetry axis midway between rows 0 and 1.
	b: If b = 0, column 0 is a zero flux boundary. If b = 1, there is a symmetry axis midway between columns 0 and 1. (b must be one in r-z geometry.)

Card Number	Description

c: If c = 0, row tt is a zero flux boundary. If c = 1, there is a symmetry axis midway between rows (tt-1) and tt.

d: If d = 0, column ss is a zero flux boundary. If d = 1, there is a symmetry axis midway between columns (ss-1) and ss. (d must be zero in r-z geometry.)

1002 B_z^2, λ_o, ϵ, χ_1, χ_2, χ_3

B_z^2: Buckling constant appearing in the diffusion equations. (B_z^2 is normally zero in r-z geometry.)

λ_o: Initial approximation to the eigenvalue.

ϵ: Convergence criterion used to terminate the iteration cycle when

$$\frac{\bar{\lambda}^{(m)} - \underline{\lambda}^{(m)}}{2\lambda^{(m)}} \leq \epsilon^2,$$

where m is the outer iteration index.

χ_i: The integrals of the fission spectrum appearing in the diffusion equations. Three values must always be specified, even if some are zero.

$$(\chi_1 = 1 \text{ for } k = 1;\ 0.95 < \sum_{i=1}^{k-1} \chi_i < 1.05,$$

$$\text{and } \chi_k = 0 \text{ for } k > 1.)$$

2101 D_1, Σ_1^a, Σ_1^R, $(\nu\Sigma^f)_1$, φ_1

The values of the above parameters for group 1, composition 1, where φ is the input flux approximation. If D = 0, φ must also be zero and Σ^a must be replaced by C. In this case, the logarithmic derivative condition is applied at the boundaries of this composition for this group.

Card Number	Description
2102 ⋮ 21(n)	As above for group 1, compositions 2 through n.
2201 ⋮ 22(n)	As above for group 2, compositions 1 through n.
2(k)01 ⋮ 2(k)(n)	D_k, Σ_k^a, $(\nu\Sigma^f)_k$, φ_k As above for the last group with $\Sigma_k^R = 0$ omitted. If k = 1, this format, rather than the one above for group 1, is to be used.
3001 3002 ⋮	H_x, cc A mesh interval in the x or r coordinate direction followed by the column number at which the interval changes from this value to the next specified. There may be as many sets (H_x, cc) per card and as many cards in this series as needed, but no set may overlap two cards. The successive values of cc must be increasing, and the last value specified must equal ss on card 1001. This mesh specification must always begin at column 0, even if there is a symmetry axis between columns 0 and 1.
4001 4002 ⋮	H_y, rr A mesh interval in the y or z coordinate direction followed by the row number at which the interval changes from this value to the next specified. There may be as many sets (H_y, rr) per card and as many cards in this series as needed, but no set may overlap two cards. The successive values of rr must be increasing, and the last value specified must equal tt on card 1001. This mesh specification must always begin at row 0, even if there is a symmetry axis between rows 0 and 1. If this series is identical to the 3000 series, it may be omitted. This implies that ss = tt on card 1001.

Card Number	Description
5001 5002 ⋮	v, ww, xx, yy, zz The material composition of a problem is described by successively laying rectangular blocks of specified composition over the mesh in an entirely arbitrary manner. Any block of composition may be laid over all or part of other blocks specified previously. For each mesh rectangle, the last specification which includes this rectangle determines its composition. It is important that every mesh rectangle be included within at least one of the composition specifications. It is not necessary that every composition for which input is specified in the 2000 series appear in the mesh. v: Number of the composition whose outer boundaries are described by the following four words ($1 \leq v \leq n$). ww: Column forming the left-hand boundary of v $\left[00 \leq ww \leq (ss-1),\ ww < xx\right]$. xx: Column forming the right-hand boundary of v ($01 \leq xx \leq ss$). yy: Row forming the upper boundary of v $\left[00 \leq yy \leq (tt-1),\ yy < zz\right]$. zz: Row forming the lower boundary of v ($01 \leq zz \leq tt$). There may be as many sets (v, ww, xx, yy, zz) per card and as many cards in this series as needed, but no set may overlap two cards.

APPENDIX II. OPERATING INSTRUCTIONS

A. Card Reader

The 72-72 Card Reader Board must be used.

B. Printer

SHARE Printer Board #2 must be used and there must be 120-column paper in the Printer.

C. Card Punch

Not used.

D. Tapes

Logical Tape 1 must be the Instruction Tape, and Tapes 2-6 must be blanks. (Tape 6 is an exception if Sense Switch 3 is depressed. See Binary Output.) All but the Instruction Tape are rewound at the beginning of each problem and restart, and all including the Instruction Tape are rewound at the end of each problem.

E. Sense Switches

1. Up: Normal.
   Down: Computer will stop at end of next unextrapolated iteration (see Removing a Problem and Excessive Running Time).
2. Up: Normal.
   Down: Restart (see Restarting).
3. Up: Normal.
   Down: Flux from previous run mounted on Tape Unit 6 used as input flux approximation (see Binary Output).

4-6. Not used.

F. Starting

Ready the input deck (no blank cards) in the Card Reader, ready the Printer, set the necessary Sense Switches and Tape Selector Switches, Clear, and Load Tape. When the computer stops with a Select on the Card Reader, press the Start Key on the Card Reader to read in the remaining cards.

G. Restarting

If a problem stops for any reason (other than input error), it may be restarted by rewinding Tape 1, depressing Sense Switch 2, and pressing Clear and Load Tape. (The Sense Switch becomes effective and may be released almost immediately.) If the problem had stopped before reaching the first restart point, it will stop again immediately (see Program Stops). In this case ready the input deck in the Card Reader and Start to begin the problem again.

If a restart is attempted with Sense Switch 2 not depressed, the computer will stop with a Select on the Card Reader. Clear the computer, press the Start Key on the Card Reader to turn out the Select Light, rewind Tape 1, depress Sense Switch 2, and Load Tape to restart again.

The information on Tapes 2-6 is used for restart purposes, and these tapes must not be changed. (Tape 4 is a possible exception. See Removing a Problem and Excessive Running Time.)

H. Removing a Problem

If it is necessary to remove a problem from the computer on short notice, rewind, remove, and label Tapes 2-6. To restart the problem, remount these tapes and follow the restart procedure above.

If more time is available for removing the problem, depress Sense Switch 1. This will cause the computer to stop after the next unextrapolated iteration is completed (see Program Stops). (Sense Switch 1 has not become effective until the computer actually stops and must not be released before that time.) At this point rewind, remove, and label Tapes 2, 3, 5, and 6. To restart, remount these tapes, mount a blank on Tape Unit 4, and follow the restart procedure above. Time-wise, this is the more efficient stopping procedure.

I. Excessive Running Time

If it appears that a problem is taking too long to converge or may be in error, depress Sense Switch 1. When the computer stops after the next unextrapolated iteration is completed (see Program Stops), Start to force a preliminary edit. At the completion of the edit, remove and label Tapes 2, 3, 5, and 6, and print Tape 4. Do not continue the problem until

the edit has been checked. To restart, remount Tapes 2, 3, 5, and 6, mount a blank on Tape Unit 4, and follow the restart procedure above.

J. Decimal Output

The entire contents of the first input card are used to identify the on-line output and each page of off-line output. Additional on-line printing includes an edit of the input, a picture of the mesh, results of the omega routine, condensed results of each outer iteration, and the integrated and averaged final flux and source.

The edit of the final flux and source is placed on Tape 4 for off-line printing. (Tape 4 will be blank if columns 61-66 of the first input card are set for no output.) The Carriage Control Switch on the off-line Printer must be set to Program, and 120-column paper must be used. An EOF defines the end of the output on Tape 4. At the completion of a problem, this tape must be replaced by a blank before another problem is started, since it is used almost immediately for temporary storage.

K. Binary Output

At the end of an edit and whenever the computer stops on Sense Switch 1, Tape 6 contains the most recent flux values in binary form with one file for each group. For this reason, intermediate or final flux values of one problem may be used as the input flux approximation for another run of the same problem or for a different problem.

In the first case, a problem may be restarted by using only Tape 6 after it has stopped on Sense Switch 1, been force edited, or edited automatically. It is necessary only to replace the value of λ_o on card 1002 by the final value of λ obtained in the first run and start the problem from the beginning with the binary output of the previous run on Tape Unit 6 and Sense Switch 3 depressed. This is not an efficient restart procedure but is superior to starting the problem from scratch if Tapes 2, 3, and 5 are not available. In the second case, the final flux values of one problem may be used as the input flux approximation for another by depressing Sense Switch 3 and mounting the binary output of the first problem on Tape Unit 6 before beginning the second problem.

In both cases, the information on Tape 6 is destroyed, since results of the later run are written over those of the former run. Hence, the binary output of one problem may not be used as the input flux approximation for two other problems unless the tape is copied. (Sense Switch 3 does not become effective for some time and should not be released until the iterations are started.)

L. Instruction Tape Preparation

To prepare an Instruction Tape, ready a blank Tape 1, ready the PDQ02 deck (WBPQ2000 - WBPQ2662) in the Card Reader, Clear, and Load Cards. After the tape is written, it will be rewound and the computer will stop at 0302. The Instruction Tape contains 38 records followed by an EOF. Since its preparation is rather lengthy, it should not be remade for each problem.

M. Program Stops

The following stops are in octal. Words in parentheses indicate the routine in which the stop occurs. Unless otherwise indicated, a restart is to be initiated at each of these stops.

0013:	Error in loading binary tape loader from Tape 1. Load Tape to try again.
0051:	Error in loading record of instructions from Tape 1. Start to try again.
0121:	Tape 2 error. (Restart)
0126:	Problem restarted before reaching first restart point. Ready input deck in Card Reader and Start to begin problem again.
0126:	Tape 5 error. (Standard Source)
0136:	Tape 6 error. (Restart)
0175:	Tape 6 error. (Standard Source)
0306:	Tape 2 error. (Standard Source)
0333:	Interchange Tapes 3 and 4 by changing their Tape Selector Switches and Start.
0365:	Tape 2 error. (Symmetry Setup)

0366:	Tape 3 error. (Symmetry Setup)
0367:	Tape 4 error. (Symmetry Setup)
0374:	Tape 2 error. (Edit)
0416:	Tape 6 error. (Edit)
0461:	Tape 2 error. (Picture)
0462:	Tape 5 error. (Picture)
0465:	Entire mesh not filled with composition. Problem cannot be run.
0607:	Input error. Columns 59-60 of first input card do not contain "XY" or "RZ," or columns 68-72 of first input card do not contain "PDQ02," or card number of card containing error has been printed before restoring paper.
0610:	Tape 2 error. (Input)
0611:	Tape 3 error. (Input)
0641:	Drum error. (Edit)
0732:	Tape 2 error. (Input Print)
0733:	Tape 3 error. (Input Print)
0747:	Tape 2 error. (Average)
0750:	Tape 5 error. (Average)
0751:	Tape 6 error. (Average)
1002:	Tape 3 error. (Omega)
1035:	Drum error. (Omega)
1045:	Expander routine has failed twice due to computer error.
1046:	Tape 2 error. (Special Source)
1057:	Tape 6 error. (Omega)
1063:	Tape 4 error. (Omega)

1070:	Tape 5 error. (Special Source)
1114:	Problem completed.
1145:	Drum error. (Special Source)
1201:	Tape 6 error. (Special Source)
1207:	Coefficient routine has had to initiate more than 10 automatic restarts due to inconsistent results within the computer.
1210:	Tape 2 error. (Coefficient)
1211:	Tape 3 error. (Coefficient)
1212:	Tape 5 error. (Coefficient)
1213:	Tape 6 error. (Coefficient)
1217:	Tape 2 error. (Expander)
1220:	Tape 3 error. (Expander)
1222:	Tape 5 error: (Expander)
1223:	Tape 6 error. (Expander)
1324:	Drum error. (Iteration)
1325:	Tape 2 error. (Iteration)
1326:	Tape 3 error. (Iteration)
1327:	Tape 4 error. (Iteration)
1330:	Tape 6 error. (Iteration)
1431:	Tape 2 error. (Omega)
2457:	Next unextrapolated iteration after Sense Switch 1 depressed now finished. Remove problem from computer or Start to force a preliminary edit.

Appendix III. SAMPLE PROBLEM

```
SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-70                    XYP     8PD002

      DEC 1001,2,5,25,25,1,1,0,0

      DEC 1002,-.0009,1.,.05,1.,0,0

      DEC 2101,1.62,0,.041,0,200.

      DEC 2102,2.00,0,0,0.1100.

      DEC 2103,1.73,.0042,.013,.0063,1100.

      DEC 2104,1.73,.0033,.014,.0046,1400.

      DEC 2105,1.70,.0490,.011,0.1200.

      DEC 2201,.238,.012,0,400.

      DEC 2202,1.00,.005,0,300.

      DEC 2203,.433,.117,.207,200.

      DEC 2204,.434,.093,.156,200.

      DEC 2205,0,.1,0,0

      DEC 3001,1.6,1,2.1,9,.8,11,.1,15,1.,17,2.1,21,.3,22,4.5,25

      DEC 5001,1.0,25,0,25,2.0,.0,.10,2.0,16,0,16,2,0,10.0,22

      DEC 5002,3,1,9,17,21,3,17,21,1,.4,1.5,11.15,3,5,9,11,15

      DEC 5003,3,11,15,11.15,3.11.15,5.0,4,11,15,1,5,4,1,5,1,9

      DEC 5004,5,6,14,9,11.5,9,11.,14,5.0,14,0.1,5,9,11,0,4

      DEC 5005,5,0,1,6,14,.,6.4,9,11.5,0,4,0,1,5,6,1,0,4
```

Running Time: 0.16 hours

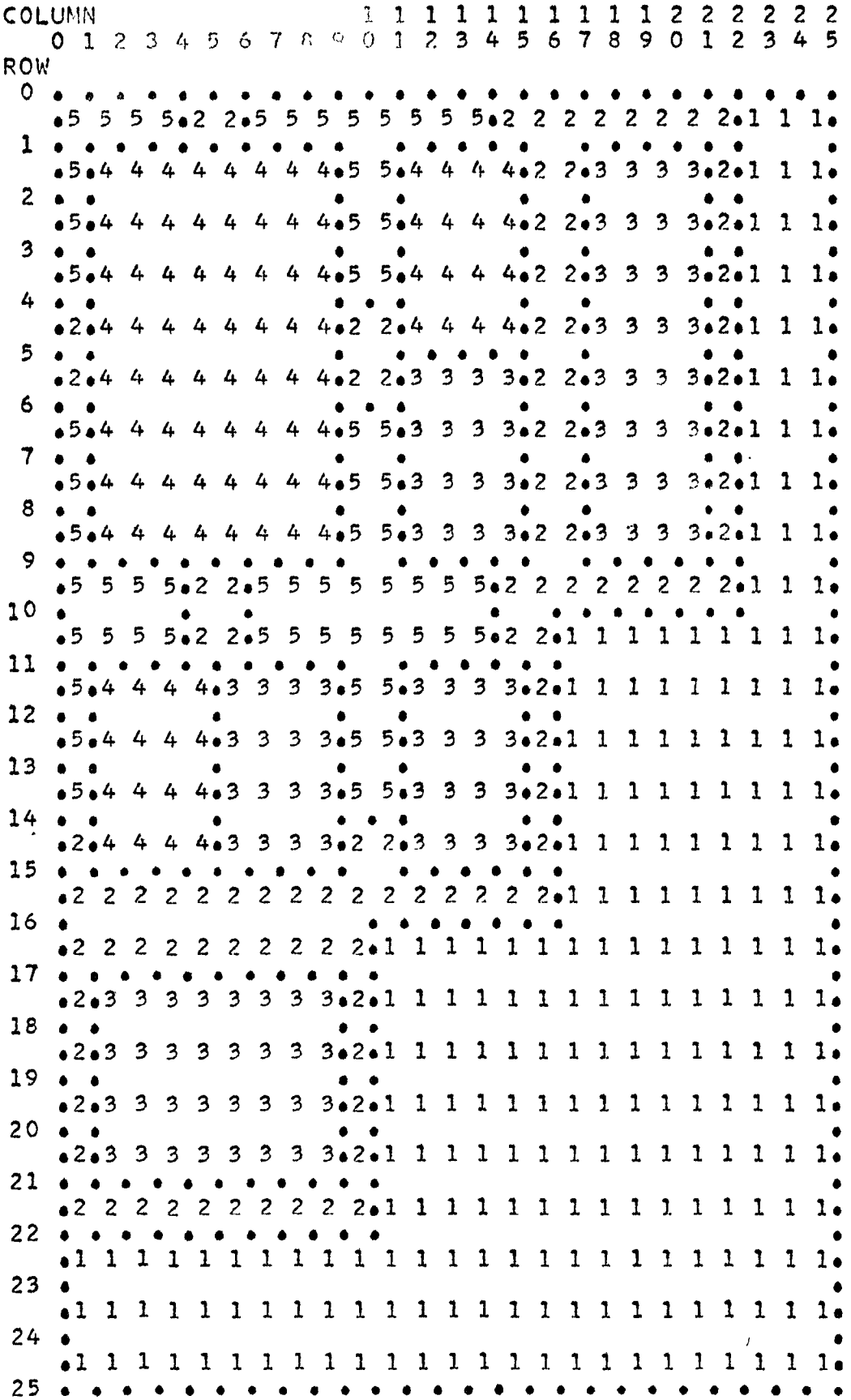
SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-70 XYP 8PDQ02

2 GROUPS 5 COMPS SS= 25 TT= 25 B.C.=1,1,0,0

B SQ	LAMBDA	EPSILON	CHI 1	CHI 2	CHI 3
-0.000900	1.000000	0.050000	1.000000	0.	0.

GROUP	COMP	D	SIGMA A	SIGMA R	NU SIGMA F	PHI
1	1	1.620000	0.	0.041000	0.	200.000
1	2	2.000000	0.	0.	0.	1100.000
1	3	1.730000	0.004200	0.013000	0.006300	1100.000
1	4	1.730000	0.003300	0.014000	0.004600	1400.000
1	5	1.700000	0.049000	0.011000	0.	1200.000
2	1	0.238000	0.012000		0.	400.000
2	2	1.000000	0.005000		0.	300.000
2	3	0.433000	0.117000		0.207000	200.000
2	4	0.434000	0.093000		0.156000	200.000
2	5	0.	0.100000		0.	0.

MESH	COL	MESH	COL	MESH	COL	MESH	COL	MESH	COL	MESH	COL
1.6000	1	2.1000	9	0.8000	11	2.1000	15	1.0000	17	2.1000	21
0.8000	22	4.5000	25								


COLUMN
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GROUP	LAMBDA	PI	SHK1	SHK2	SHK3	SHK4	WMI.	OMEGA	WMAX	NMIN	NMAX
1	0.98575551	0.98590469	0.98805390	0.	0.	0.	1.71205811	1.73293905	1.82112415	10	21
2	0.94237324	0.94290929	0.94804063	0.	0.	0.	1.49861744	1.51727882	1.60532503	5	9

GROUP	INITIAL RES.	NO. INNER IT.	FINAL RES.
1	33762.9517	16	1390.3391
2	17150.8369	8	696.5498

OUTER IT. NO.	MAX. LAMBDA	LAMBDA	MIN. LAMBDA	THETA
1	1.563233	1.019213	0.650909	0.

GROUP	INITIAL RES.	NO. INNER IT.	FINAL RES.
1	2176.6006	16	91.4113
2	1025.4861	9	38.1011

OUTER IT. NO.	MAX. LAMBDA	LAMBDA	MIN. LAMBDA	THETA
2	1.043833	1.002311	0.949656	0.

GROUP	INITIAL RES.	NO. INNER IT.	FINAL RES.
1	407.3254	18	16.9247
2	242.8321	9	11.0390

OUTER IT. NO.	MAX. LAMBDA	LAMBDA	MIN. LAMBDA	THETA
3	1.005820	0.994892	0.982806	0.

GROUP	INITIAL RES.	NO. INNER IT.	FINAL RES.
1	142.9053	17	6.4721
2	105.3831	9	4.7564

OUTER IT. NO.	MAX. LAMBDA	LAMBDA	MIN. LAMBDA	THETA	SIGMA	EPSILON	DEGREE
4	0.997083	0.993112	0.988346	0.244096	0.392407	0.001074	1

GROUP	INITIAL RES.	NO. INNER IT.	FINAL RES.
1	86.8545	17	4.0600
2	67.4249	9	2.9134

OUTER IT. NO.	MAX. LAMBDA	LAMBDA	MIN. LAMBDA	THETA
5	0.993521	0.992319	0.989141	0.

COMPOSITION-INTEGRATED AREA

0.154197E 04 0.179560E 03 0.493920E 03 0.423360E 03 0.964800E 02

COMPOSITION-INTEGRATED FLUX - GROUP 1

0.304584E 06 0.200590E 06 0.568634E 06 0.588353E 06 0.116625E 06

COMPOSITION-AVERAGED FLUX - GROUP 1

0.197529E 03 0.111712E 04 0.115127E 04 0.138972E 04 0.120880E 04

COMPOSITION-INTEGRATED FLUX - GROUP 2

0.555178E 06 0.479058E 05 0.100651E 06 0.749298E 05 0.838342E 04

COMPOSITION-AVERAGED FLUX - GROUP 2

0.360045E 03 0.266796E 03 0.203780E 03 0.176988E 03 0.868928E 02

COMPOSITION-INTEGRATED SOURCE

0. 0. 0.244171E 05 0.143955E 05 0.

COMPOSITION-AVERAGED SOURCE

0. 0. 0.494354E 02 0.340029E 02 0.

SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-70 XYP 8PDQ02 FLUX..GROUP 1 LAMBDA .992319 PAGE 1

	1	2	3	4	5	6	7	8	9	10	11	12
1	1252.056	1307.369	1361.268	1414.799	1437.505	1399.775	1331.264	1263.425	1195.267	1180.794	1184.555	1225.437
2	1307.369	1375.242	1429.016	1466.688	1476.793	1450.266	1396.630	1328.113	1247.392	1227.937	1235.444	1285.700
3	1361.268	1429.016	1478.943	1508.946	1513.998	1491.052	1443.726	1377.716	1295.945	1275.589	1282.769	1330.184
4	1414.799	1466.688	1508.946	1533.665	1536.040	1514.300	1470.692	1410.488	1341.896	1327.054	1328.683	1359.242
5	1437.505	1476.793	1513.998	1536.040	1537.077	1515.369	1473.084	1416.467	1358.845	1348.754	1347.348	1365.891
6	1399.775	1450.266	1491.052	1514.300	1515.369	1492.581	1448.120	1387.349	1319.337	1305.277	1307.628	1335.619
7	1331.264	1396.630	1443.726	1470.692	1473.084	1448.120	1399.002	1331.113	1248.011	1227.049	1232.795	1271.791
8	1263.425	1328.113	1377.716	1410.488	1416.467	1387.349	1331.113	1258.582	1172.988	1150.927	1154.319	1188.471
9	1195.267	1247.392	1295.945	1341.896	1358.845	1319.337	1248.011	1172.988	1094.344	1073.993	1071.300	1088.132
10	1180.794	1227.937	1275.589	1327.054	1348.754	1305.277	1227.049	1150.927	1073.993	1055.542	1048.708	1058.631
11	1184.555	1235.444	1282.769	1328.683	1347.348	1307.628	1232.795	1154.319	1071.300	1048.708	1043.342	1052.890
12	1225.437	1285.700	1330.184	1359.242	1365.891	1335.619	1271.791	1188.471	1088.132	1058.631	1052.890	1061.241
13	1269.313	1326.439	1363.271	1381.396	1380.541	1351.362	1291.066	1206.451	1098.782	1065.397	1056.112	1051.865
14	1320.321	1357.439	1380.758	1388.176	1380.108	1348.810	1289.697	1206.861	1101.415	1067.692	1050.422	1017.205
15	1355.136	1370.378	1379.235	1375.950	1358.873	1322.236	1261.622	1177.685	1070.027	1031.488	999.278	923.001
16	1360.567	1370.963	1375.524	1368.451	1347.050	1307.064	1244.737	1157.806	1042.804	997.481	951.250	852.605
17	1362.108	1371.641	1373.214	1362.973	1333.230	1295.782	1232.313	1143.249	1015.949	958.592	894.701	772.486
18	1340.065	1349.775	1347.049	1331.512	1301.694	1255.535	1189.740	1096.158	948.780	877.945	798.386	646.305
19	1272.199	1282.633	1277.704	1259.350	1227.123	1179.458	1112.875	1017.649	863.509	788.997	705.792	546.276
20	1148.837	1161.532	1155.981	1136.947	1104.761	1058.160	993.789	902.193	753.733	682.142	603.863	455.502
21	945.983	954.165	948.181	930.726	901.680	859.683	801.659	720.749	599.929	544.328	484.205	366.448
22	852.211	854.880	843.739	832.537	805.601	766.426	711.913	636.222	530.889	484.585	437.068	333.865
23	401.449	399.622	394.390	384.807	370.131	349.615	322.478	288.173	247.335	231.084	214.812	173.885
24	158.633	157.525	154.955	150.668	144.419	136.014	125.351	112.552	98.132	92.492	86.840	72.272

	13	14	15	16	17	18	19	20	21	22	23	24
1	1269.313	1320.321	1355.136	1360.567	1362.108	1340.065	1272.199	1148.837	945.983	852.211	401.449	158.633
2	1326.439	1357.439	1370.378	1370.963	1371.641	1349.775	1282.633	1161.532	954.166	854.880	399.622	157.525
3	1363.271	1380.758	1379.235	1375.524	1373.214	1347.049	1277.704	1155.981	948.181	848.739	394.390	154.955
4	1381.396	1388.176	1375.950	1368.451	1362.973	1331.512	1259.350	1136.947	930.726	832.537	384.807	150.668
5	1380.541	1380.108	1358.873	1347.050	1338.230	1301.694	1227.123	1104.761	901.680	805.601	370.131	144.419
6	1351.362	1340.810	1322.236	1307.064	1295.782	1255.535	1179.458	1058.160	859.683	766.426	349.615	136.014
7	1291.066	1289.697	1261.622	1244.737	1232.313	1189.740	1112.875	992.789	801.659	711.913	322.478	125.351
8	1206.451	1206.861	1177.685	1157.806	1143.249	1096.158	1017.649	902.193	720.749	636.222	288.173	112.552
9	1098.782	1101.415	1070.027	1042.804	1015.949	948.780	863.509	753.733	599.929	530.889	247.335	98.132
10	1065.397	1067.692	1031.488	997.481	958.392	877.945	788.997	682.142	544.328	484.585	231.084	92.492
11	1056.112	1050.422	999.278	951.250	894.701	798.386	705.792	603.863	484.205	437.068	214.812	86.840
12	1051.865	1017.205	923.001	852.605	772.436	646.305	546.276	455.502	366.448	333.865	173.885	72.272
13	1025.074	967.840	847.432	766.354	676.424	537.642	436.307	354.450	283.035	257.981	138.965	59.094
14	967.840	892.464	758.932	675.213	585.332	449.404	353.825	281.111	221.906	201.764	110.479	47.751
15	847.432	758.932	633.488	561.920	485.682	369.190	286.565	224.525	175.494	159.123	87.665	38.288
16	766.354	675.213	561.920	500.241	435.838	333.058	258.143	201.501	156.951	142.157	78.429	34.381
17	676.424	585.332	485.682	435.838	385.557	293.738	231.879	180.654	140.361	127.026	70.135	30.836
18	537.642	449.404	369.190	333.058	293.738	234.853	183.608	143.160	111.020	100.398	55.461	24.480
19	436.307	353.825	286.565	258.143	231.879	183.608	144.304	112.766	87.469	79.097	43.718	19.344
20	354.450	281.111	224.525	201.501	180.654	143.160	112.766	88.269	68.522	61.979	34.283	15.194
21	283.035	221.906	175.494	156.951	140.361	111.020	87.469	68.522	53.232	48.161	26.665	11.832
22	257.981	201.764	159.123	142.157	127.026	100.398	79.097	61.979	48.161	43.577	24.136	10.715
23	138.965	110.479	87.665	78.429	70.135	55.461	43.718	34.283	26.665	24.136	13.390	5.953
24	59.094	47.751	38.288	34.381	30.836	24.480	19.344	15.194	11.832	10.715	5.953	2.649

	1	2	3	4	5	6	7	8	9	10	11	12
1	101.284	129.590	144.638	161.529	175.842	160.313	141.674	125.572	97.471	.000	94.382	120.035
2	129.590	166.906	184.935	195.749	200.236	193.714	180.927	161.550	124.540	.000	120.221	153.752
3	144.638	184.935	203.189	211.846	213.984	209.360	198.346	178.250	137.857	.000	132.720	168.029
4	161.529	195.749	211.846	219.150	220.486	216.300	206.118	186.996	149.164	144.045	145.452	172.745
5	175.842	200.236	213.984	220.486	221.483	217.361	207.572	189.769	158.184	153.115	152.298	163.520
6	160.313	193.714	209.360	216.300	217.361	213.050	202.826	183.397	143.882	135.655	130.986	145.478
7	141.674	180.927	198.346	206.118	207.572	202.826	192.133	172.331	122.741	.000	104.838	130.306
8	125.572	161.550	178.250	186.996	189.769	183.397	172.331	153.858	118.257	.000	90.731	115.273
9	97.471	124.540	137.857	149.164	158.184	143.882	132.741	118.257	91.428	.000	70.472	89.496
10	.000	.000	.000	144.045	153.115	135.655	.000	.000	.000	.000	.000	.000
11	94.382	120.221	132.720	145.452	152.298	130.986	104.838	90.731	70.472	.000	69.074	88.547
12	120.035	153.752	168.029	172.745	163.520	145.478	130.306	115.273	89.496	.000	88.547	113.996
13	133.047	168.310	181.737	183.176	169.959	153.038	142.006	130.112	105.995	.000	108.775	136.887
14	148.520	175.330	184.566	184.074	171.342	157.544	152.287	151.976	157.485	168.759	167.657	187.803
15	164.044	171.614	174.335	172.957	166.565	162.041	167.740	188.738	233.120	248.389	266.378	324.030
16	165.594	168.794	170.147	168.695	164.551	162.886	171.284	198.690	258.742	281.322	314.659	384.623
17	167.001	167.087	167.294	165.736	162.553	162.120	171.225	202.062	285.697	325.658	461.059	597.542
18	176.574	168.752	165.402	162.822	160.375	160.382	169.917	209.288	342.738	392.555	575.154	781.491
19	205.553	187.653	180.827	176.850	173.771	173.364	183.059	225.884	373.172	424.166	603.669	811.208
20	277.305	248.525	238.991	233.275	228.406	225.985	233.148	271.572	409.621	456.559	609.673	777.870
21	437.340	431.523	424.121	415.370	405.595	397.324	397.317	420.737	499.475	531.953	628.559	722.039
22	499.390	494.371	486.614	476.636	465.012	454.369	451.462	470.673	536.693	575.450	636.101	696.739
23	768.119	762.804	751.182	732.868	707.785	676.566	640.934	603.849	568.555	555.472	542.254	502.002
24	434.383	431.104	423.950	412.717	397.312	377.869	354.855	329.087	301.515	290.697	279.715	250.094

	13	14	15	16	17	18	19	20	21	22	23	24
1	133.047	148.520	164.044	165.594	167.001	173.574	205.553	277.305	437.340	499.390	768.119	434.383
2	168.510	170.330	171.014	166.794	[illegible]	166.752	187.653	248.525	431.523	494.371	762.804	431.104
3	181.787	184.566	174.335	170.147	167.294	165.402	180.827	238.991	424.121	486.614	751.182	423.950
4	183.176	184.074	172.957	169.605	[illegible]	162.522	176.850	233.275	415.370	476.636	732.868	412.717
5	169.959	171.342	166.565	164.551	163.553	160.375	173.771	228.406	405.595	465.012	707.785	397.312
6	163.030	167.544	162.041	[illegible]	[illegible]	160.382	173.364	225.965	397.324	454.369	676.566	377.869
7	142.000	152.257	167.740	171.264	[illegible]	[illegible]	168.059	233.148	397.317	451.462	640.934	354.855
8	130.112	151.970	[illegible]	168.660	[illegible]	202.238	225.884	271.572	420.737	470.673	603.849	329.087
9	105.995	137.400	203.120	[illegible]	[illegible]	342.738	372.172	409.621	499.475	536.693	568.555	301.515
10	.000	100.759	248.300	[illegible]	[illegible]	372.555	424.166	456.559	531.953	575.450	555.472	290.697
11	100.775	167.007	206.370	314.009	[illegible]	375.154	603.669	609.673	628.559	636.101	542.254	279.715
12	150.087	167.805	324.030	504.625	[illegible]	[illegible]	-11.208	777.870	722.039	696.739	502.002	250.094
13	160.289	211.020	355.736	[illegible]	[illegible]	[illegible]	643.965	792.395	710.160	675.272	451.729	219.786
14	211.020	258.732	401.019	439.929	[illegible]	[illegible]	803.363	741.364	651.904	615.371	396.365	189.938
15	359.756	401.019	[illegible]	[illegible]	[illegible]	767.186	736.4[illegible]	666.134	576.783	541.567	340.892	161.700
16	422.403	459.929	532.384	579.535	[illegible]	737.082	702.057	623.184	540.012	505.841	315.591	149.103
17	634.227	643.990	654.2[illegible]	[illegible]	710.216	[illegible]	[illegible]	589.874	503.454	470.510	291.233	137.109
18	621.748	790.336	757.186	737.052	[illegible]	[illegible]	590.273	510.236	429.196	399.268	243.654	113.994
19	843.960	803.263	736.403	709.099	[illegible]	690.273	611.738	434.267	[illegible]	334.232	201.499	93.765
20	792.395	741.304	666.134	[illegible]	[illegible]	610.236	434.267	303.517	298.930	276.031	164.690	76.275
21	710.160	651.904	576.783	540.012	503.454	429.196	360.743	298.930	243.859	224.565	132.810	61.254
22	675.272	615.371	541.567	505.841	470.510	399.268	334.232	276.031	224.565	206.604	121.809	56.096
23	451.729	396.365	340.892	315.591	291.233	243.654	201.499	164.690	132.810	121.809	71.039	32.533
24	219.786	189.938	161.700	149.103	137.109	113.994	93.765	76.275	61.254	56.096	32.533	14.853

	1	2	3	4	5	6	7	8	9	10	11	12
1	21.560	26.230	28.825	31.707	34.044	31.448	28.225	25.401	20.704	.000	20.173	24.362
2	26.230	32.363	35.423	37.284	38.030	36.891	34.649	31.311	25.166	.000	24.438	29.900
3	28.825	35.423	38.501	39.989	40.346	39.519	37.583	34.145	27.467	.000	26.605	32.331
4	31.707	37.284	39.989	41.242	41.462	40.709	38.920	35.660	29.442	.000	28.802	33.201
5	34.044	38.030	40.346	41.462	41.622	40.879	39.157	36.120	30.927	.000	34.985	37.123
6	31.448	36.891	39.519	40.709	40.879	40.102	38.302	34.992	28.514	.000	35.352	38.528
7	28.225	34.649	37.583	38.920	39.157	38.302	36.408	33.007	26.449	.000	29.468	34.986
8	25.401	31.311	34.145	35.660	36.120	34.992	33.007	29.791	23.844	.000	26.054	31.349
9	20.704	25.166	27.467	29.442	30.927	28.514	26.449	23.844	19.297	.000	21.337	25.381
10	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
11	20.173	24.438	26.605	28.802	34.985	35.352	29.468	26.054	21.337	.000	20.871	24.962
12	24.362	29.900	32.331	33.201	37.123	38.528	34.986	31.349	25.381	.000	24.962	30.283
13	26.594	32.389	34.630	34.930	38.372	40.192	37.529	34.534	28.863	.000	29.170	34.962
14	29.243	33.596	35.144	35.101	38.620	41.109	39.648	39.062	39.538	.000	41.323	45.284
15	31.825	33.075	33.541	33.311	37.637	41.873	42.670	46.488	54.997	.000	61.436	72.889
16	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
17	43.150	43.228	43.281	42.894	42.100	41.722	43.207	49.029	65.540	.000	.000	.000
18	44.993	43.435	42.725	42.093	41.398	41.109	42.668	50.228	76.924	.000	.000	.000
19	50.564	46.925	45.481	44.542	43.701	43.317	44.904	53.169	82.687	.000	.000	.000
20	64.640	58.762	56.754	55.451	54.240	53.445	54.523	61.899	89.540	.000	.000	.000
21	96.489	95.336	93.767	91.845	89.639	87.662	87.295	91.633	107.171	.000	.000	.000
22	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
23	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
24	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

	13	14	15	16	17	18	19	20	21	22	23	24
1	26.594	29.243	31.825	.000	43.150	44.993	50.564	64.640	96.489	.000	.000	.000
2	32.389	33.596	33.075	.000	43.228	43.435	46.925	58.762	95.336	.000	.000	.000
3	34.630	35.144	33.541	.000	43.281	42.725	45.481	56.754	93.767	.000	.000	.000
4	34.930	35.101	33.311	.000	42.894	42.093	44.542	55.451	91.845	.000	.000	.000
5	38.372	38.620	37.637	.000	42.100	41.398	43.701	54.240	89.639	.000	.000	.000
6	40.192	41.109	41.873	.000	41.722	41.109	43.317	53.445	87.662	.000	.000	.000
7	37.529	39.648	42.670	.000	43.207	42.668	44.904	54.523	87.295	.000	.000	.000
8	34.534	39.062	46.488	.000	49.029	50.228	53.169	61.899	91.633	.000	.000	.000
9	28.863	39.538	54.997	.000	65.540	76.924	82.687	89.540	107.171	.000	.000	.000
10	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
11	29.170	41.323	51.436	.000	.000	.000	.000	.000	.000	.000	.000	.000
12	34.962	45.284	72.889	.000	.000	.000	.000	.000	.000	.000	.000	.000
13	39.638	49.779	79.808	.000	.000	.000	.000	.000	.000	.000	.000	.000
14	49.779	59.180	87.792	.000	.000	.000	.000	.000	.000	.000	.000	.000
15	79.808	87.792	105.116	.000	.000	.000	.000	.000	.000	.000	.000	.000
16	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
17	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
18	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
19	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
20	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
21	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
22	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
23	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
24	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

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1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	21.560	26.230	26.230	28.825	28.825	31.707	31.707	34.044	34.044	31.448	31.448
2	.000	26.230	32.363	32.363	35.423	35.423	37.284	37.284	38.030	38.030	36.891	36.891
	.000	26.230	32.363	32.363	35.423	35.423	37.284	37.284	38.030	38.030	36.891	36.891
3	.000	28.825	35.423	35.423	38.501	38.501	39.989	39.989	40.346	40.346	39.519	39.519
	.000	28.825	35.423	35.423	38.501	38.501	39.989	39.989	40.346	40.346	39.519	39.519
4	.000	31.707	37.284	37.284	39.989	39.989	41.242	41.242	41.462	41.462	40.709	40.709
	.000	31.707	37.284	37.284	39.989	39.989	41.242	41.242	41.462	41.462	40.709	40.709
5	.000	34.044	38.030	38.030	40.346	40.346	41.462	41.462	41.622	41.622	40.879	40.879
	.000	34.044	38.030	38.030	40.346	40.346	41.462	41.462	41.622	41.622	40.879	40.879
6	.000	31.448	36.891	36.891	39.519	39.519	40.709	40.709	40.879	40.879	40.102	40.102
	.000	31.448	36.891	36.891	39.519	39.519	40.709	40.709	40.879	40.879	40.102	40.102
7	.000	28.225	34.649	34.649	37.583	37.583	38.920	38.920	39.157	39.157	38.302	38.302
	.000	28.225	34.649	34.649	37.583	37.583	38.920	38.920	39.157	39.157	38.302	38.302
8	.000	25.401	31.311	31.311	34.145	34.145	35.660	35.660	36.120	36.120	34.992	34.992
	.000	25.401	31.311	31.311	34.145	34.145	35.660	35.660	36.120	36.120	34.992	34.992
9	.000	20.704	25.166	25.166	27.467	27.467	29.442	29.442	30.927	30.927	28.514	28.514
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
10	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
11	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	20.173	24.438	24.438	26.605	26.605	28.302	28.302	29.956	40.014	35.352	35.352
12	.000	24.362	29.900	29.900	32.331	32.331	33.201	33.201	31.792	42.454	38.528	38.528
	.000	24.362	29.900	29.900	32.331	32.331	33.201	33.201	31.792	42.454	38.528	38.528
13	.000	25.294	32.389	32.389	34.630	34.630	35.930	35.930	32.854	43.879	40.192	40.192
	.000	25.294	32.389	32.389	34.630	34.630	35.930	35.930	32.854	43.879	40.192	40.192
14	.000	29.243	[illegible]	.396	35.144	35.144	35.101	35.101	33.073	44.162	41.109	41.109
	.000	29.243	[illegible]	.396	35.144	35.144	35.101	35.101	33.073	44.162	41.109	41.109
15	.000	31.825	[illegible]	.075	33.541	33.541	33.311	33.311	32.235	43.040	41.873	41.873
	.000	.000	[illegible]	.000	.000	.000	.000	.000	.000	.000	.000	.000
16	.000	.000	[illegible]	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
17	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	43.150	43.228	43.228	43.281	43.281	42.894	42.894	42.100	42.100	41.722	41.722
18	.000	44.993	43.435	43.435	42.725	42.725	42.093	42.093	41.398	41.398	41.109	41.109
	.000	44.993	43.435	43.435	42.725	42.725	42.093	42.093	41.398	41.398	41.109	41.109
19	.000	50.564	46.925	46.925	45.481	45.481	44.542	44.542	43.701	43.701	43.317	43.317
	.000	50.564	46.925	46.925	45.481	45.481	44.542	44.542	43.701	43.701	43.317	43.317

	1		2		3		4		5		6	
20	.000	64.640	58.762	58.762	56.754	56.754	55.451	55.451	54.240	54.240	53.445	53.445
	.000	64.640	58.762	58.762	56.754	56.754	55.451	55.451	54.240	54.240	53.445	53.445
21	.000	96.489	95.336	95.336	93.767	93.767	91.845	91.845	89.639	89.639	87.662	87.662
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
22	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
23	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
24	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

	7		8		9		10		11		12	
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	28.225	28.225	25.401	25.401	20.704	.000	.000	.000	.000	20.173	24.362	24.362
2	34.649	34.649	31.311	31.311	25.166	.000	.000	.000	.000	24.438	29.900	29.900
	34.649	34.649	31.311	31.311	25.166	.000	.000	.000	.000	24.438	29.900	29.900
3	37.583	37.583	34.145	34.145	27.467	.000	.000	.000	.000	26.605	32.331	32.331
	37.583	37.583	34.145	34.145	27.467	.000	.000	.000	.000	26.605	32.331	32.331
4	38.920	38.920	35.660	35.660	28.442	.000	.000	.000	.000	28.802	33.201	33.201
	38.920	38.920	35.660	35.660	28.442	.000	.000	.000	.000	28.802	33.201	33.201
5	39.157	39.157	36.120	36.120	30.927	.000	.000	.000	.000	29.956	31.792	31.792
	39.157	39.157	36.120	36.120	30.927	.000	.000	.000	.000	40.014	42.454	42.454
6	38.302	38.302	34.992	34.992	28.514	.000	.000	.000	.000	35.352	38.528	38.528
	38.302	38.302	34.992	34.992	28.514	.000	.000	.000	.000	35.252	38.528	38.528
7	36.408	36.408	33.007	33.007	26.449	.000	.000	.000	.000	29.468	34.986	34.986
	36.408	36.408	33.007	33.007	26.449	.000	.000	.000	.000	29.468	34.986	34.986
8	33.007	33.007	29.791	29.791	23.844	.000	.000	.000	.000	26.054	31.349	31.349
	33.007	33.007	29.791	29.791	23.844	.000	.000	.000	.000	26.054	31.349	31.349
9	26.449	26.449	23.844	23.844	19.267	.000	.000	.000	.000	21.337	25.381	25.381
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
10	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
11	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	29.468	29.468	26.054	26.054	21.337	.000	.000	.000	.000	20.871	24.962	24.962
12	34.986	34.986	31.349	31.349	25.381	.000	.000	.000	.000	24.962	30.283	30.283
	34.986	34.986	31.349	31.349	25.381	.000	.000	.000	.000	24.962	30.283	30.283
13	37.529	37.529	34.534	34.534	28.880	.000	.000	.000	.000	29.170	34.962	34.962
	37.529	37.529	34.534	34.534	28.880	.000	.000	.000	.000	29.170	34.962	34.962
14	39.648	39.648	39.062	39.062	39.538	.000	.000	.000	.000	41.323	45.284	45.284
	39.648	39.648	39.062	39.062	39.538	.000	.000	.000	.000	41.323	45.284	45.284
15	42.670	42.670	46.488	46.488	54.097	.000	.000	.000	.000	61.436	72.889	72.889
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
16	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
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17	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	43.207	43.207	49.029	49.029	65.540	.000	.000	.000	.000	.000	.000	.000
18	42.668	42.668	50.228	50.228	76.924	.000	.000	.000	.000	.000	.000	.000
	42.668	42.668	50.228	50.228	76.924	.000	.000	.000	.000	.000	.000	.000
19	44.904	44.904	53.169	53.169	82.687	.000	.000	.000	.000	.000	.000	.000
	44.904	44.904	53.169	53.169	82.687	.000	.000	.000	.000	.000	.000	.000

	7		8		9		10		11		12	
20	54.523	54.523	61.899	61.899	89.540	.000	.000	.000	.000	.000	.000	.000
	54.523	54.523	61.899	61.899	89.540	.000	.000	.000	.000	.000	.000	.000
21	87.295	87.295	91.633	91.633	107.171	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
22	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
23	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
24	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

	13		14		15		16		17		18	
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	26.594	26.594	29.243	29.243	[illegible]	.000	.000	.000	.000	43.150	[illegible]	44.993
2	32.389	32.389	35.596	35.596	[illegible]	.000	.000	.000	.000	43.228	[illegible]	43.435
	32.389	32.389	35.596	35.596	[illegible]	.000	.000	.000	.000	43.228	[illegible]	43.435
3	34.630	34.630	35.144	35.144	[illegible]	.000	.000	.000	.000	43.281	42.725	42.725
	34.630	34.630	35.144	35.144	[illegible]	.000	.000	.000	.000	43.281	42.725	42.725
4	34.930	34.930	35.101	35.101	[illegible]	.000	.000	.000	.000	42.894	42.093	42.093
	34.930	34.930	35.101	35.101	[illegible]	.000	.000	.000	.000	42.894	42.093	42.093
5	32.864	32.864	33.076	33.076	[illegible]	.000	.000	.000	.000	42.100	41.398	41.398
	43.879	43.879	44.162	44.162	[illegible]	.000	.000	.000	.000	42.100	41.398	41.398
6	40.192	40.192	41.109	41.109	[illegible]	.000	.000	.000	.000	41.722	41.109	41.109
	40.192	40.192	41.109	41.109	[illegible]	.000	.000	.000	.000	41.722	41.109	41.109
7	37.529	37.529	39.648	39.648	[illegible]	.000	.000	.000	.000	43.207	42.668	42.668
	37.529	37.529	39.648	39.648	[illegible]	.000	.000	.000	.000	43.207	42.668	42.668
8	34.534	34.534	39.062	39.062	[illegible]	.000	.000	.000	.000	49.029	50.228	50.228
	34.534	34.534	39.062	39.062	[illegible]	.000	.000	.000	.000	49.029	50.228	50.228
9	28.863	28.863	39.536	39.536	[illegible]	.000	.000	.000	.000	65.540	76.924	76.924
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
10	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
11	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	29.170	29.170	41.323	41.323	[illegible]	.000	.000	.000	.000	.000	.000	.000
12	34.962	34.962	45.284	45.284	72.889	.000	.000	.000	.000	.000	.000	.000
	34.962	34.962	45.284	45.284	72.889	.000	.000	.000	.000	.000	.000	.000
13	39.638	39.638	49.779	49.779	79.808	.000	.000	.000	.000	.000	.000	.000
	39.638	39.638	49.779	49.779	79.808	.000	.000	.000	.000	.000	.000	.000
14	49.779	49.779	59.180	59.180	87.792	.000	.000	.000	.000	.000	.000	.000
	49.779	49.779	59.180	59.180	87.792	.000	.000	.000	.000	.000	.000	.000
15	79.808	79.808	87.792	87.792	105.116	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
16	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
17	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
18	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
19	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

	13		14		15		16		17		18	
20	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
21	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
22	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
23	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
24	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

	19		20		21		22		23		24	
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	50.564	50.564	64.640	64.640	96.489	.000	.000	.000	.000	.000	.000	.000
2	46.925	46.925	58.762	58.762	95.336	.000	.000	.000	.000	.000	.000	.000
	46.925	46.925	58.762	58.762	95.336	.000	.000	.000	.000	.000	.000	.000
3	45.481	45.481	56.754	56.754	93.767	.000	.000	.000	.000	.000	.000	.000
	45.481	45.481	56.754	56.754	93.767	.000	.000	.000	.000	.000	.000	.000
4	44.542	44.542	55.451	55.451	91.845	.000	.000	.000	.000	.000	.000	.000
	44.542	44.542	55.451	55.451	91.845	.000	.000	.000	.000	.000	.000	.000
5	43.701	43.701	54.240	54.240	89.639	.000	.000	.000	.000	.000	.000	.000
	43.701	43.701	54.240	54.240	89.639	.000	.000	.000	.000	.000	.000	.000
6	43.317	43.317	53.445	53.445	87.662	.000	.000	.000	.000	.000	.000	.000
	43.317	43.317	53.445	53.445	87.662	.000	.000	.000	.000	.000	.000	.000
7	44.904	44.904	54.523	54.523	87.295	.000	.000	.000	.000	.000	.000	.000
	44.904	44.904	54.523	54.523	87.295	.000	.000	.000	.000	.000	.000	.000
8	53.169	53.169	61.899	61.899	91.633	.000	.000	.000	.000	.000	.000	.000
	53.169	53.169	61.899	61.899	91.633	.000	.000	.000	.000	.000	.000	.000
9	82.687	82.687	89.540	89.540	117.171	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
10	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
11	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
12	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
13	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
14	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
15	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
16	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
17	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
18	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
19	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

	19		20		21		22		23		24	
20	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
21	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
22	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
23	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
24	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

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