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CALCULATED CRITICAL RADII OF SPHERES OF  
PLUTONIUM 239 AND URANIUM 233 WITH  
VARIOUS SPHERICAL REFLECTORS



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*Donald R. Ferguson and Donald C. Coonfield*

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# CALCULATED CRITICAL RADII OF SPHERES OF PLUTONIUM 239 AND URANIUM 233 WITH VARIOUS SPHERICAL REFLECTORS

Donald R. Ferguson and Donald C. Coonfield

**Abstract.** Calculated critical radii of spheres of 100-percent plutonium 239 and 100-percent uranium 233 are listed as a function of reflector thickness of several types of reflectors. Corresponding critical masses are calculated using the listed critical radii.

## DISCUSSION

**Calculational Method:** The computer code DTF,<sup>1</sup> converted to the Drum Scope Operated 2-Bank Control Data 3600 by Rocky Flats personnel, was used to calculate the critical radii of spheres of 100-percent plutonium 239 (<sup>239</sup>Pu) and 100-percent uranium 233 (<sup>233</sup>U) reflected by various materials of varied thicknesses. The DTF code is a later development of Carlson's  $S_n$  method<sup>2</sup> which approximates a solution to the Boltzmann transport equation. Planar, cylindrical, and spherical geometries are permitted. With the application of various boundary conditions, cell, finite plane, and finite-cylinder, critical parameters can be calculated. The code permits isotropic or linear anisotropic scattering.

Table I contains the nuclides, densities, and nuclear densities that were used. For the various reflector compositions and thicknesses, Tables II and III contain the critical radii and masses of <sup>239</sup>Pu and <sup>233</sup>U respectively. The Hanson and Roach<sup>3</sup> 16-group neutron cross sections and 5 angular directions were used in all cases. The precision<sup>4</sup> on lambda ( $\lambda$ ) was 0.0001, and a fission guess<sup>5</sup> of 1.0 for all radial intervals was used. Uniform radial intervals of approximately 0.5 centimeters (cm) were used in all fissile spheres.

<sup>1</sup> Bengt G. Carlson, William J. Worlton, Walter Gruber, and Martin Shapiro. *DTF Users Manual*. UNC Physics-Mathematics 3321. United Nuclear Corporation, White Plains, New York. Volume I, November 1963. Volume II, May 1964.

<sup>2</sup> *Methods in Computational Physics*. Statistical Physics, Volume 1. Editors, Berni Alder, et al. Academic Press, New York. 1963.

<sup>3</sup> Gordon E. Hansen and William H. Roach. *Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies*. LAMS-2543. University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico. December 1960.

<sup>4</sup> An epsilon ( $\epsilon$ ) of 0.0001 was used in all problems. Epsilon ( $\epsilon$ ) is the precision on lambda ( $\lambda$ ). A problem terminates when  $\lambda$  is within  $\epsilon$  of unity. In all cases in this report,  $\lambda$  is k<sub>effective</sub> (neutron multiplication).

<sup>5</sup> Uniform fission densities of 1.0 neutrons per cubic centimeter for each spatial interval were used for input in all problems.

Initially, the radial intervals in the reflector were varied from approximately 0.85 cm for thicknesses less than 6 inches up to 1.2 cm for the larger reflector thicknesses. Where the reflector thicknesses were 6 inches or thicker and contained carbon and hydrogen, the calculated critical masses increased slightly with increased reflector thickness. By redoing the calculations with radial intervals of approximately 0.85 cm in the reflector, the critical mass became "well behaved" as the reflector thickness increased. After reducing the radial intervals by one half in the reflector thicknesses of 6 inches and less, no significant change in critical mass was observed. The calculated critical masses of <sup>239</sup>Pu in Figure 1 and of <sup>233</sup>U as shown in Figure 2 are plotted as a function of reflector composition and thickness.

The results in choice of radial intervals indicate that one should maintain uniform radial intervals as the reflector thickness increases. In the calculations, no attempt has been made to find the "best" choice of radial intervals. Only "well behaved" graphs were obtained. The data presented have not been verified experimentally.

TABLE I. Data Relating to Densities of Material Used.

Material	Density (grams per cubic centimeter)	Atomic Number Density
Plutonium-239 Metal (100 percent)	19.70	$0.049660 \times 10^{24}$
Uranium-233 Metal (100 percent)	18.60	$0.048100 \times 10^{24}$
Aluminum (Al)	2.70	$0.060295 \times 10^{24}$
Beryllium (Be)	1.84	$0.123011 \times 10^{24}$
Carbon (C) (graphite)	1.90	$0.095316 \times 10^{24}$
Carbon Tetrachloride (CCl <sub>4</sub> )	1.59	
Carbon (C)		$0.006215 \times 10^{24}$
Chlorine (Cl)		$0.024861 \times 10^{24}$
Polyethylene (CH <sub>2</sub> )	1.19	$0.051176 \times 10^{24}$
Carbon (C)		$0.102352 \times 10^{24}$
Hydrogen (H)		
Oil (C <sub>22</sub> H <sub>46</sub> )	0.89	$0.038356 \times 10^{24}$
Carbon (C)		$0.069734 \times 10^{24}$
Hydrogen (H)		
Trichloroethylene (C <sub>2</sub> HCl <sub>3</sub> )	1.46	
Carbon (C)		$0.013348 \times 10^{24}$
Hydrogen (H)		$0.006674 \times 10^{24}$
Chlorine (Cl)		$0.020023 \times 10^{24}$
Iron (Fe)	7.87	$0.084900 \times 10^{24}$
Water (H <sub>2</sub> O)	1.00	
Hydrogen (H)		$0.066944 \times 10^{24}$
Oxygen (O)		$0.033472 \times 10^{24}$
Liquid Nitrogen (N <sub>2</sub> )	0.81	$0.017380 \times 10^{24}$
Liquid Oxygen (O <sub>2</sub> )	1.15	$0.021634 \times 10^{24}$
Uranium 238 ( <sup>238</sup> U)	18.80	$0.047592 \times 10^{24}$

TABLE II. Critical Radii and Masses of 100-Percent Plutonium-239 Metal with Various Reflectors.

Reflector Material	Reflector Density (grams per cubic centimeter)	Reflector Thickness (inches)	Critical Radius (centimeters)	Critical Mass (kilograms)
Bare Case	—	—	4.896	9.685
Aluminum (Al)	2.70	1.00	4.526	7.651
		2.00	4.387	6.967
		4.00	4.261	6.384
		6.00	4.200	6.114
		8.00	4.165	5.962
		10.00	4.142	5.864
		14.00	4.121	5.775
Beryllium (Be)	1.84	0.25	4.570	7.876
		0.50	4.349	6.788
		0.75	4.188	6.061
		1.00	4.053	5.502
		2.00	3.733	4.293
		4.00	3.417	3.292
		6.00	3.254	2.843
		8.00	3.169	2.626
		10.00	3.120	2.506
		14.00	3.073	2.395
Carbon (C) (graphite)	1.90	0.50	4.526	7.651
		1.00	4.331	6.704
		2.00	4.115	5.750
		4.00	3.908	4.925
		6.00	3.795	4.510
		8.00	3.718	4.241
		10.00	3.670	4.079
		14.00	3.616	3.902
Carbon Tetrachloride (CCl <sub>4</sub> )	1.59	1.0	4.745	8.816
		2.0	4.686	8.491
		4.0	4.630	8.190
		8.0	4.583	7.943
		10.0	4.571	7.801
		14.0	4.556	7.804
Polyethylene (CH <sub>2</sub> )	1.19	1.0	4.026	5.385
		2.0	3.716	4.234
		4.0	3.657	4.036
		6.0	3.655	4.029
		8.0	3.655	4.029
		10.0	3.654	4.026
		14.0	3.653	4.023
Oil (C <sub>22</sub> H <sub>46</sub> )	0.89	1.0	4.353	6.806
		2.0	4.065	5.543
		4.0	3.924	4.986
		6.0	3.909	4.929
		8.0	3.908	4.925
		10.0	3.907	4.921
		14.0	3.906	4.918
		—	—	—
Trichloroethylene (C <sub>2</sub> HCl <sub>3</sub> )	1.46	1.0	4.718	8.666
		2.0	4.646	8.275
		4.0	4.572	7.886
		8.0	4.513	7.585
		10.0	4.502	7.530
		14.0	4.492	7.480
Iron (Fe)	7.87	1.0	4.387	6.967
		2.0	4.207	6.144
		4.0	4.053	5.494
		6.0	3.980	5.202
		8.0	3.939	5.043
		10.0	3.912	4.940
		14.0	3.889	4.854

TABLE II. Critical Radii and Masses of 100-Percent Plutonium-239 Metal with Various Reflectors (Concluded).

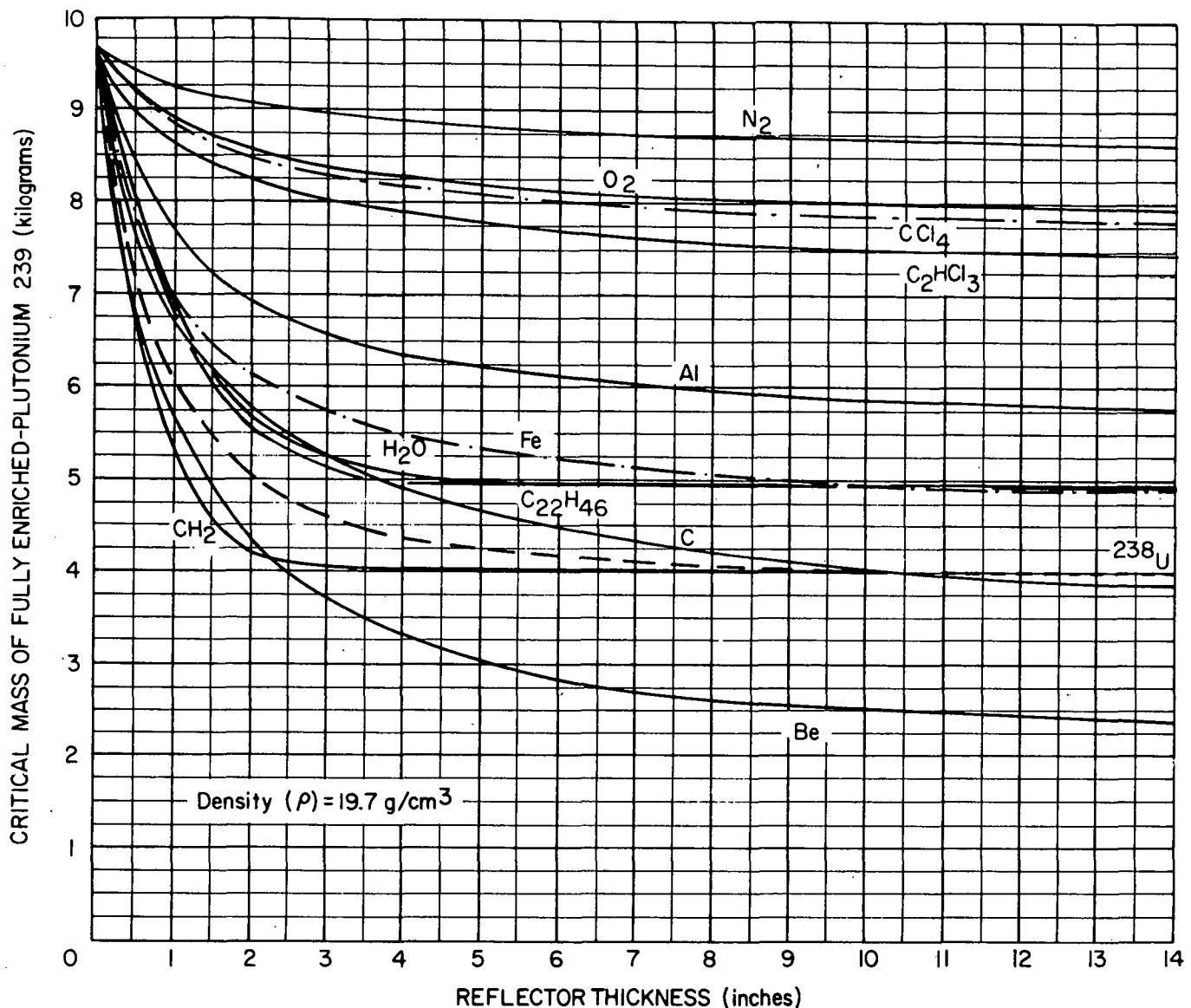
Reflector Material	Reflector Density (grams per cubic centimeter)	Reflector Thickness (inches)	Critical Radius (centimeters)	Critical Mass (kilograms)
Water ( $H_2O$ )	1.00	0.25	4.734	8.755
		0.50	4.604	8.053
		0.75	4.490	7.470
		1.00	4.385	6.958
		2.00	4.098	5.679
		4.00	3.943	5.059
		6.00	3.924	4.906
		8.00	3.924	4.986
		10.00	3.923	4.982
		14.00	3.920	4.971
Liquid Nitrogen ( $N_2$ )	0.81	1.00	4.815	9.212
		2.00	4.783	9.029
		4.00	4.753	8.860
		8.00	4.727	8.716
		10.00	4.720	8.677
		14.00	4.712	8.633
Liquid Oxygen ( $O_2$ )	1.15	1.0	4.757	8.883
		2.0	4.702	8.578
		4.0	4.650	8.297
		8.0	4.608	8.074
		10.0	4.597	8.016
		14.0	4.584	7.949
Uranium 238 ( $^{238}U$ )	18.80	1.00	4.185	6.048
		2.00	3.938	5.039
		4.00	3.752	4.359
		6.00	3.690	4.146
		8.00	3.668	4.072
		10.00	3.659	4.042
		14.00	3.658	4.039

TABLE III. Critical Radii and Masses of 100-Percent Uranium-233 Metal with Various Reflectors.

Reflector Material	Reflector Density (grams per cubic centimeter)	Reflector Thickness (inches)	Critical Radius (centimeters)	Critical Mass (kilograms)
Bare Case	—	—	5.760	14.889
Aluminum (Al)	2.70	1.00	5.303	11.619
		2.00	5.115	10.426
		4.00	4.935	9.364
		6.00	4.845	8.861
		8.00	4.791	8.568
		10.00	4.756	8.382
		14.00	4.720	8.193
Beryllium (Be)	1.84	1.00	4.681	7.991
		2.00	4.233	5.909
		4.00	3.801	4.279
		6.00	3.576	3.563
		8.00	3.456	3.216
		10.00	3.387	3.027
		14.00	3.321	2.854
Carbon (C) (graphite)	1.90	1.00	5.041	9.980
		2.00	4.796	8.595
		4.00	4.451	6.870
		6.00	4.293	6.164
		8.00	4.190	5.731
		10.00	4.119	5.445
		14.00	4.030	5.099

TABLE III. Critical Radii and Masses of 100-Percent Uranium-233 Metal with Various Reflectors (Concluded).

Reflector Material	Reflector Density (grams per cubic centimeter)	Reflector Thickness (inches)	Critical Radius (centimeters)	Critical Mass (kilograms)
Carbon Tetrachloride ( $CCl_4$ )	1.59	1.00	5.576	13.507
		2.00	5.498	12.948
		4.00	5.421	12.412
		8.00	5.353	11.951
		10.00	5.335	11.831
		14.00	5.312	11.678
Polyethylene ( $CH_2$ )	1.19	1.00	4.630	7.733
		2.00	4.194	5.748
		4.00	4.108	5.401
		6.00	4.099	5.366
		8.00	4.098	5.362
		10.00	4.097	5.358
		14.00	4.096	5.354
Oil ( $C_{22}H_{46}$ )	0.89	1.00	5.057	10.076
		2.00	4.659	7.879
		4.00	4.454	6.884
		6.00	4.434	6.792
		8.00	4.431	6.778
		10.00	4.431	6.778
		14.00	4.429	6.769
Trichloroethylene ( $C_2HCl_3$ )	1.46	1.00	5.537	13.226
		2.00	5.438	12.529
		4.00	5.333	11.817
		6.00	5.277	11.449
		8.00	5.232	11.158
		10.00	5.230	11.146
		14.00	5.216	11.056
Iron (Fe)	7.87	1.00	5.129	10.512
		2.00	4.882	9.066
		4.00	4.661	7.889
		6.00	4.554	7.358
		8.00	4.491	7.057
		10.00	4.451	6.870
		14.00	4.413	6.696
Water ( $H_2O$ )	1.00	1.00	5.093	10.293
		2.00	4.696	8.068
		4.00	4.472	6.968
		6.00	4.451	6.870
		8.00	4.441	6.834
		10.00	4.440	6.819
		14.00	4.437	6.806
Liquid Nitrogen ( $N_2$ )	0.81	1.00	5.660	14.127
		2.00	5.617	13.807
		4.00	5.575	13.500
		8.00	5.537	13.226
		10.00	5.527	13.154
		14.00	5.513	13.055
Liquid Oxygen ( $O_2$ )	1.15	1.00	5.579	13.529
		2.00	5.514	13.062
		4.00	5.440	12.543
		8.00	5.375	12.099
		10.00	5.357	11.978
		14.00	5.335	11.831
Uranium 238 ( $^{238}U$ )	18.80	1.00	4.850	8.888
		2.00	4.504	7.119
		4.00	4.236	5.922
		6.00	4.146	5.553
		8.00	4.114	5.425
		10.00	4.102	5.378
		14.00	4.101	5.374

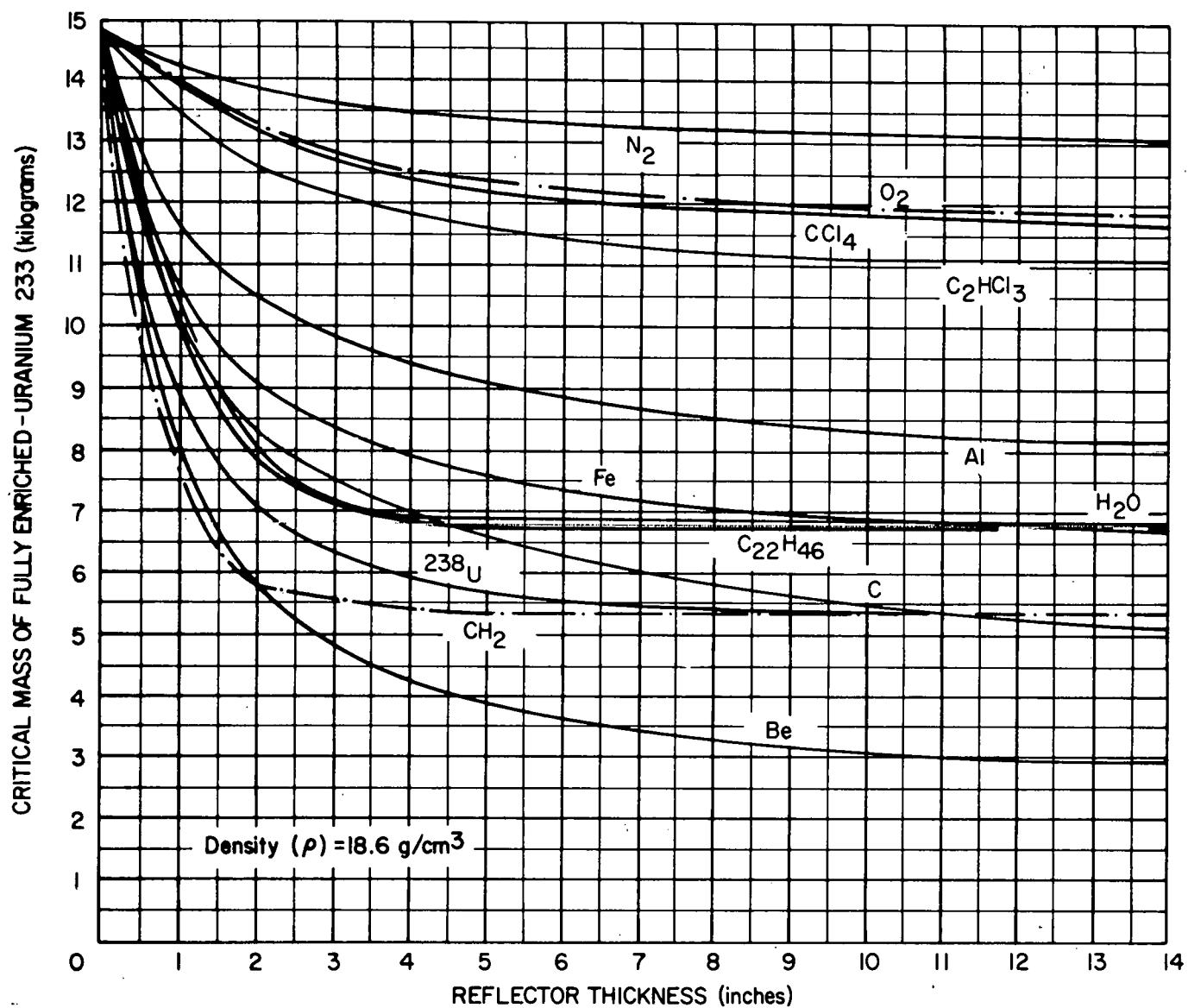


Legend

$N_2$ = Liquid Nitrogen	$H_2O$ = Water
$O_2$ = Liquid Oxygen	$C_{22}H_{46}$ = Oil
$CCl_4$ = Carbon Tetrachloride	$C$ = Carbon (graphite)
$C_2HCl_3$ = Trichloroethylene	$^{238}U$ = Uranium 238
$Al$ = Aluminum	$CH_2$ = Polyethylene
$Fe$ = Iron	$Be$ = Beryllium

(Broken graph lines have no special significance other than to clarify reading.)

FIGURE 1. Data Relating to the Function of Reflector Composition and Thickness.



Legend	
$N_2$ = Liquid Nitrogen	$H_2O$ = Water
$O_2$ = Liquid Oxygen	$C_{22}H_{46}$ = Oil
$CCl_4$ = Carbon Tetrachloride	C = Carbon (graphite)
$C_2HCl_3$ = Trichloroethylene	$^{238}U$ = Uranium 238
Al = Aluminum	$CH_2$ = Polyethylene
Fe = Iron	Be = Beryllium

(Broken graph lines have no special significance other than to clarify reading.)

FIGURE 2. Data Relating to the Function of Reflector Composition and Thickness.