

IGR-R/CA-234



UNITED KINGDOM ATOMIC ENERGY AUTHORITY
INDUSTRIAL GROUP

A HIGH-FREQUENCY INDUCTANCE LEVEL GAUGE FOR LIQUIDS IN
INACCESSIBLE LOCATIONS

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RESEARCH AND DEVELOPMENT BRANCH

CAPENHURST

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SUMMARY A new type of level gauge has been developed to give remote indications when measuring levels of corrosive and radio-active liquids in inaccessible locations. The principle of operation depends on the variation of inductance of a vertical metal rod which dips into the liquid. 7

The indications are independent of liquid density, and the instrument is capable of a high order of accuracy in suitable conditions. An instrument operated in the laboratory gave an accuracy better than 1% using 1.0 N nitric acid as the liquid; the range of level variation was 40 cm.

The probe in contact with the liquid is simple and inherently reliable. With a small number of simple electronic components close to the probe, a remote indication is obtained using a resonance method of inductance measurement.

The principle is restricted to liquids of good conductivity, viz. a resistivity of 3 ohm/cm or less. A method of calculating the change in calibration from a perfectly-conducting liquid to one of finite conductivity is shown. The possibility of applying the principle to liquid metal level measurement is considered. 1

For the attention of Chemical Plant Design Committee

UNITED KINGDOM ATOMIC ENERGY AUTHORITY
INDUSTRIAL GROUP

RISLEY

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October 1957

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1. INTRODUCTION

A new type of level gauge has been developed to give remote indications when measuring levels of corrosive and radio-active liquids in inaccessible locations. The investigation was prompted by the shortcomings of the capacitance (Davidson, 1955a) and pneumatic types of level gauge, both of which have been used for this application. The new gauge is independent of density changes in the test liquid, and the effects of wetting the measuring element are negligible.

The principle used is the variation of the inductance of a vertical metal rod which dips into the liquid. If the liquid is assumed to be a perfect conductor, then the inductance measured is that of the portion of the rod which is not covered by the liquid. Inductance is measured by the resonance method.

It is shown that the instrument can be used with most strong acids or alkalis in concentrations greater than 1.0 N. A formula is derived for calculating the 'error' caused by finite conductivity of the liquid; it shows how the 'skin depth' varies with conductivity and frequency. The rod may be made of stainless steel or any other corrosion-resistant metal. The few electronic components associated with the resonant circuit are mounted in a sealed compartment in the probe-head. A single co-axial cable joins the probe to the indicator unit, and its length is not critical.

The principle might be used with advantage to measure levels of liquid metals used as reactor coolants.

2. PRINCIPLE OF OPERATION

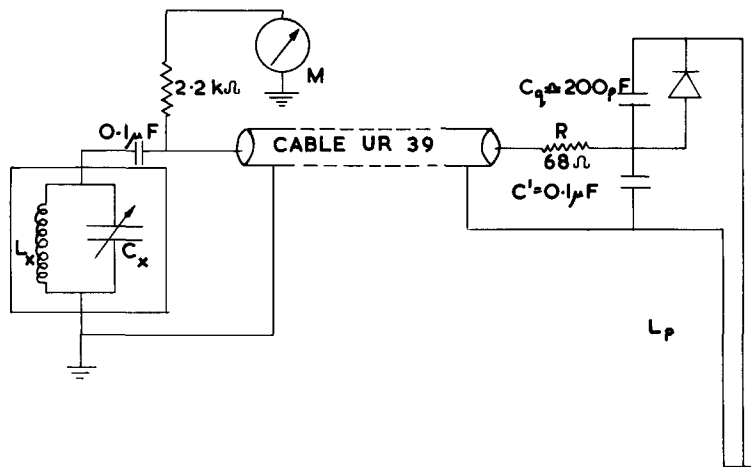
The instrument is the subject of a patent application by Davidson (1955b).

The inductance per unit length of a rod of diameter d in the centre of a tube of inside diameter D is given by:-

$$L = 2 \times 10^{-9} \ln \frac{D}{d} \text{ henries per cm} \dots\dots\dots(1)$$

Under suitably chosen operating conditions this co-axial arrangement may be regarded as a pure inductance (see Appendix 1).

When the liquid to be measured is contained in a metal tank, its wall may be regarded as the tube, but then the calibration of the instrument would depend on the size and shape of the tank. In general it is more convenient to have a self-contained probe unit consisting of a rod and a perforated tube forming a rigid co-axial line. If the line is of uniform cross-section, a linear relationship exists between its inductance and the liquid level.



SIMPLIFIED DIAGRAM OF REMOTE-INDICATING CIRCUIT

FIG. 1

Fig. 1 shows the general arrangement for remote indication. A resonant circuit is formed by the inductance L of the probe with fixed capacitors C_q and C' in series. Since the inductance of the probe varies with the liquid level, it follows that the resonant frequency of the tuned circuit will be a function of liquid depth.

The resonant circuit is excited from a capacitor-tuned oscillator, and the voltage developed across capacitor C_q is detected and its value indicated on meter M . This value will be a maximum when the resonant circuit is excited at its resonant frequency. In this condition:

$$f = \frac{1}{2\pi\sqrt{L_x C_x}} = \frac{1}{2\pi\sqrt{L_p C_p}} \dots\dots\dots(2)$$

- where f = resonant frequency of tuned circuit
 L_x = oscillator tuned circuit inductance (constant)
 C_x = oscillator tuned circuit capacitance
 L_p = probe inductance
 C_p = probe capacitance, viz. C_q and C' in series (constant)

From equation (2):

$$\frac{L_p C_p}{L_x C_x} = 1$$

$$\text{or } \frac{L_p}{C_x} = \frac{L_x}{C_p} = \text{constant}$$

$$\therefore L_p \propto C_x$$

\propto depth of liquid in the case of a probe of uniform cross-section

3. DESIGN CONSIDERATIONS

3.1 PROBE DESIGN

A probe consisting of a stainless-steel rod mounted centrally in a stainless-steel cylinder was used. This has the advantage of being readily manufactured from standard materials. The co-axial construction also reduces the problem of external field to a minimum. The level of liquid inside the cylinder must follow the general level in the main tank; to allow this, the cylinder wall was perforated at several points along its length. The factors governing the choice of dimensions for the tube can be derived from equation (1). The ratio of diameters of tube and rod should be large, to give a large inductance per unit length. The effect of wetting by a film of liquid will be to change the ratio of rod and tube diameters. This change, however, will be negligible if the conditions for mechanical strength of the rod are satisfied. Practical considerations limit the diameter of the tube.

The connections to the resonant circuit components should be kept as short and as rigid as possible, since they will have the same order of inductance per unit length as the probe itself. These components are mounted directly on top of the probe, outside the tank, and are connected to the indicator unit by a single co-axial cable. This cable serves the dual purpose of conveying energy from the oscillator to the resonant circuit and of conveying the rectified voltage from the detector back to the meter. So that the length of this cable shall not be critical, it is matched at the probe end, and in consequence only a small portion of the energy transmitted is used to energise the resonant circuit.

The components required adjacent to the probe are, therefore:-

- (i) The resonating capacitor.
- (ii) The detector (a germanium diode).
- (iii) The cable matching resistor.
- (iv) A d.c. blocking capacitor, which also serves as a coupling impedance between the cable and the resonant circuit.

These are all simple components which, being well under-run, can be expected to have a very high order of reliability.

3.2 CHOICE OF FREQUENCY

This is a matter of compromise. Different factors influence the choice of upper and lower limits of the working range of frequency, and these may be considered separately.

At high frequencies, ambiguities can arise from the resonant line effects. In a quarter-wave resonant line, the resonant frequency is given by:

$$f_o = \frac{c}{4r}$$

where c = velocity of light in air

r = length of co-axial line

To avoid these ambiguities, the maximum operating frequency (maximum liquid level) must be below the resonant line frequency corresponding to minimum liquid level.

A range of frequencies exists which is sufficiently low to avoid ambiguities but in which the effect of distributed capacitance along the line is still considerable. This range may be used when a linear scale is not essential.

When a linear scale is specified, the upper frequency limit must be further reduced so that the probe may be regarded as a pure inductance. A method of satisfying this condition is discussed in Appendix 1.

At the lower end of the frequency range, the limit is set by the magnitude of a systematic error which occurs because the effective current path is not along the surface of the liquid but at some distance below it. This 'skin depth' given by:

$$S = 2.51 \times 10^3 \sqrt{\rho/f} \dots \dots \dots (3)$$

where S = skin depth, cm

ρ = resistivity of liquid, ohm/cm

f = frequency, Mc/s

This expression, derived in Appendix 2, shows that the error due to skin depth varies with resistivity and inversely with frequency. In addition to skin depth errors, the resistivity of the liquid causes loss of sensitivity because it reduces the magnification of the probe resonant circuit; this effect is also considered in Appendix 2. It follows that the working frequency should be as high as possible within the limits which satisfy the conditions determined in Appendix 1.

4. DESCRIPTION OF THE INSTRUMENT

The instrument consists of two main parts, the indicator unit and the probe.

4.1 THE INDICATOR UNIT

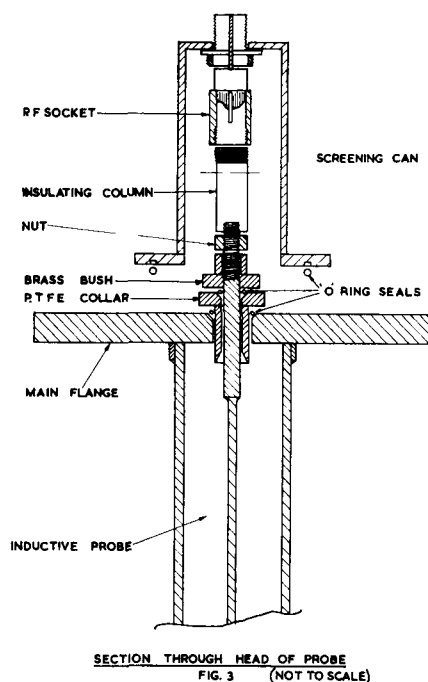
This is built on to a standard electronic chassis. It comprises an oscillator which drives the probe tuned circuit via a buffer amplifier and a meter which indicates the value of the peak voltage developed across the tuned circuit. The output from the oscillator is amplitude-stabilised so that the resonance peaks indicated on the meter are unaffected by variations in the driving voltage with frequency.

Fig. 2 is a circuit diagram of the unit. Valves $V3$ and $V4$ comprise the high frequency portion, and these with their associated circuits are built on a separate chassis as indicated by the dotted line. $V3$ is a cathode-coupled oscillator whose frequency of oscillation is controlled by a linear-law Sullivan variable capacitor. The output from the buffer amplifier $V4$ excites the resonant circuit. It is also rectified in a voltage-doubling circuit and fed back to the amplitude stabilising circuit consisting of $V2A$ and $V2B$.

[illegible]

4.2 THE PROBE UNIT

The inductive loop consists of a stainless-steel tube of inside diameter $1\frac{1}{4}$ in., with a $\frac{1}{8}$ in. diameter rod along its centre line. The rod is welded to the centre of an end-plate which closes the lower extremity of the tube. Holes along the tube prevent air-locks and so ensure that the level of the liquid in the probe will be the same as that in the main tank.

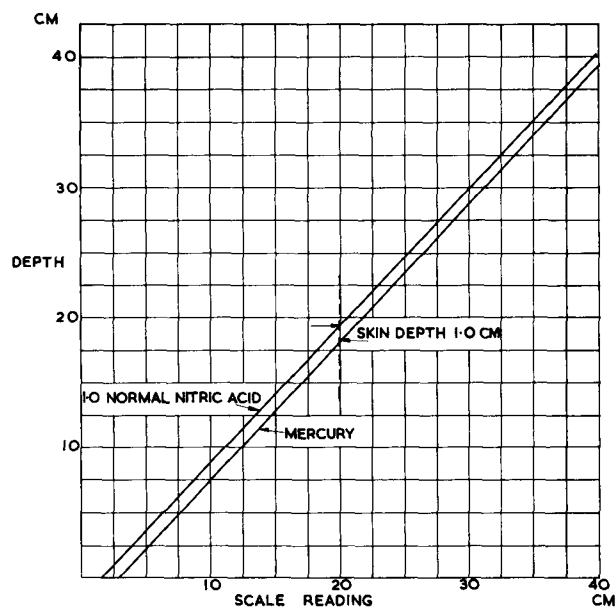


The circuit of the probe head unit is shown in Fig. 1. Capacitors C_q and C' are connected in series across the probe inductance and the exciting voltage

from the oscillator is injected across C' . Since $C' \gg C_q$, most of the voltage in the resonant circuit appears across C_q and this is rectified by the germanium rectifier. Resistance R matches the characteristic impedance of the concentric cable which connects the probe to the Oscillator Unit.

5. PERFORMANCE AND DISCUSSION

The instrument described has a probe length of 50 cm and a range of 40 cm in level. The setting accuracy is ± 0.1 cm on a 3 in. radius semi-circular scale. The two calibration curves (Fig. 4) were plotted, one for normal (1.0-N) nitric acid and the other for mercury. These illustrate the linear relation between scale reading and depth of liquid. Mercury may be regarded as a perfect conductor and therefore the skin depth for 1.0 N nitric acid is represented by the vertical distance between the two curves.



CALIBRATION CURVES

FIG 4

Normal oscillator drifts produce negligible error since the frequency range is about 3 : 1. The frequency can, in any case, be checked periodically against a standard. A simple built-in crystal calibrator was provided in one indicator unit made for test purposes. The resonating capacitor (C_q in Fig. 1) is a moulded silvered mica type and is stable to better than 0.1%.

Two factors affect the setting accuracy of this type of instrument: the magnification (or selectivity) of the resonant circuit and the scale length of the meter which indicates the peak. The selectivity decreases as the resistivity of the liquid increases, thereby reducing the setting accuracy; eqn.(A2) of Appendix 2 shows quantitatively the relationship between these two variables.

The effective scale length of the meter may be increased by backing off, i.e. by electrically suppressing the zero. As the peak output of the probe is somewhat variable over the range, however, the amount of backing off must be made adjustable, in order to bring the peak on scale, if a large increase in effective scale length is required.

A major source of error in this type of instrument arises from the fact that it is difficult to tune accurately to a peak on a meter. A null-balance form of indication would give greater accuracy. This can be obtained by cyclically varying the oscillator frequency over a small range at a suitable modulating frequency, say 50 c/s. The modulation frequency component of the probe detector output is amplified and synchronously rectified to produce a voltage which changes sign as the mean oscillator frequency is tuned through the probe resonance. The meter reads zero when the tuning is a long way from resonance on either side, but there is no ambiguity because only at resonance does the meter reading change sign.

6. APPLICATION TO LIQUID METALS

The principle is well-suited to the level measurement of liquid metals, such as sodium or potassium, since they have high conductivity. It has the advantage over other types of instrument (e.g. those depending on eddy current effects) that it is independent of temperature variations.

In practice, two problems have to be solved: the working temperature of the electrical components and the type of insulator to be used to withstand the action of sodium vapour. Electrical components are now available for use up to 100°C. Future developments may extend this upper limit considerably. In some cases, however, it is possible to arrange that the components are sufficiently far removed from the point of measurement to ensure that they do not exceed their maximum working temperature by extending the rigid co-axial line system upwards through the thermal lagging. This means, of course, a lower frequency and a smaller fractional change between minimum and maximum levels, so that the oscillator and fixed capacitor stabilities become more important. An extension of up to ten times the level range is considered possible.

An insulator which forms an effective gas-tight seal must be put in the co-axial line close to the tank. This insulator cannot be kept cool as it would then collect condensed metal vapour. It can, however, be kept out of direct contact with the liquid metal by a suitable trap for the blanket gas.

7. CONCLUSIONS

7 1 A new type of electronic liquid level gauge has been developed, and is capable of high accuracy, as shown by laboratory tests. The present design is limited to use with liquids of resistivity less than about 3 ohm/cm, such as solutions of strong acids or alkalis (about 1.0 N or greater).

7 2 The design may be modified to extend the use of the instrument to liquids of higher resistivity; to do this, higher frequencies must be used, and the resonant line mode of operation becomes necessary.

7 3 The system is of special value when the probe must be placed in an inaccessible position. This applies whether the probe is physically inaccessible or whether it is unapproachable because of contamination or radiation hazards. The probe itself is simple and robust, and will require no attention; the electronic components in the probe-head are extremely under-run and should last indefinitely.

7.4 The system is particularly attractive when the connecting link must be of considerable length, because a single co-axial line is all that is required to connect the probe to the Indicator Unit.

7.5 Most gauges, notably the capacitance type, are adversely affected when the surfaces of the measuring element are wetted. The present system is unaffected in this way.

7.6 The new system has the advantage of being independent of temperature in its application to liquid metal level measurement.

REFERENCES

DAVIDSON, D.F. (1955a). U.K.A.E.A. Report IGR-TN/CA-251. *A capacitance level gauge.*

DAVIDSON, D.F. (1955b). Provisional Patent Application 7894/55. *Single-probe liquid level detection.*

LIST OF SYMBOLS

C	=	Capacitance of probe per unit length
C_p	=	Capacitance of probe resonant circuit, farads
C_x	=	Capacitance of oscillator tuned circuit, farads
D	=	Inner diameter of cylinder, cm
d	=	Diameter of rod, cm
f	=	Frequency, c/s
G	=	Conductance of liquid per unit length, $\text{ohm}^{-1}/\text{cm}^{-1}$
j	=	The complex operator $\sqrt{-1}$
l	=	Depth of non-immersed part of probe, cm
L	=	Inductance per unit length, henries/cm
L_p	=	Probe inductance, henries
L_x	=	Inductance of oscillator tuned circuit, henries
Q	=	Magnification of tuned circuit
R	=	Series resistance per unit length, ohm/cm
S	=	Skin depth, cm
Z_o	=	Characteristic impedance of transmission line
Z_{sc}	=	Input impedance of transmission line terminated by a short-circuit
ρ	=	Resistivity of liquid, ohm/cm
ω	=	Angular frequency, rad/sec (= $2 \pi f$)

APPENDIX 1

LOW FREQUENCY INDUCTANCE

As explained in Section 2, if a linear scale is required, the probe must be a pure inductance. The maximum frequency for this condition to be satisfied is derived as follows:

From standard co-axial line theory:

$$Z_{sc} = Z_0 \tanh \gamma l \quad \text{where} \quad \gamma^2 = (R + j\omega L)(G + j\omega C)$$

In this case R and G may be neglected because

$$\omega L \gg R \quad \text{and} \quad \omega C \gg G$$

$$\text{Hence} \quad \gamma^2 = j^2 \omega^2 LC$$

$$\gamma = j\omega \sqrt{LC}$$

$$Z_{sc} = \sqrt{\frac{L}{C}} \tanh j\omega l \sqrt{LC}$$

$$= j \sqrt{\frac{L}{C}} \tan \omega l \sqrt{LC}$$

$$\text{or} \quad \omega L_p = \sqrt{\frac{L}{C}} \tan \omega l \sqrt{LC}$$

$$= \omega l L \quad \text{within 1\%, if} \quad \frac{\tan \omega l \sqrt{LC}}{\omega l \sqrt{LC}} \leq 1.01$$

$$\omega l \sqrt{LC} \leq 0.2$$

If we take the case of $l = 100 \text{ cm}$

$$\omega_{\max} = \frac{0.2}{100 \sqrt{LC}} = 6 \times 10^7$$

$$\text{or} \quad f_{\max} = \frac{60}{2\pi} \times 10^6 \approx 10 \text{ Mc/s}$$

APPENDIX 2

CALCULATION OF ERRORS CAUSED BY FINITE CONDUCTIVITY OF THE LIQUID

Because of the limited conductivity of the liquid, the effective current path is not along the surface of the liquid but at some distance below it. It follows that the inductance of the non-immersed part of the probe is increased by a small inductance whose magnitude depends on the skin depth. Since this represents a systematic error, it is of interest to assess its magnitude and also how it depends on other parameters. The liquid part of the inductive loop also introduces resistance to the probe tuned circuit and affects the sensitivity of the system.

It is convenient to regard the immersed part of the probe as a transmission line and the following treatment shows that a reasonable simplification is to regard the line as infinite.

By transmission line theory:

$$Z_{sc} = Z_0 \tanh \gamma l$$

$$\text{where } \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\text{Consider } R \rightarrow 0, \quad C \rightarrow 0$$

$$\text{or } \omega L \gg R, \quad G \gg \omega C$$

$$\text{Then } \gamma^2 = j\omega LG$$

$$\gamma = \frac{(1 + j) \cdot \sqrt{\omega LG}}{\sqrt{2}}$$

$$\text{and } Z_0 = \sqrt{\frac{j\omega L}{G}} = \sqrt{\frac{\omega L}{2G}} + j \sqrt{\frac{\omega L}{2G}} \dots\dots\dots(A1)$$

$$\text{Let } \alpha = \frac{1}{\sqrt{2}} \sqrt{\omega LG}; \quad \beta = \frac{1}{\sqrt{2}} \sqrt{\omega LG}$$

$$\begin{aligned} Z_{sc} &= Z_0 \tanh \gamma l \\ &= Z_0 \frac{\tanh \alpha l + j \tan \beta l}{1 + j \tanh \alpha l \tan \beta l} \end{aligned}$$

$$= Z_o \frac{\tanh l \sqrt{\frac{\omega L G}{2}} + j \tan l \sqrt{\frac{\omega L G}{2}}}{1 + j \tanh l \sqrt{\frac{\omega L G}{2}} \tan l \sqrt{\frac{\omega L G}{2}}}$$

Let $\omega L G \rightarrow \infty$

$$\text{Then } Z_{sc} = Z_o \left(\frac{1 + j \tan \beta l}{1 + j \tan \beta l} \right)$$

$$= Z_o$$

$$= R_1 + j\omega L_1 \text{ say}$$

$$R_1 + j\omega L_1 = \sqrt{\frac{\omega L}{2G}} + j \sqrt{\frac{\omega L}{2G}}$$

$$j\omega L_1 = j \sqrt{\frac{\omega L}{2G}} = j\omega L S$$

where $S = \text{skin depth}$

$$S = \sqrt{\frac{1}{2\omega L G}}$$

Now, when $\omega L G \rightarrow \infty$, $S \rightarrow 0$, so we can say that for small skin depths

$$Z_{sc} = Z_o$$

$$\text{We now have } Z_{sc} = Z_o = \sqrt{\frac{\omega L}{2G}} + j \sqrt{\frac{\omega L}{2G}} \dots\dots\dots(A2)$$

$$= R_1 + j\omega L_1$$

The real and imaginary parts of this expression represent, respectively, a damping resistance R_1 in the probe resonant circuit and a small inductance L_1 incremental to that of the non-immersed part of the probe.

MAGNIFICATION

The magnification of the probe tuned circuit is given by:

$$Q = \omega \frac{\text{total inductance of circuit}}{\text{total resistance of circuit}}$$

If it is assumed that R_1 is the only resistive component and that the incremental inductance due to skin effect is negligible, then:

$$\begin{aligned} Q &= \frac{l\omega L}{R_1} \\ &= l \sqrt{2\omega LG} \end{aligned} \quad \left[\text{from (A2)} \right]$$

where $G = \frac{2\pi}{\rho \ln\left(\frac{D}{d}\right)}$

$$\begin{aligned} Q &= l \sqrt{2 \cdot \frac{2\pi}{\rho \ln\left(\frac{D}{d}\right)} \cdot 2\pi f \cdot 2 \cdot 10^{-9} \cdot \ln\left(\frac{D}{d}\right)} \\ &= 3.96 \cdot 10^{-4} l \sqrt{\frac{f}{\rho}} \end{aligned}$$

SKIN DEPTH

From equation (A2), $\omega L_1 = \sqrt{\frac{\omega L}{2G}}$

The skin depth is given by:

$$S = \frac{L_1}{L}$$

$$\begin{aligned}
&= \frac{1}{\sqrt{2 \omega L G}} \\
&= 2.51 \times 10^3 \sqrt{\frac{\rho}{f}} \dots\dots\dots (A3)
\end{aligned}$$

For the case of 1.0 N nitric acid, $\rho = 3 \text{ ohm/cm}$ at $f = 15 \text{ Mc/s.}$

Whence $S = 1.12 \text{ cm}$

Experimental confirmation of this figure was obtained by comparing readings taken with mercury and 1.0 N nitric acid (Fig. 4).