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HIGH ENERGY FFAG RING MAGNET WITH SPIRALLY RIDGED FIELD*

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In the first reports on the focussing properties of magnetic fields having the so called spirally ridged form

$$H = Ho(r/ro)^k \quad (1 + f \sin\left[\frac{x}{\lambda} - N\theta\right]) \text{ or}$$

$$H = Ho(r/ro)^k \quad (1 + f \sin\left[1/w \text{ Ln } r/ro - N\theta\right])$$

The motion was expanded about a circular reference circle and the inhomogeneous term was ignored in the discussion of the stability of the motion[1,2,3]. It was pointed out at the end of reference 1 that the inhomogeneous term must be considered. This is because the scalloped motion of the closed equilibrium orbit causes the orbit to spend more time in the negative field gradient regions and less time in the positive field gradient regions than would be spent if the equilibrium orbit were circular. The treatment with a scalloped orbit as a reference orbit should give stronger vertical focussing.

The differential equation for the motion about the scalloped orbit has been derived by Laslett and the motion exhibits the expected effect. This is described in a con-

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current report by Laslett.

The result is that the radial focussing of a negative momentum compaction magnet is practically lacking and that a positive momentum compaction machine is easier to achieve. We will apply Laslett's results to a revision of the example of a spiral ridge 20 Bev ring magnet as discussed in 1. Collecting his formulae : the closed orbit Laslett found is

$$r = r_1 \left[1 - \frac{f}{N^2 - (k+1)} \sin \left[N\theta + \arc\tan \left\{ (k-2)w \right\} \right] \right]$$

$r_1 v$ is the radial displacement from this reference orbit.

$r_1 y$ is the axial displacement from this reference orbit.

For large k and very small $\delta = \tan^{-1}\left[(k+2)w\right]$.

$$v'' + \left\{ 1 + k - \frac{1}{2}\left(\frac{f}{w}\right)^2 \left(1/\left[N^2-k\right]\right) + \underbrace{\frac{f}{w}\cos N\theta + \frac{1}{2}(f/w)^2\left(1/\left[N^2-k\right]\right)\cos 2N\theta}_{g(r,\theta)} \right\} v = o$$

$$y'' - \left\{ k - \frac{1}{2}(f/w)^2\left(1/\left[N^2-k\right]\right) + \frac{f}{w}\cos N\theta + \frac{1}{2}(f/w)^2\left(1/\left[N^2-k\right]\right)\cos 2N\theta \right\} y = o$$

We see that the usual constant term, k, is decreased when the reference orbit is scalloped.

Using g (r,θ) in Symon's smooth approximation we have

$$A.G. = \left[\int g(r,\theta)d\theta + c\right]^2$$ for the alternating gradient term in the equation for the smooth motion:

$$V'' + \left(\left[k - \frac{1}{2}\left(\frac{f}{w}\right)^2 \frac{1}{N^2 - k}\right] + A.G.\right) V = 0$$

$$Y'' + \left(-\left[k - \frac{1}{2}\left(\frac{f}{w}\right)^2 \frac{1}{N^2 - k}\right] + A.G.\right) Y = 0 \qquad (2)$$

We must have

$$\int g(\theta)\, d\theta + c = 0 = \frac{f}{wN} \sin N\theta + \frac{1}{4N}\left(\frac{f}{w}\right)^2 \frac{1}{(N^2-k)} \sin 2N\theta + C$$

So $C = 0$

$$\left[\int g(\theta)d\theta + c\right]^2 = \left(\frac{f}{w}\right)^2 \frac{1}{N^2} \sin^2 N\theta + \frac{1}{2N^2}\left(\frac{f}{w}\right)^3 \frac{1}{N^2-k} \sin N\theta \times$$

$$\times \sin 2N\theta + \frac{1}{16N^2}\left(\frac{f}{w}\right)^4 \frac{\sin^2 2N\theta}{(N^2-k)^2}$$

$$\overline{\left[\int g(\theta)d\theta + c\right]^2} = \left(\frac{f}{w}\right)^2 \frac{1}{N^2}\frac{1}{2} + \frac{1}{32}\left(\frac{f}{w}\right)^4 \frac{1}{N^2}\frac{1}{(N^2-k)^2} \qquad (3)$$

or

$$A.G. \cong \left(\frac{f}{\sqrt{2}wN}\right)^2$$ which is the same alternating gradient term which one gets when a circle is used for the reference orbit.

The usual rule

$$\nu_r^2 = \left\{1 + K\right\} + (A.G.) \qquad (4)$$

$$\nu_y^2 = \left\{-K\right\} + AG,$$ where K is the non-alternating term in the differential equation (1), becomes

$$\nu_r^2 \doteq \left\{ 1 + k - \left(\frac{f}{\sqrt{2}w} \frac{1}{\sqrt{N^2 - k}} \right)^2 \right\} + \left(\frac{f}{\sqrt{2}wN} \right)^2$$

$$\nu_y^2 = \left\{ - k + \left(\frac{f}{\sqrt{2}w} \frac{1}{\sqrt{N^2 - k}} \right)^2 \right\} + \left(\frac{f}{\sqrt{2}\, wN} \right)^2 \tag{5}$$

or

$$\nu_r^2 \doteq 1 + k$$

$$\nu_y^2 = - k + \left(\frac{f}{wN} \right)^2 = - k + 2 \text{ A.G.}$$

Thus in this approximation the radial focussing is determined mainly by the variation of average field with radius, not by the alternating gradients while the axial focussing is enhanced. We have taken ridges which make a small angle with the orbit to find this result. That is, $w \lll 1$. With $w \longrightarrow \infty$ as in Mark I, the radial focussing depends on the A.G. term.

As an example of a magnet choose:

$KE_{maximum} = 20$ Bev, $\overline{H_o} = 14{,}000$ Gausses

$Ke_{injection} = 5$ Mev, $\overline{H_i} = 64.$ Gausses

$r_o = 5000$ cm

$r_i = 4825$ cm

$d \cong r_o - r_i = 175$ cm

Then

$k \cong 150$

$\nu_r = \sqrt{151} = 12.3$ radial betatron oscillations around the machine.

Choose

$\nu_y = 6.2$ axial oscillations around the machine.

Choose $N > 2\nu_r$ so $\sigma_r < \pi$.

$N = 37$ gives $\begin{cases} \sigma_r \cong \dfrac{2\pi}{3} \\ \sigma_y \cong \dfrac{\pi}{3} \end{cases}$

So by (5)

$\frac{f}{w} = \sqrt{\nu_y^2 + k} = 504$. We can have $f = \frac{1}{4}$ so $2\pi w \doteq 15$ cm radial spacing between ridges, and there would be about twelve ridges across the pole, each spiraling about one third of the way around the machine. We know the gap can be $\sim \frac{1}{4}\, 2\pi w \cong 4$ cm without critical points in the shape of the iron pole surface.