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A STUDY OF THE PRIMARY SHIELD FOR THE PRDC REACTOR

for

ATOMIC POWER DEVELOPMENT ASSOCIATES

by

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Temperature distributions, irradiation effects, stacking arrangements, voidage, and economics for the borated-graphite shield of the PRDC reactor were investigated.

Of the shield systems considered, four are reported here. System 1 contains 30 in. of 1 per cent borated graphite, with either ordinary graphite or a cement as a filler for the remainder of the volume. The maximum temperature at the flex plates in this system was calculated to be 500 F. Systems 2 and 3 consist of 2 in. of 5 per cent borated graphite near the core vessel and 1/2 in. of Boral at the primary-shield tank. A filler material of carbon blocks is used in System 2 and graphite in System 3. The calculated maximum temperatures were 700 F and 350 F, respectively. System 4 consists of a laminated structure of Boral and graphite near the primary-shield tank and carbon-block filler. It was calculated to have a maximum temperature of 600 F at the flex plates. The maximum temperature at the flex plates recommended by APDA is 500 F.

Energy storage and radiation damage were found to be within permissible limits in all four systems. However, these conclusions are based on experimental data from the Hanford reactor in which the neutron-energy spectrum differs considerably from the PRDC spectrum.

A porosity of less than 740 cu ft is required in order that a sodium leak from the core vessel does not expose the core. The voidages in any of the systems mentioned above is about 400 cu ft excluding absorption effects. These are believed to be small.

The systems containing Boral were found to be less expensive than the ones using only borated graphite. Over-all material costs range between $230,000 for Boral systems and $350,000 for borated-graphite systems.

INTRODUCTION

Atomic Power Development Associates is developing a fast-reactor system, with the first stage of neutron shielding located between the reactor-core vessel and the primary-container vessel. This shield should attenuate the neutron fluxes to about 10^9 n/(cm^2)(sec). Following this primary shield, a secondary shield surrounding the primary vessel and sodium piping is used to further reduce the flux to about 10^4 n/(cm^2)(sec). Finally, a biological shield is planned which encloses the steel building that contains the reactor.

The first stage of shielding is some form of borated graphite or its equivalent. Neutrons will be slowed down in the graphite to permit absorption by the boron. Some studies performed by APDA have indicated that sufficient shielding can be obtained by using a homogeneous 1 per cent borated graphite annulus 30 in. thick at the lower reactor vessel and 18 in. thick at the upper reactor vessel. However, some questions regarding irradiation effects, temperature distributions throughout the shield, and choice of a suitable stacking arrangement for the shield remained to be answered. In addition, the economics of the first-stage shield design required further considerations.

Battelle was asked to study these problems. In the course of this study, especially with respect to the economics of the system, a number of alternate primary-shield designs were considered. This report presents the results of Battelle's investigations. Subsequent shield calculations and corrosion studies appear to be necessary before the final shield specifications can be written.

DESCRIPTION OF THE SHIELD SYSTEMS

The following four systems represent possible configurations which might satisfy the shield requirements.

System 1. (Figure 1)

Around the lower reactor vessel, the first 30 in. from the core vessel contains 1 per cent borated graphite. (The economics of using up to 5 per cent boration are also considered.) A 4-in. layer of thermal insulation after the first 6 in. of borated graphite returns the heat generated in this region to the reactor vessel and the sodium coolant. The rest of the volume about the lower reactor vessel (extending to the top of the flex plates which support the reactor vessel) is filled with ordinary graphite. Four in. of thermal insulation are placed around the first 6 in. of borated graphite in the region about the upper vessel also, but the total thickness of borated graphite is decreased to 18 in. Two filler materials are considered in this upper region. The first (System 1a) is a concrete made of one part calcium aluminate cement, two parts pure silicon sand, and four parts large aggregate. Samples made of Alcoa high-purity calcium aluminate cement and Universal-Atlas Lumnite cement are currently being tested in molten sodium at APDA. The second filler material considered is carbon block (System 1b), which could be used if the Lumnite should prove to be unsatisfactory.

System 2. (Figure 2)

Around the lower reactor vessel, the 6 in. inside the thermal insulation contains 4 in. of ordinary graphite followed by 2 in. of 5 per cent borated graphite. Around the upper vessel, 2 in. of 3 per cent borated graphite are substituted for the 2 in. of 5 per cent borated graphite. Carbon block serves as both filler material and moderator outside of the insulation throughout the vessel, except for 1 ft. of ordinary graphite at the bottom of the primary shield tank. Two 1/4-in. sheets of Boral are located just inside the primary-shield tank.

System 3. (Figure 3)

The carbon block of System 2 is replaced by graphite from the top of the flex plates down.

System 4. (Figure 4)

This system has a laminar arrangement of three 1/4-in. Boral sheets separated by 2-in. layers of graphite adjacent to the primary-shield tank. The 4 in. of thermal

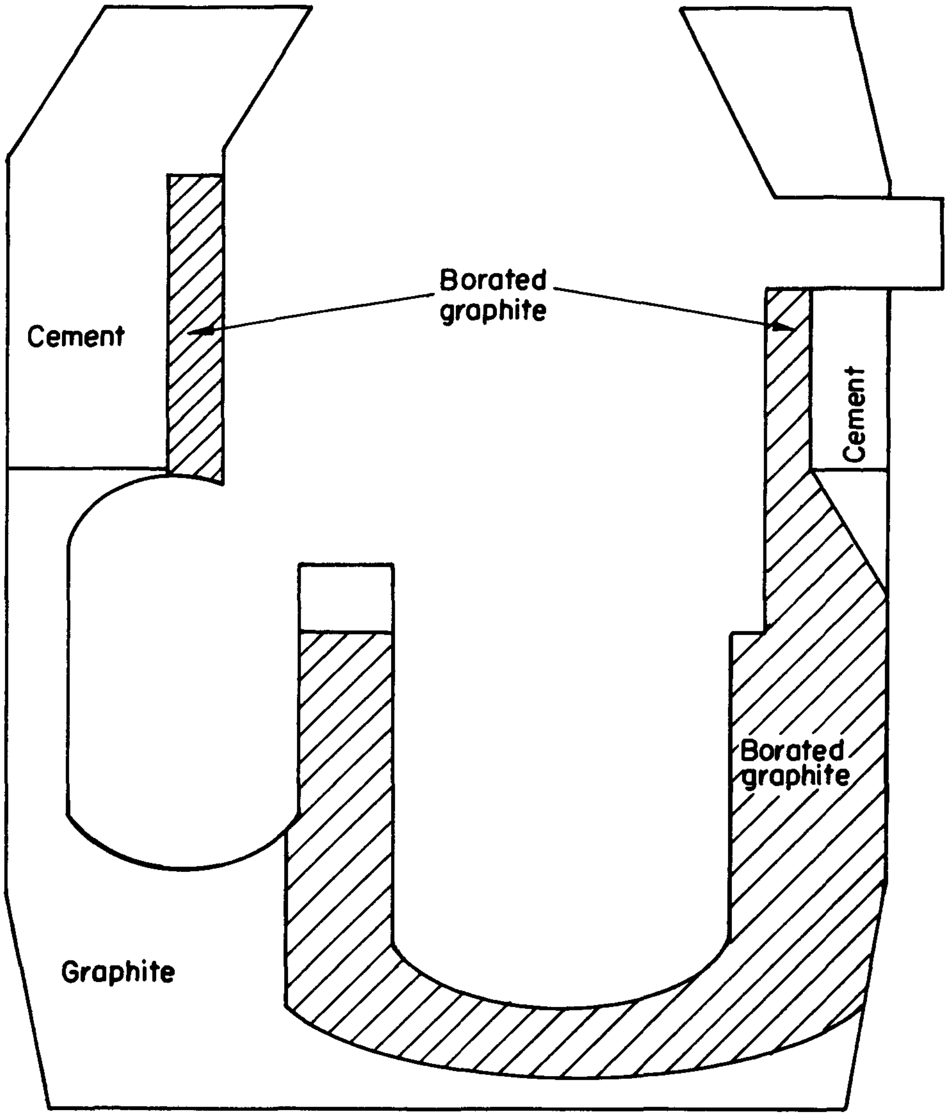

FIGURE 1. SYSTEM 1

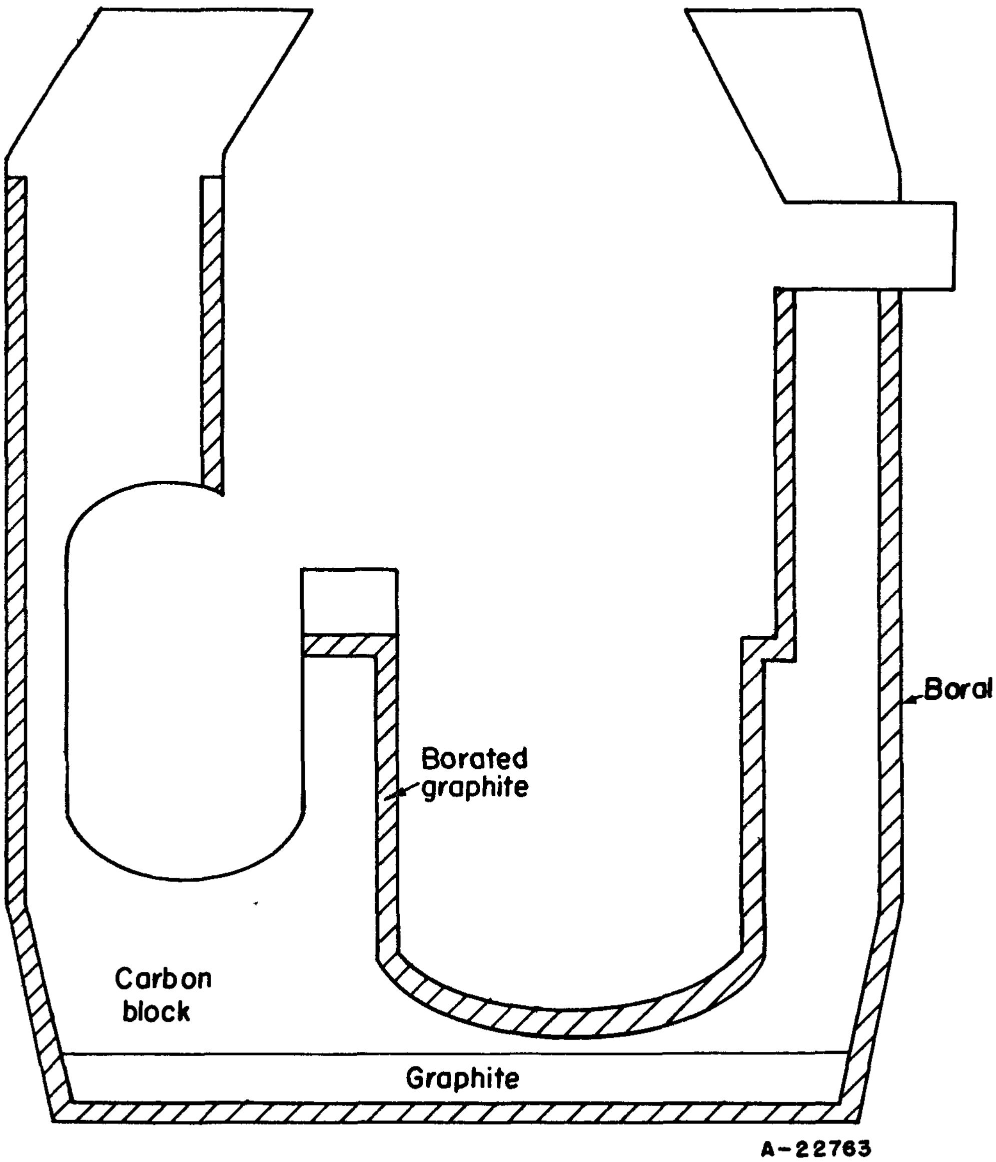

FIGURE 2. SYSTEM 2

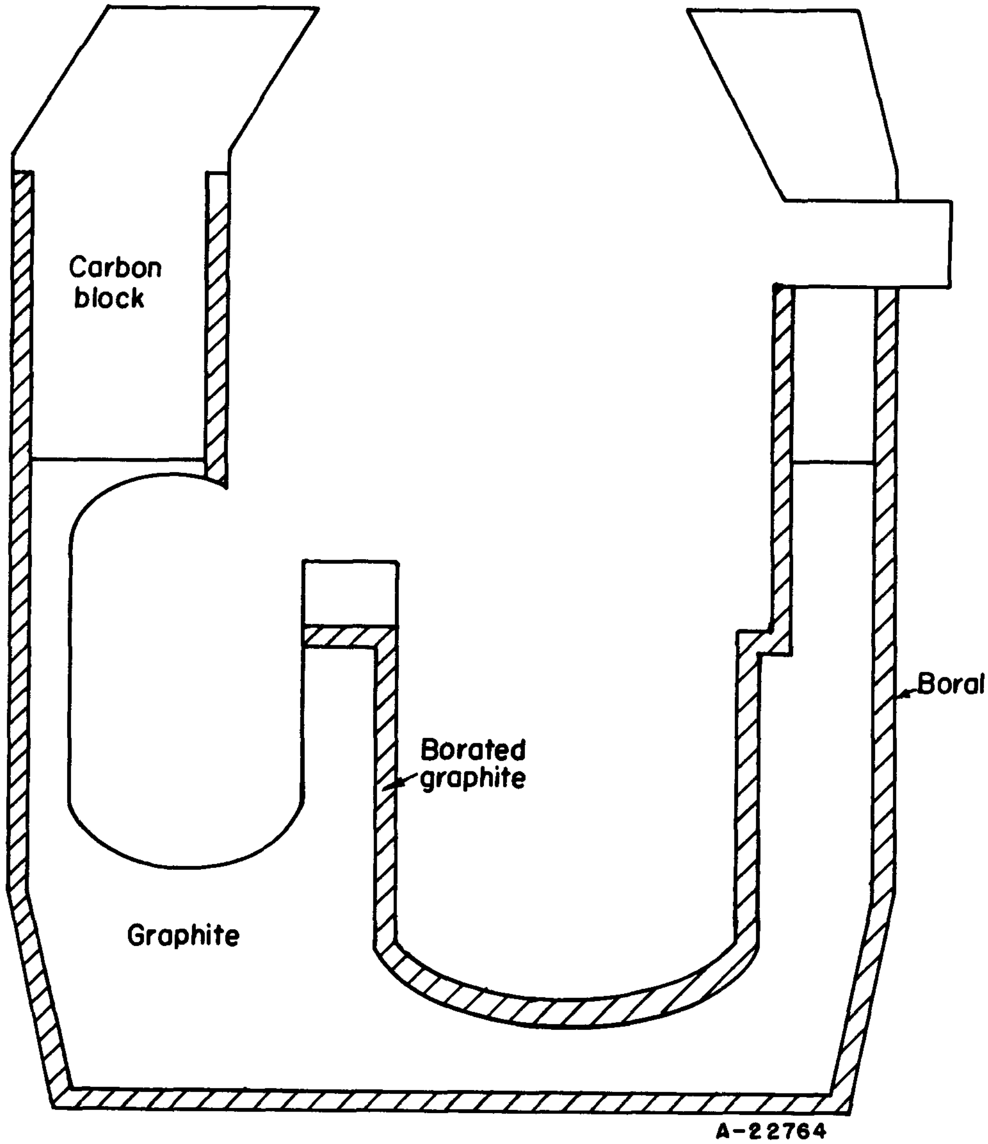

FIGURE 3. SYSTEM 3

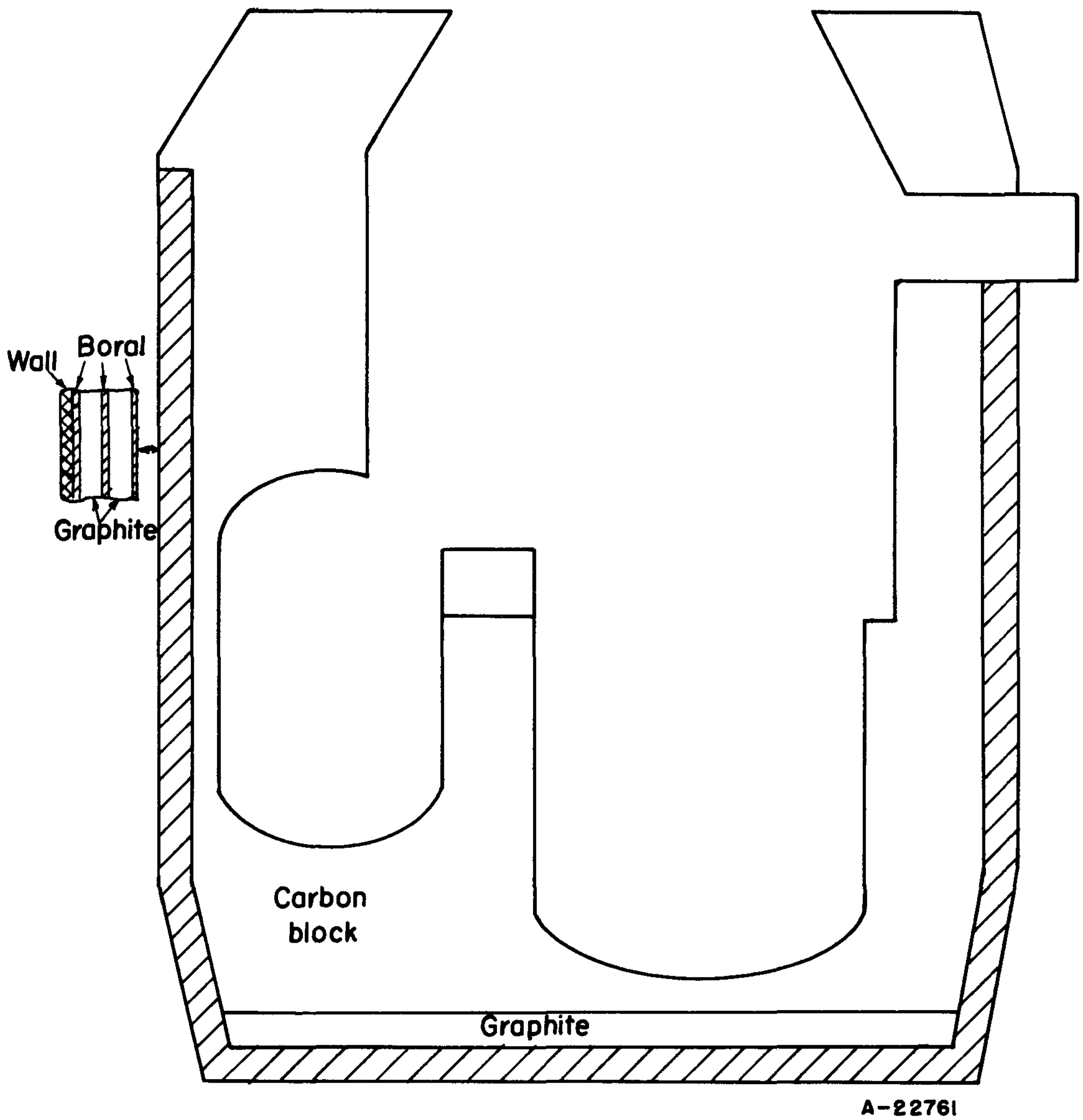

FIGURE 4. SYSTEM 4

insulation is directly against the reactor vessel, with no borated graphite included. With the exception of 1 ft of graphite on the bottom, and the laminations, carbon block is used throughout as the filler material.

The first system is the only one which to date has been shown to give satisfactory neutron shielding. The other systems are included in the analysis in case the less expensive systems using Boral are shown to be applicable. Systems using pebble-type filler were not included because the porosity of a pebble bed is too great. A porosity of less than 10 per cent is required so that a sodium leak in the core vessel will not expose the core.

Considerations of the above systems include the demonstration of their practicability in regard to temperature distributions, irradiation effects, stacking arrangements with allowances for thermal expansion and radiation distortion, voidage, sodium reactions, and economics.

TEMPERATURES IN THE SHIELD SYSTEMS

A two-dimensional analogue was set up to determine maximum temperatures at the flex plates and the temperature distributions to be expected in each of the shield systems. A complete description of this analogue is in Appendix A. Temperature distributions in the horizontal plane through the core center are shown for each of the shield systems in Figures 5 through 8. An average thermal conductivity of 10 Btu/(hr)(ft)(F) was used for graphite in this analysis. This value allows a factor of four for maximum irradiation effects (see irradiation of materials), and a factor of two for the estimated maximum loss of thermal bonding at the joints. The joints were taken to be a combination of actual contact, radiation, and gas gap. The heat-transfer coefficient for a gas gap of .01 in. is 28 Btu/(hr)(ft^2)(F), and the heat-transfer coefficient obtained from pure radiation at an ambient temperature of 500 F through a gap of any thickness is 6.2 Btu/(hr)(ft^2)(F). From considerations of these values and the expected actual contact, it seems reasonable to assume that an average thermal conductivity of 10 Btu/(hr)(ft)(F) can be obtained. The average conductivity of the carbon block was taken to be 1 Btu/(hr)(ft)(F) by similar considerations. The surface of the material adjacent to the primary shield tank was assumed to be an isotherm in each case.

In System 1, the temperature of the graphite adjacent to the primary-shield tank was calculated to be 200 F. This temperature was obtained by assuming radiation to the tank across the 2-in. gap and conduction to the 125 F nitrogen coolant outside the primary vessel. The temperature on the inside of the thermal insulation around the reactor vessel was calculated to be 900 F, and the temperature around the transfer rotor container was taken to be 700 F inside the thermal insulation. The heat-transfer coefficient for the thermal insulation was obtained from APDA's insulation specifications. The analogue solution for the temperatures in System 1 is shown in Figure 5. The maximum flex-plate temperature is about 500 F for this system, which is the maximum temperature recommended for the flex plates by APDA.

In System 2, the temperature of the carbon blocks adjacent to the Boral sheets was calculated to be 250 F. This temperature allows for about 1/16-in. loss of bond at the Boral-steel interface and at the Boral-Boral interface. Since very little heat leaks through the insulation or is generated in the carbon block, the outside carbon surface,

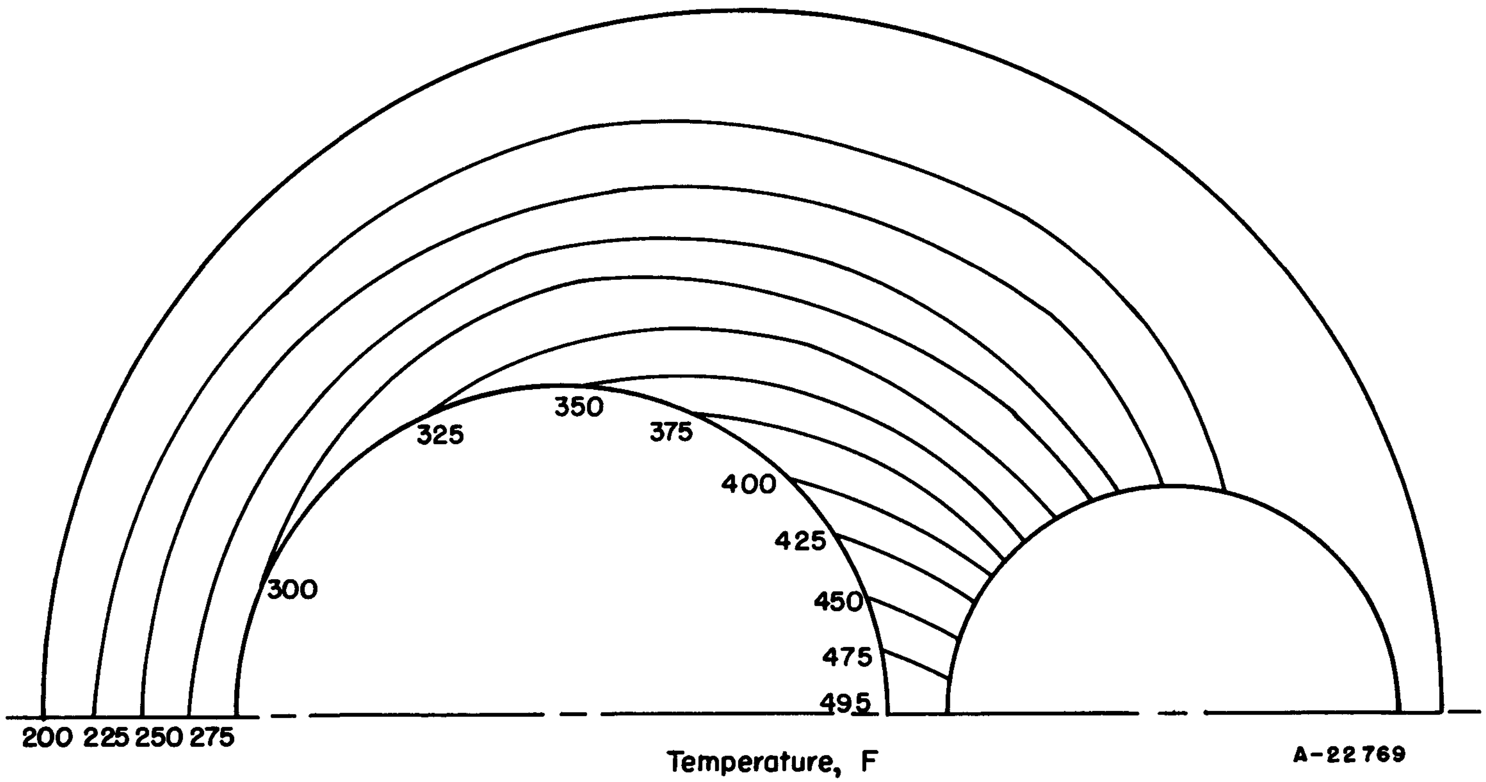

FIGURE 5. TEMPERATURE DISTRIBUTION IN SYSTEM 1

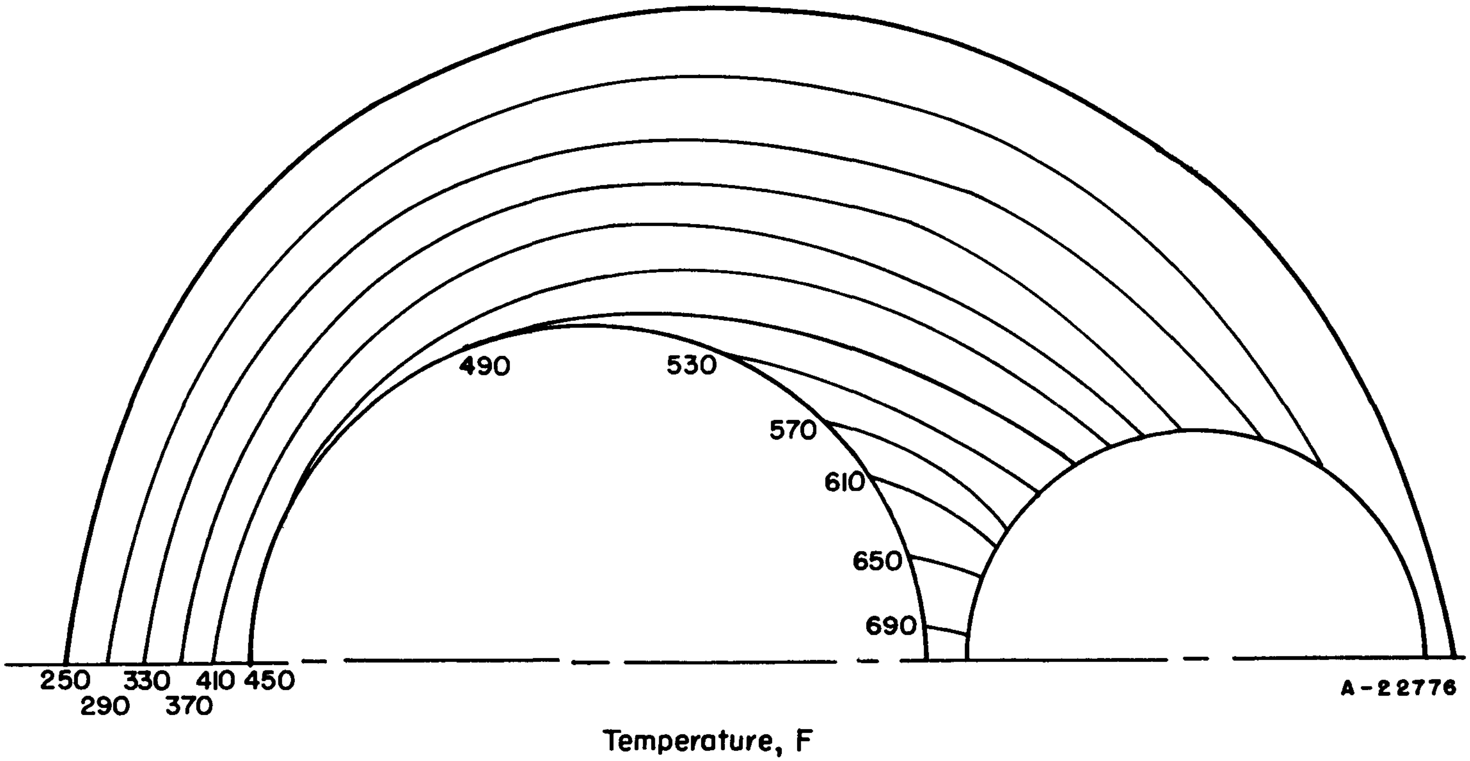

FIGURE 6. TEMFERATURE DISTRIBUTION IN SYSTEM 2

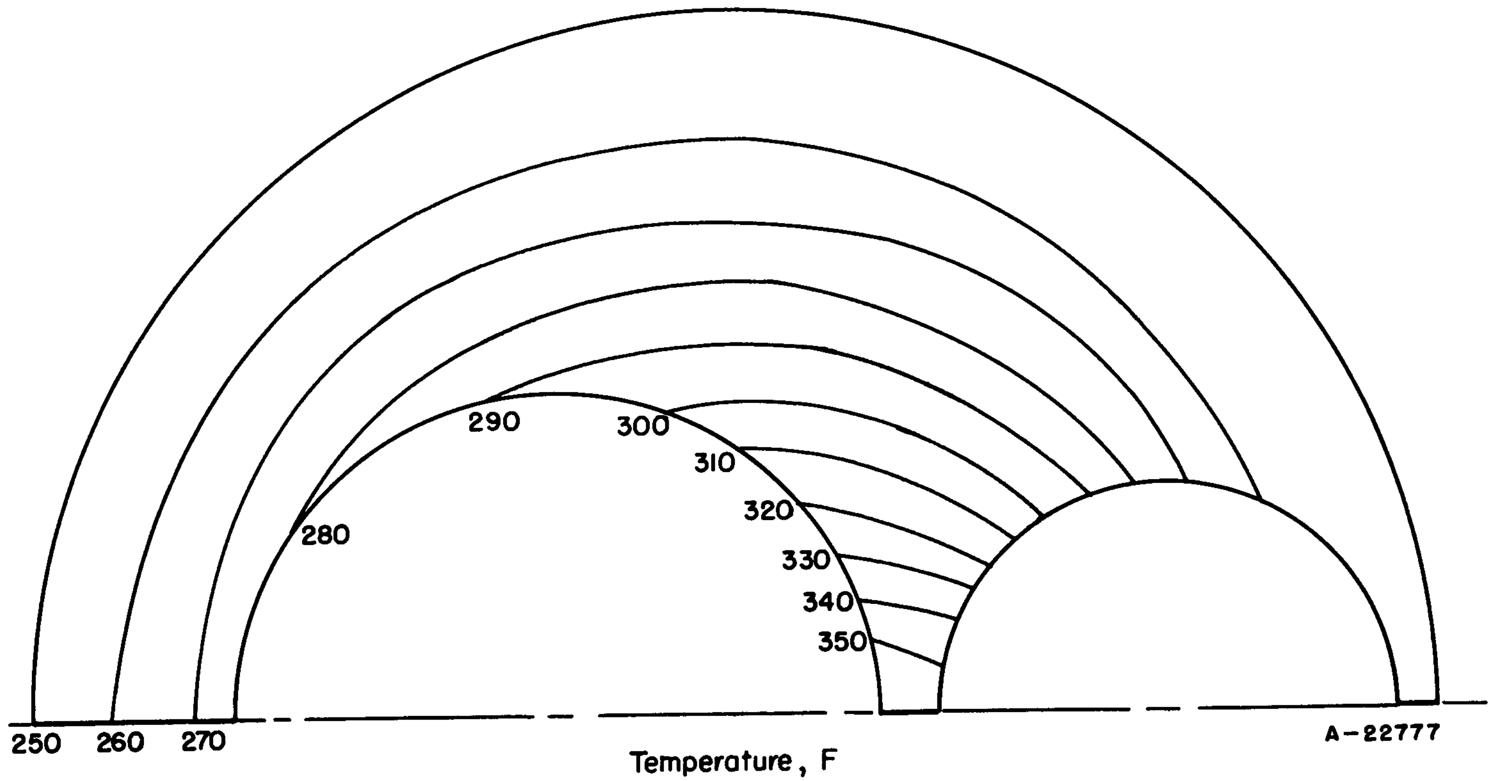

FIGURE 7. TEMPERATURE DISTRIBUTION IN SYSTEM 3

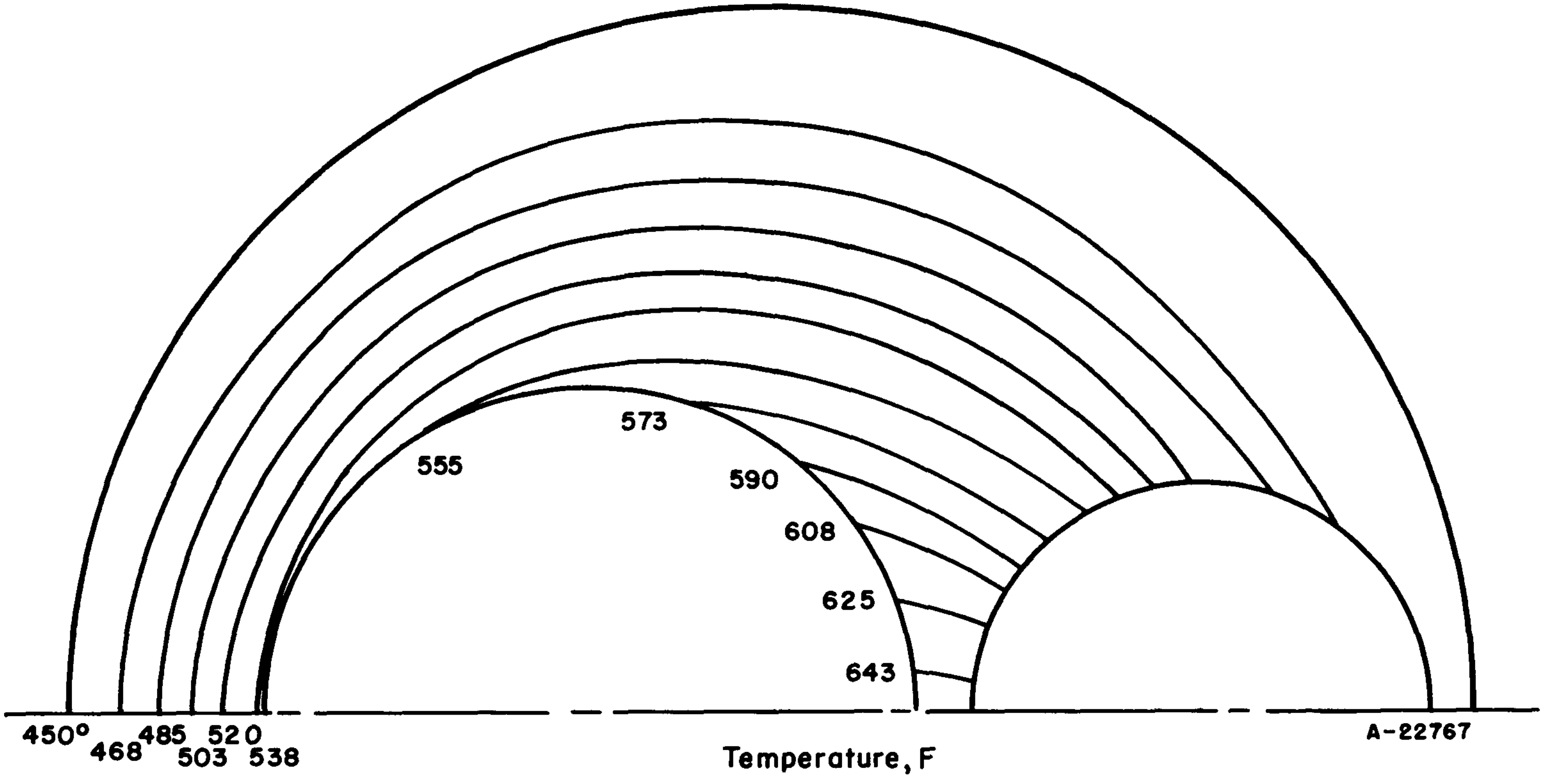

FIGURE 8. TEMPERATURE DISTRIBUTION IN SYSTEM 4

which is separated from the Boral by a 2-in. gap, is only slightly hotter than the Boral surface. The maximum flex-plate temperature calculated from the analogue is nearly 700 F (Figure 6). This is somewhat higher than the recommended 500 F. However, the tops of the flex plates will be about 700 F in any system because of their intimate thermal contact with the upper reactor vessel. The temperature directly inside the insulation is 930 F, which is slightly higher than in System 1.

The boundary conditions in System 3 are the same as those in System 2, but the lower thermal conductivity of graphite decreases the maximum flex-plate temperature to only 350 F (Figure 7).

In System 4, the temperature of the carbon block adjacent to Boral laminations was calculated to be 450 F. This calculation allowed for approximately a 10-mil separation at the Boral-graphite and Boral-steel interfaces with a 2-in. gap between the inside Boral face and the carbon brick. A temperature of 750 F was calculated at the inside of the insulation, next to the core vessel. The analogue solution to this system shows the maximum flex-plate temperature to be about 650 F (Figure 8). Since no borated graphite is used around the core vessel in this system, none of the heat generated in the shield is returned to the sodium. Consequently, the removal of this extra heat increases the load on the external nitrogen cooling system. Also, the neutron fluxes throughout the filler are much higher in this system than in the others.

Figure 9 shows the temperature distribution in a system similar to System 1 but without thermal insulation. These temperatures go as high as 1100 F and show the effect of extensive insulation failures.

Since the worst forseeable values of thermal conductivity were used everywhere, and since vertical conduction was neglected, the temperatures reported here should be higher than those to be expected in the physical systems. Temperatures around the upper vessel are lower than the maximum shown here for all systems. It was assumed that the heat production around the upper vessel would be 0.01 the production in the plane of the core center.

IRRADIATION EFFECTS

The maximum integrated neutron-flux irradiation received by any part of the graphite is approximately 2×10^{21} nvt during 10 years of operation. Under this exposure, the strength of graphite increases and the graphite becomes more brittle. However, in this application, embrittlement is not an important factor.

Figure 10 shows the effect of irradiation on thermal conductivity. Although the data is not complete, a factor of four should be sufficient to allow for the decrease in thermal conductivity at the temperatures encountered in the systems considered here.

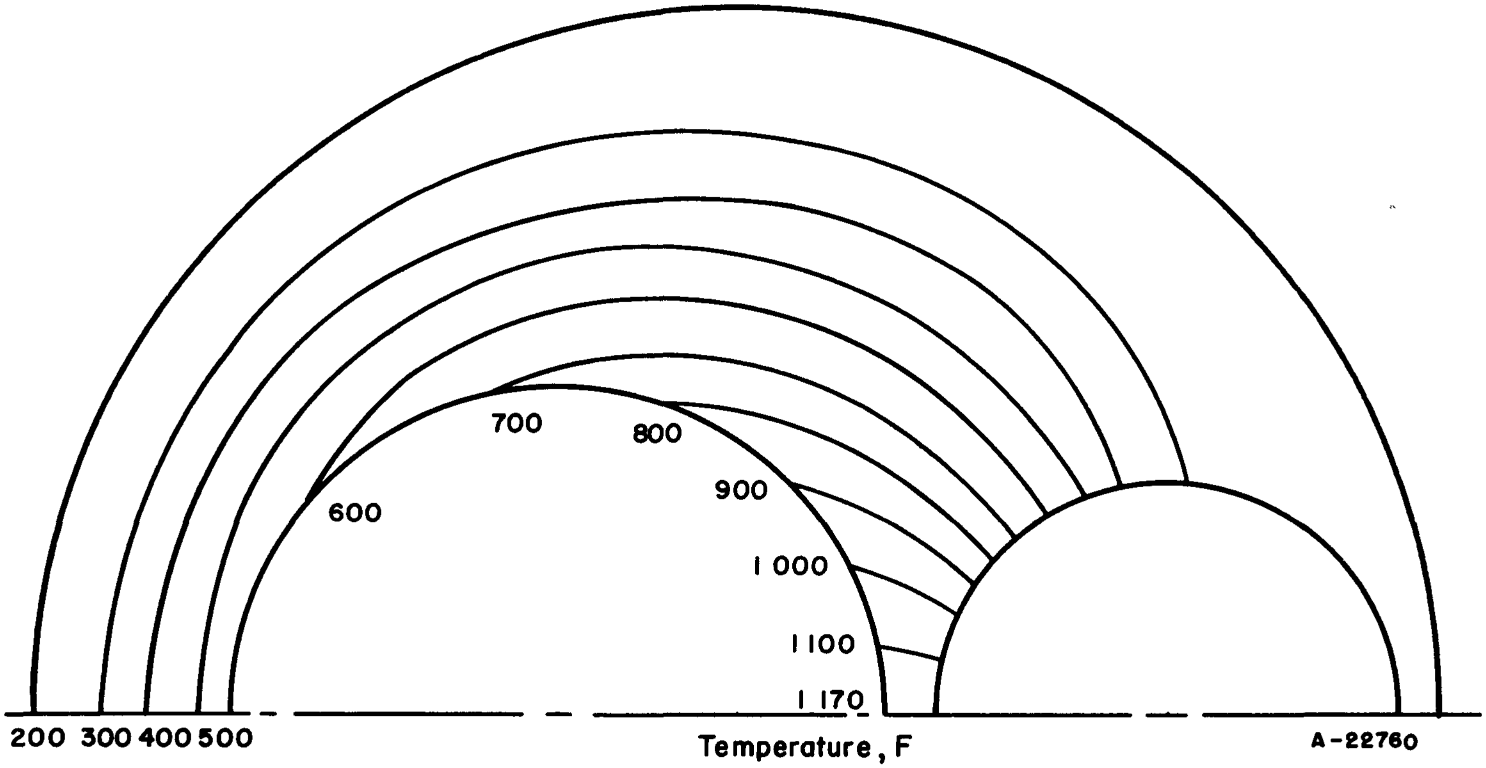

FIGURE 9. TEMPERATURE DISTRIBUTION IN SYSTEM I WITHOUT INSULATION

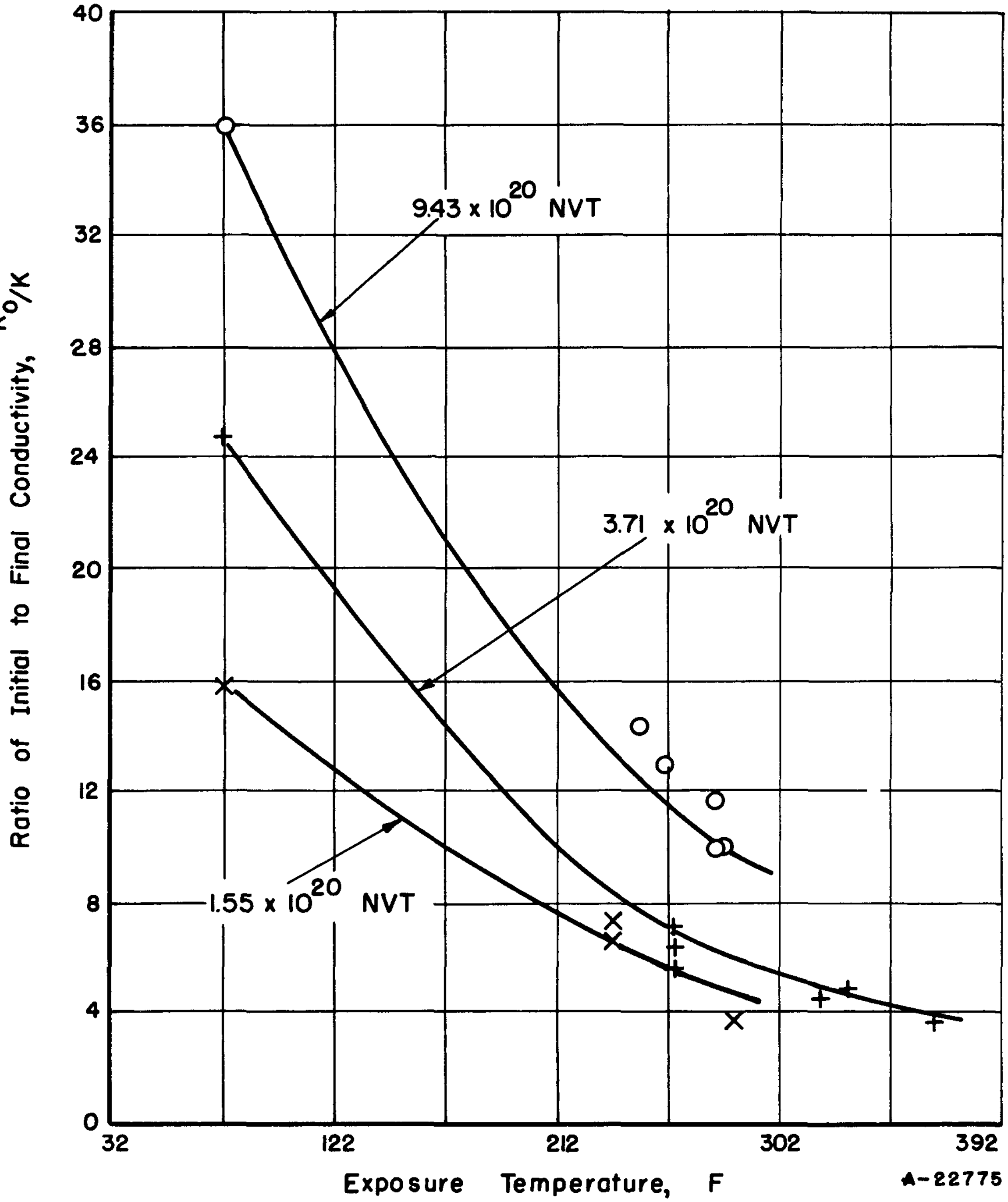

FIGURE 10. VARIATIONS IN THERMAL CONDUCTIVITY OF TRANSVERSE CUT CSF GRAPHITE WITH EXPOSURE TEMPERATURE

Energy Storage

Energy storage in the graphite becomes a problem only when this energy affords the possibility of a spontaneous rise in the graphite temperature. This condition occurs if the effective specific heat of the graphite becomes negative in any temperature range (i.e., the solid curves in Figures 11 and 12 fall above the dashed curve). Figure 11 shows the effect of irradiation temperature on the stored-energy release at an integrated flux of 1.25 x 10^{20} nvt. Figure 12 shows the effect of flux on stored-energy release at an irradiation temperature of 86 F. Several conclusions can be reached by utilizing the information contained in these figures and discussion in Reference 1. It appears that irradiation at a temperature of 327 F eliminates completely the energy-storage peak; consequently, integrated fluxes in the range of 10^{21} nvt may be considered safe. Irradiation temperatures as low as 230 F are safe for integrated fluxes of 1.25 x 10^{20} nvt. Energy storage for fluxes of about 10^{19} nvt can be neglected even for cold irradiation because a temperature rise of only about 30 F would result (Figure 13) if all the stored energy were spontaneously released. Furthermore, extrapolation of Figure 12 strongly indicates that no spontaneous energy release will occur at this low flux.

To sum up the energy-storage problem, none of the systems considered here present any problems according to the above criteria. The data used in drawing the above conclusions represent a limited amount of experimental work done in the Hanford reactor at a considerably different flux energy spectrum than will be obtained in the PRDC reactor. However, the general conclusions should be valid.

Distortion

Figure 14 shows the distortion of graphite under two irradiation conditions as a function of temperature. The higher irradiation would correspond to the flux received within the first few inches of the lower core vessel. In the PRDC reactor the temperatures are higher than those considered in the curves. However, the largest part of the graphite or carbon block receives a flux much lower than 1.55 x 10^{20} nvt. Consequently, a 0.1 per cent length change or 0.3 per cent volume change due to irradiation (these changes are allowed in the stacking arrangements presented here) would seem sufficient. The effect of radiation on carbon block are less severe than on graphite, but little experimental work has been done with carbon.

STACKING ARRANGEMENTS

The basic block sizes for the plain and the borated graphite are shown in Figure 15. A combination of the wedge and straight blocks may be used to obtain a circle of any required diameter. Two methods of assembling the blocks have been considered. In the first of these, 2-ft-thick horizontal layers are planned, with clamps around the total circumference and the circumference of the borated graphite in each layer. A cross section of one of the layers is shown in Figure 16. Block sizes of about 50 and 100 lb were chosen because it was felt that this size is still light enough to be manageable. This method of stacking in circles, concentric with the reactor vessel, is chosen because it requires very little cutting of the borated graphite and should afford a rapid,

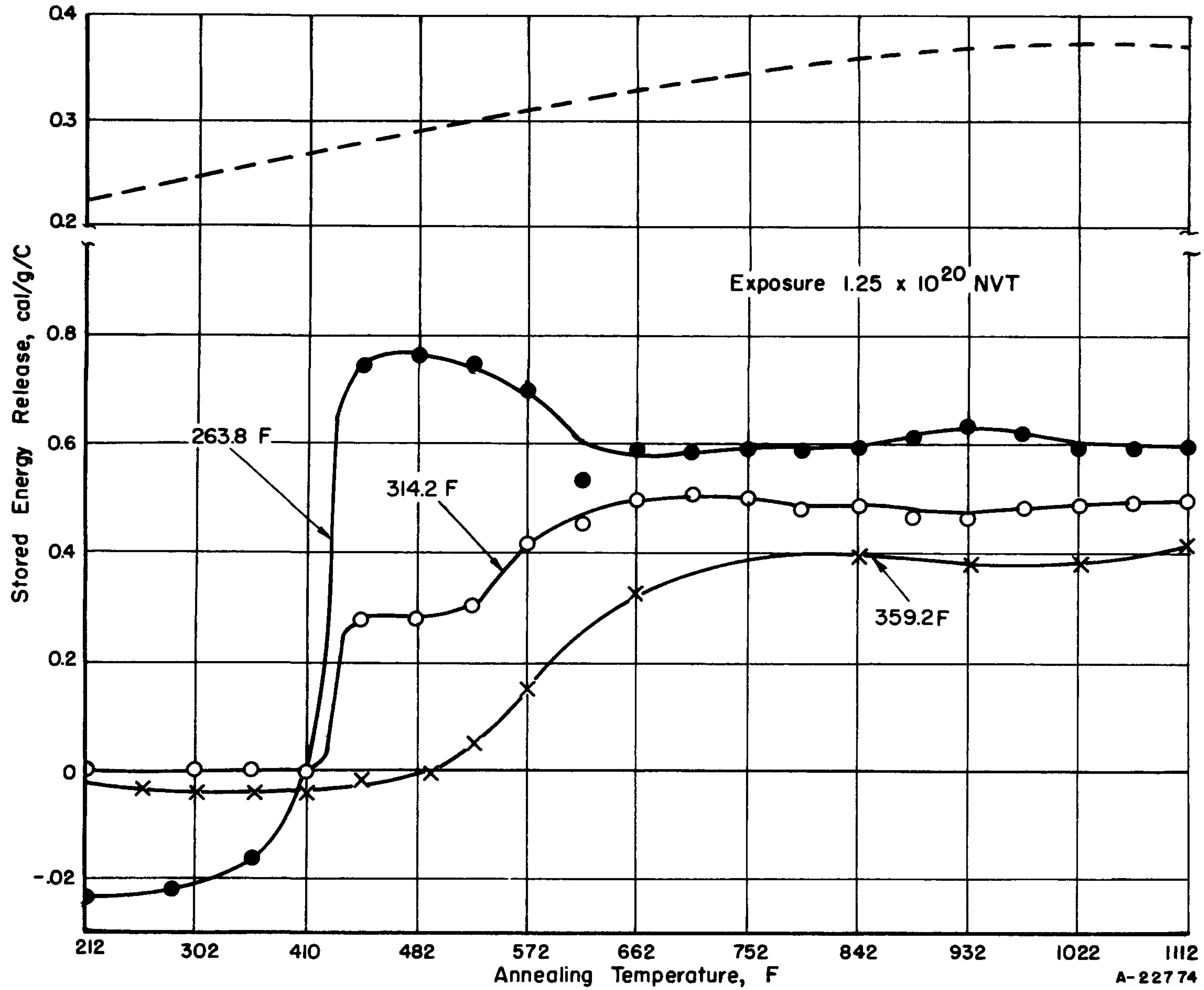

FIGURE 11. EFFECT OF EXPOSURE TEMPERATURE ON STORED ENERGY

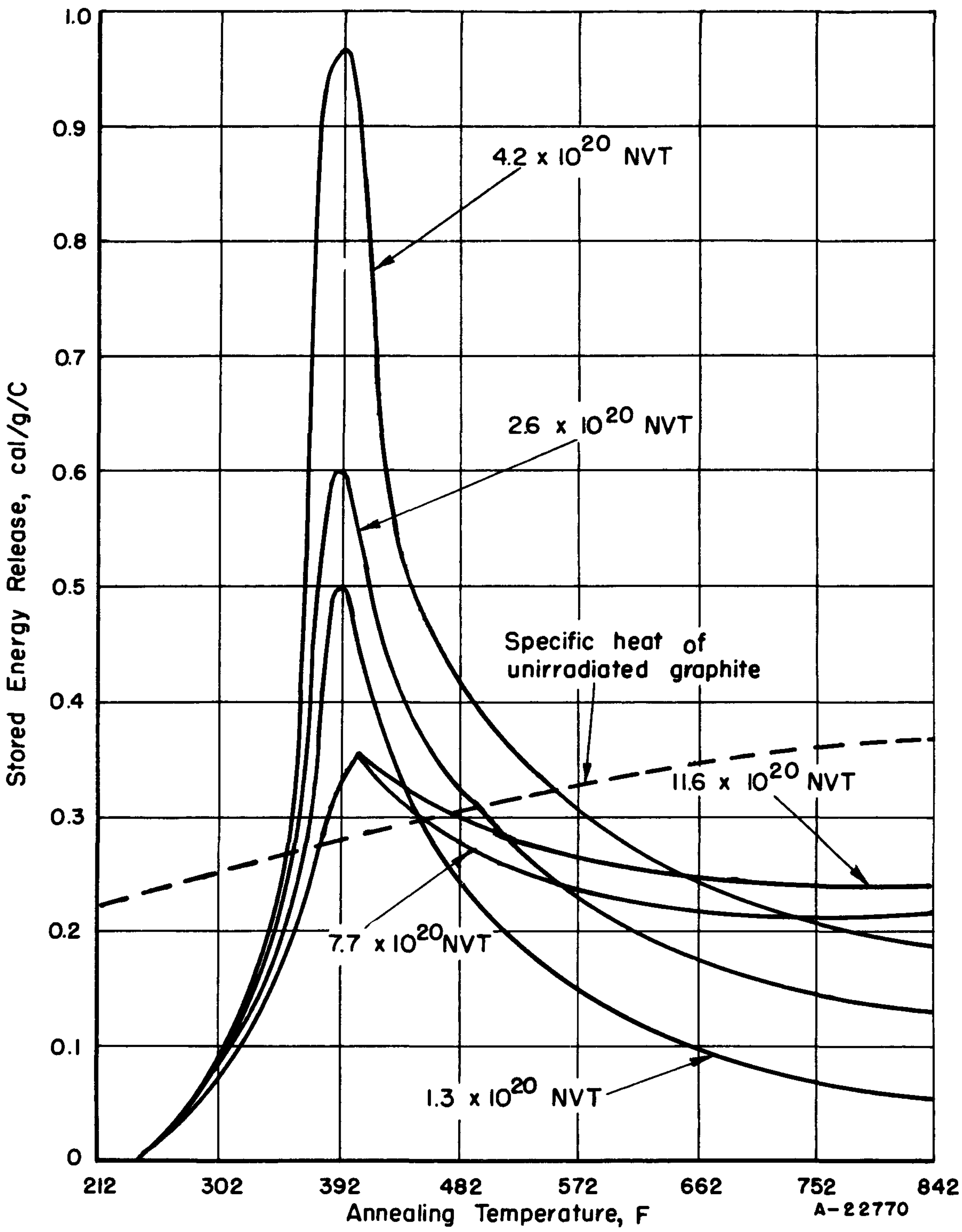

FIGURE 12. STORED ENERGY ANNEALING SPECTRA IN GRAPHITE IRRADIATED AT 80 F

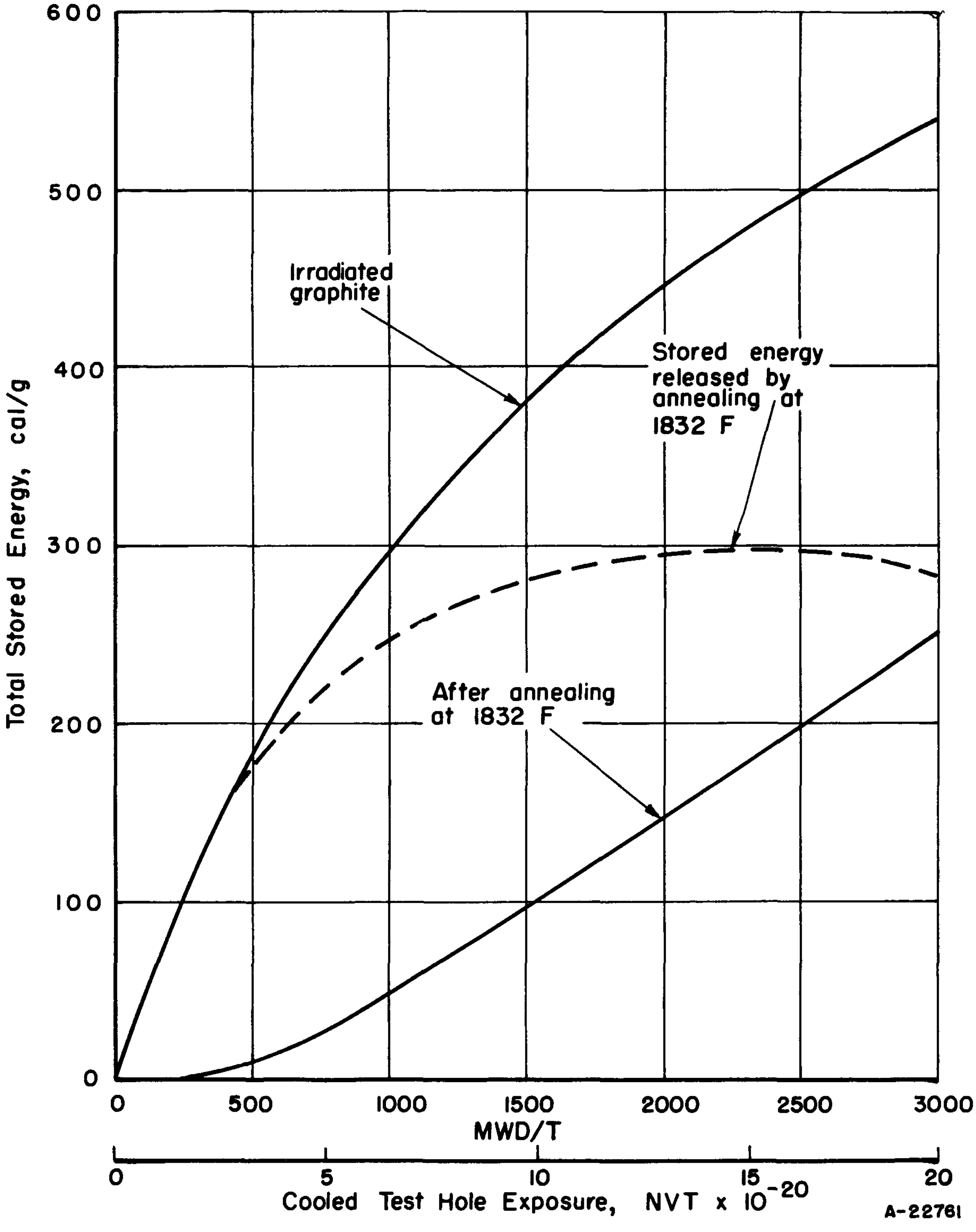

FIGURE 13. BUILD-UP OF TOTAL STORED ENERGY IN IRRADIATED GRAPHITE

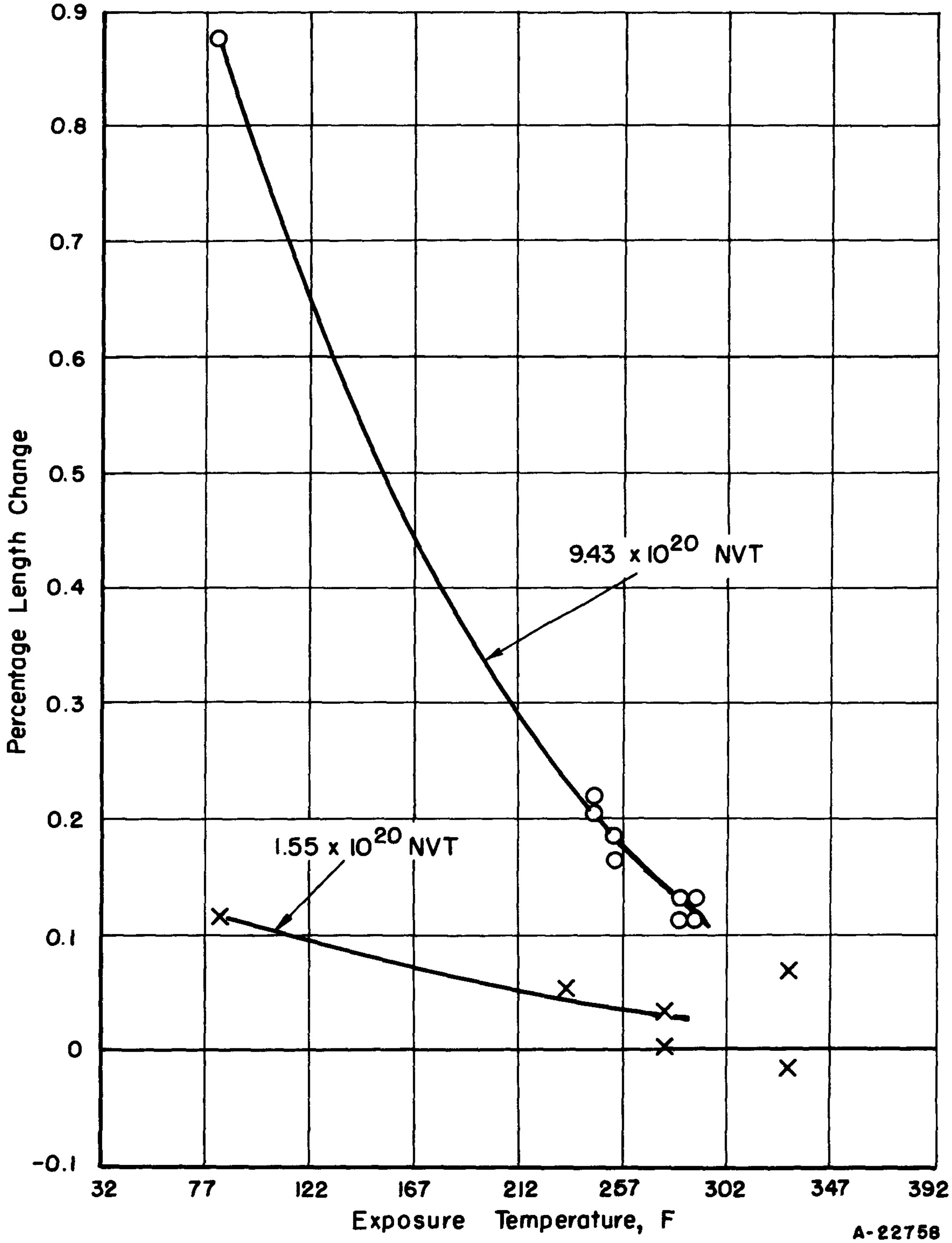

FIGURE 14. EFFECTS OF TEMPERATURE ON PHYSICAL EXPANSION

Transverse Cut CSF

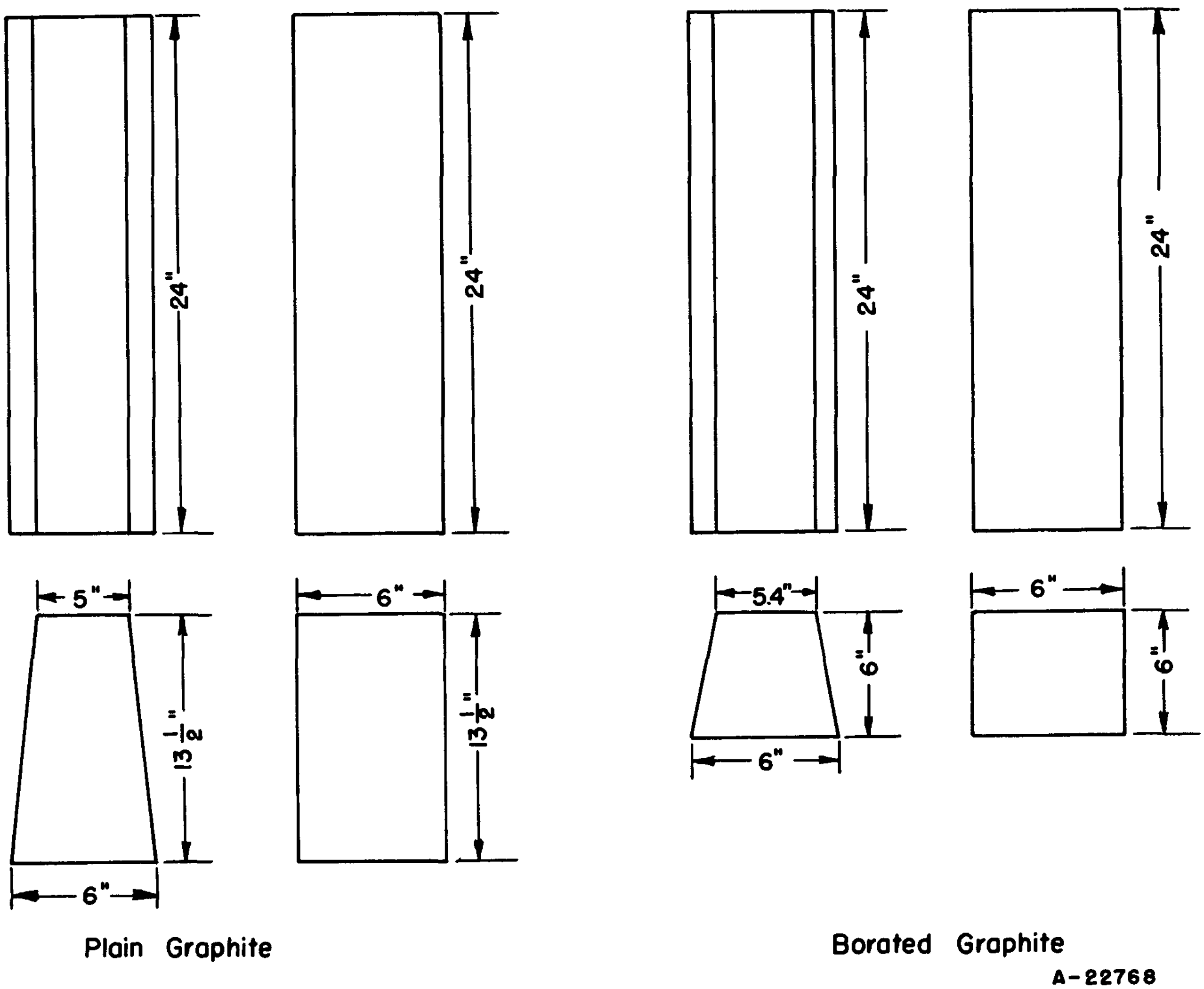

FIGURE 15. APPROXIMATE BLOCK SIZES

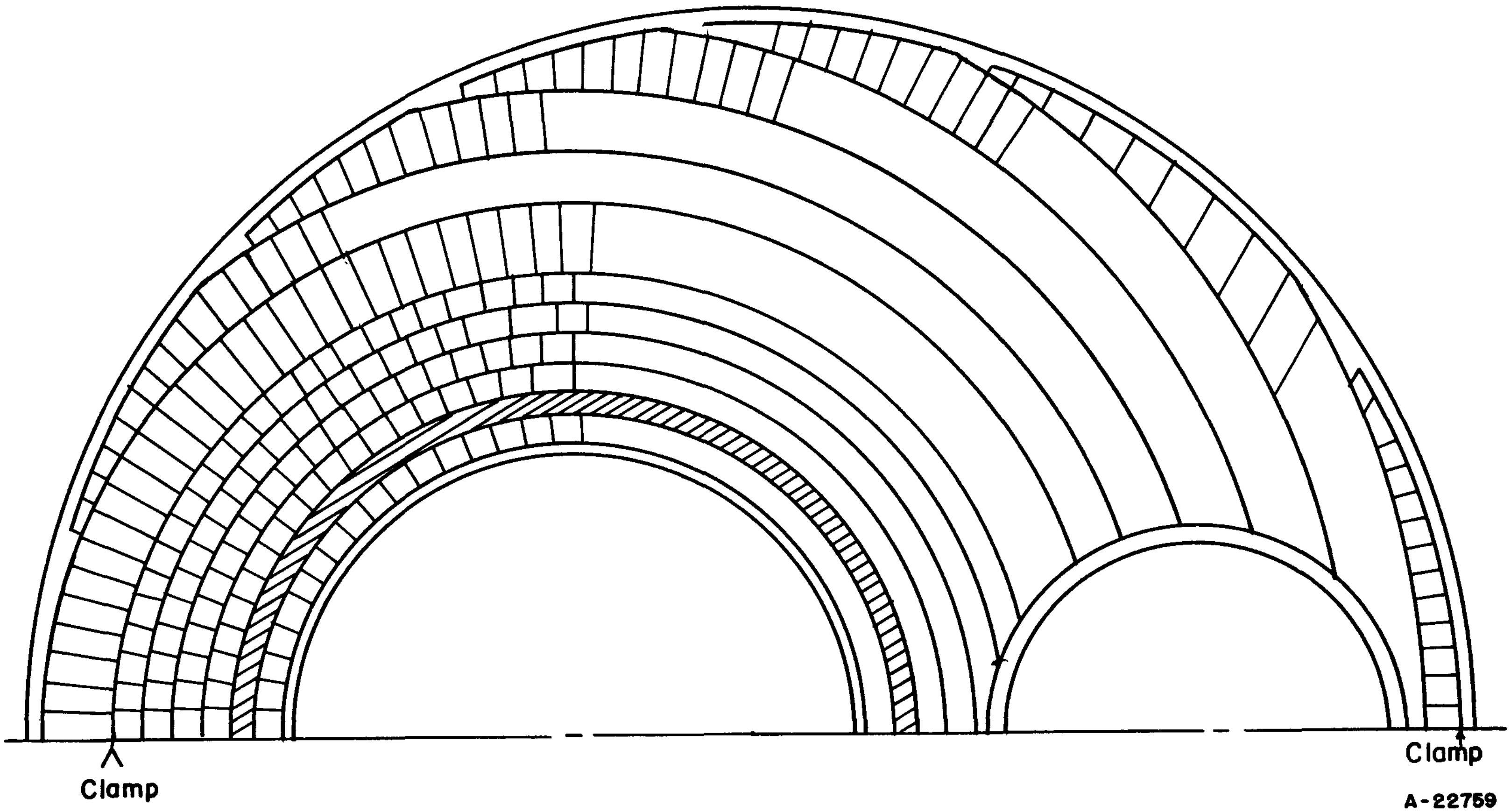

FIGURE 16. STACKING ARRANGEMENT FOR SYSTEM I

systematic installation. A clamp consisting of approximately 1/8 by 1-in. steel strips with some type of spring clamps at the joints can be used. Since only a 2-in. space is left between the primary-shield tank and the graphite, a special type of clamp will be required here.

The alternate method of assembling was cementing instead of clamping and is discussed in the installation section of this report.

A space of about 1/2 in. between the lower core vessel and the graphite, and about 3/4 in. between the upper vessel and the graphite must be allowed for thermal expansion of the stainless steel vessels. A space of approximately 1 in. between the primary-shield tank and the filler is required for thermal expansion and radiation distortion of the graphite or carbon block. However, a 2-in. space is allowed to facilitate clamping, should clamping be used. In the systems where Boral shielding is placed against the primary-shield tank, additional allowance must be made for circumferential expansion.

VOIDAGE AND SODIUM REACTIONS ON GRAPHITE

All of the systems considered result in a voidage of approximately 400 cu ft below the center of the 30-in. outlet pipe. Of this voidage, the 2-in. gap at the shield tank or Boral sheet represents 300 cubic feet. If necessary, this can be reduced to 1 in. without major difficulty.

The information in References 2 and 3 on the absorption of sodium by graphite indicates that this is a negligible effect. Liquid sodium wets graphite at temperatures above 840 F with an absorption of 10^{-4} a/o into the pores. This absorption causes a very slight volume expansion of the graphite (about 0.8 per cent). Since the temperature of the sodium must be at least 840 F before wetting occurs, this effect probably can be neglected in the shield. However, all tests were conducted on reactor-grade graphite, which has a density of approximately 1.67, instead of regular graphite, which has a density of about 1.55.

Graphite resists corrosion by sodium up to 1450 F in static systems, according to References 2 and 3. Corrosion occurs only when a metal surface is available to remove the carbon from the sodium through carburization of the metal. Since carburization is a slow process and a sodium temperature of at least 840 F is required, the effect of this reaction can be neglected here.

ECONOMICS

Materials

The approximate costs and quantities of materials required, up to the center of the 30-in. outlet pipe, in each of the four systems are shown in Table 1.

TABLE 1. SUMMARY OF COSTS[a]

	Borated Graphite Containing Indicated Percentage of Boron[b]							
	1	3	5	Graphite	Carbon Brick	Boral	Lumnite	Total
				System 1a				
Amount	1800 ft^3	--	--	5600 ft^3	--	--	1300 ft^3	
Cost	$162, 000	--	--	$156, 000	--	--	$3, 000	$333, 000
				System 2				
Amount	--	110 ft^3	60 ft^3	910 ft^3	6400 ft^3	4500 ft^2	--	
Cost	--	$15, 000	$12, 000	$25, 500	$96, 000	$63, 000	--	$224, 000
				System 3				
Amount	--	110 ft^3	60 ft^3	5600 ft^3	2800 ft^3	4500 ft^2	--	
Cost	--	$15, 000	$12, 000	$156, 000	$42, 000	$63, 000	--	$300, 000
				System 4				
Amount	--	--	--	2470 ft^3	5400 ft^3	6800 ft^2		
Cost	--	--	--	$69, 000	$81, 000	$95, 000	--	$245, 000
				System 1b				
Amount	1800 ft^3	--	--	5600 ft^3	1300 ft^3	--	--	
Cost	$162, 000	--	--	$156, 000	$19, 400	--	--	$349, 000

(a) A copy of the quotation received by Battelle from National Carbon Company is on the reverse side of this sheet.
(b) A tooling charge of about $12, 000 is required for processing borated graphite. This has been added into the totals.

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ITEM	LOT	DESCRIPTION	PRICE
A	150, 000 lbs.	6" x 6" x 12" Graphite Blocks	$ 850.00 Per 1000 lbs.
	1, 000, 000 lbs.	containing 1% boron	$ 775.00 " " "
B	150, 000 lbs.	6" x 6" x 12" Graphite Blocks	$1, 050.00 " " "
	1, 000, 000 lbs.	containing 2% boron	$ 975.00 " " "
C	150, 000 lbs.	6" x 6" x 12" Graphite Blocks	$1, 250.00 " " "
	1, 000, 000 lbs.	containing 3% boron	$1, 175.00 " " "
D	150, 000 lbs.	6" x 6" x 12" Graphite Blocks	$1, 575.00 " " "
	1, 000, 000 lbs.	containing 4% boron	$1, 450.00 " " "
E	150, 000 lbs.	6" x 6" x 12" Graphite Blocks	$1, 800.00 " " "
	1, 000, 000 lbs.	containing 5% boron	$1, 675.00 " " "

Note: An alteration to equipment charge of $12, 220.00 is also to be quoted in addition to the above unit prices.

The 6" x 6" x 12" blocks to have a taper over the 6" length from 6" to approx. 5-4/10". Tolerances on all dimensions to be Plus or Minus 1/16".

Delivery: Would be 38 to 42 weeks for the first 150, 000 lbs. and 16 to 18 weeks for each additional 150, 000 lbs. This delivery applicable to any single boron density order. Delivery will be from Niagara Falls, N. Y.

Copy of National Carbon Company Quotation on Borated Graphite

The volumes of graphite listed in Table 1 allow for a 30 per cent loss in machining operations. A machining cost of $5 per cu ft is also included in the nonborated-graphite costs listed in the table. These figures are based on the costs of milling graphite blocks of about the same size as those in the Battelle Research reactor. A charge of $6 per man-hour to cover the cost of labor and equipment was used. The equipment required included an ordinary saw and a large milling machine.

The National Carbon Company quotations (reverse side of Table 1) include the costs for machining the borated graphite. Consequently, the $5 per cu ft machining cost and the 30 per cent volume loss are not applied to this material.

No machining charges can be reasonably applied to the carbon-block systems because carbon block requires diamond machine tools; extensive machining would not be advisable. For this reason, a filler paste would be preferable for use around obstacles. Detailed specifications on a suitable paste may be obtained from National Carbon Company.

It should be pointed out that National Carbon will furnish machined unborated graphite. However, based on these estimates, the costs of the graphite would amount to 20 to 50 per cent more than the combined material and machining costs estimated here.

Although System 1 is based on 1 per cent of boron in the graphite, the improvement in shielding properties may make the use of higher boron contents desirable. In System 1a, the total material cost would be increased by 11.4 per cent if 2 per cent borated graphite were used, by 23.1 per cent if 3 per cent were used, by 41.7 per cent if 4 per cent were used, and by 54.9 per cent if 5 per cent were used. Various combinations, which use smaller volumes of 2 to 5 per cent borated graphite to obtain the same shielding properties as the 1 per cent system, are also possible.

Installation

Installation costs of the graphite in the Battelle reactor thermal column (aside from the machining costs previously mentioned) were about 1/5 the cost of the graphite, or $9 per cu ft. Approximately 20 per cent of the blocks had to be cut to a close fit, for which an ordinary bandsaw was used. The remainder were put in as machined. This cost is also based on a labor and equipment cost of $6 per man-hour. The installation cost per cu ft of Lumnite should be low compared with that of graphite. In the discussion concerning the shield installation, costs for engineering have been overlooked. Both engineering and clamping will increase the above costs.

An alternate method of installation which eliminates clamping would be to use a carbon cement in the joints between blocks. This cement can also be used to fill in around obstacles. Carbon cements have a thermal conductivity from 2 to 7 Btu/(hr)(ft)(F). Thus a 1/8-in. joint of this type would have a somewhat better conductivity than that assumed for the joints previously. This cement works like mortar and would cost about 8 cents per lb.

REFERENCES

(1) Geneva Conference, A/Conf. 8/P 746, USA (July 8, 1955).

(2) Liquid Metals Handbook, Second Edition, U. S. Atomic Energy Commission, Department of the Navy (July, 1952), p 149.

(3) Liquid Metals Handbook, Sodium (NaK) Supplement, TID-5277, U. S. Atomic Energy Commission, Department of the Navy (July 1, 1955), pp 152-155.

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APPENDIX A

THE ELECTRIC ANALOGUE NETWORK

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THE ELECTRIC ANALOGUE NETWORK

Although the longitudinal temperature gradients cannot be considered negligible in the shield, the magnitude of these gradients is sufficiently smaller than the radial gradients to make a two-dimensional analogue reasonable. It is convenient to consider a cross-sectional volume element of unit thickness in the longitudinal direction. The flow of heat can be represented by the two-dimensional equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -\frac{Q}{K} \quad , \qquad \text{(A-1)}$$

where x and y are rectangular coordinates, Q is the rate of heat generation per unit volume, K is the thermal conductivity of the region, and T is the temperature.

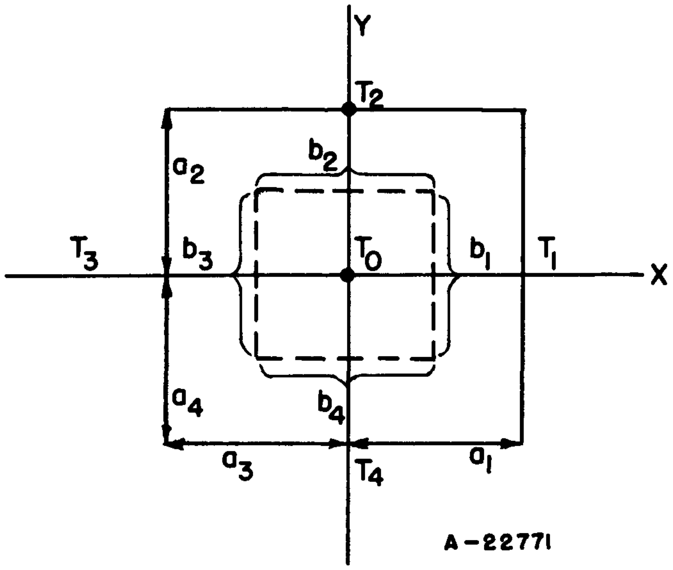

FIGURE A-1. TEMPERATURE NETWORK

Figure A-1 is a rectangular region of unit thickness, where K and Q are constant. A rectangular network of points is chosen in which temperatures T_0, T_1, T_2, T_3, and T_4 are to be determined. The derivatives in Equation (A-1) may be replaced by first-order differences between the temperatures at the network points. This assumes that the temperature T_0 is affected only by the four adjacent network points, rather than by the continuous temperature field.

When the derivatives in Equation (A-1) are replaced by first-order differences based on the region of Figure A-1, the equation becomes

$$\left[\frac{T_1 - T_0}{a_1} + \frac{T_3 - T_0}{a_3}\right] \frac{2}{a_1 + a_3} + \left[\frac{T_2 - T_0}{a_2} + \frac{T_4 - T_0}{a_4}\right] \frac{2}{a_2 + a_4} = \frac{Q}{K}$$

or

$$(T_1 - T_0)\frac{b_1}{a_1} + (T_2 - T_0)\frac{b_2}{a_2} + (T_3 - T_0)\frac{b_3}{a_3} + (T_4 - T_0)\frac{b_4}{a_4} = \frac{QA_0}{K}, \quad .$$

which is

$$\sum_{i=1}^{4} K(T_i - T_0)\frac{b_i}{a_i} = QA_0 , \qquad \text{(A-2)}$$

where

$$b_1 = b_3 = \frac{a_2 + a_4}{2}$$

$$b_2 = b_4 = \frac{a_1 + a_3}{2}$$

and

$$b_1 b_2 = A_0$$

where A_0 is the area enclosed by the dotted lines on Figure A-1.

If a rectangular network can be superimposed on a region, then Equation (A-2) can be applied. A region with nonrectangular boundaries could be represented by a nonrectangular network. However, the difference approximation to Equation (A-1) for the nonrectangular network would lead to expressions for equivalent dimensions a′ and b′. The a′ and b′ are, in general, not simply the average distance between points. However, for a nearly rectangular network, average distances corresponding to the a and b of Figure A-1 are a reasonable approximation.

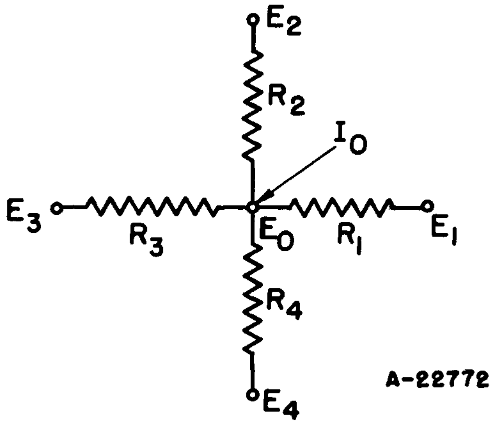

FIGURE A-2. ELECTRICAL ANALOGUE OF TEMPERATURE NETWORK

Figure A-2 illustrates a network of electrical resistors analogous to Figure A-1. The source of current, I_0, corresponds to the source of heat QA_0. The equation representing this current is

$$I_0 = \sum_{i=1}^{4} \frac{E_i - E_0}{R_i} \quad . \tag{A-3}$$

The electrical analogue for the thermal problem is based on the analogy between Equations (A-2) and (A-3). A current I_0 is chosen proportional to QA_0, and the resistors R_i are made proportional to the corresponding quantities $\frac{a_i}{b_i K}$. For example,

$$I_0 = \alpha Q A_0 \tag{A-4}$$

and

$$R_i = \beta \frac{a_i}{b_i K} \quad , \tag{A-5}$$

with α and β being constants of proportionality. It follows that

$$\alpha Q A_0 = \sum_{i=1}^{4} \frac{b_i K}{\beta a_i} (E_i - E_0) \quad ,$$

so that

$$(E_i - E_0) = \alpha\beta \, (T_i - T_0) \quad . \tag{A-6}$$

That is, the voltage differences in the electrical case are proportional to the corresponding temperature differences in the thermal case. From Equation (A-5), a network of electrical resistors can be chosen to represent the thermal conductivity in the shield.

For a point on the thermal-insulation boundary, one of the conduction resistors, R_i, must be replaced by a heat-transfer resistor, R_h. Figure A-3 shows the electrical analogy to a boundary point on the insulation, with no heat generation. The temperature distribution inside the insulation was solved as a separate problem. The insulation around the core vessel and transfer-rotor container was treated by an equivalent heat-transfer coefficient instead of as a separate region.

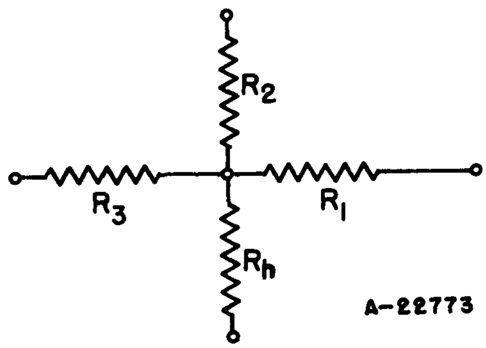

FIGURE A-3. ELECTRICAL ANALOGUE OF BOUNDARY POINT ON THE INSULATION

The electrical and thermal equations for Figure A-3 are, respectively,

$$\sum_{i=1}^{3} \frac{E_i - E_0}{R_i} = \frac{E_0}{R_h}$$

and

$$\sum_{i=1}^{3} K(T_i - T_0)\frac{b_i}{a_i} = h\ T_0 b_h \ ,$$

where h is the heat-transfer coefficient of the insulation. The heat-transfer resistor, R_h, is chosen so that

$$R_h = \frac{\beta}{b_h h} \ . \qquad \text{(A-7)}$$

To control the currents to points on the network, resistors R_g are inserted between the battery and the points. If V_b is the battery voltage, then

$$R_g = \frac{V_b}{I_0} = \frac{V_b}{\alpha Q A_0} \ .$$

Figure A-4 is a cross section of the shield. Superimposed on this is the grid used to subdivide the region.

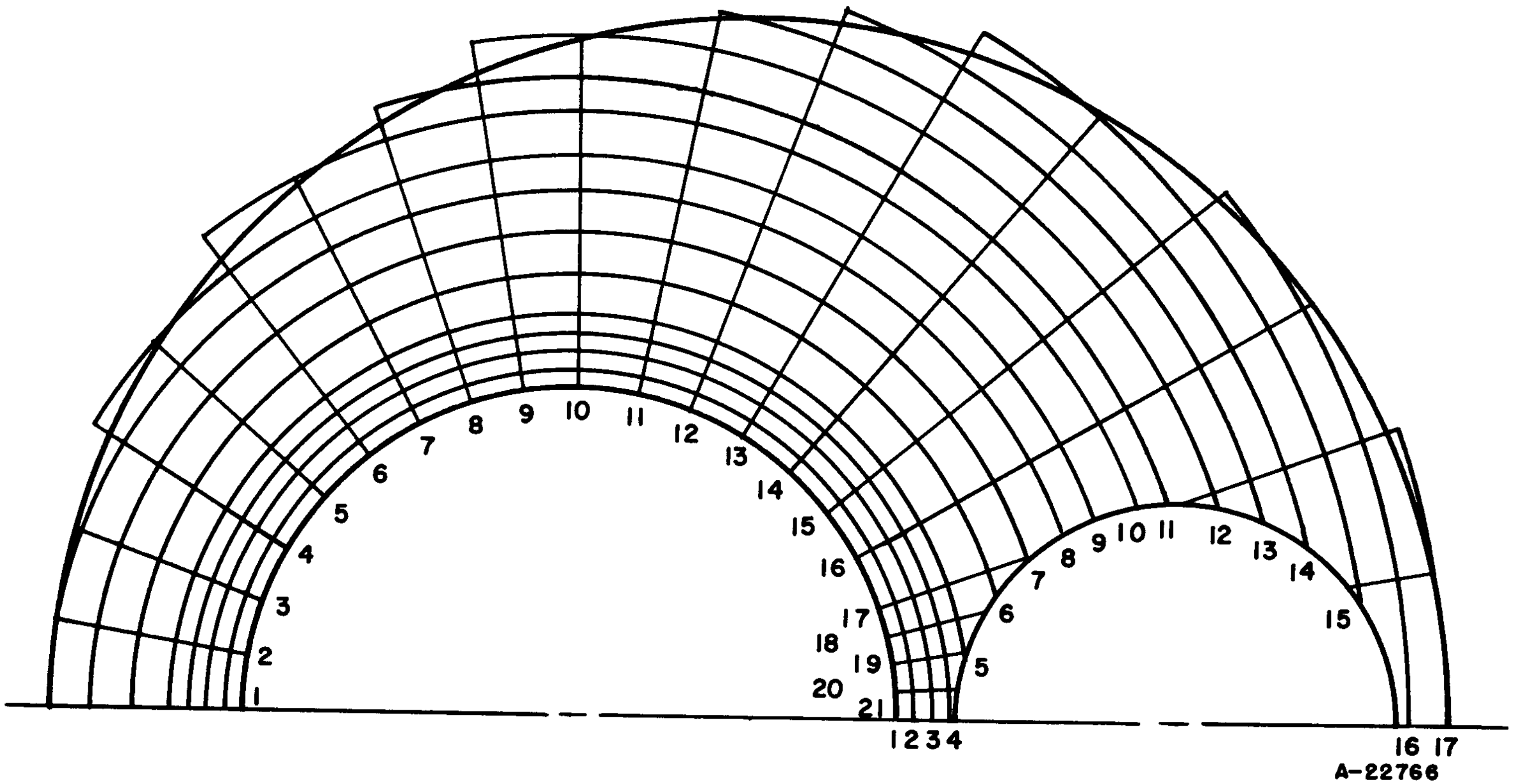

FIGURE A-4. ANALOGUE GRID